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基于信号量化处理的随机时滞网络化系统的分析与设计

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摘要: 研究一类时滞分布依赖的网络化系统的量化控制问题. 首先, 在考虑信号量化处理的影响下, 建立包含时延区间概率分布信息和信号量化信息的新的网络化系统模型; 然后, 运用 Lyapunov 稳定性理论、矩阵函数的凸性、自由权技术和 Jensen 不等式等分析方法给出系统渐近稳定和镇定的条件; 最后, 运用线性矩阵不等式 (LMI) 技术求解量化控制器. 仿真结果和横向比较结果验证了所述方法的有效性.

关键词: 网络控制系统; 量化; 凸性

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Analysis and design for networked systems with random delay based on signal quantization

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Abstract: This paper considers the problem of quantized control for a class of networked control system(NCS). The new model of NCS are derived considering the delay, packet dropout, quantization and delay probability distribution. Then by using of the analysis method of Lyapunov stability theory, convexity of matrix function, free-weighting technology and Jensen inequalities etc, the stability and stabilization criteria are given. Quantized controller is designed by using of LMI technology. Finally, the results of the simulation and the comparison with related paper show the effectiveness of the proposed method.

Key words: networked control system; quantization; convexity

1 引言

由通讯网络构建的闭环系统称为网络化系统. 网络化系统由于其成本低廉、安装简便、易于维护且可靠性高而得到了广泛应用. 但是, 通讯网络带来的传输时滞、网络丢包和错序以及量化等的影响又将降低系统的性能, 甚至使得系统不稳定. 近年来, 关于网络化系统的稳定性研究和控制器设计已经形成了一个研究热点, 其中针对时滞的研究方法主要包括时滞依赖的方法和时滞独立的方法, 而时滞依赖的方法由于利用了时滞信息, 得到了保守性较小的结果. Zhang 等人^[1]假设网络诱导时延小于 1 个采样周期, 并在假设不存在数据包丢失和错序的情况下, 利用混杂系统技术研究了网络化系统的稳定性. Yue 等人^[2]在同时考虑网络延迟、数据丢包情况下, 建立了

一种新型网络控制系统 (NCS) 模型, 并进行了稳定性分析和稳定化设计. Peng 等人^[3]则研究了时滞在区间范围内变化的网络化系统的状态反馈控制器设计.

上述研究都仅利用了网络时滞的区间信息, 而对于网络化控制系统, 人们亦可得到时滞分布的概率分布信息, 即小概率大时滞, 或大时滞小概率. 文献 [4] 将概率分布应用于时滞系统的分析, 在时滞小于一定范围、概率非常大的情况下, 得到了较大的保证系统稳定的时滞上界. 文献 [5-7] 将时滞概率分布应用于网络化系统的分析, 假设时滞落在 2 个区间的概率已知, 且在时滞第 1 区间的下界为 0 的情况下, 利用矩阵函数的凸性研究了系统的稳定和镇定问题.

以上研究均假设系统的输出能够直接传送到控制器, 控制信号能够直接传送到驱动器, 而且假设这

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种传输是无损传输. 然而, 在网络化系统中, 考虑到网络传输能力的限制, 这样的假设通常是不成立的, 因此量化处理是必要的^[8-12]. 但是, 至今尚未见到对时滞分布依赖的网络化系统量化控制的研究.

本文在考虑量化的影响下, 建立了概率分布时滞的网络化系统的新模型, 利用矩阵函数的凸性、Jessen 不等式和自由权方法等对系统进行分析和设计, 通过在分析中加入新的变量, 大大降低了分析的难度和复杂度. 最后通过数值例子验证了所述方法的有效性.

2 系统描述和预备知识

2.1 时变量化器

考虑到网络通道的通信能力的限制, 同时为减小网络传输的数据量^[13], 网络传输的信号由 2 个量化器先量化再传输. 通常, 量化器 $q: R^l \rightarrow p$ 为一分段常函数, 其中 p 为 R^l 的一有限子集. 量化器满足下列条件:

条件 1 若 $\|z\| \leq F$, 则 $\|q(z) - z\| \leq \Delta$;

条件 2 若 $\|z\| > F$, 则 $\|q(z)\| > F - \Delta$.

条件 1 和条件 2 中的 F 和 Δ 分别为量化器的量化范围和误差.

本文采用时变量化器 $\mu q(\mu^{-1}z)$, 变量 μ 会随着变量 z 的变化而变化, 因此有

若 $\|z\| \leq \mu F$, 则 $\|\mu q(\mu^{-1}z) - z\| \leq \mu \Delta$;

若 $\|z\| > \mu F$, 则 $\|\mu q(\mu^{-1}z)\| > \mu(F - \Delta)$.

由此可见, z 和 μ 的大小将改变量化范围和量化误差, 从而对于小信号加快量化速度, 而对于大信号则增加量化精度.

2.2 系统描述

考虑如下线性网络化系统:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ x(t) &= \varphi(t), t \in [-\tau_M, 0]. \end{aligned} \quad (1)$$

其中: $x(t) \in R^n$ 和 $u(t) \in R^m$ 分别为状态向量和控制输入向量, A 和 B 为适当维数的时常矩阵. 考虑到网络时滞和量化的影响, 传递至控制器的状态量为 $\bar{x}(t) = \mu_1 q_1(\mu_1^{-1}x(t - \tau(t)))$, 则信号经量化后的相应的控制器为

$$u(t) = \mu_2 q_2(\mu_2^{-1}K\mu_1 q_1(\mu_1^{-1}x(t - \tau(t))))$$

其中 $\tau(t) \in [\tau_m, \tau_M]$, τ_m 和 τ_M 为传输时滞的上下界. 则闭环系统模型为

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\mu_2 q_2(\mu_2^{-1}K\mu_1 q_1(\mu_1^{-1}x(t - \tau(t)))) = \\ & Ax(t) + BKx(t - \tau(t)) - B\mu_2 \delta(\mu_1, \mu_2), \\ x(t) &= \varphi(t), t \in [-\tau_M, 0]. \end{aligned} \quad (2)$$

其中

$$\begin{aligned} \delta(\mu_1, \mu_2) &= \\ & \mu_2^{-1}Kx(t - \tau(t)) - q_2(\mu_2^{-1}K\mu_1 q_1(\mu_1^{-1}x(t - \tau(t)))) \\ \tau(t) &\in [\tau_m, \tau_M] \text{ 随时间随机变化, 且 } \tau(t) \in [\tau_m, \tau_1] \text{ 和 } \\ \tau(t) &\in [\tau_1, \tau_m] \text{ 的概率已知. 定义如下 2 个子集:} \end{aligned}$$

$$\Omega_1 = \{t : \tau(t) \in [\tau_m, \tau_1]\},$$

$$\Omega_2 = \{t : \tau(t) \in [\tau_1, \tau_M]\}.$$

并定义如下 2 个时延函数:

$$\tau_1(t) = \begin{cases} \tau(t), & t \in \Omega_1; \\ \frac{\tau_1}{2}, & t \in \Omega_2. \end{cases} \quad (3)$$

$$\tau_2(t) = \begin{cases} \tau(t), & t \in \Omega_2; \\ \tau_1, & t \in \Omega_1. \end{cases} \quad (4)$$

假设存在如下随机变量:

$$\beta(t) = \begin{cases} 1, & t \in \Omega_1; \\ 0, & t \in \Omega_2. \end{cases}$$

且 $\text{Prob}\{\beta(t) = 1\} = \beta_0$, $\text{Prob}\{\beta(t) = 0\} = 1 - \beta_0$. 又因为量化参数 δ 本身是个很小的量, 所以不考虑 δ 中 $\tau(t)$ 的概率分布. 则系统模型 (2) 可改写如下形式:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \beta(t)BKx(t - \tau_1(t)) + \\ & (1 - \beta(t))BKx(t - \tau_2(t)) - B\mu_2 \delta(\mu_1, \mu_2), \\ x(t) &= \varphi(t), t \in [-\tau_M, 0]. \end{aligned} \quad (5)$$

定义 1 如果存在常数 $\alpha > 0$ 和 $\beta > 0$ 使得系统 (5) 的解满足

$$E\{\|x(t)\|^2\} \leq \alpha e^{-\beta t} \sup_{-\tau_M \leq s \leq 0} E\{\|\varphi(s)\|^2\}, \quad (6)$$

则称系统 (5) 在均方意义上指数渐近稳定.

引理 1 假设 Ξ_1, Ξ_2 和 Ω 为时常矩阵, $\tau_i(t)$ ($i = 1, 2$) 如式 (3) 和 (4) 所定义, 则对于 $t \in R^+$, 有 $[(\tau_1(t) - \tau_m)\Xi_{11} + (\tau_1 - \tau_1(t))\Xi_{12}] + [(\tau_2(t) - \tau_1)\Xi_{21} + (\tau_M - \tau_2(t))\Xi_{22}] + \Omega < 0$ 成立当且仅当如下 4 个不等式成立:

$$(\tau_1 - \tau_m)\Xi_{11} + (\tau_M - \tau_1)\Xi_{12} + \Omega < 0, \quad (7)$$

$$(\tau_1 - \tau_m)\Xi_{11} + (\tau_M - \tau_1)\Xi_{22} + \Omega < 0, \quad (8)$$

$$(\tau_1 - \tau_m)\Xi_{21} + (\tau_M - \tau_1)\Xi_{12} + \Omega < 0, \quad (9)$$

$$(\tau_1 - \tau_m)\Xi_{21} + (\tau_M - \tau_1)\Xi_{22} + \Omega < 0. \quad (10)$$

引理 2 设 $\tau_m \leq \tau(t) \leq \tau_M$ 且 $\dot{x}(t+) : [-\tau_M, -\tau_m] \rightarrow R^n$, 则对于任意正定矩阵 $R > 0$, 下列不等式成立:

$$\begin{aligned} & -(\tau_M - \tau_m) \int_{t-\tau_M}^{t-\tau_m} \dot{x}^T(s) R \dot{x}(s) ds \leq \\ & \begin{bmatrix} x(t - \tau_m) \\ x(t - \tau_M) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t - \tau_m) \\ x(t - \tau_M) \end{bmatrix}. \end{aligned} \quad (11)$$

3 稳定性分析

定理 1 给定常数 $\tau_m, \tau_1, \tau_M, \beta_0$ 和矩阵 K , 如果存在正定对称矩阵 $P, Q_1, Q_2, Q_3, T, R_1, R_2, R_3$ 以及矩阵 N_i^j ($i = 1, 2, j = 1, 2, \dots, 7$), M_i^j ($i = 1, 2, j =$

1, 2, \dots, 7) 和 $S_j (j=1, 2, 3, 4)$, 使得下列 LMIs 有解:

$$\begin{bmatrix} \Gamma_{11} + T & * \\ \Gamma_{21}^l & \Gamma_{22} \end{bmatrix} < 0, \quad l = 1, 2, 3, 4. \quad (12)$$

则系统 (5) 是均方意义上指数渐近稳定的, 且量化器参数满足

$$2\Delta \|SB\| \|T^{-1}\| \leq \frac{\|x(t)\|}{\mu_2} \leq F_1, \\ F_2 \geq \|K\| (\Delta_1 + F_1).$$

其中

$$\Gamma_{11} = \Gamma + \Xi_1 + \Xi_1^T + \Xi_2 + \Xi_2^T, \\ \Gamma = \begin{bmatrix} \Phi_{11} & * & * & * & * & * & * \\ R_1 & \Phi_{22} & * & * & * & * & * \\ 0 & 0 & -Q_2 & * & * & * & * \\ 0 & 0 & 0 & -Q_3 & * & * & * \\ P^T & 0 & 0 & 0 & \Phi_{55} & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{11} = Q_1 + Q_2 + Q_3 - R_1, \quad \Phi_{22} = -Q_1 - R_1, \\ \Phi_{55} = \tau_m^2 R_1 + (\tau_1 - \tau_m)R_2 + (\tau_M - \tau_1)R_3, \\ \Xi_1 = [SA \ 0 \ 0 \ 0 \ -S \ \beta_0 SBK \ (1 - \beta_0)SBK], \\ \Xi_2 = \\ [0 \ N_1 \ -M_1 + N_2 \ -M_2 \ 0 \ -N_1 + M_1 \ -N_2 + M_2],$$

$$\Gamma_{21}^1 = \begin{bmatrix} (\tau_1 - \tau_m)M_1^T \\ (\tau_M - \tau_1)M_2^T \end{bmatrix}, \quad \Gamma_{21}^2 = \begin{bmatrix} (\tau_1 - \tau_m)M_1^T \\ (\tau_M - \tau_1)N_2^T \end{bmatrix}, \\ \Gamma_{21}^3 = \begin{bmatrix} (\tau_1 - \tau_m)N_1^T \\ (\tau_M - \tau_1)M_2^T \end{bmatrix}, \quad \Gamma_{21}^4 = \begin{bmatrix} (\tau_1 - \tau_m)N_1^T \\ (\tau_M - \tau_1)N_2^T \end{bmatrix}, \\ \Gamma_{22} = \text{diag}[-(\tau_1 - \tau_m)R_2 \quad -(\tau_M - \tau_1)R_3].$$

证明 选取如下的 Lyapunov-Krasovskii 泛函:

$$V(x_t) = \\ x^T(t)Px(t) + \int_{t-\tau_m}^t x^T(s)Q_1x(s)ds + \\ \int_{t-\tau_1}^t x^T(s)Q_2x(s)ds + \int_{t-\tau_M}^t x^T(s)Q_3x(s)ds + \\ \tau_m \int_{t-\tau_m}^t \int_s^t \dot{x}^T(v)R_1\dot{x}(v)dvds + \\ \int_{t-\tau_1}^{t-\tau_m} \int_s^t \dot{x}^T(v)R_2\dot{x}(v)dvds + \\ \int_{t-\tau_M}^{t-\tau_1} \int_s^t \dot{x}^T(v)R_3\dot{x}(v)dvds. \quad (13)$$

系统的无穷小算子定义为

$$\mathcal{L}V(x_t) = \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} \{E(V(x_{t+\Delta})|x_t) - V(x_t)\}, \quad (14)$$

则根据式 (13) 和 (14) 及利用自由权矩阵法可得

$$\mathcal{L}V(x_t) = \\ 2x^T(t)P\dot{x}(t) - x^T(t - \tau_m)Q_1x(t - \tau_m) + \\ x^T(t)(Q_1 + Q_2 + Q_3)x(t) -$$

$$x^T(t - \tau_1)Q_2x(t - \tau_1) - x^T(t - \tau_M)Q_3x(t - \tau_M) + \\ \dot{x}^T(t)[\tau_m^2 R_1 + (\tau_1 - \tau_m)R_2 + (\tau_M - \tau_1)R_3]\dot{x}(t) - \\ \tau_m \int_{t-\tau_m}^t \dot{x}(s)R_1\dot{x}(s)ds - \int_{t-\tau_1}^{t-\tau_m} \dot{x}(s)R_2\dot{x}(s)ds - \\ \int_{t-\tau_M}^{t-\tau_1} \dot{x}(s)R_3\dot{x}(s)ds + \\ 2\xi^T(t)S[Ax(t) - B\mu_2\delta + \beta_0BKx(t - \tau_1(t)) + \\ (1 - \beta_0)BKx(t - \tau_2(t)) - \dot{x}(t)] + \\ 2\xi^T(t)N_1 \left[x(t - \tau_m) - x(t - \tau_1(t)) - \int_{t-\tau_1(t)}^{t-\tau_m} \dot{x}(s)ds \right] + \\ 2\xi^T(t)M_1 \left[x(t - \tau_1(t)) - x(t - \tau_1) - \int_{t-\tau_1}^{t-\tau_1(t)} \dot{x}(s)ds \right] + \\ 2\xi^T(t)N_2 \left[x(t - \tau_1) - x(t - \tau_2(t)) - \int_{t-\tau_2(t)}^{t-\tau_1} \dot{x}(s)ds \right] + \\ 2\xi^T(t)M_2 \left[x(t - \tau_2(t)) - x(t - \tau_M) - \int_{t-\tau_M}^{t-\tau_2(t)} \dot{x}(s)ds \right].$$

其中

$$\xi^T(t) = [x^T(t) \ x^T(t - \tau_m) \ x^T(t - \tau_1) \ x^T(t - \tau_M) \rightarrow \\ \leftarrow \dot{x}^T(t) \ \dot{x}^T(t - \tau_1(t)) \ \dot{x}^T(t - \tau_2(t))], \\ N_1^T = [N_1^{1T} \ N_1^{2T} \ N_1^{3T} \ N_1^{4T} \ N_1^{5T} \ N_1^{6T} \ N_1^{7T}], \\ M_1^T = [M_1^{1T} \ M_1^{2T} \ M_1^{3T} \ M_1^{4T} \ M_1^{5T} \ M_1^{6T} \ M_1^{7T}], \\ N_2^T = [N_2^{1T} \ N_2^{2T} \ N_2^{3T} \ N_2^{4T} \ N_2^{5T} \ N_2^{6T} \ N_2^{7T}], \\ M_2^T = [M_2^{1T} \ M_2^{2T} \ M_2^{3T} \ M_2^{4T} \ M_2^{5T} \ M_2^{6T} \ M_2^{7T}], \\ S^T = [S_1^T \ 0 \ 0 \ 0 \ S_2^T \ S_3^T \ S_4^T].$$

由引理 2 得

$$- \tau_m \int_{t-\tau_m}^t \dot{x}(s)R_1\dot{x}(s)ds \leq \\ \begin{bmatrix} x(t) \\ x(t - \tau_m) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau_m) \end{bmatrix},$$

且由矩阵不等式性质知下列不等式成立:

$$- 2\xi^T(t)N_1 \int_{t-\tau_1(t)}^{t-\tau_m} \dot{x}(s)ds \leq \\ (\tau_1(t) - \tau_m)\xi^T(t)N_1R_2^{-1}N_1^T\xi(t) + \\ \int_{t-\tau_1(t)}^{t-\tau_m} \dot{x}^T(s)R_2\dot{x}(s)ds, \\ - 2\xi^T(t)M_1 \int_{t-\tau_1}^{t-\tau_1(t)} \dot{x}(s)ds \leq \\ (\tau_1 - \tau_1(t))\xi^T(t)M_1R_2^{-1}M_1^T\xi(t) + \\ \int_{t-\tau_1}^{t-\tau_1(t)} \dot{x}^T(s)R_2\dot{x}(s)ds, \\ - 2\xi^T(t)N_2 \int_{t-\tau_2(t)}^{t-\tau_1} \dot{x}(s)ds \leq \\ (\tau_2(t) - \tau_1)\xi^T(t)N_2R_3^{-1}N_2^T\xi(t) + \\ \int_{t-\tau_2(t)}^{t-\tau_1} \dot{x}^T(s)R_3\dot{x}(s)ds, \\ - 2\xi^T(t)M_2 \int_{t-\tau_M}^{t-\tau_2(t)} \dot{x}(s)ds \leq \\ (\tau_M - \tau_2(t))\xi^T(t)M_2R_3^{-1}M_2^T\xi(t) + \\ \int_{t-\tau_M}^{t-\tau_2(t)} \dot{x}^T(s)R_3\dot{x}(s)ds.$$

则可得

$$\begin{aligned}
 & \mathcal{L}V(x_t) \leq \\
 & 2x^T(t)P\dot{x}(t) + x^T(t)[Q_1 + Q_2 + Q_3]x(t) - \\
 & x^T(t - \tau_m)Q_1x(t - \tau_m) - x^T(t - \tau_1)Q_2x(t - \tau_1) - \\
 & x^T(t - \tau_M)Q_3x(t - \tau_M) + \\
 & \dot{x}^T(t)[\tau_m^2R_1 + (\tau_1 - \tau_m)R_2 + (\tau_M - \tau_1)R_3]\dot{x}(t) + \\
 & \begin{bmatrix} x(t) \\ x(t - \tau_m) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau_m) \end{bmatrix} + \\
 & 2\xi^T(t)N_1[x(t - \tau_m) - x(t - \tau_1(t))]+ \\
 & 2\xi^T(t)M_1[x(t - \tau_1(t)) - x(t - \tau_1)]+ \\
 & 2\xi^T(t)N_2[x(t - \tau_1) - x(t - \tau_2(t))]+ \\
 & 2\xi^T(t)M_2[x(t - \tau_2(t)) - x(t - \tau_M)]+ \\
 & (\tau_1(t) - \tau_m)\xi^T(t)N_1R_2^{-1}N_1^T\xi(t)+ \\
 & (\tau_1 - \tau_1(t))\xi^T(t)M_1R_2^{-1}M_1^T\xi(t)+ \\
 & (\tau_2(t) - \tau_1)\xi^T(t)N_2R_3^{-1}N_2^T\xi(t)+ \\
 & (\tau_M - \tau_2(t))\xi^T(t)M_2R_3^{-1}M_2^T\xi(t)+ \\
 & 2\xi^T(t)S[Ax(t) + \beta_0BKx(t - \tau_1(t)) + \\
 & (1 - \beta_0)BKx(t - \tau_2(t)) - B\mu_2\delta - \dot{x}(t)]. \tag{15}
 \end{aligned}$$

由引理 1 可知, 不等式 (15) 等价于如下 4 个不等式:

$$\begin{aligned}
 & \mathcal{L}V(x_t) \leq \\
 & \xi^T(t)\Gamma_{11}\xi(t) + (\tau_1 - \tau_m)\xi^T(t)M_1R_2^{-1}M_1^T\xi(t) + \\
 & (\tau_M - \tau_1)\xi^T(t)M_2R_3^{-1}M_2^T\xi(t) - 2\xi^T(t)SB\mu_2\delta, \\
 & \mathcal{L}V(x_t) \leq \\
 & \xi^T(t)\Gamma_{11}\xi(t) + (\tau_1 - \tau_m)\xi^T(t)M_1R_2^{-1}M_1^T\xi(t) + \\
 & (\tau_M - \tau_1)\xi^T(t)N_2R_3^{-1}N_2^T\xi(t) - 2\xi^T(t)SB\mu_2\delta, \\
 & \mathcal{L}V(x_t) \leq \\
 & \xi^T(t)\Gamma_{11}\xi(t) + (\tau_1 - \tau_m)\xi^T(t)N_1R_2^{-1}N_1^T\xi(t) + \\
 & (\tau_M - \tau_1)\xi^T(t)M_2R_3^{-1}M_2^T\xi(t) - 2\xi^T(t)SB\mu_2\delta, \\
 & \mathcal{L}V(x_t) \leq \\
 & \xi^T(t)\Gamma_{11}\xi(t) + (\tau_1 - \tau_m)\xi^T(t)N_1R_2^{-1}N_1^T\xi(t) + \\
 & (\tau_M - \tau_1)\xi^T(t)N_2R_3^{-1}N_2^T\xi(t) - 2\xi^T(t)SB\mu_2\delta.
 \end{aligned}$$

由式 (12) 得

$$\mathcal{L}V(x_t) \leq -\frac{\|\xi(t)\|}{\|T^{-1}\|}(\xi(t) - 2\|SB\|\mu_2\delta\|T^{-1}\|).$$

由 δ 的定义, 可将 δ 表示成

$$\begin{aligned}
 \delta = & \\
 & \mu_2^{-1}\mu_1K[\mu_1^{-1}x(t) - q_1(\mu_1^{-1}x(t))] + \\
 & [\mu_2^{-1}K\mu_1q_1(\mu_1^{-1}x(t)) - q_2(\mu_2^{-1}K\mu_1q_1(\mu_1^{-1}x(t)))] \leq \\
 & \|K\|\Delta_1 + \Delta_2 = \Delta,
 \end{aligned}$$

则

$$E\{\mathcal{L}V(x_t)\} \leq -\lambda(T)E\{\sqrt{\xi^T(t)\xi(t)}\}.$$

定义函数 $W(t) = e^{\varepsilon t}V(x_t)$, 则 $W(t)$ 的无穷小算子为

$$\mathcal{L}W(t) = \varepsilon e^{\varepsilon t}V(x_t) + e^{\varepsilon t}\mathcal{L}V(x_t), \tag{16}$$

$$E\{W(x_t)\} - E\{W(x_0)\} =$$

$$\int_0^t \varepsilon e^{\varepsilon s}E\{V(x_s)\}ds + \int_0^t e^{\varepsilon s}E\{\mathcal{L}V(x_s)\}ds. \tag{17}$$

结合式 (17) 可以证明, 当 $\varepsilon > 0$ 充分小时, 可以找到一个常数 $\rho > 0$, 使得

$$E\{V(x_t)\} \leq \rho \sup_{-\tau_M \leq s \leq 0} E\{\|\phi(s)\|^2\} e^{-\varepsilon t}. \tag{18}$$

由式 (13) 可得

$$V(x_t) \geq \{\lambda_{\min}(P)\}x^T(t)x(t), \tag{19}$$

$$E\{x^T(t)x(t)\} \leq$$

$$\sqrt{\lambda_{\min}^{-1}(P)\rho} e^{-\varepsilon t} \sup_{-\tau_M \leq s \leq 0} E\{\|\phi(s)\|^2\}. \tag{20}$$

由此, 定理 1 得证. \square

4 状态反馈控制器设计

定理 2 给定常数 $\tau_m, \tau_1, \tau_M, \lambda_j (j=2, 3, 4), \beta_0$, 如果存在正定对称矩阵 $\tilde{P}, \tilde{Q}_i (i=1, 2, 3), \tilde{T}, \tilde{R}_1, \tilde{R}_2, \tilde{R}_3$ 以及矩阵 $X, Y, \tilde{N}_i^j (i=1, 2, j=1, 2, \dots, 7), \tilde{M}_i^j (i=1, 2, j=1, 2, \dots, 7)$, 使得下列 LMIs 有解:

$$\begin{bmatrix} \tilde{T}_{11} + \tilde{T} & * \\ \tilde{T}_{21} & \tilde{T}_{22} \end{bmatrix} < 0, \quad l = 1, 2, 3, 4, \tag{21}$$

且量化器参数满足

$$2\Delta \| [X^{-T} \ 0 \ 0 \ 0 \ \lambda_2 X^{-T} \ \lambda_3 X^{-T} \ \lambda_4 X^{-T}]^T B \| \times$$

$$\| \tilde{T}^{-1} \| \leq \frac{\|x(t)\|}{\mu_2} \leq F_1,$$

$$F_2 \geq \|YX^{-T}\| (\Delta_1 + F_1).$$

则系统 (5) 渐近稳定, 且控制器为 $K = YX^{-T}$. 其中

$$\tilde{T}_{11} = \tilde{T} + \tilde{\Xi}_1 + \tilde{\Xi}_1^T + \tilde{\Xi}_2 + \tilde{\Xi}_2^T,$$

$$\tilde{T} = \begin{bmatrix} \tilde{\Phi}_{11} & * & * & * & * & * & * \\ 0 & \tilde{\Phi}_{22} & * & * & * & * & * \\ 0 & 0 & -\tilde{Q}_2 & * & * & * & * \\ 0 & 0 & 0 & -\tilde{Q}_3 & * & * & * \\ \tilde{P}^T & 0 & 0 & 0 & \tilde{\Phi}_{55} & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Phi}_{11} = \tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 - \tilde{R}_1, \quad \tilde{\Phi}_{22} = -\tilde{Q}_1 - \tilde{R}_1,$$

$$\tilde{\Phi}_{55} = \tau_m^2 \tilde{R}_1 + (\tau_1 - \tau_m) \tilde{R}_2 + (\tau_M - \tau_1) \tilde{R}_3,$$

$$\tilde{T}_{21}^1 = \begin{bmatrix} (\tau_1 - \tau_m) \tilde{M}_1^T \\ (\tau_M - \tau_1) \tilde{M}_2^T \end{bmatrix}, \quad \tilde{T}_{21}^2 = \begin{bmatrix} (\tau_1 - \tau_m) \tilde{M}_1^T \\ (\tau_M - \tau_1) \tilde{N}_2^T \end{bmatrix},$$

$$\tilde{T}_{21}^3 = \begin{bmatrix} (\tau_1 - \tau_m) \tilde{N}_1^T \\ (\tau_M - \tau_1) \tilde{M}_2^T \end{bmatrix}, \quad \tilde{T}_{21}^4 = \begin{bmatrix} (\tau_1 - \tau_m) \tilde{N}_1^T \\ (\tau_M - \tau_1) \tilde{N}_2^T \end{bmatrix},$$

$$\begin{aligned} \tilde{\Xi}_1 = & \text{diag}(\overbrace{X \cdots X}^7) [SA \ 0 \ 0 \ 0 \ -S \rightarrow \\ & \leftarrow \beta_0 SBK \ (1-\beta_0) SBK] \text{diag}(\overbrace{X^T \cdots X^T}^7), \\ \tilde{\Xi}_2 = & \text{diag}(\overbrace{X \cdots X}^7) [0 \ N_1 \ -M_1+N_2 \ -M_2 \ 0 \rightarrow \\ & \leftarrow -N_1+M_1 \ -N_2+M_2] \text{diag}(\overbrace{X^T \cdots X^T}^7), \\ \tilde{\Gamma}_{22} = & \text{diag}(-(\tau_1 - \tau_m)\tilde{R}_2 - (\tau_M - \tau_1)\tilde{R}_3). \end{aligned}$$

证明 将不等式(12)展开, 并令 $S_1 = X^{-1}, S_j = \lambda_j X^{-1}, j = 2, 3, 4$. 对式(12)两边分别左乘和右乘 $\text{diag}(\overbrace{X \cdots X}^9)$ 及其转置, 并令 $XPX^T = \tilde{P}, XTX^T = \tilde{T}, XQ_iX^T = \tilde{Q}_i (i = 1, 2, 3), XR_iX^T = \tilde{R}_i (i = 1, 2, 3), XN_i^jX^T = \tilde{N}_i^j (i = 1, 2, j = 1, 2, \dots, 7), XM_i^jX^T = \tilde{M}_i^j (i = 1, 2, j = 1, 2, \dots, 7)$, 同时令 $Y = KX^T$, 则定理 2 得证. \square

5 数值仿真

例 1 考虑如下系统:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t). \quad (22)$$

对于一给定的线性控制器 $u(t) = [-3.75 \ -11.5]x(t)$, 表 1 列出了保证系统渐近稳定的时滞上界. 在相同条件 $\beta_0 = 1, \tau_m = 0$ 的情况下, 定理 1 求得的时滞上界为 1.0432, 文献 [2] 为 0.8695.

表 1 给定 β_0, τ_1 , 假设 $\tau_m = 0$ 的最大时滞

τ_1	β_0		
	0.5	0.7	0.9
0.1	3.171	5.305	15.850
0.5	2.482	3.868	10.630
0.8	1.965	2.763	6.662

例 2 考虑如下系统:

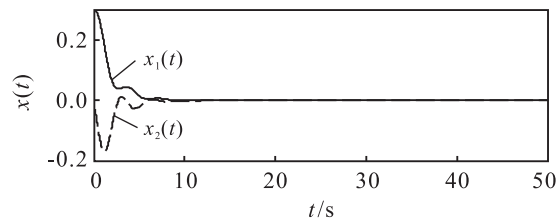
$$\dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & 0.99 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u(t). \quad (23)$$

考虑量化的影响, 通过定理 2, 利用 LMI 工具箱, 选择 $\lambda_2 = 20, \lambda_3 = 0.2, \lambda_4 = 0.2, \tilde{T} = I, \tau_m = 0, \tau_1 = 0.2, \beta_0 = 0.5$, 并取

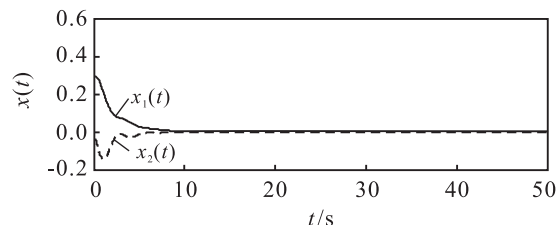
$$\begin{aligned} \tau_M = 0.5, \quad K = [-0.1642 \quad -0.2598], \\ X = \begin{bmatrix} 0.0013 & -0.0018 \\ -0.0007 & 0.0016 \end{bmatrix}, \end{aligned}$$

则 $F_1 \geq 4.0472\Delta, F_2 \geq 0.0777(\Delta_1 + F_1)$. 取 $\Delta_1 = \Delta_2 = 0.1$, 则 $\|x(t)\|/\mu_1 \geq 15.54, \delta = 1.077$. 时变量化器变量 $\mu_1 = \mu_2$ 随着 $x(t)$ 的变化而变化, 假设 $x(0) = [0.3 \ 0]^T$. 图 1 为有量化和无量化作用下的系统状态响应曲线和控制输入曲线, 图 2 为时滞随机变化的特

性. 从图 1 可以看出, 在状态量快速变化期间, 量化作用最大; 在状态量趋于稳定期间, 量化精度大.



(a) 考虑信号量化情况下系统的状态响应



(b) 不考虑信号量化情况下系统的状态响应

图 1 考虑量化和不量化情况下系统(23)状态响应

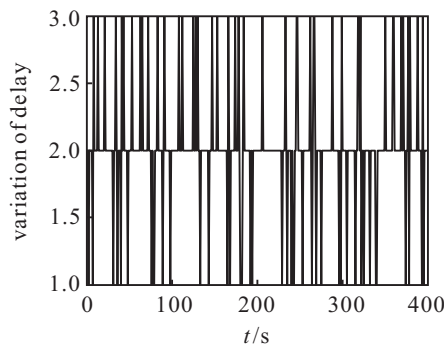


图 2 时滞随机变化特性

6 结论

本文首先通过引入一个随机变量建立了新的时滞分布依赖的具有量化的网络化系统模型; 然后利用 Lyapunov 稳定性理论, 借助于矩阵函数的凸性、自由权方法和 Jessen 不等式技术对新模型进行了稳定性分析和控制器设计; 最后通过求解 LMI 不等式, 利用 2 个例子的仿真结果验证了所提出方法的有效性. 因为利用了时滞的更多信息, 所以得到了更好的结果.

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