

科目：工程數學 A(3003)

校系所組：中央大學電機工程學系(電子組、系統與生醫組)

交通大學電子研究所(甲組、乙 A 組、乙 B 組)

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清華大學電機工程學系(甲組)

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清華大學工程與系統科學系(丁組)

陽明大學生物醫學工程學系(醫學電子組)

參考用

1. (15%) Let  $V$  be the set of all 2-by-2 real symmetric matrices, i.e.,

$$V = \left\{ \begin{pmatrix} a & b \\ b & c \end{pmatrix} : a, b, c \in \mathbb{R}, \text{ where } \mathbb{R} \text{ the set of real number} \right\}$$

and let  $W$  be the set of all 2-by-2 real matrices. Let  $T : V \rightarrow W$  be a linear transformation defined by

$$T \left( \begin{pmatrix} a & b \\ b & c \end{pmatrix} \right) = \begin{pmatrix} a-c & -b+c \\ -2a+b+c & -a+c \end{pmatrix}.$$

(a) (5%) Find a basis  $\gamma$  for the range space of  $T$ . (Please show details of your derivations.)

(b) (5%) Consider the Frobenius inner product where the inner product between

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \text{ and } B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \in W$$

is defined by  $\langle A, B \rangle = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$ .

Find an orthonormal basis  $\alpha$  for the range space of  $T$  using the Gram-Schmidt process on the basis  $\gamma$  obtained in (a). (Please show details of your derivations.)

(c) (5%) Let

$$\beta = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}$$

be an ordered basis for  $V$  and let  $\alpha$  be the orthonormal basis obtained in (b). Find the matrix representation of  $T$  in the ordered basis  $\beta$  and  $\alpha$ , i.e.,  $[T]_{\beta}^{\alpha}$ .

2. (10%) Suppose that

$$\det \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = -5$$

and  $b_1c_2 = -b_2c_1 = 1$ . Compute the determinant of the matrix  $M$ . Please show your derivations.

$$M = \begin{pmatrix} b_1 + 2c_1 & b_2 + 2c_2 & b_3 + 2c_3 \\ a_1 & a_2 & a_3 + 2 \\ 3b_1 + c_1 & 3b_2 + c_2 & 3b_3 + c_3 \end{pmatrix}$$

注意：背面有試題

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3. (25%) In the s-domain, via Laplace transform, one has

$$Y(s) = \frac{1}{s^2 + 16} + \frac{s}{(s^2 + 16)^2} - \frac{s}{(s^2 + 16)^2} e^{-\pi s}$$

(a) (9%) If this  $Y(s)$  is used to describe a 2nd-order Ordinary Differential Equation:

$$y''(t) + p(t)y'(t) + q(t)y(t) = r(t),$$

with the initial conditions:  $y(0) = 0$  and  $y'(0) = 1$ , find  $p(t)$ ,  $q(t)$  and  $r(t)$ , respectively.

(b) (5%) Plot  $r(t)$ .

(c) (5%) Find the solution  $y(t)$ .

(d) (6%) Plot  $y(t)$ .

4. (15%) Assume  $x \neq 0$ , for a 2nd-order differential equation:  $x^2 y'' + xy' + y = 4 \sin(\ln x)$ .

(a) (6%) Please find the general solution of the corresponding homogeneous ODE in real function format.

(b) (9%) Please find the particular solution that satisfies the given non-homogeneous ODE.

5. (15%) Legendre equation meets the Sturm-Liouville problem on the interval:  $-1 \leq x \leq 1$  and the solution can be expressed as a generalized Legendre-Fourier series:

$$y(x) = \sum_{n=0}^{\infty} a_n y_n(x)$$

where  $y_n(x)$  is the corresponding eigenfunction of the Legendre equation, then

$$\int_{-1}^1 y_m(x) y_n(x) dx = \delta_{mn} = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

If  $y(x) = x^2 + 2x + 3$  is a solution for this Legendre equation, please express  $y(x)$  as a generalized Legendre-Fourier series. (Please show details of your derivations.)

6. (20%) For a complex function  $f(z) = (1 + z^3)^{-1}$ ,

(a) (6%) Please find the residues of  $f(z)$ .

(b) (14%) Applying the results in (a) to evaluate the integral by contour integration in the complex plane

$$I = \int_0^{\infty} \frac{dx}{1 + x^3}$$