國　立　清　華　大　學　命　題　紙

九十三學年度 電子工程研究所 系（所） 組碩士班入學考試

科目 近代物理 科號 2904 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Suppose two reference frames are moving with a relative speed v along the x direction, where the origins of the two frames are supposed to coincide at the instant $t = 0$. A space-time transformation is given in a symmetric way by

$$\begin{cases} x_2 = \gamma(x_1 - \tau_1 v/c) \\ \tau_2 = \gamma(\tau_1 - x_1 v/c) \end{cases}$$

where $\tau = ct$. The inverse transformation is then given by $x_1 = \gamma(x_2 + \tau_2 v/c)$, and $\tau_1 = \gamma(\tau_2 + x_2 v/c)$. Suppose the transformation is self-consistent, find the factor γ in terms of v and c. (14%)

2. The commutator between two operators A and B is given by

$$[A, B] = AB - BA$$

Show that $\left[x, \frac{d^2}{dx^2}\right] = c_1 \frac{d}{dx}$, and $\left[\frac{d}{dx}, f\right] = c_2 \frac{df}{dx}$, and find the constants c_1 and c_2, where f is an arbitrary scalar function of space. (16%)

3. Show that the de Broglie wavelength of a particle of rest mass m and kinetic energy KE is given by

$$\lambda = \frac{hc}{\sqrt{\mathrm{KE}\left(\mathrm{KE} + 2mc^2\right)}},$$

where h is the Planck constant, c is the speed of light. (7%)

4. A photon of energy $h\nu$ collides with a stationary electron of rest mass m. Show that it is not physically possible for the photon to impart all its energy to the electron. (8%)

5. The frequency of oscillation of a harmonic oscillator of mass m and spring constant C is $\nu = \frac{1}{2\pi}\sqrt{\frac{C}{m}}$. The energy of the oscillator is $E = p^2/2m + Cx^2/2$, where p is its momentum when its displacement from the equilibrium position is x. In classical physics the minimum energy of the oscillator is $E_{min} = 0$. Use the uncertainty principle to show that the minimum energy is actually $E_{min} = h\nu/2$. (8%)

6. Consider the operator $\hat{C}$,

$$\hat{C}\varphi(x) = \varphi^*(x).$$

$\varphi^*(x)$ is the complex conjugate of $\varphi(x)$.

(a) What are the eigenvalues of $\hat{C}$? (6%)
(b) What are the eigenfunctions of $\hat{C}$? (6%)

7. A one dimensional infinite potential well extends from $x = -L$ to $x = +L$ and is divided into three sections by rigid wall of infinite potential at $x = -x_0$ to $x = +x_0$, where $x_0 < L$. Assume each section contains one particle with mass m in its ground state.

(a) Draw the potential diagram of this potential well and draw the wavefunction of each particle in ground state clearly. (4%)
(b) Write down the expression of the total energy of this three particle system in terms of x_0 and L. (4%)
(c) Determine the value of x_0 and for which the total energy (found in part (c)) is minimized. (4%)
(d) What is the value of this total energy? (4%)

8. Eight identical noninteracting Fermions (each with mass m) are placed in a cubical box of size L.

(a) Find the lowest energy of the system. (4%)
(b) List the quantum number of the occupied states. (3%)
(c) List the quantum number of the occupied states if the particles are Boson and the system is in the ground state. (3%)

9. An electron in an atom is in $4F_{5/2}$ state.

(a) Find the value of quantum number n, ℓ, and j. (3%)
(b) What is the magnitude of the electron's total angular momentum? (3%)
(c) What are the possible values for the z component of the electron's total angular momentum? (3%)