What floats your boat?
Preference revelation from lotteries over complex goods

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Idaho's Four Rivers Lottery

Objective and contribution

Basic premise:

- Win rates contain information about preferences over the attributes of the lottery goods.

Objectives:

- We derive a complete vNM utility index map over lottery good attributes based on endogenous win rates.
- We estimate a vNM utility function using 4 Rivers Lottery application data for 2000-2010.

Literature in a nutshell:

- Several papers estimate WTP from lottery-allocated goods based on applicant attributes.
- No one has exploited the information content of win rates to characterize preferences in a lottery structure like this.

Lottery structure

- Applicant pays fee C for the chance to win one discrete complex good $\mathbf{x}_i$ among alternatives $\mathbf{X}$.
- A_i applicants for each of Q_i units of good $\mathbf{x}_i$.
- Probability of winning $\mathbf{x}_i$ is $\pi_i = Q_i/A_i$; not known with certainty at application time.
- $\hat{A}_i$ is the expected number of applicants for $\mathbf{x}_i$.
- $\hat{\pi}_i \propto Q_i/\hat{A}_i$ is the expected probability of winning good $\mathbf{x}_i$ upon which applicants base their decision.
- Losers receive reservation good x_0, and C is lost.
- Nonapplicants retain C, receive x_0 with certainty.

Applicant's decision

- Applicant $I's$ utility for $\mathbf{x}_i$ is $U_I(\mathbf{x}_i) = U(\mathbf{x}_i)v_{Ii}$; $v_{Ii} \sim (1, \sigma^2)$.
- Reservation utility is normalized to zero: $U_I(\mathbf{x}_i) = 0$.
- Applicants choose among $N+1$ gambles:

$$g_i \equiv \{\hat{\pi}_i \circ \mathbf{x}_i | \, i \in (1, \cdots, N)\}$$
$$g_0 \equiv \{(\mathbf{x}_0, C) | \, i = 0\}$$

- For each good: $E[U_I(\mathbf{x}_i)] = \hat{\pi}_i U_I(\mathbf{x}_i)$.
- Applicant I chooses $\mathbf{x}_i$ such that

$$\widehat{EU}_I(\mathbf{X}) = \sup\{(\hat{\pi}_i U_I(\mathbf{x}_i), \ C) : i = 0, 1, \cdots N\}$$

Lottery equilibrium

Equilibrium conditions for application

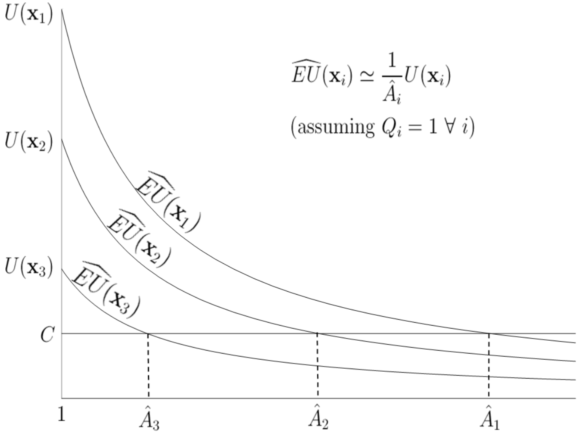
- Applicants accept lower odds for more preferred goods.
- $\hat{\pi}_i$ is the *ex ante* predicted win probability based on the representative applicant $U_l(\mathbf{x}_i) = U(\mathbf{x}_i)$ ($E[v_{li}] = 1$ for all i, l).
- In equilibrium, expected probabilities $\hat{\pi}_i =$ are such that expected utility of the representative applicant equals C.

$$\hat{\pi}_i(\mathbf{x}_i, Q_i)U(\mathbf{x}_i) = \hat{\pi}_j(\mathbf{x}_j, Q_j)U(\mathbf{x}_j) = C$$

Equilibrium expected application demands

$$\hat{\mathbf{A}}(\mathbf{X}, \mathbf{Q}) = \begin{bmatrix} \hat{A}_1(\mathbf{x}_1, Q_1) & \hat{A}_2(\mathbf{x}_2, Q_2) & \cdots & \hat{A}_N(\mathbf{x}_N, Q_N) \end{bmatrix}'$$

Lottery equilibrium



Utility tradeoffs: attributes and odds of winning

- Recast applicant decision as a choice over continuous $\mathbf{x} \in \mathbf{X}$.
- $\mathbf{Q}$ set by lottery authority, can vary over $\mathbf{x}$.
- Define $\mathbf{Q}(\mathbf{x})$, known at application time.
- Allow $\mathbf{Q}$ to affect U directly: e.g. congestion.
- Application demands $\mathbf{A}(\mathbf{x}, \mathbf{Q}(\mathbf{x}))$.

Applicant decision revisited

$$C = \frac{\mathbf{Q}(\mathbf{x})}{\mathbf{A}(\mathbf{x}, \mathbf{Q}(\mathbf{x}))} U(\mathbf{x}, \mathbf{Q}(\mathbf{x})).$$

MU and MRS

define $u = \ln(U)$, $a = \ln(A)$, and $q = \ln(Q)$.

First-order conditions provide MU:

$$\partial u(\mathbf{x})/\partial x^j = \partial a(\mathbf{x})/\partial x^j$$
$$\partial u(\mathbf{x})/\partial q = \partial a(\mathbf{x})/\partial q - 1$$

MRS follows directly:

$$\mathrm{MRS}^{jk} = \frac{\partial a(\mathbf{x})/\partial x^j}{\partial a(\mathbf{x})/\partial x^k}; \qquad \mathrm{MRS}^{jq} = \frac{\partial a(\mathbf{x})/\partial x^j}{\partial a(\mathbf{x})/\partial q - 1} \qquad j \neq q.$$

All elements recoverable from estimable regression for application demand $\mathbf{A}(\mathbf{X}, \mathbf{Q})$.

vNM utility maps

- Assuming the vNM Utility Theorem holds, vNM utility index can be derived from equilibrium win rates.

vNM utility

$$U(\mathbf{x}_i) = \frac{\pi(\mathbf{x}_1)}{\pi(\mathbf{x}_i)} = \frac{A(\mathbf{x}_i)}{A(\mathbf{x}_1)} \frac{Q_1}{Q_i},$$

where $\mathbf{x}_1$ is the most preferred alternative identified by the lowest probability of success.

- This value is estimable and therefore mappable over the characteristics in $\mathbf{x}$.

Empirical strategy

Applicants apply based on expectations about river characteristics.

Two-stage estimation approach

Stage 1: Estimate model(s) of river/weather characteristics over the permit season based on information available at application time.

Stage 2: Use predicted values from these models as proxies for expectations in an application demand model.

Data

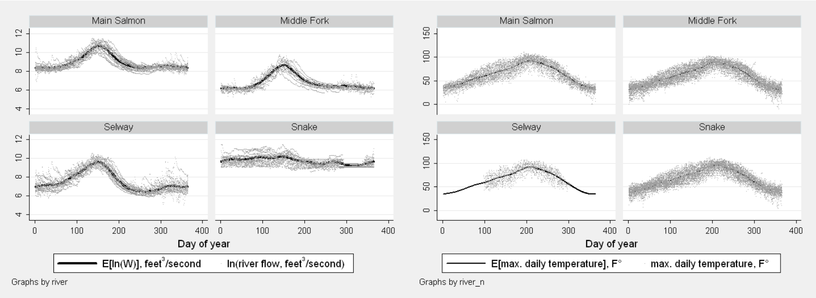
- River flows and temperature by river/day of year, and a January forecast based on snowpack.
- Application and permit numbers by river/day, 2000-2010 seasons.

Data descriptions

w	$\ln(\mathit{flow}) \equiv \ln(\text{feet}^3/\text{second})$.
t	Daily Maximum temperature, (F°).
f	January 1 flow forecast % of average for April-July.
$A; a$	# applicants by river/date; $a = \ln(A)$.
$Q; q$	# of available permits by river/date; $q = \ln(Q)$.
d	Day of year.
y	Year.

First stage: Water and weather

Trigonometric regression used for flexibility.



Second Stage: model of application rates

- Application numbers (by river/day) are count data.
- Negative binomial regression used to account for empirical overdispersion: Contagion due to group coordination of applications?
- Implied regression relationship: $\ln(A) = \mathbf{X}\boldsymbol{\beta} + \varepsilon$.
- $\mathbf{x}\boldsymbol{\beta} =$ quadratic in $\hat{w}$ and $\hat{t}$ to allow second-order curvature; river dummies, and q.

Second Stage Regression results

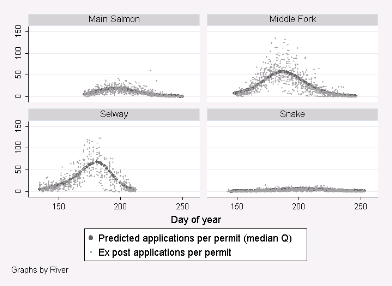
var	β
q	0.26***
$\hat{w}$	13.30***
$\hat{w}\cdot$ MF	0.98
$\hat{w}\cdot$ SE	-0.22
$\hat{w}\cdot$ SN	-18.51**
$\hat{w}^2\cdot$ MN	-0.65***
$\hat{w}^2\cdot$ MF	-0.81***
$\hat{w}^2\cdot$ SE	-0.65***
$\hat{w}^2\cdot$ SN	0.38
$\hat{t}$	0.90***
$\hat{t}^2$	-0.01***
constant	-110.5***

$^*10\%;^{**}5\%;^{***}1\%.$

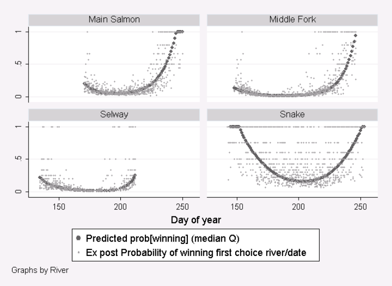
Notes

- Negative binomial regression; Dep. var. $= A$.
- $N = 3,458$. Dispersion param. sig. at $< 1\%$.
- Base Case: $y = 2000$, $r =$ Main Salmon.
- Annual dummies & non-interacting river dummies omitted for space.

Application & win rates (2007 prediction)



(c) A/Q; $\hat{A}_i/Q$ (2007).



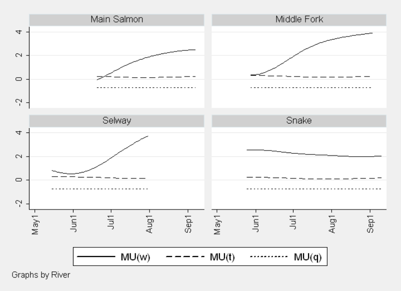
(d) π, $\hat{\pi}$ (2007).

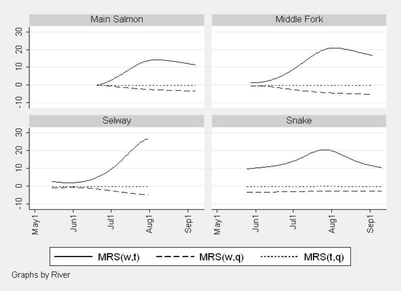
Three perspectives on preferences

- Marginal utility.
- Marginal rates of substitution.
- vNM utility maps.

MU and MRS

Figure : MU and MRS between w, t, and q. 2007 base.

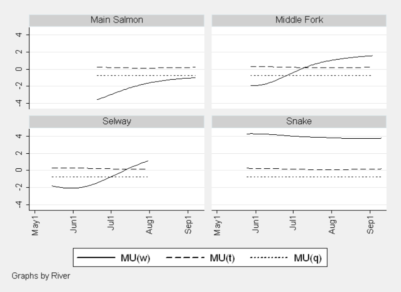


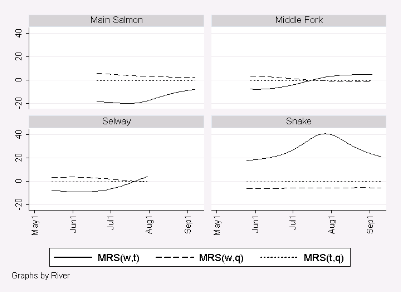


- MU(w) positive and increasing through the season.
- MU(q) negative: signs of congestion effects.
- MRS(w,t) driven largely by w.

MU and MRS: high water year

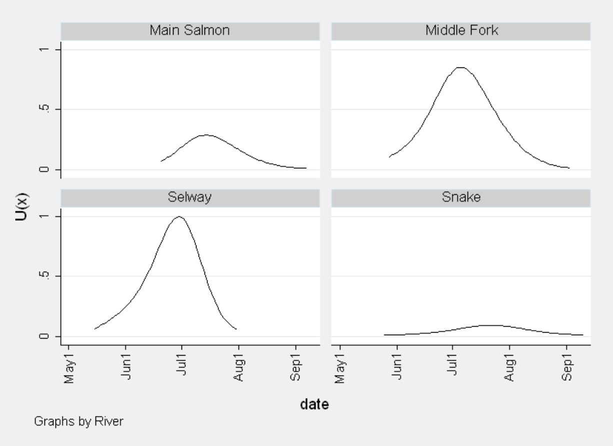
Figure : MU and MRS between w, t, and q. Jan. forecast 125% normal.





- increase expected water levels — MU(w) becomes negative for the early season.

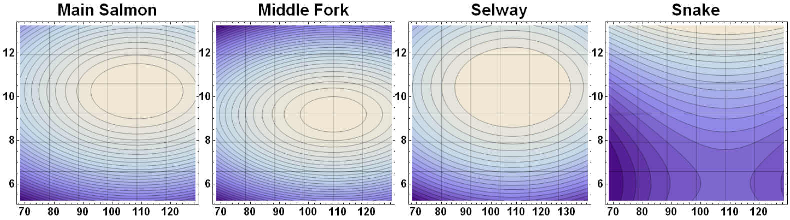
vNM utility. example 2007



- Selway around July 1 is the most preferred for 2007.

Utility maps

Figure : vNM utility index in w,t space based on median Q for each river.



- There can be too much water and too much heat.
- Snake river shows little empirical variation and results are odd and weak (curvature not statistically significant).

Conclusions and extensions

- Win rates provide sufficient information to fully characterize vNM utility index for the set of available alternatives.
- They provide a mechanism for estimation MRS as well as implicit hedonic prices over the characteristics of the goods.
- Can be useful for estimating the benefits of policy changes over available goods.

Contribution to literature in more detail

- Big literature on choice and equilibrium under uncertainty, but essentially all assumes exogenous outcomes.
- Exception: endogenous lottery probabilities: Rapoport (2002).
- Empirical: Scrogin (2005), Scrogin & Berrens (2003) and others; WTP and welfare estimation.
  - S & SB: $\hat{\pi}_t = f(\pi_{t-1})$; use $\hat{\pi}_t$ in RUM to estimate $U(\mathbf{x})$. Problematic if lottery goods or structure changes.
  - Nickerson (1990) estimates application demand $a(\mathbf{x})$ attributes.
- Nutshell: we extend Nickerson and Scrogin papers to more fully utilize information content of application rates.