文章编号:1007-4708(2010)05-0839-06

不均匀磁电弹板中的厚度扭曲波的传播

孔艳平*， 郭树起， 刘金喜

(石家庄铁道大学 工程力学系,石家庄 050043)

摘　要:压电-压磁复合材料或结构的许多应用是与弹性波的传播密切相关的,这要求人们首先从理论的角度弄清楚弹性波的传播规律。本文研究由多个不均匀磁电弹介质组成的薄板厚度扭曲波的传播性质,从磁电弹全耦合场三维方程出发得到了其精确解,根据所得到的解分析了波的传播特征,这些结果对于理解和设计谐振器、滤波器以及声波元件提供了有价值的理论基础。

关键词:磁电弹材料;压电材料;压磁材料;波;振动

中图分类号:O346　　　**文献标识码:**A

1 引　言

压电材料具有力-电耦合效应,而压磁材料具有力-磁耦合效应,即分别能够实现机械能和电能、机械能和磁能之间的相互转换,因而这两类材料得到了广泛的工程应用。当将压电材料和压磁材料按照一定的方式制作成复合材料(或结构)时,这种复合材料不仅具有力-电和力-磁耦合性能,而且具有磁电耦合效应。磁电效应的存在意味着当施加电场时,压电-压磁复合材料(也称作磁电弹材料)会产生磁极化;反之,在施加磁场时,压电-压磁复合材料或结构会产生电极化。磁电之间的耦合是一种新的积效应,来源于压电和压磁相之间的相互作用。压电-压磁复合材料可以用于制作磁电控制装置[1]、传感器[2,3]、微波器件[4,5]和其他电子产品等[6]。这些应用引起了凝聚态物理、材料科学和固体力学等领域的研究人员对压电-压磁复合材料的磁电性能以及相关基本力学问题的极大兴趣。压电-压磁复合材料或结构的许多应用是与弹性波的传播密切相关的[1-6],这要求人们首先从理论的角度弄清楚弹性波的传播规律,即几何构形、尺寸、材料性能和边界条件对传播特征的影响。Alahits等[7]最早定性地研究了具有压电、压磁和磁电效应各向异性弹性半空间的表面波的存在性问题。Soh和Liu[8]分析了由半无限大压电介质和半无限大压磁介质组成的双材料空间中界面SH波的存在性,给出了SH波存在时压电和压磁材料的性能所应满足的条件。Zhou等[9]利用Schmidt方法分析了两种不同压电压磁复合材料界面可导通单裂纹和共线裂纹对反平面简谐波的散射问题。Feng等[10]研究了压电-压磁复合材料中圆弧型界面裂纹对SH波的散射问题,通过数值算例讨论了入射角和裂纹张开角对动应力强度因子的影响,这对于界面性能的无损检测是有意义的。

以晶体薄板的厚扭振动模态为工作方式,石英和极化陶瓷可以做成共振器、滤波仪和声波传感器。近年来,由于AIN和ZnO薄膜制成薄膜共振器而引起了大家的兴趣[11]。当6 mm晶体的6次对称轴平行于其主平面时,在无界薄板中发现厚度扭曲波[12]。对面内极化的6 mm晶体,在有限矩形板中也发现了厚度扭曲波[13]。Yang等[14]研究了无限的、不均匀的压电介质中的厚度扭曲振动。本文从磁-电-弹全耦合场出发,对于由不均匀交替横观各向同性压电-压磁材料介质中波的传播特性进行探索性的研究,试图得到一些有价值的理论结果。

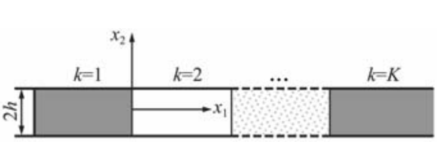

图1　横观各向同性磁电弹性板
Fig. 1　An inhomogeneous magneto-electro-elastic plate

收稿日期:2008-11-28;**修改稿收到日期:**2009-05-19.
基金项目:国家自然科学基金(10972147)资助项目.
作者简介:孔艳平*(1977-),女,硕士,讲师
(E-mail:kongyanping1977@163.com);
郭树起(1972-),男,博士,副教授;
刘金喜(1961-),男,博士,教授,博士生导师.

2 问题的描述与求解

考虑一横观各向同性的不均匀的磁电弹性板，极化轴沿 x_3 方向，板的上下表面均是应力自由表面，每一部分的材料与它相邻部分的材料不同，仅考虑反平面变形状态，此时厚度扭曲运动由如下位移 u_1,u_2 和 u_3，电势 φ_e 及磁势 φ_m 控制：

$$u_1=u_2=0,u_3=u_3(x_1,x_2,t)$$
$$\varphi_e=\varphi_e(x_1,x_2,t),\varphi_m=\varphi_m(x_1,x_2,t) \tag{1}$$

模态的控制方程为

$$\begin{cases} c_{44}^{(k)}\nabla^2 u_3+e_{15}^{(k)}\nabla^2\varphi_e+h_{15}^{(k)}\nabla^2\varphi_m=\rho u_{3,tt}\\ e_{15}^{(k)}\nabla^2 u_3-\varepsilon_{11}^{(k)}\nabla^2\varphi_e-\alpha_{11}^{(k)}\nabla^2\varphi_m=0\\ h_{15}^{(k)}\nabla^2 u_3-\alpha_{11}^{(k)}\nabla^2\varphi_e-\mu_{11}^{(k)}\nabla^2\varphi_m=0 \end{cases} \tag{2}$$

式中 $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ 为二维 Laplace 算子；c_{44},e_{15},h_{15} 和 α_{11} 分别为弹性、压电、压磁和磁电常数；ε_{11} 和 μ_{11} 分别为介电常数和磁通率，上标 k 表示研究材料的位置，t 表示时间，下标中的逗号“,”表示对其后的变量求偏导数。

引入电位移势 Ψ_e 及磁感势 Ψ_m，可以由文献[13,14]得出：

$$\begin{cases} \Psi_e=\varepsilon_{11}^{(k)}\varphi_e+\alpha_{11}^{(k)}\varphi_m-e_{11}^{(k)}u_3\\ \Psi_m=\alpha_{11}^{(k)}\varphi_e+\mu_{11}^{(k)}\varphi_m-h_{15}^{(k)}u_3 \end{cases} \tag{3}$$

即

$$\begin{bmatrix}\varphi_e\\ \varphi_m\end{bmatrix}=\frac{1}{\varepsilon_{11}^{(k)}\mu_{11}^{(k)}-(\alpha_{11}^{(k)})^2}\left\{\begin{bmatrix}\mu_{11}^{(k)}\\ -\alpha_{11}^{(k)}\end{bmatrix}\Psi_e+\begin{bmatrix}-\alpha_{11}^{(k)}\\ \varepsilon_{11}^{(k)}\end{bmatrix}\Psi_m+\begin{bmatrix}e_{15}^{(k)}\mu_{11}^{(k)}-h_{15}^{(k)}\alpha_{11}^{(k)}\\ h_{15}^{(k)}\varepsilon_{11}^{(k)}-e_{15}^{(k)}\alpha_{11}^{(k)}\end{bmatrix}u_3\right\} \tag{4}$$

则控制方程(2)可以简化为

$$(c_{SH}^{(k)})^2\nabla^2 u_3=u_{3,tt},\nabla^2\Psi_e=0,\nabla^2\Psi_m=0 \tag{5}$$

式中

$$c_{SH}^{(k)}=\sqrt{\tilde{c}_{44}^{(k)}/\rho^{(k)}}$$

$$\tilde{c}_{44}^{(k)}=c_{44}^{(k)}+\frac{\mu_{11}^{(k)}(e_{15}^{(k)})^2+\varepsilon_{11}^{(k)}(h_{15}^{(k)})^2-2e_{15}^{(k)}h_{15}^{(k)}\alpha_{11}^{(k)}}{\mu_{11}^{(k)}\varepsilon_{11}^{(k)}-(\alpha_{11}^{(k)})^2}$$

此时的应力 $\sigma_{3\beta}$、电位移 D_β 和磁感应强度 B_β 为

$$\begin{cases} \sigma_{3\beta}=\tilde{c}_{44}^{(k)}u_{3,\beta}+a^{(k)}\Psi_{e,\beta}+b^{(k)}\Psi_{m,\beta}\\ D_\beta=-\Psi_{e,\beta},B_\beta=-\Psi_{m,\beta} \end{cases} \tag{6}$$

式中 $\beta=1,2$，系数 $a^{(k)}$ 和 $b^{(k)}$ 分别为

$$\begin{cases} a^{(k)}=\dfrac{e_{15}^{(k)}\mu_{11}^{(k)}-h_{15}^{(k)}\alpha_{11}^{(k)}}{\varepsilon_{11}^{(k)}\mu_{11}^{(k)}-(\alpha_{11}^{(k)})^2}\\ b^{(k)}=\dfrac{h_{15}^{(k)}\varepsilon_{11}^{(k)}-e_{15}^{(k)}\alpha_{11}^{(k)}}{\varepsilon_{11}^{(k)}\mu_{11}^{(k)}-(\alpha_{11}^{(k)})^2} \end{cases} \tag{7}$$

考虑磁电弹性板的主要边界，即上下表面的边界条件，其力学边界条件为应力自由，即 $\sigma_{32}(x_1,\pm h)=0$，电磁学边界条件为电绝缘 $D_2(x_1,\pm h)=0$ 和磁短路 $B_2(x_1,\pm h)=0$，代入方程(6)，有

$$u_{3,2}=0,\Psi_{e,2}=0,\Psi_{m,2}=0 \tag{8}$$

由方程(5)和方程(8)可以得到关于 x_2 的对称模态和反对称模态的波，由文献[14]得

$$\begin{cases} u_3^{(k,n)}=[A_1^{(k,n)}\exp(i\xi_1^{(k,n)}x_1)+A_2^{(k,n)}\exp(-i\xi_1^{(k,n)}x_1)]\cos\xi_2^{(n)}x_2\exp(i\omega t)\\ \Psi_e^{(k,n)}=[B_1^{(k,n)}\exp(\xi_2^{(n)}x_1)+B_2\exp(-\xi_2^{(n)}x_1)]\cos\xi_2^{(n)}x_2\exp(i\omega t)\\ \Psi_m^{(k,n)}=[C_1^{(k,n)}\exp(\xi_2^{(n)}x_1)+C_2\exp(-\xi_2^{(n)}x_1)]\cos\xi_2^{(n)}x_2\exp(i\omega t) \end{cases} \tag{9}$$

$$\xi_2^{(n)}=\frac{n\pi}{2h},\ n=0,2,4,6\cdots$$

$$\begin{cases} u_3^{(k,n)}=[A_1^{(k,n)}\exp(i\xi_1^{(k,n)}x_1)+A_2^{(k,n)}\exp(-i\xi_1^{(k,n)}x_1)]\sin\xi_2^{(n)}x_2\exp(i\omega t)\\ \Psi_e^{(k,n)}=[B_1^{(k,n)}\exp(\xi_2^{(n)}x_1)+B_2\exp(-\xi_2^{(n)}x_1)]\sin\xi_2^{(n)}x_2\exp(i\omega t)\\ \Psi_m^{(k,n)}=[C_1^{(k,n)}\exp(\xi_2^{(n)}x_1)+C_2\exp(-\xi_2^{(n)}x_1)]\sin\xi_2^{(n)}x_2\exp(i\omega t) \end{cases} \tag{10}$$

$$\xi_2^{(n)}=\frac{n\pi}{2h},\ n=1,3,5,7\cdots$$

式中

$$\xi_1^{(k,n)}=\sqrt{\frac{\rho^{(k)}\omega^2}{\tilde{c}_{44}^{(k)}}-(\xi_2^{(n)})^2}=\sqrt{\frac{\rho^{(k)}}{\tilde{c}_{44}^{(k)}}}\sqrt{\omega^2-(\frac{n\pi}{2h})^2\frac{\tilde{c}_{44}^{(k)}}{\rho^{(k)}}}=\frac{1}{v_T^{(k)}}\sqrt{\omega^2-(\omega_0^{(k,n)})^2} \tag{11}$$

$$v_T^{(k)}=\sqrt{\frac{\tilde{c}_{44}^{(k)}}{\rho^{(k)}}},(\omega_0^{(k,n)})^2=(\frac{n\pi}{2h})^2\frac{\tilde{c}_{44}^{(k)}}{\rho^{(k)}} \tag{12}$$

式(9,10)中的 A_1,A_2,B_1,B_2,C_1 和 C_2 是未知系数，$v_T^{(k)}$ 是第 k 部分材料的剪切波的波速，应用界面连续条件，可以得到未知系数。

$$\begin{aligned}\varphi_e = &\frac{1}{\varepsilon_{11}^{(k)}\mu_{11}^{(k)} - (\alpha_{11}^{(k)})^2}\{\mu_{11}^{(k)}[B_1^{(k,n)}\exp(\xi_2^{(n)}x_1) + \\ &B_2\exp(-\xi_2^{(n)}x_1)] - \alpha_{11}^{(k)}[C_1^{(k,n)}\exp(\xi_2^{(n)}x_1) + \\ &C_2\exp(-\xi_2^{(n)}x_1)] + (e_{15}^{(k)}\mu_{11}^{(k)} - h_{15}^{(k)}\alpha_{11}^{(k)})[A_1^{(k,n)} \cdot \\ &\exp(i\xi_1^{(k,n)}x_1) + A_2^{(k,n)}\exp(-i\xi_1^{(k,n)}x_1)]\} \cdot \\ &\cos\xi_2^{(n)}x_2\exp(i\omega t)\end{aligned} \tag{13}$$

$$\begin{aligned}\varphi_m = &\frac{1}{\varepsilon_{11}^{(k)}\mu_{11}^{(k)} - (\alpha_{11}^{(k)})^2}\{-\alpha_{11}^{(k)}[B_1^{(k,n)}\exp(\xi_2^{(n)}x_1) + \\ &B_2\exp(-\xi_2^{(n)}x_1)] + \varepsilon_{11}^{(k)}[C_1^{(k,n)}\exp(\xi_2^{(n)}x_1) + \\ &C_2\exp(-\xi_2^{(n)}x_1)] + (h_{15}^{(k)}\varepsilon_{11}^{(k)} - e_{15}^{(k)}\alpha_{11}^{(k)})[A_1^{(k,n)} \cdot \\ &\exp(i\xi_1^{(k,n)}x_1) + A_2^{(k,n)}\exp(-i\xi_1^{(k,n)}x_1)]\} \cdot \\ &\cos\xi_2^{(n)}x_2\exp(i\omega t)\end{aligned} \tag{14}$$

$$\begin{aligned}\sigma_{31} = &\tilde{c}_{44}^{(k)}\{[A_1^{(k,n)}i\xi_1^{(k,n)}\exp(i\xi_1^{(k,n)}x_1) - A_2^{(k,n)}i\xi_1^{(k,n)} \cdot \\ &\exp(-i\xi_1^{(k,n)}x_1)]\cos\xi_2^{(n)}x_2\exp(i\omega t)\} + a^{(k)} \cdot \\ &\{[B_1^{(k,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x_1) - B_2\xi_2^{(n)}\exp(-\xi_2^{(n)}x_1)] \cdot \\ &\cos\xi_2^{(n)}x_2\exp(i\omega t)\} + b^{(k)}\{[C_1^{(k,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x_1) - \\ &C_2\xi_2^{(n)}\exp(-\xi_2^{(n)}x_1)]\cos\xi_2^{(n)}x_2\exp(i\omega t)\}\end{aligned} \tag{15a}$$

$$\begin{aligned}D_1^{(k,n)} = &-[B_1^{(k,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x_1) - \\ &B_2\xi_2^{(n)}\exp(-\xi_2^{(n)}x_1)]\cos\xi_2^{(n)}x_2\exp(i\omega t)\end{aligned} \tag{15b}$$

$$\begin{aligned}B_1^{(k,n)} = &-[C_1^{(k,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x_1) - \\ &C_2\xi_2^{(n)}\exp(-\xi_2^{(n)}x_1)]\cos\xi_2^{(n)}x_2\exp(i\omega t)\end{aligned} \tag{15c}$$

$x_1 = x^{(m)}$ 处第 k 部分的界面与第$(k+1)$部分的界面处位移、电场以及磁场符合连续条件，可以得到

$$\begin{aligned}u_3^{(k,n)}(x_1 = x^{(k)-}) = &[A_1^{(k,n)}\exp(i\xi_1^{(k,n)}x^{(k)}) + A_2^{(k,n)} \\ &\cdot \exp(-i\xi_1^{(k,n)}x^{(k)})] = [A_1^{(k+1,n)}\exp(i\xi_1^{(k+1,n)}x^{(k)}) + \\ &A_2^{(k+1,n)}\exp(-i\xi_1^{(k+1,n)}x^{(k)})] = u_3^{(k,n)}(x_1 = x^{(k)-})\end{aligned} \tag{16}$$

$$\begin{aligned}\varphi_e^{(k,n)}(x_1 = x^{(k)-}) = &\frac{1}{\varepsilon_{11}^{(k)}\mu_{11}^{(k)} - (\alpha_{11}^{(k)})^2}\{\mu_{11}^{(k)}[B_1^{(k,n)} \cdot \\ &\exp(\xi_2^{(n)}x^{(k)}) + B_2\exp(-\xi_2^{(n)}x^{(k)})] - \alpha_{11}^{(k)}[C_1^{(k,n)} \cdot \\ &\exp(\xi_2^{(n)}x^{(k)}) + C_2\exp(-\xi_2^{(n)}x^{(k)})] + \\ &(e_{15}^{(k)}\mu_{11}^{(k)} - h_{15}^{(k)}\alpha_{11}^{(k)}) \cdot [A_1^{(k,n)}\exp(i\xi_1^{(k,n)}x^{(k)}) + \\ &A_2^{(k,n)}\exp(-i\xi_1^{(k,n)}x^{(k)})]\} = \frac{1}{\varepsilon_{11}^{(k+1)}\mu_{11}^{(k+1)} - (\alpha_{11}^{(k+1)})^2} \\ &\{\mu_{11}^{(k+1)}[B_1^{(k+1,n)}\exp(\xi_2^{(n)}x^{(k)}) + B_2\exp(-\xi_2^{(n)}x^{(k)})] - \\ &\alpha_{11}^{(k+1)}[C_1^{(k+1,n)}\exp(\xi_2^{(n)}x^{(k)}) + \\ &C_2 \cdot \exp(-\xi_2^{(n)}x^{(k)})] + (e_{15}^{(k+1)}\mu_{11}^{(k+1)} - \\ &h_{15}^{(k+1)}\alpha_{11}^{(k+1)})[A_1^{(k+1,n)} \cdot \exp(i\xi_1^{(k+1,n)}x^{(k)}) + \\ &A_2^{(k+1,n)}\exp(-i\xi_1^{(k+1,n)}x^{(k)})]\} = \\ &\varphi_e^{(k+1,n)}(x_1 = x^{(k)+})\end{aligned} \tag{17}$$

$$\begin{aligned}\varphi_m^{(k,n)}(x_1 = x^{(k)-}) = &\frac{1}{\varepsilon_{11}^{(k)}\mu_{11}^{(k)} - (\alpha_{11}^{(k)})^2} \\ &\{-\alpha_{11}^{(k)}[B_1^{(k,n)} \cdot \exp(\xi_2^{(n)}x^{(k)}) + \\ &B_2\exp(-\xi_2^{(n)}x^{(k)})] + \varepsilon_{11}^{(k)}[C_1^{(k,n)} \cdot \exp(\xi_2^{(n)}x^{(k)}) + \\ &C_2\exp(-\xi_2^{(n)}x^{(k)})] + (h_{15}^{(k)}\varepsilon_{11}^{(k)} - \\ &e_{15}^{(k)}\alpha_{11}^{(k)})[A_1^{(k,n)}\exp(i\xi_1^{(k,n)}x^{(k)}) + \\ &A_2^{(k,n)}\exp(-i\xi_1^{(k,n)}x^{(k)})]\} = \\ &\frac{1}{\varepsilon_{11}^{(k+1)}\mu_{11}^{(k+1)} - (\alpha_{11}^{(k+1)})^2}\{-\alpha_{11}^{(k+1)}[B_1^{(k+1,n)} \cdot \\ &xp(\xi_2^{(n)}x^{(k)}) + B_2\exp(-\xi_2^{(n)}x^{(k)})] + \varepsilon_{11}^{(k+1)}[C_1^{(k+1,n)} \cdot \\ &\exp(\xi_2^{(n)}x^{(k)}) + C_2\exp(-\xi_2^{(n)}x^{(k)})] + (h_{15}^{(k+1)}\varepsilon_{11}^{(k+1)} - \\ &e_{15}^{(k+1)}\alpha_{11}^{(k+1)})[A_1^{(k+1,n)}\exp(i\xi_1^{(k+1,n)}x^{(k)}) + A_2^{(k+1,n)} \cdot \\ &\exp(-i\xi_1^{(k+1,n)}x^{(k)})]\} = \varphi_m^{(k+1,n)}(x_1 = x^{(k)+})\end{aligned} \tag{18}$$

$$\begin{aligned}D_1^{(k,n)}(x_1 = x^{(k)-}) = &-[B_1^{(k,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x^{(k)}) - \\ &B_2^{(k,n)}\xi_2^{(n)}\exp(-\xi_2^{(n)}x^{(k)})] = \\ &-[B_1^{(k+1,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x^{(k)}) - \\ &B_2^{(k+1,n)}\xi_2^{(n)}\exp(-\xi_2^{(n)}x^{(k)})] = D_1^{(k+1,n)}(x_1 = x^{(k)+}) \\ B_1^{(k,n)}(x_1 = x^{(k)-}) = &-[C_1^{(k,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x^{(k)}) - \\ &C_2^{(k,n)}\xi_2^{(n)}\exp(-\xi_2^{(n)}x^{(k)})] = \\ &-[C_1^{(k+1,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x^{(k)}) - \\ &C_2^{(k+1,n)}\xi_2^{(n)}\exp(-\xi_2^{(n)}x^{(k)})] = B_1^{(k+1,n)}(x_1 = x^{(k)+})\end{aligned} \tag{19}$$

$$\begin{aligned}\rho\sigma_{(31)}^{(k,n)}(x_1 = x^{(k)-}) = \\ &\tilde{c}_{44}^{(k)}\{[A_1^{(k,n)}i\xi_1^{(k,n)}\exp(i\xi_1^{(k,n)}x^{(k)}) - \\ &A_2^{(k,n)}i\xi_1^{(k,n)}\exp(-i\xi_1^{(k,n)}x^{(k)})]\} + a^{(k)}\{[B_1^{(k,n)}\xi_2^{(n)} \cdot \\ &\exp(\xi_2^{(n)}x^{(k)}) - B_2^{(k,n)}\xi_2^{(n)}\exp(-\xi_2^{(n)}x^{(k)})]\} + \\ &b^{(k)}\{[C_1^{(k,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x^{(k)}) - \\ &C_2^{(k,n)}\xi_2^{(n)}\exp(-\xi_2^{(n)}x^{(k)})]\} = \\ &\tilde{c}_{44}^{(k+1)}\{[A_1^{(k+1,n)}i\xi_1^{(k+1,n)}\exp(i\xi_1^{(k+1,n)}x^{(k)}) - \\ &A_2^{(k+1,n)}i\xi_1^{(k+1,n)}\exp(-i\xi_1^{(k+1,n)}x^{(k)})]\} + \\ &a^{(k+1)}\{[B_1^{(k+1,n)}\xi_2^{(n)} \cdot \exp(\xi_2^{(n)}x^{(k)}) - \\ &B_2^{(k+1,n)}\xi_2^{(n)}\exp(-\xi_2^{(n)}x^{(k)})]\} + \\ &b^{(k+1)} \cdot \{[C_1^{(k+1,n)}\xi_2^{(n)}\exp(\xi_2^{(n)}x^{(k)}) - \\ &C_2^{(k+1,n)}\xi_2^{(n)}\exp(-\xi_2^{(n)}x^{(k)})]\} = \sigma_{(31)}^{(k+1,n)}(x_1 = x^{(k)+})\end{aligned} \tag{20}$$

可以写成矩阵形式：

$$[\boldsymbol{S}]_{x^{(m)}}^{(k,n)}\{\boldsymbol{X}\}^{(k,n)} = [\boldsymbol{S}]_{x^{(m)}}^{(k+1,n)}\{\boldsymbol{X}\}^{(k+1,n)} \quad (21))$$

矩阵$[\boldsymbol{S}]_{x^{(m)}}^{(k,n)}$，$\{\boldsymbol{X}\}^{(k,n)}$，$[\boldsymbol{S}]_{x^{(m)}}^{(k+1,n)}$ 和$\{\boldsymbol{X}\}^{(k+1,n)}$ 见附录。方程(21) 可以进一步写成：

$$\{\boldsymbol{X}\}^{(k+1,n)} = [\boldsymbol{T}]^{(k,n)}\{\boldsymbol{X}\}^{(k,n)} \quad (22)$$

$$[\boldsymbol{T}]^{(k,n)} = ([\boldsymbol{S}]_{x^{(m)}}^{(k+1,n)})^{-1}[\boldsymbol{S}]_{x^{(m)}}^{(k,n)} \quad (23)$$

假设该磁-电-弹介质是由 K 部分组成，根据方程(22,23) 可以写成：

$$\{\boldsymbol{X}\}^{(K,n)} = [\boldsymbol{T}]^{(K-1,n)}[\boldsymbol{T}]^{(K-2,n)}[\boldsymbol{T}]^{(K-3,n)}\cdots [\boldsymbol{T}]^{(2,n)}[\boldsymbol{T}]^{(1,n)}\{\boldsymbol{X}\}^{(1,n)} \quad (24)$$

方程(24) 表示了该介质第一部分与最后一部分的关系，它是由包含 6 个未知数 $A_1^{(k,n)}$，$B_1^{(k,n)}$，$C_1^{(k,n)}$，$A_2^{(k,n)}$，$B_2^{(k,n)}$ 和 $C_2^{(k,n)}$ 的方程组成。

3 算例与讨论

作为例子，首先考虑由压电介质和压磁介质交替组成的复合板，该介质由四部分组成，如图 2 所示。

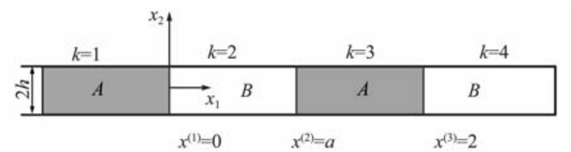

图 2 四部分材料组成的磁电弹性板

Fig. 2 A four-sectioned plate

选择 A 为压电材料 PZT-7，B 为压磁材料 $CoFe_2O_4$，材料常数[15] 列入表 1。

表 1 材料常数

Tab. 1 Material constant

材料	c_{44}	κ_{11}	μ_{11}	e_{15}	h_{15}	ρ
PZT-7	25	17.1	5	13.5	0	5.3
$CoFe_2O_4$	45.3	0.08	100	0	550	5.3

注：c_{44} 的单位为 10^9 N/m^2，κ_{11} 的单位为 10^{-9} C^2/Nm，μ_{11} 的单位为 10^{-6} NS^2/C^2，e_{15} 的单位为 C/m^2，h_{15} 的单位为 N/Am，ρ 的单位为 kg/m^3。

薄板的厚度 $2h = 2$ mm，中间材料的宽度 $a = 5$ mm，选择 $k = 1$，由方程(12) 可以得到

$$\omega_0^A = \frac{\pi}{2h}\sqrt{\frac{\widetilde{c}_{44}^{(A)}}{\rho^{(A)}}} < \frac{\pi}{2h}\sqrt{\frac{\widetilde{c}_{44}^{(B)}}{\rho^{(B)}}} = \omega_0^B \quad (25)$$

波从 x 负向入射，在第一部分材料 A 中，由于对称性选择 $A_2^{(1,n)}$，$B_2^{(1,n)}$，$C_2^{(1,n)}$ 为零，其他材料的系数可以通过方程(24) 可以得到。讨论三种情况：(1) $\omega > \omega_0^B > \omega_0^A$，在该条件下，选择最后一部分的材料中，$A_1^{(4,k)} = B_1^{(4,k)} = C_1^{(4,k)} = 0$，由方程(22) 可以得出第一部分材料的系数 $A_1^{(1,k)}$，$B_1^{(1,k)}$ 和 $C_1^{(1,k)}$，由图 3 发现，波能在材料 B 中传播，在最后一部分材料中，仅有向右的波存在，四部分的波形均为正弦波，各部分材料中的波长不同，各个部分材料用点画线分开。

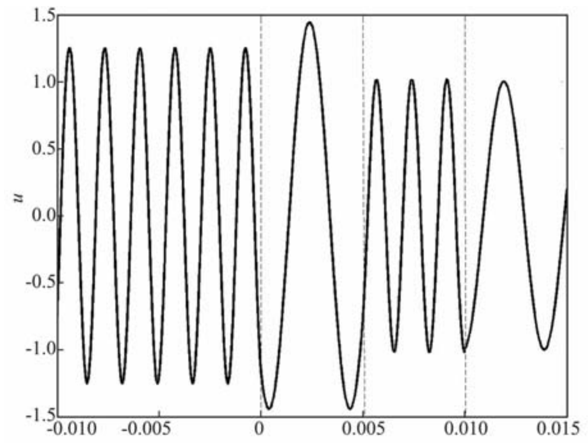

图 3 当 $\omega = 1.12\omega^{(B)}$ 位移分布

Fig. 3 Displacement distribution when $\omega = 1.12\omega^{(B)}$

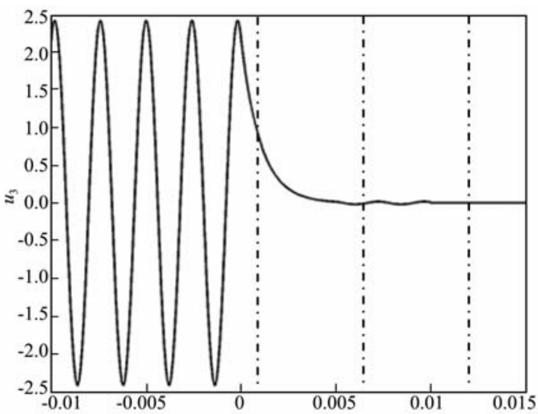

图 4 当 $\omega = 0.95\omega^{(B)}$ 位移分布

Fig. 4 Displacement distribution when $\omega = 0.95\omega^{(B)}$

(2) $\omega_0^B > \omega > \omega_0^A$，最后一部分的材料的系数选择与第一种情况相同，由图 4 发现，波从 x 负向入射，在材料 A 中按正弦形式传播，但是在材料 B 中按指数形式衰减，然后很快衰减到零。

(3) $\omega_0^B > \omega_0^A > \omega$，在该条件下，波既不能在材料 A 中传播，也不能在材料 B 中传播。

4 结 论

本文研究了由多个不均匀磁电弹介质组成的薄板厚度扭曲波的传播性质。从磁电弹全耦合场三维方程出发得到了其精确解，根据所得到的解分析了波的传播性质。主要结论为：(1) 当 $\omega > \omega_0^B > \omega_0^A$，四部分的波形均为正弦波，各部分材料中的波长不

同；(2) 当 $\omega_0^B > \omega > \omega_0^A$，在开始的材料 A 中按正弦形式传播，但是在材料 B 中按指数形式衰减；(3) $\omega_0^B > \omega_0^A > \omega$ 时，波既不能在材料 A 中传播，也不能在材料 B 中传播。这些结果对于理解和设计谐振器、滤波器以及声波元件提供了有价值的理论基础。

附　　录：

$$[\boldsymbol{S}]_{x^{(m)}}^{(k,n)} = \begin{bmatrix} \beta_1^{(k)} & 0 & 0 & \beta_2^{(k)} & 0 & 0 \\ \beta_5^{(k)}\beta_1^{(k)}/\beta_0^{(k)} & \mu_{11}^{(k)}\beta_3^{(k)}/\beta_3^{(k)} & \alpha_{11}^{(k)}\beta_3^{(k)}/\beta_0^{(k)} & \beta_5\beta_2^{(k)}/\beta_0^{(k)} & \mu_{11}^{(k)}\beta_3^{(k)}/\beta_0^{(k)} & \alpha_{11}^{(k)}\beta_4/\beta_0^{(k)} \\ \beta_6^{(k)}\beta_1^{(k)}/\beta_0^{(k)} & -\alpha_{11}^{(k)}\beta_3^{(k)}/\beta_0^{(k)} & \varepsilon_{11}^{(k)}\beta_3^{(k)}/\beta_0^{(k)} & \beta_6\beta_2^{(k)}/\beta_0^{(k)} & -\alpha_{11}^{(k)}\beta_4^{(k)}/\beta_0^{(k)} & \varepsilon_{11}^{(k)}\beta_4/\beta_0^{(k)} \\ \tilde{c}_{44}^{(k)} i\xi_1^{(k,n)}\beta_1^{(k)} & a^{(k)}\xi_2^{(n)}\beta_3^{(k)} & b^{(k)}\xi_2^{(n)}\beta_3^{(k)} & -\tilde{c}_{44}^{(k)} i\xi_1^{(k,n)}\beta_2^{(k)} & -a^{(k)}\xi_2^{(n)}\beta_4^{(k)} & b'\xi_2 \\ 0 & -\xi_2^{(n)}\beta_3^{(k)} & 0 & 0 & \xi_2^{(n)}\beta_4^{(k)} & 0 \\ 0 & 0 & -\xi_2^{(n)}\beta_3^{(k)} & 0 & 0 & \xi_2^{(n)}\beta_4^{(k)} \end{bmatrix}_{6\times 6}$$

上述矩阵中，各系数如下表示：

$\beta_0^{(k)} = \varepsilon_{11}^{(k)}\mu_{11}^{(k)} - (\alpha_{11}^{(k)})^2 \beta_1^{(k)} = \exp(i\xi_1^{(k,n)}x^{(k)})$

$\beta_2^{(k)} = \exp(-i\xi_1^{(k,n)}x^{(k)})$，$\beta_3^{(k)} = \exp(\xi_2^{(k,n)}x^{(k)})$

$\beta_4^{(k)} = \exp(-\xi_2^{(k,n)}x^{(k)})$，$\beta_5^{(k)} = e_{15}^{(k)}\mu_{11}^{(k)} - h_{15}^{(k)}\alpha_{11}^{(k)}$

$\beta_6^{(k)} = h_{15}^{(k)}\varepsilon_{11}^{(k)} - e_{15}^{(k)}\alpha_{11}^{(k)}$

$\{\boldsymbol{X}\}^{(k,n)} = [A_1^{(k,n)}, B_1^{(k,n)}, C_1^{(k,n)}, A_2^{(k,n)}, B_2^{(k,n)}, C_2^{(k,n)}]^{\mathrm{T}}$

式中 T 表示矩阵转置。

式(21)中的 $\{\boldsymbol{X}\}^{(k+1,n)}$，$[\boldsymbol{S}]_{x^{(m)}}^{(k+1,n)}$ 的各元素均是将附录 $\{\boldsymbol{X}\}^{(k,n)}$，$[\boldsymbol{S}]_{x^{(m)}}^{(k,n)}$ 的各元素的 k 更换为 $k+1$。

参考文献(References)：

[1] Prieto J L, Aroca C, Lopez E, et al. Magnetostrictive-piezoelectric magnetic sensor with current excitation[J]. *J Magnet Magn Mater*, 2000, **215-216**(2): 756-758.

[2] Ueno T, Higuchi T. High sensitive and heat-resistant magnetic sensor using magnetostrictive/piezoelectric laminate composite[J]. *IEEE Trans Magn*, 2005, **41**(10): 3670-3672.

[3] Ueno T, Qiu J, Tani J. Magnetic force control with composite of giant magnetostrictive and piezoelectric materials[J]. *IEEE Trans Magn*, 2003, **39**(6): 3534-3540.

[4] Bichurin M I, Petrov V M, Petrov R V, et al. Magnetoelectric microwave devices[J]. *Ferroelectrics*, 2002, **280**(2): 211-218.

[5] Srinivasan G, Bichurin M I, Mantese J V. Ferromagnetic-ferroelectric layered structures: Magnetoelectric interactions and devices[J]. *Integrated Ferroelectrics*, 2005, **71**(1): 45-57.

[6] Fiebig M. Topical review: Revival of the magnetoelectric effect[J]. *J Phys D: Appl Phys*, 2005, **38**(8): R123-R148.

[7] Alahits V I, Darinskii A N, Lothe J. On the existence of surface waves in half-infinite anisotropic elastic media with piezoelectric and piezomagnetic properties[J]. *Wave Motion*, 1992, **16**(3): 265-283.

[8] Soh A, Liu J. Interfacial shear horizontal waves in a piezoelectric-piezomagnetic bi-material[J]. *Philos Mag Lett*, 2006, **86**(1): 31-35.

[9] Zhou Z G, Wu L Z, Wang B. The dynamic behavior of two collinear interface cracks in magneto-electro-elastic materials[J]. *Euro J Mech Solids*, 2005, **24**(2): 253-262.

[10] Feng W J, Su R K L, Liu Y Q. Scattering of SH waves by an arc-shaped interface crack between a cylindrical magneto-electro-elastic inclusion and matrix with the symmetry of 6 mm[J]. *Acta Mech*, 2006, **183**(1-2): 81-102.

[11] Pang W, Zhang H, Kim E S. Micromachined acoustic wave resonator isolated from substrate[J]. *IEEE Trans Ultrason Ferroelectr Freq Control*, 2005, **52**(8): 1239-1246.

[12] Bleustein J L. Some simple modes of wave propagation in an infinite piezoelectrice plate[J]. *J Acoust Soc Am*, 1969, **45**(3): 614-620.

[13] Yang J S, Guo S H. Thickness-twist modes in a rectangular piezoelectric resonator of hexagonal crystals[J]. *Appl Phys Lett*, 2006, **88**(15): 153506.

[14] Yang J, Chen Z, Hu Y. Trapped thickness-twist modes in an inhomogeneous piezoelectric plate[J]. J *Philosophical Magazine Letters*, 2006, **86**(11): 699-705.

[15] Liu TJ C, Chue, C H. On the singularities in a bimaterial magneto-electro-elastic composite wedge under antiplane deformation[J]. *Compos Struct*, 2006, **72**(15): 254-265.

Propagation of thickness-twist waves in a inhomogeneous magneto-electro-elastic plate

KONG Yan-ping*, GUO Shu-qi, LIU Jin-xi

(Department of Engineering Mechanics, Shijiazhuang Tiedao University, Shijiazhuang 050043, China)

Abstract: Many application of piezoelectric and piezomagnetic compound materials have close connection with the propagation of elastic waves. This needs us firstly gain clear of the laws of elastic waves propagation theoretically. We study thickness-twist vibrations and waves in unbounded, multi-sectioned magneto-electro-elastic plate. An exact solution from the three-dimensional equations of magneto-electro-elastic material is obtained. Wave propagation characteristics are calculated based on the solution. The results are useful in the understanding and design of plate resonators, filters and acoustic wave sensors.

Key words: magneto-electro-elastic material; piezoelectric material; piezomagnetic material; wave; vibrate

(上接第 838 页)

Chaotic characteristic analysis of seismic dynamic reponses of SDOF systems

YANG Pi-xin, YANG Di-xiong*

(Department of Engineering Mechanics, State Key Laboratory for Structural Analysis of Industrial Equipment, Dalian University of Technology, Dalian 116024, China)

Abstract: Nonlinear dynamics theory and chaotic time series analysis are suggested to examine the chaotic characteristic of dynamic responses of single degree of freedom (SDOF) system subjected to earthquake ground motions in this paper. The typical near-fault ground motion records are selected as the seismic input. Then, the chaotic time series analysis is applied to calculate quantitatively the nonlinear characteristic parameters of acceleration responses of elastic and inelastic SDOF systems with representative periods. Numerical results show that the correlation dimension of these acceleration responses is fractal dimension, and their maximal Lyapunov exponent is larger than 0. Moreover, it is illustrated that the seismic dynamic responses of SDOF system under earthquake excitation present the chaotic character rather than the pure random signal, which provides us new approach and new perspective for understanding the irregularity and complexity of the seismic dynamic responses of structures.

Key words: SDOF systems; seismic dynamic responses; chaotic time series analysis; correlation dimension; maximal Lyapunov exponent