Production and Operation Managements

Inventory Control
Subject to Known Demand

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Contents

- Introduction
- Types of Inventories
- Motivation for Holding Inventories;
- Characteristics of Inventory System;
- Relevant Costs;
- The EOQ Model;
- EOQ Model with Finite Production Rate
- Quantity Discount Models
- Resource-constrained multiple product system
- EOQ models for production planning
- Power-of-two policies

The EOQ Model-Basic Model

- Express the average annual cost as a function of the lot size
  - Order cost in each order cycle: C(Q)=K+cQ;
  - The average holding cost during one order cycle is hQ/2;
  - The average annual cost (suppose there are n cycles in a year) :

$$G(Q)=\frac{(K+cQ)n}{nT}+\frac{hQ}{2}=\frac{K+cQ}{Q/\lambda}+\frac{hQ}{2}=\frac{K\lambda}{Q}+\lambda c+\frac{hQ}{2}$$

$$G'(Q)=-\frac{K\lambda}{Q^2}+\frac{h}{2};\quad G''(Q)=\frac{2K\lambda}{Q^3}>0 \quad for\ Q>0;$$

$$G'(Q)=0\rightarrow Q^*=\sqrt{\frac{2K\lambda}{h}}$$

The EOQ Model-Basic Model

- Example 4.1
  - ✓ Pencils are sold at a fairly steady rate of 60 per week;
  - ✓ Pencils cost 2 cents each and sell for 15 cents each;
  - ✓ Cost $12 to initiate an order, and holding costs are based on annual interest rate of 25%.
  - ✓ Determine the optimal number of pencils for the book store to purchase each time and the time between placement of orders

- **Solutions**
  - ✓ Annual demand rate $\lambda=60\times 52=3{,}120$;
  - ✓ The holding cost is the product of the variable cost of the pencil and the annual interest-h=0.02 ×0.25=0.05

$$Q^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2\times 12\times 3{,}120}{0.05}} = 3{,}870$$

$$T = \frac{Q}{\lambda} = \frac{3{,}870}{3{,}120} = 1.24\ yr$$

The EOQ Model-Considering Lead Time

- Since there exits lead time τ (4 moths for Example 4.1), order should be placed some time ahead of the end of a cycle;
- Reorder point R -determines when to place order in terms of inventory on hand, rather than time.

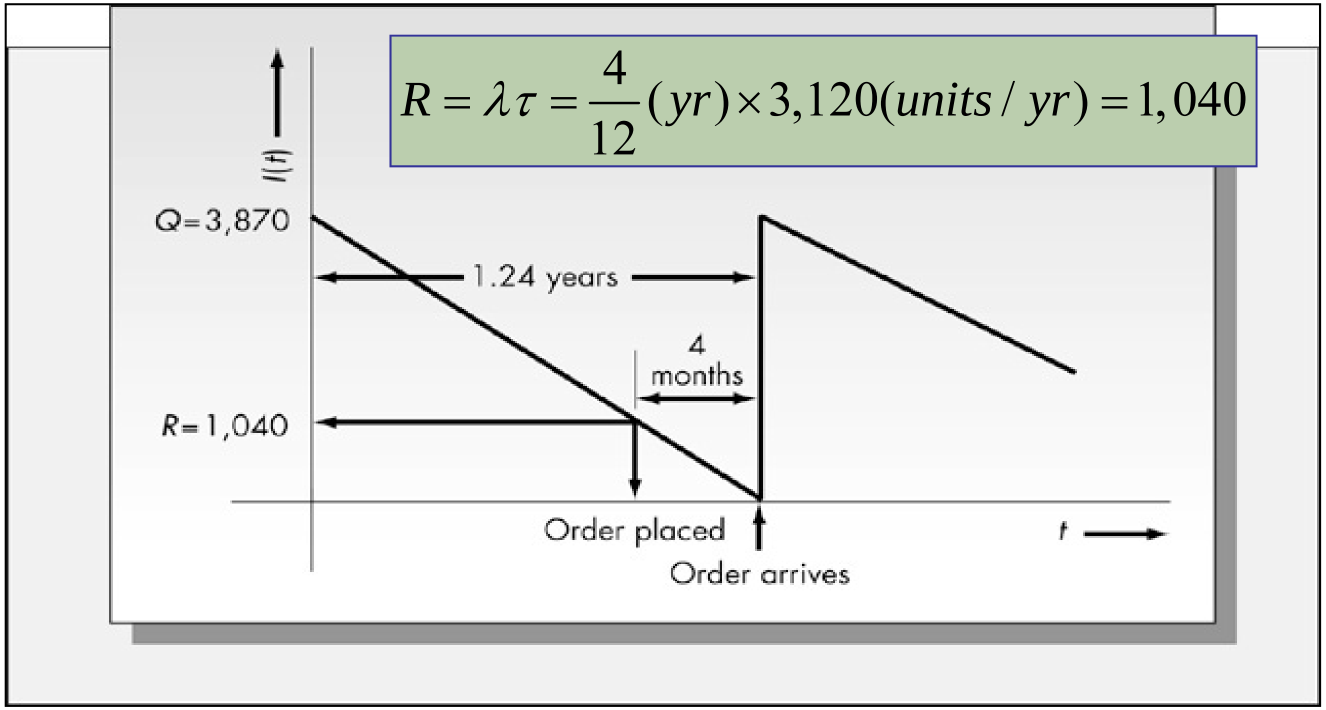

Fig. 4-6 Reorder Point Calculation for Example 4.1

The EOQ Model-Considering Lead Time

- Determine the reorder point when the lead time exceeds a cycle.

Computing R for placing order 2.31 cycles ahead is the same as that 0.31 cycle ahead.

Example:

- EOQ=25;
- λ=500/yr;
- τ=6 wks;
- T=25/500=2.6 wks;
- τ/T=2.31---2.31 cycles are included in LT.
- Action: place every order 2.31 cycles in advance.

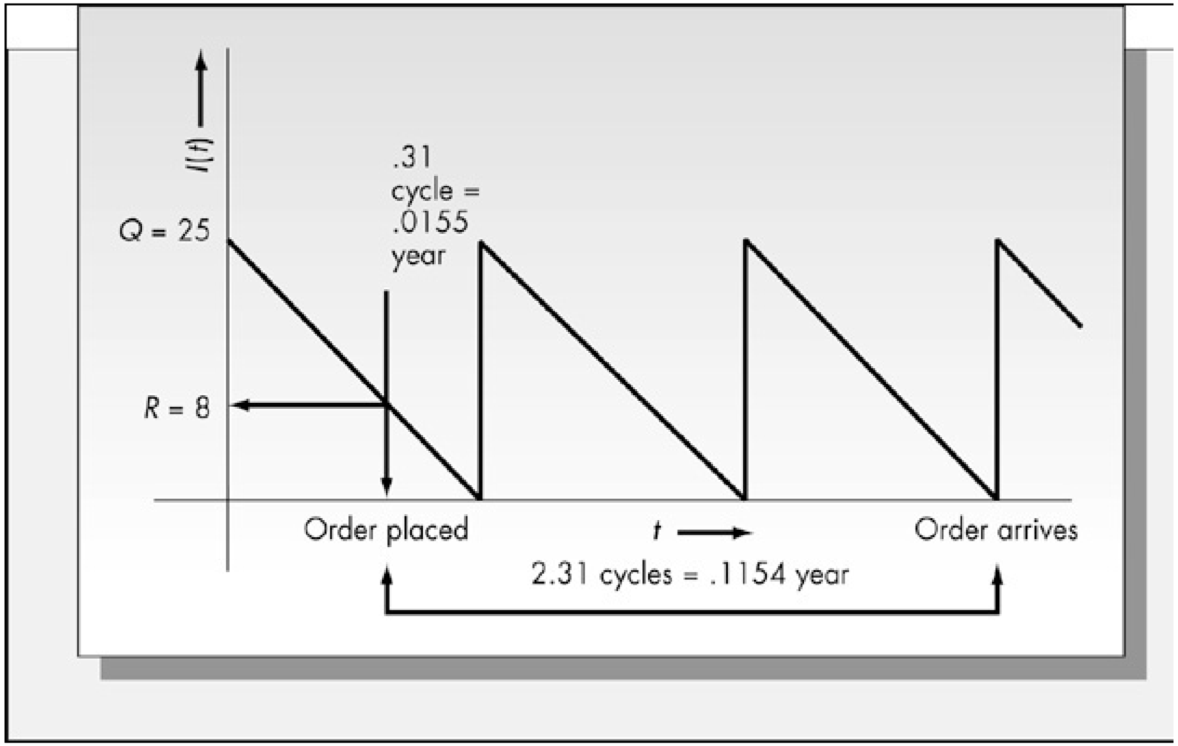

Fig 4-7 Reorder Point Calculation for Lead Times Exceeding One Cycle

The EOQ Model- Sensitivity

- How sensitive is the annual cost function to errors in the calculation of Q?

➢Considering Example 4.1. Suppose that the bookstore orders pencils in batches of 1,000, rather than 3,870 as the optimal solution indicates. What additional cost is it incurring by using a suboptimal solution?

By substituting Q=1,000, we can find the average annual cost for this lot size.

$$
\begin{aligned}
G(Q) &= K\lambda / Q + hQ/2 \\
&= (12)(3{,}120)/1{,}000 + (0.005)(1{,}000)/2 \\
&= \$39.94
\end{aligned}
$$

Which is considerably larger than the optimal cost of \$19.35.

The EOQ Model- Sensitivity

- Let's obtain a universal solution to the sensitivity problem.

  Let G* be the average annual holding and setup cost at the optimal solution. Then

$$
\begin{aligned}
G^* &= K\lambda / Q^* + hQ^*/2 \\
&= \frac{K\lambda}{\sqrt{2K\lambda/h}} + \frac{h}{2}\sqrt{\frac{2K\lambda}{h}} \\
&= 2\sqrt{\frac{K\lambda h}{2}} \\
&= \sqrt{2K\lambda h}
\end{aligned}
$$

It follows that for any Q,

$$
\begin{aligned}
\frac{G(Q)}{G^*} &= \frac{K\lambda/Q + hQ/2}{\sqrt{2K\lambda h}} \\
&= \frac{1}{2Q}\sqrt{\frac{2K\lambda}{h}} + \frac{Q}{2}\sqrt{\frac{h}{2K\lambda}} \\
&= \frac{Q^*}{2Q} + \frac{Q}{2Q^*} \\
&= \frac{1}{2}\left[\frac{Q^*}{Q} + \frac{Q}{Q^*}\right]
\end{aligned}
$$

The EOQ Model- Sensitivity

- To see how one would use this result, consider using a suboptimal lot size in Example 4.1.
- The optimal solution was $Q^*=3,870$, and we wished to evaluate the cost error of using $Q=1,000$. Forming the ratio Q^*/Q gives 3.87. Hence,

$$G(Q)/G^*=(0.5)(3.87+1/3.87)=(0.5)(4.128)=2.06.$$

This says that the average annual holding and setup cost with $Q=1,000$ is 2.06 times the optimal average holding and setup cost.

The EOQ Model- Sensitivity

- In general, the cost function G(Q) is relative insensitive to errors in Q. For example,

  if Q is twice as large as Q*, then G/G(Q*)＝1.25 , meaning that **an error of 100% in Q will generate an error of 25% in annual average cost**.

- And suppose that the order quantity differed from the optimal by ΔQ units. A value of Q=Q*+ ΔQ would result in a *lower* average annual cost than a value of Q=Q*- ΔQ. ---**Not symmetric**.

The EOQ Model- Sensitivity

$$
\begin{aligned}
\frac{G(Q^*+\Delta Q)}{G^*}-\frac{G(Q^*-\Delta Q)}{G^*} &= \frac{1}{2}\left[\frac{Q^*}{Q^*+\Delta Q}+\frac{Q^*+\Delta Q}{Q^*}\right]-\frac{1}{2}\left[\frac{Q^*}{Q^*-\Delta Q}+\frac{Q^*-\Delta Q}{Q^*}\right] \\
&= \frac{1}{2}\left[\frac{Q^*}{Q^*+\Delta Q}-\frac{Q^*}{Q^*-\Delta Q}\right]+\frac{1}{2}\left[\frac{Q^*+\Delta Q}{Q^*}-\frac{Q^*-\Delta Q}{Q^*}\right] \\
&= \frac{1}{2}\left[\frac{-2Q^*\Delta Q}{(Q^*+\Delta Q)(Q^*-\Delta Q)}\right]+\frac{1}{2}\left[\frac{2\Delta Q}{Q^*}\right] \\
&= -\frac{Q^*\Delta Q}{(Q^*)^2-(\Delta Q)^2}+\frac{\Delta Q}{Q^*} \\
&= \frac{\Delta Q}{Q^*-(\Delta Q)^2/Q^*}+\frac{\Delta Q}{Q^*}\le 0
\end{aligned}
$$

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The EOQ Model for Finite Production Rate

- The simple EOQ model is based on assumption that the items are obtained from an outside supplier, and thus entire lot is delivered at the same time;
- EOQ model is also effective when units are internally produced, based on assumption that production rate is infinite , such that the entire lots are delivered at the same time.
- If the production rate is finite and comparable to the rate of demand, the simple EOQ model will be ineffective.

The EOQ Model for Finite Production Rate

- Assumption:
  - ✓ Items are produced at a rate P during a production run;
  - ✓ P> λ for feasibility (comparable to the rate of demand);
- Let
  - ✓ Q is the lot size of each production run;
  - ✓ T is the cycle length, the time between successive startups. $T=T_1+T_2$, where T_1 is production time, while T_2 is the downtime (no production);
- Note that the maximum level of on-hand inventory during a cycle is no longer Q.

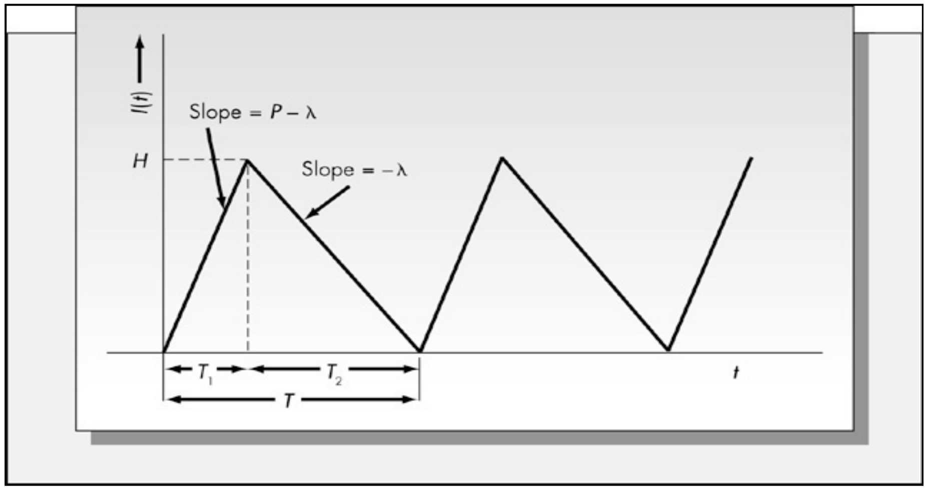

Fig. 4-8 Inventory Levels for Finite Production Rate Model

The EOQ Model for Finite Production Rate

- The number of units consumed in each cycle is λT;
- The number of units produced at rate P in a production time T_1 is $Q=T_1P$;
- The number of produced units during the production time should satisfies the demand for the cycle time: $\lambda T=T_1P=Q \rightarrow T_1=Q/P$;
- The maximum level of inventory on hand is $H=T_1(P-\lambda)=Q(1-\lambda/P)$;
- Since average inventory level is H/2, thus the annual average inventory cost follows:

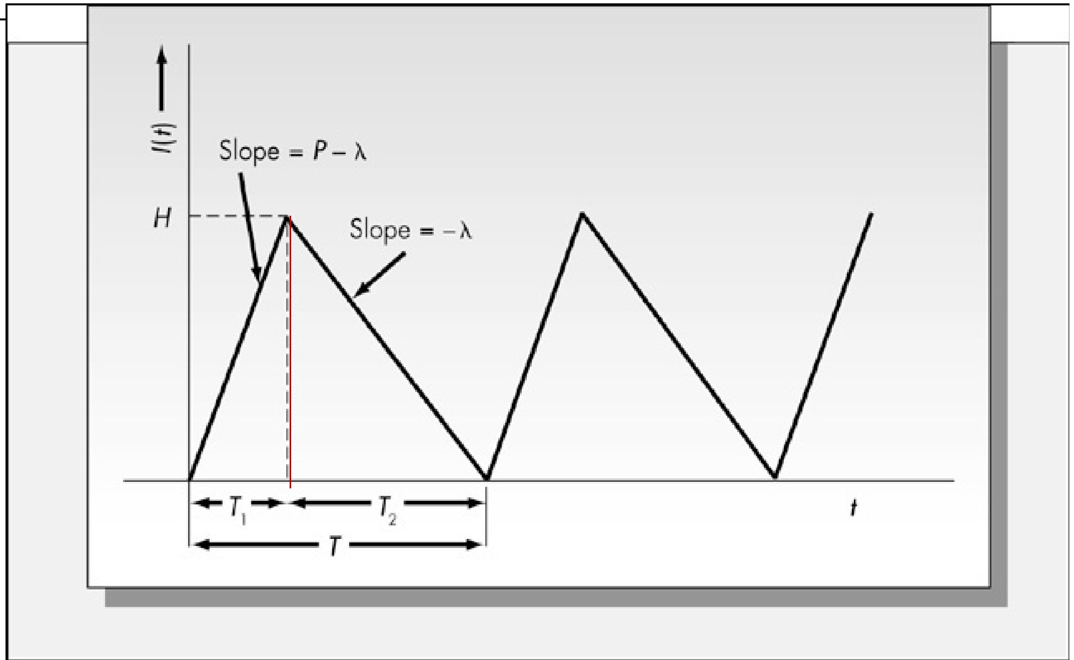

Fig. 4-8 Inventory Levels for Finite Production Rate Model

$$\boxed{G(Q)=\frac{K}{T}+\frac{hH}{2}=\frac{K\lambda}{Q}+h\frac{Q}{2}(1-\lambda/p)}$$

$$Let\ \ h'=h(1-P/\lambda),\ \ Then\ \ \ Q^*=\sqrt{\frac{2k\lambda}{h'}}$$

The EOQ Model for Finite Production Rate

Example 4.3

•Given

- ✓ P=10,000 units/yr;
- ✓ K=\$50;
- ✓ λ=2,500 units/yr;
- ✓ $h=\$2\times 0.3=0.6$

•Determine:

- ✓Optimized size of a production run Q;
- ✓The length of each production run T;
- ✓The average annual cost of holding cost and setup;
- ✓Maximum level of inventory on hand.

- Solutions:

- ✓ $h' = h(1-\lambda / P) = 0.6(1-2{,}500/10{,}000) = 0.45$;
- ✓ $Q^* = (2K\lambda/h')^{1/2} = 745$;
- ✓ $T = Q^*/\lambda = 745/2{,}500 = 0.298$ yr;
- ✓ The production time (uptime) $T_1 = Q^*/P = 0.0745$ yr;
- ✓ The downtime is $T_2 = T - T_1 = 0.2235$ yr;
- ✓ $G(Q^*) = K\lambda/Q^* + h'Q^*/2 = 335.41$;
- ✓ $H = Q^*(1-\lambda/P) = 559$ units;

In-class quiz

- **Given**
  - ✓ $c=\$2.4$/ unit;
  - ✓ $K=\$45$;
  - ✓ $\lambda=280$ units/yr;
  - ✓ $I=0.2$

- **Determine:**
  - ✓EOQ;
  - ✓The length of each order cycle T;
  - ✓The average annual cost of holding cost and setup;
  - ✓Maximum level of inventory on hand.

- Solutions:
  - ✓ $h=Ic=0.48$;
  - ✓ $Q^*=(2K\lambda/h)^{1/2}=229$;
  - ✓ $T=Q^*/\lambda=229/280=.8179$ yrs. (= 9.81 months);
  - ✓ $G(Q^*)=K\lambda/Q^*+hQ^*/2=\$109.98$;
  - ✓ $H=Q^*=229$ units;

The End!