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高速小展弦比机翼颤振的微分求积法分析

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摘 要:引入微分求积法,分析高速小展弦比机翼的气动弹性问题。将小展弦比机翼等效为悬臂板,基于一阶活塞气动力理论建立机翼颤振偏微分方程,采用微分求积法将偏微分方程转化为常微分方程,根据频率重合理论对颤振问题进行求解。分析了机翼的固有频率及颤振速度,并与有限元软件计算结果进行比较,误差在2%以内,很好的验证了微分求积法求解小展弦比机翼颤振问题的有效性。分析了机翼面积、展弦比及厚度对颤振速度的影响,结果表明,小展弦比机翼的颤振速度受结构尺寸的影响较大,颤振速度随面积和展弦比的增大而减小,随机翼厚度的增大而增大。

关键词:微分求积法;气动弹性;小展弦比;机翼;颤振

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1 引 言

随着航空航天技术的发展,高速飞行器逐渐成为各个国家研制的新一代飞行器。这类飞行器为了降低阻力一般采用小展弦比薄机翼,这就使得机翼的弦向变形无法忽略,结构变形不能再简单的采用弯曲和扭转两个模态来描述,给气动弹性问题的研究带来了新的使命要求。

小展弦比机翼的气动弹性问题研究不能忽略弦向变形而简化为二元机翼[1-5],也无法等效为薄壁梁[6-9],但可以简化为二维模型,如板或壳[10]。颤振问题的分析方法一般有Galerkin[11-14]等经典方法和有限元法[15]。本文采用微分求积法进行颤振分析。微分求积法是将偏微分方程离散为常微分方程的另一种方法,其将方程的偏微分项等效为多项式求和的形式,求解思路简单。其相对于Galerkin等经典方法而言处理复杂边界条件更加灵活,而相对于有限元方法更容易实现,计算效率更高。

微分求积法(DQM)最早由Bellman和Casti[16]于1971年提出,现已广泛应用于动力学研究领域[17-22]。K M Liew[23]采用微分求积法研究了对称性复合材料平板的振动问题。Francesco Tornabene[24]采用广义微分求积法对球形结构的振动问题进行了分析。S T Choi[25]采用微分求积法分析了非圆形的曲板的振动问题。陈大林[26]采用微分求积法分析了四边简支二维薄板的非线性气动弹性响应问题。

本文将微分求积法应用于高速小展弦比机翼的颤振问题。首先将小展弦比机翼等效为悬臂板,基于一阶活塞气动力理论及弹性理论建立机翼的颤振偏微分方程,再根据频率重合理论对颤振问题进行求解,最后分析机翼面积、厚度及展弦比等不同参数对颤振速度的影响。微分求积法的引入为小展弦比机翼的颤振分析设计及基础理论研究提供了另一种思路,同时也拓展了微分求积法的求解领域。

2 微分求积法

微分求积法是用全域上全部节点的函数值进行加权求和表示函数及其导数在给定节点处的值,因而可以将微分方程变成以节点处的函数值为未知数的代数方程组。对于给定点 $x=x_i$,函数 $f(x)$ 对 x 的 r 阶导数可以表示为

$$\partial^r f(x)/\partial x=\sum_{j=1}^{N} A_{ij}^{(r)} f(x_j)$$
$$i=1,2,\cdots,N;\ r=1,2,\cdots,N-1 \quad (1)$$

式中 N 为求解域上离散点的个数,$A_{ij}^{(r)}$ 为权系数。微分求积法的关键问题是如何确定其中的权系数,本文采用拉格朗日插值多项式来构造权系数,非对角线元素的一阶权系数表示为

$$A_{i,j}^{(1)}=M^{(1)}(x_i)/(x_i-x_j)M^{(1)}(x_j)$$
$$i,j=1,2,3,\cdots,N \quad (2)$$

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式中

$$M^{(1)}(x_i)=\prod_{j=1,j\neq i}^{N}(x_i-x_j),\ j=1,2,\cdots,N \quad (3)$$

非对角元素高阶权系数可利用下列递推关系确定：

$$A_{i,j}^{(r)}=r[A_{i,i}^{(r-1)}A_{i,j}^{(1)}-A_{i,j}^{(r-1)}/(x_i-x_j)]$$
$$i,j=1,2,3,\cdots,N;i\neq j \quad (4)$$

式中 $2\leqslant r\leqslant(N-1)$，$i=j$ 时，对角线上元素权系数用以下关系表示：

$$A_{i,i}^{(r)}=-\sum_{j=1,j\neq i}^{N}A_{i,j}^{(r)}$$
$$i,j=1,2,3,\cdots,N;\ r=1,2,\cdots,N-1 \quad (5)$$

3　模型与动力学方程

3.1　机翼气动弹性方程

高速飞行器为减小阻力，一般采用小展弦比机翼，如图1所示，其中 x 方向为来流方向，即机翼的弦向。

在气动弹性分析时，可以将小展弦比机翼简化为悬臂板。本文以矩形悬臂板为例，分析机翼的颤振问题。根据弹性薄板理论，可以得到机翼的气动弹性方程及其边界条件为

$$D\left[\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]+\rho h\frac{\partial^2 w}{\partial t^2}+Q_L=0 \quad (6)$$

边界条件为

$$w|_{y=0}=0,\quad \left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)\Big|_{y=b}=0$$
$$\frac{\partial w}{\partial y}\Big|_{y=0}=0,\quad \frac{\partial^3 w}{\partial y^3}+(2-\nu)\frac{\partial^3 w}{\partial x^2\partial y}\Big|_{y=b}=0 \quad (7a)$$

$$\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)\Big|_{x=0,a}=0,\quad \frac{\partial^2 w}{\partial x\partial y}\Big|_{x=0,y=b}=0$$
$$\left(\frac{\partial^3 w}{\partial x^3}+(2-\nu)\frac{\partial^3 w}{\partial x\partial y^2}\right)\Big|_{x=0,a}=0,\quad \frac{\partial^2 w}{\partial x\partial y}\Big|_{x=a,y=b}=0 \quad (7b)$$

式中 D 为机翼的弯曲刚度，ν 为材料的泊松比，ρ 为机翼材料密度，h 为机翼的厚度，w 为机翼横向位移，Q_L 为机翼受到的非定常气动力。

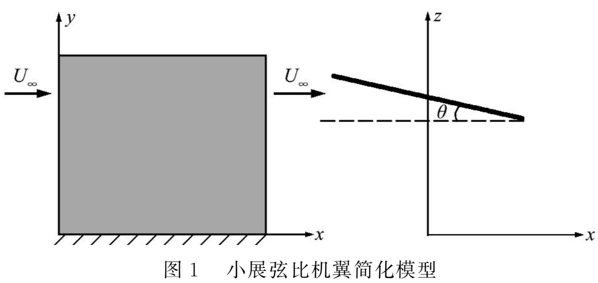

图1　小展弦比机翼简化模型

Fig.1　Calculating sketch of low-aspect-ratio wing

3.2　非定常气动力

活塞气动力理论是一种超音速非定常气动力理论，其适用范围为马赫数2～5，本文计算分析的小展弦比机翼的飞行马赫数为5，在活塞气动力的适用范围之内，因此可以采用此气动力理论进行分析。本文采用一阶活塞理论[27]计算高速气动力：

$$p/p_\infty=1+k(v_z/c_\infty) \quad (8)$$

式中 p 为当地气流压强，p_∞ 为来流压强，v_z 为翼面法向速度，c_∞ 为来流音速，k 为比热比。

对上表面：

$$v_{zu}=\left(V\frac{\partial}{\partial x}+\frac{\partial}{\partial t}\right)w(x,y,t)+V\frac{\partial H(x,y)}{\partial x} \quad (9a)$$

对下表面：

$$v_{zl}=-\left(V\frac{\partial}{\partial x}+\frac{\partial}{\partial t}\right)w(x,y,t)+V\frac{\partial H(x,y)}{\partial x} \quad (9b)$$

式中 $w(x,y,t)$ 为机翼中线上任意一点的挠度，V 为飞行器飞行速度，$H(x,y)$ 为机翼的厚度函数。作用在机翼上的气动力可由下式求得

$$Q_L=p_u-p_l=p_\infty k\frac{v_{zu}-v_{zL}}{c_\infty}=\rho_\infty\frac{V}{Ma}(v_{zu}-v_{zL}) \quad (10)$$

式中 Ma 为飞行马赫数，ρ_∞ 为来流空气密度。

3.3　微分求积法求解方程

将气动弹性方程进行无量纲化，引入无量纲量：

$$\eta=a/b,\ X=x/a,\ Y=y/b \quad (11)$$

$\lambda=1/\eta$ 即为机翼的展弦比。则机翼无量纲气动弹性方程为

$$D\left[\frac{\partial^4 w}{\partial X^4}+2\eta^2\frac{\partial^4 w}{\partial X^2\partial X^2}+\eta^4\frac{\partial^4 w}{\partial Y^4}\right]+\rho h a^4\frac{\partial^2 w}{\partial t^2}+$$
$$2a^4\rho_k\frac{V^2}{Ma}\left(\frac{1}{V}\frac{\partial w}{\partial t}+\frac{\partial w}{\partial x}\right)=0 \quad (12)$$

采用微分求积法[28-30]对微分方程及其边界条件进行离散得

$$D\left(\sum_{l=1}^{n}A_{i,l}^{(4)}w_{l,j}+2\eta^2\sum_{l=1}^{n}A_{i,l}^{(2)}\sum_{m=1}^{M}B_{j,m}^{(2)}w_{l,m}+\right.$$
$$\left.\eta^4\sum_{m=1}^{M}B_{j,m}^{(4)}w_{i,m}\right)+a^4\rho h\ddot{w}_{i,j}+2a^4\rho_k\frac{V}{Ma}\gamma\dot{w}_{i,j}+$$
$$2a^3\rho_k\gamma\frac{V^2}{Ma}\sum_{l=1}^{n}A_{i,l}^{(1)}w_{l,j}=0 \quad (13)$$

边界条件：

$$w_{i,1}=0,\ \sum_{m=1}^{M}B_{1,m}^{(1)}w_{i,m}=0 \quad (14a)$$

$$\sum_{m=1}^{M}B_{M,m}^{(2)}w_{i,m}+\nu\sum_{l=1}^{n}A_{i,l}^{(2)}w_{l,j}=0$$
$$\sum_{m=1}^{M}B_{M,m}^{(3)}w_{i,m}+(2-\nu)\sum_{l=1}^{n}A_{i,l}^{(2)}\sum_{m=1}^{M}B_{M,m}^{(1)}w_{l,m}=0 \quad (14b)$$

$$\sum_{l=1}^{n} A_{1,l}^{(2)} w_{l,j} + \nu \sum_{m=1}^{M} B_{1,m}^{(2)} w_{i,m} = 0$$
$$\sum_{l=1}^{n} A_{1,l}^{(3)} w_{l,j} + (2-\nu) \sum_{l=1}^{n} A_{1,l}^{(1)} \sum_{m=1}^{M} B_{M,m}^{(2)} w_{l,m} = 0$$
$$\sum_{l=1}^{n} A_{n,l}^{(2)} w_{l,j} + \nu \sum_{m=1}^{M} B_{n,m}^{(2)} w_{i,m} = 0$$
$$\sum_{l=1}^{n} A_{n,l}^{(3)} w_{l,j} + (2-\nu) \sum_{l=1}^{n} A_{n,l}^{(1)} \sum_{m=1}^{M} B_{M,m}^{(2)} w_{l,m} = 0 \quad (14c)$$
$$\sum_{l=1}^{n} A_{1,l}^{(1)} \sum_{m=1}^{M} B_{M,m}^{(1)} w_{l,m} = 0$$
$$\sum_{l=1}^{n} A_{n,l}^{(1)} \sum_{m=1}^{M} B_{M,m}^{(1)} w_{l,m} = 0 \quad (14d)$$

式中 $A_{i,j}^{(m)}$，$B_{i,j}^{(m)}$ 为微分求积法中函数 $w(x,y)$ 对变量 x，y 的 m 阶导数的权函数。

将边界条件式(14)代入机翼颤振方程(13)，并设 $w(x,y,t)=w(x,y)\exp(\Omega t)$，可得到代数特征值问题：

$$(\Omega^2 M + \Omega H + K) w = 0 \quad (15)$$

式中 Ω 的虚部即为给定飞行速度下机翼的固有频率，且随飞行速度的增大而发生变化，当其中两阶频率发生重合时，机翼发生颤振，此时的飞行速度即为颤振速度。

4　计算结果与讨论

在飞行马赫数 Ma > 2 的情况下，飞行器的气动弹性分析需要考虑气动热的影响。由于热传导时间远大于结构的响应时间，因此气动热与气动弹性的相互影响为弱耦合，只需考虑气动热对气动弹性的影响，在分析过程中可以将气动热视为外载。而气动热对气动弹性的影响主要有两方面：一是温度升高使得结构的力学性能下降，二是会在结构内部产生热应力，影响结构的固有特性，从而影响气动弹性的临界速度。

如果考虑气动热的影响，必然需要考虑结构的非线性热变形，使问题复杂化；而本文的研究旨在引入一种新的气动弹性分析求解方法，气动热的考虑与否对方法的适用性没有影响。因此，本文未考虑气动热的影响。

4.1　方法验证

气动弹性是弹性结构与空气动力之间的相互耦合，从气动弹性动力学方程(6)可知，当气流速度为0时，气动力等于0，此时得到的方程各阶频率即为机翼的固有频率。本文采用大型有限元软件MSC. NASTRAN 对机翼的固有频率及颤振速度进行分析，并与微分求积法求得的解进行比较，验证程序的合理性。机翼的基本参数见表1。

表1　小展弦比机翼基本参数

Tab. 1　Baseline parameters of low-aspect-ratio wing

参数	数值
弦长 a	0.4 m
翼厚 h	0.01 m
展弦比 λ	1.0
密度 ρ	2700 kg/m^3
弹性模量 E	6.76×10^{10} N/m^2
泊松比 ν	0.3

微分求积法在求解之前需要给定网格点，其布置方式一般有两种：即均匀网格和非均匀网格。本文采用非均匀网格对机翼进行划分，在边界点采用 δ 邻近处理，δ 为小量，本文 $\delta=10^{-5}$，网格点布置如下所示：

$$x_1 = y_1 = 0,\ x_2 = y_2 = \delta$$
$$x_i = \frac{1}{2}\left[1-\cos\left(\frac{i-2}{N-3}\pi\right)\right],\ i=3,4,\cdots,N-2$$
$$y_j = \frac{1}{2}\left[1-\cos\left(\frac{j-2}{M-3}\pi\right)\right],\ j=3,4,\cdots,M-2$$
$$x_{N-1} = y_{M-1} = 1-\delta,\ x_N = y_M = 1$$

根据频率重合理论，采用微分求积法及有限元方法分析小展弦比机翼颤振速度的曲线图如图2所示。表2比较了两种方法在分析固有频率及颤振速度时的计算结果及误差。

从图2可以看出，随着飞行速度的增大，机翼各阶频率均发生变化，而其前两阶频率逐渐接近。当飞行速度达到3400 m/s时，前两阶频率重合，此时机翼发生颤振问题。而采用有限元方法计算得到的颤振速度为3370 m/s，误差为0.88%。表2为机翼固有频率及颤振速度的计算结果，可以看出，采用微分求积法计算的机翼固有频率与有限元方法一致，误差在1.05%以内。可见，微分求积方法对求解颤振偏微分方程有很好的效果。

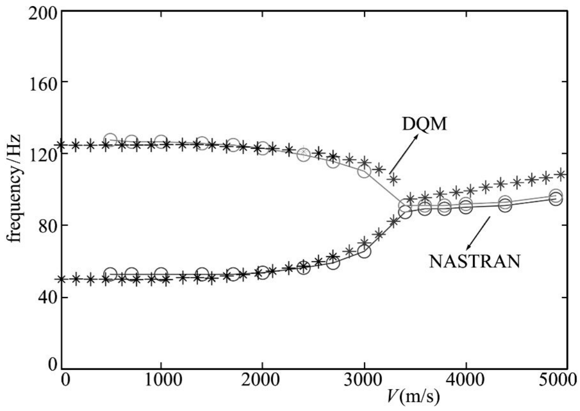

图2　小展弦比机翼颤振速度

Fig. 2　Flutter velocity of low-aspect-ratio wing

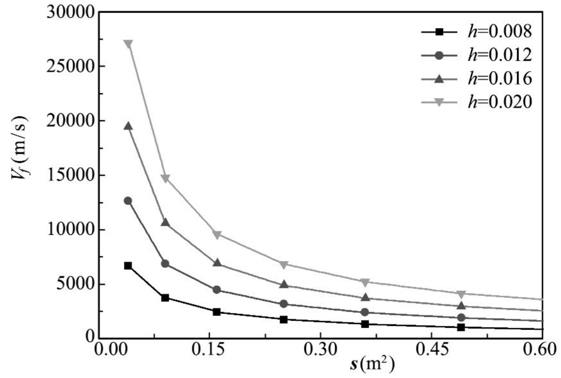

图 3　颤振速度随面积的变化

Fig. 3　The influence of area on flutter velocity

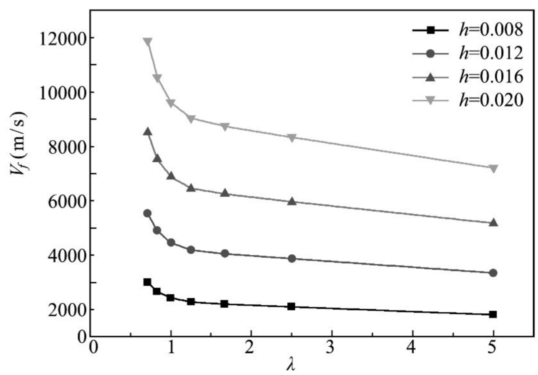

图 4　机翼展弦比对颤振速度的影响

Fig. 4　The influence of aspect ratio on flutter velocity

表 2　DQM 方法与 MSC. NASTRAN 计算结果比较

Tab. 2　Results compare of DQM and MSC. NASTRAN

		DQM	NASTRAN	误差
固有频率/Hz	一阶	50.91	51.21	0.6%
	二阶	124.75	126.08	1.05%
	三阶	322.93	320.16	0.9%
	四阶	409.80	406.85	0.8 %
颤振速度(m/s)		3400	3370	0.88%

4.2　机翼颤振参数影响

气动-结构耦合的机翼颤振问题是飞行器设计中必须予以考虑的问题。本节采用微分求积法对小展弦比机翼的颤振速度进行分析，研究机翼几何参数对颤振速度的影响。

图 3 为展弦比为 1(λ=1) 时，机翼颤振速度随面积的变化曲线。可以看出，机翼颤振速度随面积的增大而明显减小。特别是在面积小于 0.2 m² 时，颤振速度随机翼面积的增大而急剧减小，当面积大于 0.2 m² 时，颤振速度变化相对而言比较平缓。厚度 h 对机翼的颤振有很大的影响，厚度的增大，会增大机翼结构的刚度，提高各阶固有频率，提高颤振速度。

图 4 为相同机翼面积($S=a\times b=0.16$ m²)情况下，颤振速度随展弦比 λ 的变化曲线。可以看出，机翼颤振速度随展弦比的增大而减小，且随着展弦比的增大，变化趋势逐渐平缓。

5　结　论

本文应用微分求积法，对高速小展弦比机翼的颤振问题进行研究，得出以下结论。

(1) 微分求解法对高速小展弦比机翼的颤振问题的求解有很好的效果。分析了机翼的固有频率及颤振速度，并与有限元软件计算结果进行比较，误差在 2%以内。

(2) 小展弦比机翼的颤振速度受结构尺寸的影响较大。颤振速度随面积、展弦比的增大而减小，随机翼厚度的增大而增大。

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Flutter analysis of low-aspect-ratio wing by differential quadrature method

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Abstract: This paper introduces the differential quadrature method to solve the aeroelastic problem of low-aspect-ratio wing. Low aspect ratio wing flutter partial differential equations are established based on the first order piston theory, and solved based on the frequency coincidence theory. The natural frequency and flutter velocity are analyzed by DQM and FEM, the result shows that the relative errors between DQM and FEM are within 2%. So the differential quadrature method is very effective to the solve flutter problems of low aspect ratio wings. Analysis of the effects of wing area, aspect ratio and thickness on flutter speed, the results showed that: flutter speed of the low-aspect-ratio wing greatly influenced by the structure size, with the wing flutter speed area, the aspect ratio of the decreases, with the wing thickness increases.

Key words: differential quadrature method; aeroelastic; low-aspect-ratio; wing; flutter

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Optimal searching algorithm for non-probabilistic reliability

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Abstract: With the consideration of interval variables as uncertain parameters, the paper emphasizes the iterate algorithm of optimal searching path in order to get non-probabilistic reliability index. For the case of the non-linear performance function, the intersection line of the tangent plane and $G=0$ was achieved so as to linearize the performance function. The intersection point as an iterate point was determined by isocline and equivalent linear performance function. The most probable failure point (MPP) was obtained when $G=0$ after optimal searching. Furthermore, the reliability index could be calculated according to MPP. Two numerical examples were applied for testing and verifying the proposed algorithm and good agreement was achieved.

Key words: interval model; non-probabilistic reliability; reliability index; most probable failure point; optimal searching algorithm