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求解矩形夹层板在集中载荷下的稳定问题

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摘 要:在考虑了横向切应力和横向正应力对夹层板稳定影响的情况下,给出了矩形夹层板结构屈曲失稳的控制方程、基本解以及边界条件。应用功的互等定理求解了在均布载荷作用下的矩形夹层板的屈曲失稳问题。

关键词:稳定问题;功的互等定理;夹层板;边界条件;均布载荷

中图分类号:O327+.3 **文献标志码**:A doi:10.7511/jslx201304009

1 引 言

夹层板最早的理论是由 Reissner[1] 在 1947 年提出来的,它的主要特征是将表层看作为薄膜应力状态,即认为其主要承受抗弯作用,而夹心则认为主要承受横向剪切的作用。这种理论是在工程中夹层板分析及设计中最常用的理论。在 1950 年 Reissner[4] 用此观点又建立了大挠度的夹层板理论,在这同时 N. J. Hoff[5] 提出了另一种把表层看作为普通的薄板而夹心仍认为主要承受剪切作用的夹层板理论,这两种理论都是针对反对称型的弯曲或总体失稳的波形而讨论的。对于对称型的变形或局部失稳形式的分析模型则是由 PycakoB 和杜庆华[6] 提出的,除了以上三种理论模型外,还有一些更为简单和复杂的计算方法以及处理各向异性(夹心和表层)问题的基本理论模型,关于这方面的文章和论述可参见文献[7-10]。

功的互等法是求解板壳力学及弹性力学平衡、稳定和振动问题的系统的方法。应用它可直接得到挠曲面方程,较之求解相应的微分方程要简单的多。为求解夹层板的稳定问题开辟了一条方便快捷的新途径,并且还可以用于实际应用和指导工程设计。

2 矩形板稳定的功的互等定理[11]

在图 1 所示矩形板稳定基本系统和图 2 所示广义支承边矩形板之间应用功的互等定理,则得

$$
\begin{aligned}
& f(\zeta,\eta)-\int_0^a V_{1y0} f(x,0)\,\mathrm{d}x+\int_0^a V_{1yb} f(x,b)\,\mathrm{d}x- \\
& \int_0^b V_{1x0} f(0,y)\,\mathrm{d}y+\int_0^b V_{1xa} f(a,y)\,\mathrm{d}y+ \\
& \int_0^a\int_0^b\left(-F_x\frac{\partial^2 f_{j1}}{\partial x^2}-F_y\frac{\partial^2 f_{j1}}{\partial y^2}\right) f(\zeta,\eta)\,\mathrm{d}x\,\mathrm{d}y- \\
& R_{j100}k_1+R_{j1a0}k_2-R_{j1ab}k_3+R_{j10b}k_4= \\
& \int_0^a M_{y0} f_{j1,y0}\,\mathrm{d}x-\int_0^a M_{yb} f_{j1,yb}\,\mathrm{d}x+ \\
& \int_0^b M_{x0} f_{j1,x0}\,\mathrm{d}y-\int_0^b M_{xa} f_{j1,xa}\,\mathrm{d}y- \\
& \int_0^a\int_0^b\left(F_x\frac{\partial^2 f}{\partial x^2}+F_y\frac{\partial^2 f}{\partial y^2}\right) f_{j1} f(x,y,\zeta,\eta)\,\mathrm{d}x\,\mathrm{d}y
\end{aligned}
\tag{1}
$$

式中 $V_{1x0}, V_{1xa}, V_{1y0}, V_{1yb}$ 为基本系统的等效切力,如 $V_{1x0}=-\dfrac{Eh^3}{12(1-\nu^2)}\left[\dfrac{\partial^3 f_{j1}}{\partial x^3}+(2-\nu)\dfrac{\partial^3 f_{j1}}{\partial x\partial y}\right]_{x=0}$,…,而 $()_{j1}$ 为基本系统的其他相关量。

另外,注意到:

$$
\int_0^b\left[\left(-F_x\frac{\partial f_{j1}}{\partial x}\right)f\right]_0^a\mathrm{d}y+\int_0^a\int_0^b F_x\frac{\partial f_{j1}}{\partial x}\frac{\partial f}{\partial x}\mathrm{d}x\,\mathrm{d}y=\int_0^a\int_0^b -F_x\frac{\partial^2 f_{j1}}{\partial x^2}f\,\mathrm{d}x\,\mathrm{d}y \tag{2}
$$

$$
\int_0^a\left[\left(-F_y\frac{\partial f_{j1}}{\partial y}\right)f\right]_0^b\mathrm{d}x+\int_0^a\int_0^b F_y\frac{\partial f_{j1}}{\partial y}\frac{\partial f}{\partial y}\mathrm{d}x\,\mathrm{d}y=\int_0^a\int_0^b -F_y\frac{\partial^2 f_{j1}}{\partial y^2}f\,\mathrm{d}x\,\mathrm{d}y \tag{3}
$$

$$
\int_0^b\left[\left(-F_x\frac{\partial f}{\partial x}\right)f_{j1}\right]_0^a\mathrm{d}y+\int_0^a\int_0^b F_x\frac{\partial f_{j1}}{\partial x}\frac{\partial f}{\partial x}\mathrm{d}x\,\mathrm{d}y=\int_0^a\int_0^b -F_x\frac{\partial^2 f}{\partial x^2}f_{j1}\,\mathrm{d}x\,\mathrm{d}y \tag{4}
$$

$$
\int_0^a\left[\left(-F_y\frac{\partial f}{\partial y}\right)f_{j1}\right]_0^b\mathrm{d}x+\int_0^a\int_0^b F_y\frac{\partial f_{j1}}{\partial y}\frac{\partial f}{\partial y}\mathrm{d}x\,\mathrm{d}y=\int_0^a\int_0^b -F_y\frac{\partial^2 f}{\partial x^2}f_{j1}\,\mathrm{d}x\,\mathrm{d}y \tag{5}
$$

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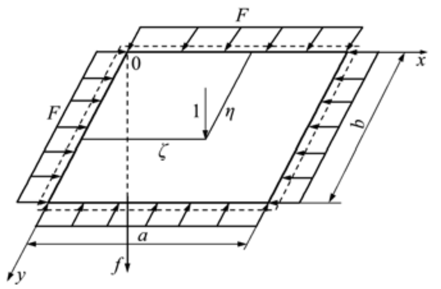

图 1　基本系统

Fig. 1　Basic system

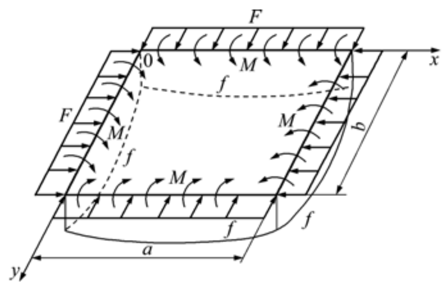

图 2　广义支承边矩形板

Fig. 2　Rectangular plates with four generalized supported edge

注意到稳定基本系统的边界值为 0，且将式(2～5)代入式(1)中，则得求解稳定问题的基本方程。

$$f(\zeta,\eta)=\int_0^a V_{j1y0}f(x,0)\,\mathrm{d}x-\int_0^a V_{j1yb}f(x,b)\,\mathrm{d}x+\int_0^b V_{j1x0}f(0,y)\,\mathrm{d}y-\int_0^b V_{j1xa}f(a,y)\,\mathrm{d}y+\int_0^a M_{y0}f_{j1,y0}\,\mathrm{d}x-\int_0^a M_{yb}f_{j1,yb}\,\mathrm{d}x+\int_0^b M_{x0}f_{j1,x0}\,\mathrm{d}y-\int_0^b M_{xa}f_{j1,xa}\,\mathrm{d}y+R_{j100}k_1-R_{j1a0}k_2+R_{j1ab}k_3-R_{j100\,b}k_4 \tag{6}$$

3　面板和夹心层的平衡方程和应力-应变关系

夹层板的单元示意图如图 3 所示。由图 3 可得两个面板的平衡微分方程分别为

$$\frac{\partial F_{x\pm}}{\partial x}+\frac{\partial H_{\pm}}{\partial y}\mp\tau_{x\pm}=0 \tag{7}$$

$$\frac{\partial H_{\pm}}{\partial x}+\frac{\partial F_{y\pm}}{\partial y}\mp\tau_{y\pm}=0 \tag{8}$$

$$\frac{\partial}{\partial y}\left[F_{y\pm}\frac{\partial f_{\pm}}{\partial y}\right]+p_{\pm}\pm\frac{\partial f_{\pm}}{\partial y}\mp\sigma_{z\pm}\mp\tau_{x\pm}\frac{\partial f_{\pm}}{\partial x}\mp\tau_y=0 \tag{9}$$

在假设忽略与面板相平行的夹心层应力的情况下，夹心层平衡方程为

$$\frac{\partial\tau_x}{\partial x}=0,\ \frac{\partial\tau_y}{\partial y}=0 \tag{10}$$

$$\frac{\partial\tau_x}{\partial x}+\frac{\partial\tau_y}{\partial y}+\frac{\partial\sigma_z}{\partial z}=0 \tag{11}$$

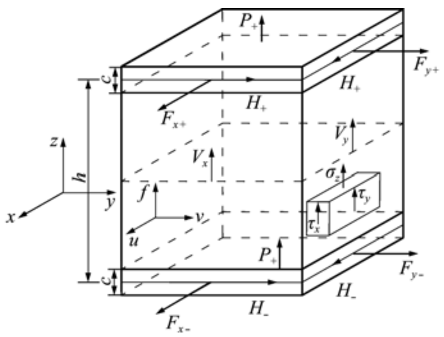

图 3　夹层板单元

Fig. 3　Element of sandwich plate

面板的应变-位移关系及应变-内力关系为

$$\varepsilon_{x\pm}=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial f_{\pm}}{\partial x}\right)^2=\frac{1}{E_f\,c}(F_{x\pm}-v_fF_{y\pm}) \tag{12}$$

$$\varepsilon_{y\pm}=\frac{\partial v_{\pm}}{\partial y}+\frac{1}{2}\left(\frac{\partial f_{\pm}}{\partial y}\right)^2=\frac{1}{E_f\,c}(F_{y\pm}-v_fF_{x\pm}) \tag{13}$$

$$r_{\pm}=\frac{\partial u_{\pm}}{\partial y}+\frac{\partial v_{\pm}}{\partial y}+\frac{\partial f_{\pm}}{\partial x}\frac{\partial f_{\pm}}{\partial y}=\frac{1}{G_f\,c}S_{\pm} \tag{14}$$

夹心层的应力-应变及应变-应力关系为

$$\varepsilon_z=\frac{\partial f}{\partial z}=\frac{\sigma_z}{E_c},\ r_x=\frac{\partial u}{\partial z}+\frac{\partial f}{\partial x}=\frac{\tau_x}{G_c} \tag{15,16}$$

$$r_y=\frac{\partial v}{\partial z}+\frac{\partial f}{\partial y}=\frac{\tau_y}{G_c} \tag{17}$$

4　夹层板稳定的控制方程

为导出夹层板的方程，定义相关的变量如下：

$$\alpha=\frac{1}{h}(u_+-u_-),\ \beta=\frac{1}{h}(v_+-v_-) \tag{18}$$

除了横向切应力的合力 V_x 和 V_y 外，还定义夹层板的合力和力偶分别为

$$F_x=F_{x+}+F_{x-},\ F_y=F_{y+}+F_{y-} \tag{19}$$

$$H=H_++H_-,\ B=(H_+-H_-)\frac{h}{2} \tag{20}$$

$$M_x=(F_{x+}-F_{x-})\frac{h}{2},\ M_y=(F_{y+}-F_{y-})\frac{h}{2} \tag{21}$$

最终可得夹层板的挠曲线控制方程：

$$\frac{ch^2E_f}{2(1-\nu_f^2)}\nabla^4 f=\left[1-\frac{ch}{2(1-\nu_f^2)}\nabla^2\right]\times\left[p+F_x\frac{\partial^2 f}{\partial x^2}+2H\frac{\partial^2 f}{\partial x\partial y}+F_y\frac{\partial^2 f}{\partial y^2}\right] \tag{22}$$

5　矩形夹层板稳定的基本解及其边界值

考虑四边简支矩形板的两个基本系统。一个是图 4 所示的基本系统，它是矩形静力弯曲问题的基本系统；另一个是图 5 所示的基本系统，它是在中面力 N_x、N_y 和单位横向集中力作用下的四边简支矩形板，它是矩形板稳定问题的基本系统，夹层

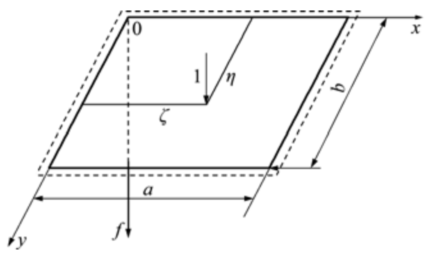

图 4　弯曲矩形板的基本系统

Fig. 4　Basic system of bending rectangular plate

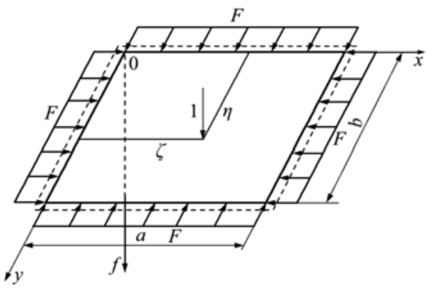

图 5　矩形板稳定的基本系统

Fig. 5　Stability basic system of rectangular plate

板稳定问题的基本方程为

$$\frac{ch^2 E_f}{2(1-\nu_f^2)}\left(\frac{\partial^4 f}{\partial x^4}+2\frac{\partial^4 f}{\partial x^2\partial y^2}+\frac{\partial^4 f}{\partial y^4}\right)=\left(\frac{chE_f\nabla^2}{2(1-\vartheta^2)}-1\right)\left(F_x\frac{\partial^2 f}{\partial x^2}+F_y\frac{\partial^2 f}{\partial y^2}\right)\tag{23}$$

夹层板稳定问题的基本解的方程为

$$\frac{ch^2 E_f}{2(1-\nu_f^2)}\left(\frac{\partial^4 f_1}{\partial x^4}+2\frac{\partial^4 f_1}{\partial x^2\partial y^2}+\frac{\partial^4 f_1}{\partial y^4}\right)+\left(1-\frac{chE_f\nabla^2}{2(1-\vartheta^2)}\right)\times\left(F_x\frac{\partial^2 f_1}{\partial x^2}+F_y\frac{\partial^2 f_1}{\partial y^2}\right)=\delta(x-\zeta,y-\eta)\tag{24}$$

6　两邻边简支一边固定一边自由的夹层矩形板的稳定

考虑两邻边简支一边固定一边自由矩形板，如图 6(a)所示。在中面力作用下该板处于临界状态，解除固定边 $y=0$ 的约束代以弯矩 M_{y0}，则得图 6(b)所示，并假设

$$M_{y0}=\sum_{m=1,2}^{\infty}C_m\sin(m\pi x/a)\tag{25}$$

$$f_{xa}=\sum_{m=1,2}^{\infty}A_n\sin(n\pi y/b)\tag{26}$$

7　临界状态的挠曲面方程和特征方程

在基本系统图 5 和实际系统图 6(b)之间应用功的互等定理，则得

$$f(\zeta,\eta)+\int_0^b V_{j1xa}f_{xa}\,dy=\int_0^a M_{y0}f_{j1,y0}\,dx\tag{27}$$

其中

$$V_{j1xa}=\left\{-Q\left[\frac{\partial^3 f_{j1}}{\partial x^3}+(2-v)\frac{\partial^3 f_{j1}}{\partial x\partial y^2}\right]-F\frac{\partial f_{j1}}{\partial x}\right\}_{x=a}=-\frac{2}{b}\sum_{n=1,2}^{\infty}\frac{1}{\alpha_n^2-\beta_n^2}\sin\frac{n\pi y}{b}\sin\frac{n\pi\eta}{b}\times\left\{\left[\alpha_n^2-(2-v)\left(\frac{m\pi}{a}\right)^2+\frac{F}{Q}\right]\frac{\sinh\alpha_n\zeta}{\sinh\alpha_n a}-\left[\beta_n^2-(2-v)\left(\frac{n\pi}{b}\right)^2+\frac{F}{Q}\right]\frac{\sinh\beta_n\zeta}{\sinh\beta_n a}\right\}\tag{28}$$

$$f_{j1,y0}=-\frac{2}{Qa}\sum_{n=1,2}^{\infty}\frac{1}{\alpha_m^2-\beta_m^2}\sin\frac{n\pi y}{b}\sin\frac{n\pi\eta}{b}\times\left[\frac{\sinh\alpha_m(b-\eta)}{\sinh\alpha_m b}-\frac{\sinh\beta_m(b-\eta)}{\sinh\alpha_m b}\right]\tag{29}$$

将式(25,26)和式(28,29)代入式(27)中，经过整理计算得

$$f(\zeta,\eta)=-\frac{1}{Q}\sum_{m=1,2}^{\infty}\frac{1}{\alpha_m^2-\beta_m^2}\sin\frac{m\pi\zeta}{a}(C_m)\times\left[\frac{\sinh\alpha_m(b-\eta)}{\sinh\alpha_m b}-\frac{\sinh\beta_m(b-\eta)}{\sinh\alpha_m b}\right]+\sum_{n=1,2}^{\infty}\frac{1}{\alpha_n^2-\beta_n^2}\sin\frac{n\pi\eta}{b}(A_n)\times\left\{\left[\alpha_n^2-(2-v)\left(\frac{n\pi}{b}\right)^2+\frac{F}{Q}\right]\frac{\sinh\alpha_n\zeta}{\sinh\alpha_n a}-\left[\beta_n^2-(2-v)\left(\frac{n\pi}{b}\right)^2+\frac{F}{Q}\right]\frac{\sinh\beta_n\zeta}{\sinh\beta_n a}\right\}\tag{30}$$

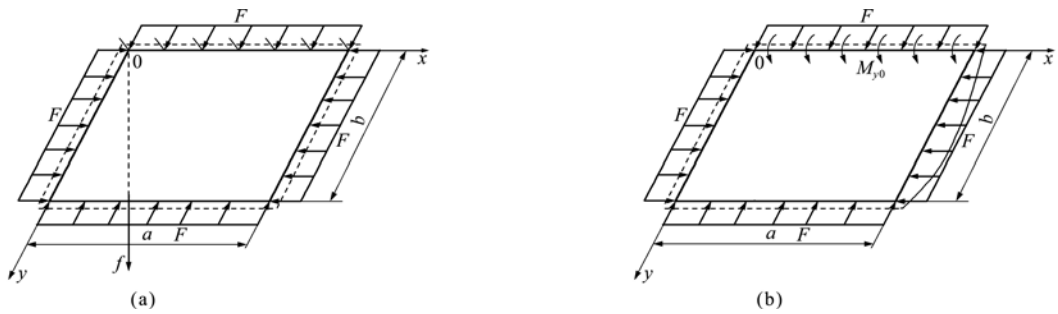

图 6　两邻边简支一边固定一边自由的夹层矩形板的实际系统

Fig. 6　Actual system of rectangular sandwich of the two adjacent edges simply supported, the third edge fixed and the fourth edge free

需满足的边界条件:

$$f_{\zeta 0}=0,\ V_{j\zeta 0}=0 \qquad (31,32)$$

将式(30)代入式(31,32),经过计算得

$$\sum_{m=1}^{\infty}\frac{1}{Q}\frac{\alpha_m\coth\alpha_m b-\beta_m\coth\beta_m b}{\alpha_m^2-\beta_m^2}\sin\frac{m\pi\zeta}{a}(C_m)-\frac{2}{a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^m}{K_{jmn}^2}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\sin\frac{m\pi\zeta}{a}(A_n)\times\left[\left(\frac{m\pi}{a}\right)^2+(2-\nu)\left(\frac{n\pi}{b}\right)^2-\frac{F}{Q}\right]=0 \qquad (33)$$

$$\frac{1}{Q}\frac{2}{b}\sum_{m=1}^{\infty}\frac{(-1)^m}{K_{jmn}^2}\left[\left(\frac{m\pi}{a}\right)^2+(2-\nu)\left(\frac{n\pi}{b}\right)^2-\frac{F}{Q}\right](C_m)\times\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\sin\frac{n\pi\eta}{b}-\sum_{n=1}^{\infty}\frac{1}{\alpha_n^2-\beta_n^2}\sin\frac{n\pi\eta}{b}(A_n)\times\left\{\left[\alpha_n^2-(2-\nu)\left(\frac{n\pi}{b}\right)^2+\frac{F}{Q}\right]^2\alpha_n\coth\alpha_n a-\left[\beta_n^2-(2-\nu)\left(\frac{n\pi}{b}\right)^2+\frac{F}{Q}\right]^2\beta_n\coth\beta_n a\right\}=0 \qquad (34)$$

利用傅里叶级数的性质,以加权余量法对式(33,34)进行积分,并进行双曲函数往三角级数的转换,进一步可以求得相应边界条件的执行方程为

$$\frac{1}{Q}\frac{\alpha_m\coth\alpha_m b-\beta_m\coth\beta_m b}{\alpha_m^2-\beta_m^2}(C_m)-\frac{2}{a}\sum_{n=1,2}^{\infty}\frac{(-1)^m}{K_{jmn}^2}\times\frac{mn\pi^2}{ab}\left[\left(\frac{m\pi}{a}\right)^2+(2-\nu)\left(\frac{n\pi}{b}\right)^2-\frac{F}{Q}\right](A_n)=0 \qquad (35)$$

$$\frac{1}{Q}\frac{2}{b}\sum_{m=1}^{\infty}\frac{(-1)^m}{K_{jmn}^2}\left[\left(\frac{m\pi}{a}\right)^2+(2-\nu)\left(\frac{n\pi}{b}\right)^2-\frac{F}{Q}\right]\times\frac{mn\pi^2}{ab}(C_m)-\frac{1}{\alpha_n^2-\beta_n^2}(A_n)\times\left\{\left[\alpha_n^2-(2-\nu)\left(\frac{n\pi}{b}\right)^2+\frac{F}{Q}\right]^2\alpha_n\coth\alpha_n a-\left[\beta_n^2-(2-\nu)\left(\frac{n\pi}{b}\right)^2+\frac{F}{Q}\right]^2\beta_n\coth\beta_n a\right\}=0 \qquad (36)$$

对式(35),m 取无穷多项,而 n 分别取 1,2,3,4,…;相类似的,对式(36),n 取无穷多项,而 m 分别取 1,2,3,4,…。最终,式(35,36)形成一组以 C_1、C_2、C_3、C_4,…和 A_1、A_2、A_3、A_4,…为未知数的(4×4)维的齐次超越方程组。这一齐次超越方程组非零解的存在说明板的平面状态受到破坏,式(30)所确定的临界状态的挠曲面方程有非零值。该齐次线性方程组的系数行列式为零是该稳定方程的特征方程。

8 数值计算及有限元模拟

作为数值算例,考虑一两邻边简支一边固定一边自由的矩形的夹层板,取夹层板的面板厚度为 $c=0.3048$ mm,夹心层的厚度为 $d=6.197$ mm,板的边长 $b=1$ m,面板的弹性模量 $E=6.826\times10^{10}$ Pa,夹心层的剪切模量 $G_c=9.281\times10^6$ Pa。引入临界荷载系数 $F_{cr}=[12(1-\nu^2)Fa^2]/Eh^3\pi^2$;另外,根据长宽比的不同和对板的泊松比 ν 取值的不同,当 $a/b=1,1.5,2,2.5,3$ 时,分别对 ν 取值为 $\nu=4\times10^{-2},8\times10^{-3},4\times10^{-3},4\times10^{-4}$,级数 m 和 n 分别取到 50 项,用 Matlab 软件编程计算可求得其临界荷载值。用临界荷载系数公式对所求结果进一步简化计算。表 1 和表 2 给出了长宽比和板的泊松比相对应的临界荷载的系数。

为了验证解析解的准确性,本算例应用 ANSYS 有限元软件对两邻边简支一边固定一边自由的矩形夹层板进行了模拟分析。本算例中采用 shell 91 单元进行建模计算。在 ANSYS 模拟计算中对纵横比 a/b 和板的泊松比 ν 分别取不同的值进行计算,当 $a/b=1,1.5,2,2.5,3$,分别对 ν 取值为 $\nu=4\times10^{-2},8\times10^{-3},4\times10^{-3},4\times10^{-4}$。在建模,加载完成后,经过 ANSYS 计算之后,可得到临界荷载值。进一步引入临界荷载系数 $F_{cr}=[12(1-\nu^2)Fa^2]/Eh^3\pi^2$,计算结果列入表 1 和表 2。

表 1 两邻边简支一边固定一边自由夹层矩形板的临界荷载系数(F_{cr})

Tab. 1 Critical load coefficient of rectangular sandwich with the two adjacent edges simply supported, the third edge fixed and the fourth edge free(F_{cr})

a/b \ ν	4×10^{-2}	8×10^{-3}	4×10^{-3}	4×10^{-4}
1	0.9326	1.6486	1.8788	2.2658
1.5	2.0552	3.6576	4.1485	4.9232
2	3.6418	6.4528	7.3662	8.8521
2.5	5.6457	10.1262	11.5289	13.8119
3	8.1036	14.5163	16.5388	19.6540

表 2 应用 ANSYS 计算的临界荷载系数(F_{cr})

Tab. 2 Critical load coefficient with ANSYS (F_{cr})

a/b \ ν	4×10^{-2}	8×10^{-3}	4×10^{-3}	4×10^{-4}
1	0.9231	1.6218	1.8344	2.1825
1.5	2.0414	3.6232	4.0889	4.8799
2	3.6127	6.3906	7.2533	8.6720
2.5	5.6291	9.9678	11.3174	13.5592
3	8.0787	14.3207	16.2599	19.3025

9 结果分析

表1给出了对纵横比 a/b 和夹层板的泊松比 ν 取不同值时的本文解析解和ANSNS有限元的数值解。依据表1中的数值解和有限元解析解，可以分析出两种计算之间的误差分别在2.42%、1.61%、2.36%和3.67%之内，均在误差允许的范围内，从而验证了本文计算结果的准确性，进而说明了功的互等法求解夹层板稳定问题的正确性。

图7为两邻边简支一边固定一边自由矩形夹层板的临界载荷系数的分布图，从图7可以非常直观的对数值解与解析解进行比较，两者能够较好的拟合。

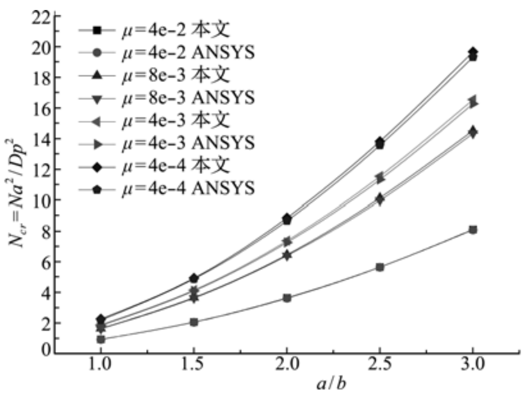

图7　两邻边简支一边固定一边自由夹层矩形板的临界荷载系数分布曲线

Fig. 7　Critical load coefficient distribution curve of rectangular sandwich with the two adjacent edges simply supported, the third edge fixed and the fourth edge free

10 结　论

功的互等法利用基本解构成位移解，采用对偶变量，可直接得到位移方程或挠曲线方程，较之求解相应的控制方程要简单的多；与其他计算方法相比具有计算简便，能有效地求解特殊载荷作用的和具有复杂边界条件的问题，解题过程程序化，概念清晰，应用广泛等优点。本文应用功的互等法对两邻边简支一边固定一边自由的矩形夹层板的稳定问题进行了理论推导和计算，给出了均布载荷作用下矩形夹层板的挠曲面方程。利用Matlab编程软件进行了数值计算，并应用ANSYS有限元软件对其进行了模拟分析计算，验证了数值计算结果的准确性。比较表明：应用功的互等法所求得的结果与有限元解比较一致。

参考文献(References)：

[1] E Reissner. On the bending of elastic plates[J]. *Math Physic*, 1947(5): 755-760.

[2] E Reissner. On the theory of bending of elastic plates [J]. *Math Physic*, 1944(23): 1-9.

[3] E Reissner. The effect of transverse shear deformation on the bending of elastic plates[J]. *Solids Structures*, 1945(12): 269-276.

[4] M H Ghayesh, S E Khadem. Rotary inertia and teno-erature effects on nonlinear vibration, steady state response and stability of an axially moving beam with time dependent velocity[J]. *International Journal of Mechanical Sciences*, 2008(50): 389-404.

[5] Hoff N J. Bending and buckling of rectangular sandwich plates [J]. *Journal of Composite Materials*, 1950, **6**(10): 325-332.

[6] 杜庆华. 三合板的一般弹性理论[J]. 物理学报, 1954, **10**(4): 25-32. (DU Qing-hua. General equations of sandwich plates under transverse loads and edgewise shears and compressions [J]. *Acta Physica Sinica*, 1954, **10**(4): 25-32. (in Chinese))

[7] Li N, S Mirza. Buckling analysis of clamped sandwich plates by the reciprocal theorem method[J]. *Journal of Computers and Structuers*, 1994, **51**(2): 137-141.

[8] 李华东, 朱　锡, 梅志远, 等. 分布载荷作用下简支功能梯度夹层板的弯曲分析[J]. 复合材料学报, 2012, **29**(2): 213-217. (LI Hua-dong, ZHU Xi, MEI Zhi-yuan, et al. Bending analysis of the simply supported functionally graded sandwich plate under distributed load [J]. *Acta Materiae Compositae Sinica*, 2012, **29**(2): 213-217. (in Chinese))

[9] 石　勇, 朱　锡, 李海涛, 等. 考虑表层抗弯刚度的夹层板静力学等效分析[J]. 海军工程大学学报, 2006, **18**(6): 106-111. (SHI Yong, ZHU Xi, LI Hai-tao, et al. Equivalent analysis for static behavior of sandwich plates with flexural stiffness of surface Layer [J]. *Journal of Naval University of Engineering*, 2006, **18**(6): 106-111. (in Chinese))

[10] 丁淑蓉. 夹层板的稳定性分析方程[J]. 机床与液压, 2005, (1): 187-189. (DING Shu-rong. Stability analysis equation of sandwich Plates[J]. *Machine Tool & Hydraulics*, 2005(1): 187-189. (in Chinese))

[11] 付宝连. 弹性力学中的能量原理及其应用[M]. 北京: 科学出版社, 2004. (FU Bao-lian. *The Energy Principle and the Application in Elasticity* [M]. Science Press, 2004. (in Chinese))

Solution to stability problem of clamped sandwich plates under concentrated load

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Abstract: Considering the influence of lateral shear deformation towards the stability of the sandwich plate, the lateral shear governing equation, fundamental solution, boundary condition as well as the static force condition of buckling instability of the rectangular sandwich plate are presented in this paper. The power of reciprocal method is applied to solve the rectangular sandwich board buckling instability problems.

Key words: stability problem; reciprocal method; clamped sandwich plates; boundary conditions; uniformly distributed load

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A high accurate combination-difference time-integration scheme for problems in structural dynamics

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Abstract: A two-step self-starting algorithm for solving structural dynamic problems called Combination-Difference Time-Integration algorithm (CDTI for short) which combines the backward difference scheme of velocity with the central difference scheme of displacement and acceleration based on Taylor series expansion is proposed. The stability and accuracy of CDTI are thoroughly analyzed by the amplification matrix and the associated eigenvalues and the results indicated that although it is conditionally stable, CDTI has high-order accuracy with no amplitude decay as well as little period elongation rate. The high accuracy of CDTI was also validated by numerical analysis.

Key words: structural dynamic problems; time integration method; combination difference; high accuracy; stability