Eigenvalues Detection Based Spectrum Sensing Algorithm for Cognitive Radio*

LEI Kejun[1], *YANG Xi*[1,2*], *PENG Shengliang*[2], *CAO Xiuying*[2]

(1. *College of Information Science and Engineering*, *Jishou University*, *Jishou Hunan* 416000, *China*;
2. *National Mobile Communications Research Laboratory*, *Southeast University*, *Nanjing* 210096, *China*)

Abstract: The task of spectrum sensing is to use the data collected by the sensing nodes (wireless sensors or cognitive users) to decide whether the spectrum holes exist or not. Recently, the maximum eigenvalue detection (MED) and the smallest eigenvalue detection (SED) methods have been proposed for spectrum sensing. Both of them perform well for the correlated signals, which is usually the case in realistic applications. However, the determinations of the thresholds for both the MED and the SED are quite involved, which limits their applications in practical sensing situations in cognitive radio (CR). Using all eigenvalues of the sample covariance matrix (SCM), a new algorithm based on the eigenvalues detection (ESD) is introduced. Multivariate statistical theories are used to obtain the decision threshold. The proposed ESD method can execute spectrum sensing without the information about the primary signal and the wireless channel. Meanwhile, it keeps the same computation complexity as that of the MED and the SED methods. More importantly, the ESD method relaxes the calculation requirement of the decision threshold by using a simple closed-form expression. Simulation results verify the effectiveness of the proposed method.

Key words: cognitive radio (CR); spectrum sensing; eigenvalues detection (ESD); maximum eigenvalue detection (MED); smallest eigenvalue detection (SED); the sample covariance matrix (SCM)

EEACC: 6140 **doi: 10.3969/j.issn.1004-1699.2012.06.011**

认知无线电中基于特征值检测的频谱感知算法*

雷可君[1], 杨 喜[1,2*], 彭盛亮[2], 曹秀英[2]

(1. 吉首大学信息科学与工程学院, 湖南 吉首 416000; 2. 东南大学移动通信国家重点实验室, 南京 210096)

摘 要:频谱感知的任务在于利用感知节点(无线传感器或者认知用户)采集的数据判断频谱空洞是否存在。基于最大特征值检测(MED)和最小特征值检测(SED)的方法最近被应用到频谱感知当中。这两种算法在检测实际应用当中普遍存在的相关信号时表现出良好的检测性能。然而,MED 和 SED 算法对应的判决门限求解非常复杂,从而限制了它们在实际的认知无线电频谱感知中的应用。该文利用取样协方差矩阵的所有特征值,提出了一种新的基于特征值检测(ESD)的算法。利用多元统计理论获得了相应的判决门限。ESD 算法无需主信号和无线信道信息参与感知过程。与此同时,它保留了与 MED 和 SED 相同的计算复杂度。更重要的是 ESD 算法对应的判决门限可以通过一个简单的闭合表达式进行求解,其计算复杂度低。仿真结果验证了新算法的有效性。

关键词:认知无线电;频谱感知;特征值检测;最大特征值检测;最小特征值检测;取样协方差矩阵

中图分类号:**TN92** **文献标识码**:**A** **文章编号**:**1004-1699(2012)06-0771-07**

In recent years, the governments and researchers have become increasingly interested in CR (Cognitive Radio), which is considered as one of the most promising solutions to deal with the conflict between the enormous spectrum demands of cognitive users (unlicensed users) and the scarcity of radio spectrum resources used by primary users (licensed users)[1-4]. In fact, IEEE has formed a working group on wireless regional area networks (IEEE 802.22) whose goal is to develop a standard for cognitive users to access the TV spectrum holes[3].

Spectrum sensing plays a fundamental role in CR, and its task is to use the data collected by wireless

项目来源:国家自然科学基金项目(61102089);湖南省教育厅科研项目(11C1058);吉首大学新开课程建设项目(2011KCB03)
收稿日期:2012-01-17 **修改日期**:2012-04-13

sensors to decide whether the spectrum holes exist or not. However, detecting the presence of the primary signal is practically difficult due to the low signal to noise ratio (SNR), deep fading and hidden nodes problem[5-6]. There are many types of basic sensing algorithms presented in the literature. Among them, the MED[7], also called the blindly combined energy detection (BCED) method in Ref[8], is a preferred technique that can achieve a high probability of detection (P_d) for the correlated primary signal, which is usually the case in most sensing scenarios[6-8]. The MED detects the existence of the primary signal in terms with the maximum eigenvalue of the SCM of the received signal. Recently, a new SED sensing algorithm based on the smallest eigenvalue detection has been introduced in Ref[9]. The simulation results of Ref[9] also show the SED can perform well for the correlated received signal. However, the determination of the decision thresholds poses a big problem for the applications of the MED and the SED. Firstly, the decision thresholds for them are derived by using the random matrix theory (RMT) under the assumption that both the sample size and the sample dimension are infinite[7,9], which results in that the threshold becomes inaccurate in realistic applications with limited sample size and sample dimension. The results of Ref[10] indicate that the inaccurate threshold may lead to poor performance. Further, the calculation of the asymptotical threshold involves the solving of the inverse cumulative distribution function (CDF) of Tracy-Widom distribution of order 1 (for the real data) or order 2 (for the complex data)[7,9-12], which requires complicated numerical computation and then cannot meet the real-time requirement in many applications.

If the primary signal is present, then the determinant (i.e., the product of all eigenvalues) of the SCM for the received signal samples is usually different from that of the statistical covariance matrix of the noise samples. Based on this fact, an alternative sensing algorithm called the eigenvalues detection (ESD) is proposed in this paper. Using all eigenvalues of the SCM as a test statistic, the proposed ESD can execute spectrum sensing without the information about the primary signal and the wireless channel. Besides, the proposed method keeps the same computation complexity as the MED and the SED, while it relaxes the calculation requirement of the threshold by using a simple closed-form expression. Simulation results verify the effectiveness of the ESD.

The notations conform to the following conventions. Vectors are column vectors denoted in lower case bold, e.g., $\mathbf{x}$. Matrices are denoted by upper case bold, e.g., $\boldsymbol{A}$. $\boldsymbol{l}_P$ and $\boldsymbol{I}_P$ are the $P\times P$ all-one matrix and identity matrix, respectively. The superscript "T" means transpose operator. det($\boldsymbol{A}$) is the determinant of $\boldsymbol{A}$. $E\{\cdot\}$ denotes the statistical expectation operator. "$\sim$" and "$\overset{a.s.}{\sim}$" mean respectively "distributed as" and "asymptotically distributed as". $W_P(N,\boldsymbol{R})$ denotes a $P\times P$ Wishart distribution with N degrees of freedom (DOF) and covariance matrix $\boldsymbol{R}$. χ_n^2 denotes a chi-square distribution with n DOF.

1 Spectrum Sensing Algorithm for Cognitive Radio Based on Eigenvalues Detection (ESD)

Considering that there are M antennas at the sensing node and N time samples can be obtained at each antenna for spectrum sensing, the $P\times 1$ received signal sample vector can be written as $x_m(n)=s_m(n)+\eta_m(n)$, $m=1,\cdots,M$, $n=1,\cdots,N$, where P denotes the number of consecutive samples of a sample vector, $s_m(n)$ and $\eta_m(n)$ denote the $P\times 1$ sample vectors of the primary signal and noise, respectively. The hypothesis testing problem for spectrum sensing can be represented as

$$\begin{cases} H_0: x_m(n)=\eta_m(n) \\ H_1: x_m(n)=s_m(n)+\eta_m(n) \end{cases} \quad (1)$$

where H_0 indicates primary signal does not exist while H_1 indicates primary signal exists. Note that $s_m(n)$ denotes the received signal after the primary signal passes through the wireless channel. Without loss of generality, we assume that $\eta_m(n)$ is a zero mean white Gaussian process with statistical covariance matrix $\sigma_\eta^2\boldsymbol{I}_P$. Assuming that the primary signal and noise are statistical independent, the $P\times P$ statistical covariance matrix of the received signal can then be written as

$$\boldsymbol{R}_x \triangleq E\{x_m(n)x_m^{\mathrm{T}}(n)\}=\boldsymbol{R}_s+\sigma_\eta^2\boldsymbol{I}_P \quad (2)$$

where $\boldsymbol{R}_s \triangleq E\{s_m(n)s_m^{\mathrm{T}}(n)\}$ is the statistical covariance matrix of the primary signal. If the primary signal is present, then we have

$$\frac{\det \boldsymbol{R}_x}{\det \sigma_\eta^2 \boldsymbol{I}_P}=\frac{\det(\boldsymbol{R}_s+\sigma_\eta^2 \boldsymbol{I}_P)}{\det \sigma_\eta^2 \boldsymbol{I}_P}>1 \tag{3}$$

where we use $\det \boldsymbol{R}_x=\det(\boldsymbol{R}_s+\sigma_\eta^2\boldsymbol{I}_P)>\det \sigma_\eta^2\boldsymbol{I}_P$ due to the non-negative definite property of $\boldsymbol{R}_s$ and the positive property of $\sigma_\eta^2\boldsymbol{I}_P$. If the primary signal is absent, then we have

$$\frac{\det \boldsymbol{R}_x}{\det \sigma_\eta^2 \boldsymbol{I}_P}=\frac{\det \sigma_\eta^2 \boldsymbol{I}_P}{\det \sigma_\eta^2 \boldsymbol{I}_P}=1 \tag{4}$$

Therefore, the quotient $\det \boldsymbol{R}_x/\det \sigma_\eta^2\boldsymbol{I}_P$ can be viewed as an indicator to decide whether the primary signal is present or not. In practical applications, the exact statistical covariance matrix can only be approximated by the SCM defined as

$$\hat{\boldsymbol{R}}_x=\frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}x_m(n)x_m^{\mathrm{T}}(n) \tag{5}$$

Hence, a new test statistic can be proposed as

$$\Lambda=\frac{\det \hat{\boldsymbol{R}}_x}{\det \sigma_\eta^2 \boldsymbol{I}_P} \tag{6}$$

Based on the above analysis, the hypothesis testing problem in Equ(1) can be re-expressed as

$$\begin{cases} H_0: \dfrac{\det \hat{\boldsymbol{R}}_x}{\det \sigma_\eta^2 \boldsymbol{I}_P}<\gamma \\ H_1: \dfrac{\det \hat{\boldsymbol{R}}_x}{\det \sigma_\eta^2 \boldsymbol{I}_P}>\gamma \end{cases} \tag{7}$$

where γ denotes the decision threshold. Denote $\lambda_1, \lambda_2, \cdots, \lambda_P$ as the eigenvalues of $\hat{\boldsymbol{R}}_x$ ordered in decreasing order. Using the equation $\det \hat{\boldsymbol{R}}_x=\prod_{i=1}^{P}\lambda_i$, the new statistic can then be equivalently rewritten as

$$\Lambda=\frac{\lambda_1}{\sigma_\eta^2}\frac{\lambda_2}{\sigma_\eta^2}\cdots\frac{\lambda_P}{\sigma_\eta^2} \tag{8}$$

From Equ (8), the proposed statistic uses all the eigenvalues of the SCM as an indicator to detect whether the primary signal is present or not. Consequently, the new sensing algorithm based on the eigenvalues detection can be summarized as follows

Algorithms 1: Spectrum Sensing Algorithm for Cognitive Radio Based on Eigenvalues Detection(ESD)

Input: $x_m(n)$, σ_η^2, M, N, P, and the target P_f

Output: "yes" if the primary signal is present, otherwise "no"

Step 1　Compute the $\hat{\boldsymbol{R}}_x$ using Equ(5);

Step 2　Calculate the statistic Λ using Equ(6);

Step 3　Determine the decision threshold γ using Equ(20)(to be given in the next section);

Step 4　If $\Lambda>\gamma$, return "yes"; If $\Lambda<\gamma$, return "no".

Remarks: (a) Different from the SED, the proposed ESD uses all eigenvalues of the SCM to construct the test statistic. If all eigenvalues of the SCM are equal, then the ESD reduces to the SED. (b) If the signal subspace is rank-one, i. e., rank ($\boldsymbol{R}_s$) = **1**, then the smallest $P-1$ eigenvalues of $\hat{\boldsymbol{R}}_x$ will be approximately equal to σ_η^2 and the proposed algorithm reduces to the MED. In this sense, the MED can be viewed as a special case of the ESD, (c) The main implementation complexity for the MED, SED, and ESD lies in the computing of the SCM defined in Equ (5) and the eigenvalue decomposition of it. Obviously, the propose ESD has the same computation complexity as the MED and the SED.

2　Analysis of the Probability of False Alarm and the Decision Threshold

Usually, the decision threshold is determined according to P_f. Therefore, the distribution function of the test statistic under H_0 should be firstly derived. When $MN\to+\infty$ and P is very small, an asymptotic distribution can be given by[13]

$$\ln \Lambda | H_0 \overset{a.s.}{\sim} N(0.2P/MN) \tag{9}$$

Noting that $\ln(x)$ is a monotonically increasing function with respect to $x>0$. Therefore, the false alarm probability can be expressed as

$$P_f=\mathrm{Prob}(\ln \Lambda | H_0>\ln \gamma) \tag{10}$$

Given a target probability of false alarm, say P_{FA}, the asymptotic threshold can then be calculated by combining Equ(9) and Equ(10)

$$\gamma_{\mathrm{asy}}\simeq\exp\left(Q^{-1}(P_{\mathrm{FA}})\sqrt{2P/MN}\right) \tag{11}$$

where $\exp(x)$ and $Q^{-1}(x)$ denote the exponential function and the inverse Marcum Q function, respectively. As mentioned above, γ_{asy} is valid for the applications with a very large sample size and a very small sample dimension. However, it becomes not accurate enough in the practical application with a large sample dimension and would cause the loss of the detection performance (see Table 1 and Fig. 1 in Sec. 4). In the following, we will give an improved decision threshold for the proposed ESD.

Denote $\hat{\boldsymbol{R}}_\eta \triangleq \hat{\boldsymbol{R}}_x | H_0$, we have $\hat{\boldsymbol{R}}_\eta \sim W_P\left(MN, \dfrac{\sigma_\eta^2\boldsymbol{I}_P}{MN}\right)$.

Using the property of Wishart distribution gives

$$MN\sigma_\eta^{-2}\hat{\boldsymbol{R}}_\eta \sim W_P(MN, \boldsymbol{I}_P) \tag{12}$$

Applying the theorem of Bartlett decomposition yields[13]

$$MN\sigma_\eta^{-2}\hat{\boldsymbol{R}}_\eta=\boldsymbol{T}^{\mathrm{T}}\boldsymbol{T} \tag{13}$$

where $\boldsymbol{T}$ is a upper-triangular matrix with diagonal elements $t_{ii}^2 \sim \sigma_{MN-i+1}^2$ ($i=1,\cdots,P$), which are independent of each other. Using Equ(13), the test statistic under H_0 can be rewritten as

$$\Lambda|H_0=(MN)^{-P}\det(\boldsymbol{T}^{\mathrm{T}}\boldsymbol{T}) \tag{14}$$

Taking natural logarithm on both sides of Equ(14) yields

$$\ln\Lambda|H_0=\ln\det(\boldsymbol{T}^{\mathrm{T}}\boldsymbol{T})-P\ln(MN) \tag{15}$$

Using the equation $\det(\boldsymbol{T}^{\mathrm{T}}\boldsymbol{T})=\prod_{i=1}^{P}t_{ii}^2$, we have

$$\ln\Lambda|H_0=\sum_{i=1}^{P}\left(\ln t_{ii}^2-\ln(MN)\right) \tag{16}$$

Define $v_i\triangleq\ln t_{ii}^2-\ln(MN-i+1)$, we obtain Equ(17)

Table 1 Actual for different sample sizes and sample dimensions when and are used

P		2	3	4	5	6	7	8	9	10
$MN=1000$	γ_{imp}	0.0917	0.0836	0.0815	0.0835	0.0895	0.0799	0.079	0.0775	0.0788
	γ_{asy}	0.0939	0.0836	0.0788	0.0753	0.0778	0.063	0.0585	0.0511	0.0467
$MN=2000$	γ_{imp}	0.0941	0.0969	0.0927	0.0832	0.0889	0.0878	0.0889	0.0848	0.0841
	γ_{asy}	0.0966	0.0969	0.0900	0.0791	0.0807	0.0742	0.0727	0.065	0.0613
$MN=3000$	γ_{imp}	0.0963	0.0900	0.0921	0.0925	0.0897	0.0856	0.0896	0.0876	0.0911
	γ_{asy}	0.0972	0.0901	0.0891	0.0862	0.083	0.0751	0.0772	0.0721	0.0716

$$\ln\Lambda|H_0=\sum_{i=1}^{P}v_i+\sum_{i=1}^{P}\ln\frac{MN-i+1}{MN} \tag{17}$$

We can prove that the following asymptotic distribution holds as the sample size MN is large(see Appendix)

$$v_i\overset{a.s.}{\sim}N\left(\frac{1}{MN-i+1},\frac{2}{MN-i+1}\right) \tag{18}$$

Using the fact that $v_i(i=1,\cdots,P)$ are all independent of each other, from (17), we can obtain the following distribution

$$\ln\Lambda|H_0\overset{a.s.}{\sim}N(\mu,\sigma^2) \tag{19}$$

where

$$\mu=\sum_{i=1}^{P}\left(\ln\frac{MN-i+1}{MN}+\frac{1}{MN-i+1}\right)$$

$$\sigma^2=\sum_{i=1}^{P}\frac{2}{MN-i+1}$$

Given the target P_{FA}, the improved decision threshold can then be determined by combining Equ(10) and Equ(19)

$$\gamma_{\mathrm{imp}}\simeq\exp(Q^{-1}(P_{\mathrm{FA}})\sqrt{\sigma^2}+\mu) \tag{20}$$

Remarks: (a) When $MN\to+\infty$ and P is very small, we have $\mu\to0$ and $\sigma^2\to2P/MN$, and then $\gamma_{\mathrm{imp}}\to\gamma_{\mathrm{asy}}$, which means that both γ_{imp} and γ_{asy} are accurate enough in the scenario with a large sample size and a very small sample dimension. However, if P becomes large, then the values of μ and σ^2 would deviate from the asymptotic ones, and then the asymptotic decision threshold γ_{asy} would become invalid while the proposed γ_{imp} is still valid. (b) As mentioned before, the determinations of the decision thresholds for both the SED and the MED need to solve the inverse Tracy-Widom distribution. Unfortunately, this distribution is defined by a complex nonlinear Painleve Ⅱ differential equation[7], and the solving heavily relies on either complicated programming techniques or a commercial statistical software package[14-15]. Compared with the MED and the SED, the determination of the threshold for the ESD does not need complex numerical computation and can meet the real-time requirement in spectrum sensing. (c) Obviously, the computation of the threshold in Equ(20) does not need any information of the primary signal and the wireless channel. For a practical application, the calculation of the threshold is needed only once for given values of M,N,P and P_{FA}.

3 Simulations

In this section, the proposed ESD is evaluated numerically and compared with the other two eigenvalue based method including the MED and the SED. For illustration, the received primary signal is assumed to be a Gaussian distribution with a statistical covariance matrix $\rho_s\boldsymbol{l}_P+(1-\rho_s)\boldsymbol{I}_P$, where ρ_s denotes the correlation coefficient between the primary signal samples. For the real signal, the decision thresholds for the MED and the SED can be respectively computed as[7,9]①.

① In reference [9], the threshold of the SED is derived for the complex signal. For the real signal, the threshold can be calculated by simply replacing $F_2^{-1}(\cdot)$ with $F_1^{-1}(\cdot)$ in the complex one (see reference [11] for details).

$$\gamma_{\text{MED}} = \frac{(\sqrt{MN}+\sqrt{P})^2}{MN}\left(1+\frac{(\sqrt{MN}+\sqrt{P})^{-2/3}}{(MNP)^{1/6}}F_1^{-1}(1-P_f)\right)$$

$$\gamma_{\text{SED}} = \frac{(\sqrt{MN}-\sqrt{P})^2}{MN}\left(1-\frac{(\sqrt{MN}-\sqrt{P})^{-2/3}}{(MNP)^{1/6}}F_1^{-1}(1-P_f)\right)$$

where $F_1^{-1}(\cdot)$ denotes the inverse CDF of Tracy-Widom distribution of order 1.

Firstly, the actual probabilities of false alarm of the proposed ESD for different sample sizes and sample dimensions are given in Table 1, where we set $P_{\text{FA}} = 0.1$ and then obtain the thresholds, i. e., γ_{asy} and γ_{imp}, using the formulae derived in Equ (11) and Equ (20). Comparing the target $P_{\text{FA}} = 0.1$ with the simulated results, we see that both γ_{asy} and γ_{imp} become more accurate with the increasing sample dimension MN, while the latter is more robust to the sample dimension P. We also see that the theoretical threshold is a little bit higher than the expected, which causes the actual P_f to be slightly lower than $P_{\text{FA}} = 0.1$. The effects of the thresholds on the detection probability are demonstrated in Fig. 1, where we fix $MN = 1\ 000$ and $\rho_s = 0.5$. It can be seen that better detection performance can be achieved by using the improved threshold γ_{imp}, especially for a large value of P.

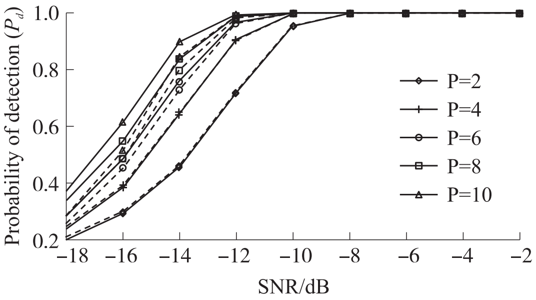

Fig. 1 The effects of the theoretical thresholds on the detection performance (dashed lines: γ_{asy}, solid lines: γ_{imp})

Secondly, the detection performance of the ESD compared with the MED and the SED for different correlation coefficients is presented in Fig. 2, where the improved threshold in Equ(20) is used for the ESD. As can be seen, compared with the MED, the ESD shows better sensing performance under low ($\rho_s = 0.1$) and moderate ($\rho_s = 0.5$) correlation coefficients. When the received signals are highly correlated ($\rho_s = 0.9$), the ESD shows better sensing performance in the low SNR region and slightly worse performance in the high SNR region. Compared with the SED, the proposed ESD can achieve higher detection probability in the high SNR region, especially for the highly correlated signal. On the other hand, from the point of view of the false-alarm probability, the SED yields a far higher P_f (about) than the presetting $P_{\text{FA}} = 0.1$, which indicates the asymptotic threshold is far lower than the true one. The lower threshold results in the unreliability of the detection performance for the SED and also the reduction of the actual spectral utilization for the cognitive user. Obviously, the proposed ESD almost achieves the desired, which implies the threshold given by Equ(20) is very accurate in practical applications.

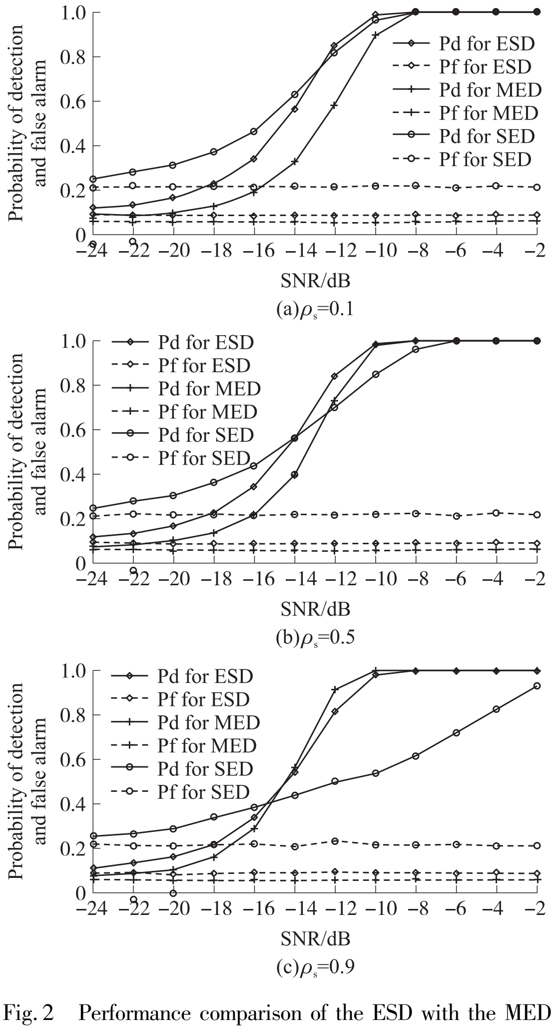

Fig. 2 Performance comparison of the ESD with the MED and the SED for different correlation coefficients ($MN = 1\ 000$, $P = 3$)

Finally, the effects of the sample size and the sample dimension are investigated in Fig. 3 and Fig. 4, respectively. At first, the detection performance of the new algorithm for different sample sizes is presented in Fig. 3, where we fix $\rho_s = 0.5$, $P = 3$ or $P = 5$, while the sample sizes vary from 100 to 1 000. As expected, the sensing performance for the ESD increases significantly with the increasing sample size. At the same time, the sensing performance of the new algorithm with different sample dimensions is investigated in Fig. 4. As expected, we observe that the sensing performance of the new algorithm can be further enhanced via increasing the sample dimension of the received signal vector. For example, the amounts of performance improvement for both $MN=200$ and $MN=1\ 000$ are about 2 dB when the sample dimension increases from the increasing sample size.

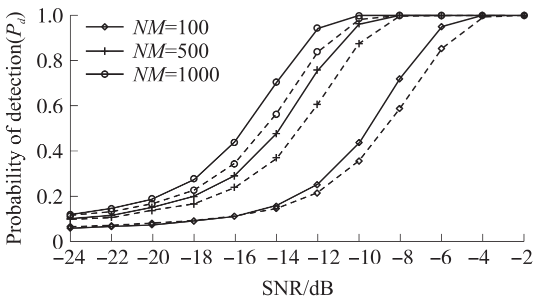

Fig. 3 Performance of the ESD for different sample sizes ($\rho_s=0.5$, dased lines: $P=3$, solid lines: $P=5$)

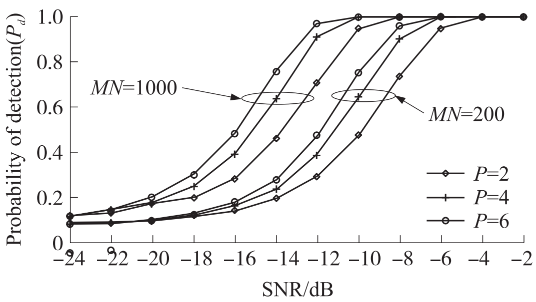

Fig. 4 Performance of the ESD for different sample dimensions($\rho_s=0.5$)

4 Conclusion

A spectrum sensing algorithm based on the eigenvalues detection has been introduced in this paper. Correspondingly, the probability of false alarm and the decision threshold are analyzed by using the multivariate statistical theories. The proposed ESD can be used for the sensing scenarios without the information about the primary signal and the wireless channel. More importantly, the proposed ESD keeps the same computation complexity as the MED and the SED, while it relaxes the calculation requirement of the decision threshold by using a simple closed-form expression. Simulation results verify the effectiveness of the proposed sensing method.

5 Appendix

Asymptotic Distribution of v_i

Define $z_i=\dfrac{MN-i+1}{2}$ and introduce a new variable $v_i'=\sqrt{z_i}\,v_i$. Using the property of the moment of the chi-square random variable[16], we get

$$E\{(t_{ii}^2)^k\}=\frac{2^k\Gamma(z_i+k)}{\Gamma(z_i)} \qquad (21)$$

The characteristic function of v_i' can hence be given as

$$E\{e^{jtv_i'}\}=(z_i)^{-j\sqrt{z_i}t}E\{(t_{ii}^2)^{j\sqrt{z_i}t}\}=(z_i)^{-j\sqrt{z_i}t}\frac{\Gamma(z_i+j\sqrt{z_i}\,t)}{\Gamma(z_i)}$$

where $j=\sqrt{-1}$ is the imaginary unit. Equivalently, we have

$$\ln E\{e^{jtv_i'}\}=-jt\sqrt{z_i}\ln z_i+\ln\Gamma(z_i+j\sqrt{z_i}\,t)-\ln\Gamma(z_i) \qquad (22)$$

Note that $z_i(1\leqslant i\leqslant P)$ is very large due to the fact that usually MN is large while P is small in practical applications. We can then use the asymptotic expansion of the log gamma function[13] to expand the second and third terms on the right hand side of Equ(22) according to z_i to get

$$\ln\Gamma(z_i+a)\simeq\left(z_i+a-\frac{1}{2}\right)\ln z_i-z_i+\frac{\ln 2\pi}{2}+\frac{B_2(a)}{2}z_i^{-1}+O(z_i^{-2})$$

$$\ln\Gamma(z_i)\simeq\left(z_i-\frac{1}{2}\right)\ln z_i-z_i+\frac{\ln 2\pi}{2}+\frac{1}{12}z_i^{-1}+O(z_i^{-2})$$

where $a=j\sqrt{z_i}\,t$ and $B_2(a)=a^2-a+1/6$. Substituting the above two equations into Equ (22), after some manipulations, we can obtain

$$\ln E\{e^{itv_i'}\}\simeq-\frac{j}{2}\sqrt{\frac{1}{z_i}}\,t-\frac{t^2}{2}=-\frac{j}{2}\sqrt{\frac{2}{MN-i+1}}\,t-\frac{t^2}{2} \qquad (23)$$

Noting that the right hand side of Equ(23) is just the characteristic function of a Gaussian random variable, we then have

$$v_i'\overset{a.s.}{\sim}\left(\frac{1}{2}\sqrt{\frac{2}{MN-i+1}},1\right) \qquad (24)$$

Using the property of the Gaussian random variable, we can easily obtain Equ(18).

References:

[1] Haykin S. Cognitive Radio: Brain-Empowered Wireless Communications[J]. IEEE Journal on Selected Areas in Communications, 2005, 23(2): 201-220.

[2] Zhang J W, Zhao Q, Zou J Y. Algorithm of Spectrum Allocation for Wireless Sensor Network Based on Dynamic Spectrum Sensing [J]. Chinese Journal of Sensors and Actuators, 2009, 22(10): 1481-1485.

[3] WANG B, Liu K J R. Advances in Cognitive Radio Networks: A Survey[J]. IEEE Journal of Selected Topics in Signal Processing, 2011, 5(1): 5-23.

[4] Wei M, Lian Q S. The Wideband Spectrum Covariance Sensing Algorithm Based on Compressed Sensing[J]. Chinese Journal of Sensors and Actuators, 2011, 24(7): 1022-1026.

[5] Unnikrishnan J, Veeravalli V V. Cooperative Spectrum Sensing and Detection for Cognitive Radio [C]//Proc. IEEE Global Communications Conference (GLOBECOM). Washington, DC, USA, 2007: 2972-2976.

[6] Zeng Y H, Liang Y C. Spectrum-Sensing Algorithms for Cognitive Radio Based on Statistical Covariances[J]. IEEE Transactions on Vehicular Technology, 2009, 58(4): 1804-1815.

[7] Zeng Y H, Koh C L, Liang Y C. Maximum Eigenvalue Detection: Theory and Application[C]//Proc. IEEE International Conference on Communications(ICC). Beijing, China, 2008: 4160-4164.

[8] Zeng Y H, Liang Y C, Zhang R. Blindly Combined Energy Detection for Spectrum Sensing in Cognitive Radio[J]. IEEE Signal Processing Letters, 2008, 15: 649-652.

[9] Cao K T, Yang ZH. Novel Cooperative Spectrum Sensing Algorithm Based on the Smallest Eigenvalue[J]. Chinese Journal of Scientific Instrument, 2011, 32(4): 736-741.

[10] Penna F, Garello R, Spirito M A. Cooperative Spectrum Sensing Based on the Limiting Eigenvalue Ratio Distribution in Wishart Matrices [J]. IEEE Communications Letters, 2009, 13(7): 507-509.

[11] FEldheim O N, Sodin S. A Universality Result for the Smallest Eigenvalues of Certain Sample Covariance Matrices [EB/OL]. Online, http://arxiv. org/PS _ cache/arxiv/pdf/0812/0812. 1961v4. pdf.

[12] Yang X, Lei K J, Peng S L, et al. Blind Detection for Primary User Based on the Sample Covariance Matrix in Cognitive Radio[J]. IEEE Communications Letters, 2011, 15(1): 40-42.

[13] Muirhead R J. Aspects of Multivariate Statistical Theory [M]. Hoboken, New Jersey, USA: John Wiley & Sons, Inc, 1982.

[14] Dieng M. RMLab Version 0. 02[EB/OL]. http://math. arizona. edu/ ~ momar/, 2006.

[15] Andrei Bejan. Largest Eigenvalues and Sample Covariance Matrices [EB/OL]. http://www. cl. cam. ac. uk/ ~ aib29/MScdssrtnWrwck. pdf.

[16] Krishnamoorthy K. Handbook of Statistical Distributions with Applications [M]. Boca Raton, FL, USA: Chapman & Hall/ CRC, 2006.

LEI Kejun (1979 -), received bachelor degree and master degree from Jishou University and Hunan University in 2002 and 2007, respectively. Now she is a lecturer in the school of the information science and engineering, Jishou University. Her research interests include digital signal processing and cognitive radio, leikejun - 123@163. com;

YANG Xi (1968 -), is an associate professor in the school of the information science and engineering, Jishou University. His research interests include wireless communications, statistical signal processing and cognitive radio, yangxi1978 @ gmail. com.