A Dynamic Model Compensation Orbit Determination Method for Maneuvering Satellite

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Abstract: The uncertainty of thrust acceleration will cause enormous error to model and track the accelerating spacecraft, it's reasonable to assume that the measurements mainly reflect the uncertainty of the thrust acceleration. This paper focuses on modeling and building on-line filter to identify the continuous thrust acceleration to compensate the uncertainty of the force model during the maneuver process, an elegant differential equation is derived for variable acceleration, an Extended Kalman Filter (EKF) is developed for dynamic system to identify forwardly the variable acceleration in real time, also the variational equation for the measurement vector to acceleration is detailed to linearise the discrete-time measurement equations. The mission flight scenery shows the algorithm developed here can estimate the acceleration and determine the orbital parameters precisely during the continuous thrust maneuver process.

Key words: Accelerating spacecraft; Parameter estimation; Orbit determination; Orbital maneuver
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动力学补偿受控卫星轨道确定算法

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摘 要：受控卫星动力学模型中推力加速度的量级远远高于其他摄动的误差量级，观测量主要反映受控卫星动力学模型的误差。本文以跟踪和精确定位空间机动目标为目的，给出基于地面雷达观测，实时估计推力加速度，修正卫星动力学模型的轨道确定算法。通过建立连续推力控制过程变质量动力学模型，给出常推力变加速度满足的运动学微分方程；建立变加速度估计系统状态方程，和扩展卡尔曼滤波轨道确定算法；并给出连续推力控制卫星运动状态关于推力加速度的变分运动方程；实际飞行控制应用表明：利用地面测量数据，实时估计推力加速度并补偿系统动力学模型，解决了连续受控卫星轨道精确确定问题。

关键词：受控卫星；参数估计；轨道确定；轨道机动

0 Introduction

A special concern of this paper will focus on tracking and navigating the satellites with continuous thrust acceleration, which has become common in current space research activities, due to its efficiency and flexibility in realizing the maneuver of space vehicles. According to the magnitude of maneuver increments, whose scope is from several kilometers to multimillion kilometers in semi-major axis, the maneuver duration will last from several minutes to several years. The issues about tracking and navigating accelerating targets,

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are the hotspots in space navigation, control and dynamics in recent years[4-16]. It's an indicative different problem to position the satellites with or without thrust acceleration, for latter case, the order of magnitude of dynamics model can reach a precision as 10^{-12}[1-3] (suppose the center force is 1), it is reasonable that the measurement data only reflect the aberrancy of prio estimation of orbital parameters. Both batch and sequential estimators are powerful methods that have successfully been applied to orbit determination problems without thrust acceleration[2-3]. In consideration of maneuvering process, the uncertainty of thrust acceleration will cause enormous error to model and track the accelerated motion during orbit transfer process, it's reasonable to assume that the measurements mainly reflect the uncertainty of the thrust acceleration. There are many reasons to arouse interests in this issue. Firstly, being the requirement of space defense, tracking and positioning the maneuvering accelerating space objective requires the smart filter arithmetic to process the measure data in real time. Hough M. E, Heppner S. and Fowler J. etc. focus on this background, a series of effective algorithms have been developed to track maneuvering vehicles during reentry and boost[4-15]. Secondly, the varying orbital parameters and thrust acceleration are required momentarily for the purpose of picking up the accelerating satellites and revising the control process in case of emergency. This paper focuses on this background, specially focus on building on-line filter to estimate the continuous thrust acceleration to compensate the uncertainty of the dynamics model during the maneuver process. In first section spacecraft variable mass motion equations during thrust acceleration transfer process are provided, which respect to the position and velocity in the ECI coordinate system. An elegant differential equation is derived for variable acceleration, which is chosen as a state variable, and measurement equation is built to reflect the relationship between the state space and measurement sampling space in section 2. An extended Kalman filter (EKF) is developed for dynamic system in section 3 to estimate forwardly the variable acceleration in real time. Also the variational equation for the measurement vector of acceleration is put forward in order to linearising the discrete-time measurement equations. The real flight mission scenery is introduced in section four to ascertain the algorithm developed here can estimate the acceleration precisely and determine the orbital parameters during the continuous thrust maneuver process. The algorithm has been applied successfully to maneuver process in commanding LEO satellite into Geo-stationary orbit.

1 The variable mass motion equations with continuous thrust acceleration

Consider spacecraft motion states of position and velocity vectors under the influence of gravitational acceleration (consider The Earth's oblate ness effects J_2) and thrust acceleration. The three dimensional of freedom equations describing the motion of the spacecraft are given in the Earth Centered Inertial (ECI) coordinate frame as

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \dot{\boldsymbol{r}} \\ \dfrac{\mathrm{d}\dot{\boldsymbol{r}}}{\mathrm{d}t} = g_r \boldsymbol{r} + g_w \boldsymbol{w}_e + a\boldsymbol{p} \end{cases} \tag{1}$$

For the duration which the acceleration is estimated, equation (1) is sufficient accurate to model the gravitational acceleration using the central body and oblateness effects J_2, ignore the high order term of the Earth's nonspherical force and other perturbations, consequently

$$\begin{cases} g_r = -\dfrac{\mu}{r^3}\left[1 + \dfrac{3}{2}J_2\left(\dfrac{R_e}{r}\right)^2\left(1 - 5\left(\dfrac{z}{r}\right)^2\right)\right] \\ g_w = -2\dfrac{\mu}{r^2}\left(\dfrac{3}{2}\right)J_2\left(\dfrac{R_e}{r}\right)^2\left(\dfrac{z}{r}\right) \end{cases} \tag{2}$$

Where J_2 (1.08263×10^{-3}) is the second order zonal term, μ ($3.986005 \times 10^{14}\mathrm{m}^3/\mathrm{s}^2$) is the Earth's gravitational constant, R_e (6378140.0m) is the radius of the Earth's equatorial plane, $\boldsymbol{r}, \dot{\boldsymbol{r}}$ are the position and velocity vector in the ECI coordinate frame, $\boldsymbol{w}_e = (0, 0, 1)^{\mathrm{T}}$ is the polar direction, p is the thrust direction, that relies on the coordinate frame upon which the attitude is stabilized and surveyed during the maneuver process, a is the variable acceleration, which is related to the magnitude of thrust force and remained mass of the satellite.

Actually, we have no knowledge of these varibles or have only designed values. The uncertainty of thrust acceleration will cause enormous error to model and track the accelerating motion of maneuvering target during orbit transfer process. It's reasonable to assume that the measurements reflect the uncertainty of the thrust acceleration. So we choose the acceleration as an unknown parameter to be estimated by conducting discrete radar measurements, and once the right side of the motion equation (1) to be compensated, the three dimensional freedom equation (1) could be predicted during the maneuver process so as to track and position the accelerating satellites.

2 The variable acceleration differential equation

Suppose that we know the initial conditions of the differential equation (1) as $\boldsymbol{r}|_{t=t_0} = \boldsymbol{r}_0$, $\dot{\boldsymbol{r}}|_{t=t_0} = \dot{\boldsymbol{r}}_0$ the position and velocity at anytime during transfer process can be expressed to be

$$\begin{cases} \boldsymbol{r}(t) = \boldsymbol{r}(t, a, \boldsymbol{r}_0, \dot{\boldsymbol{r}}_0) \\ \dot{\boldsymbol{r}}(t) = \dot{\boldsymbol{r}}(t, a, \boldsymbol{r}_0, \dot{\boldsymbol{r}}_0) \end{cases} \tag{3}$$

In this section, an elegant differential equation will be derived for variable acceleration, which is chosen as a state variable, and measurement equation is built to reflect the relationships between the state space and measurement sampling space. Assume that the mass of satellite is m_0 (kg), when the propellant burns on, the mass flow rate is $\dot{m}$ (kg/s), the specific impulse is I_{sp} (m/s), then the variable acceleration induced by the propellant is

$$a(t) = \frac{F}{m(t)} = \frac{I_{sp}\dot{m}}{m_0 - \dot{m}(t - t_0)} \tag{4}$$

Formula (4) indicates that the acceleration varies not only with the duration of the propellant burn, but also relates to the initial mass of spacecraft, the mass flow rate and specific impulse of propellant. The uncertainties of each of factor above will cause the excursion of acceleration. Apply differential operator to two sides of equation (4), an elegant differential equation is derived for variable acceleration

$$\frac{da}{dt} = \frac{d}{dt}\left(\frac{I_{sp}\dot{m}}{m_0 - \dot{m}(t - t_0)}\right) = \frac{(I_{sp}\dot{m})^2}{(m_0 - \dot{m}(t - t_0))^2}\frac{1}{I_{sp}} = \frac{a^2}{I_{sp}} \tag{5}$$

The differential equation (5) indicates that the rate of thrust acceleration only relates to the specific impulse of the propellant, and does not correlate with the magnitude of mass and mass flow rate totally. The greater the specific impulse, the lesser the rate of thrust acceleration, (see fig. 1). Because of its almost linear property, some researches take the continuous thrust acceleration as a linear or quadratic curve, whose coefficients are estimated by conducting discrete radar measurements[16]. The drawback is that even the system is observable, the relatively more state space dimensions will debase the observability of dynamics system.

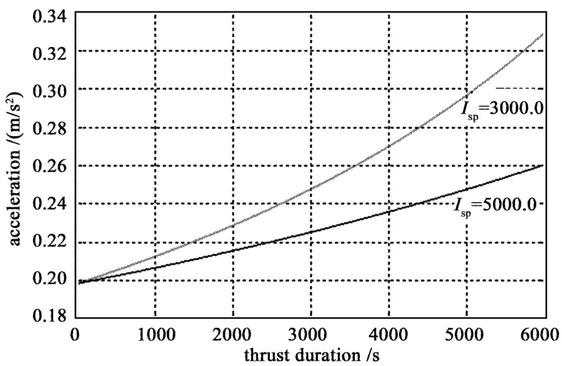

Fig. 1 the variable acceleration with different specific impulse

Therefore, the dynamics system takes the continuous thrust acceleration as the state variable, and takes the discrete radar measurements as the system observation equations. this comprises a continuous state-discrete observations system, such that:

$$\begin{cases} \dfrac{da}{dt} = \dfrac{1}{I_{sp}}a^2 + u(t) \\ z_k = z(t_k, a_k) + \boldsymbol{v}_k \end{cases} \tag{6}$$

Where $u(t)$, $\boldsymbol{v}_k$ are dynamics model and measurement noise respectively, which are zero mean Gauss noise, and satisfy

$$\mathrm{E}\{u(t)\} = 0, \mathrm{E}\{u(t_i)u^{\mathrm{T}}(t_j)\} = \boldsymbol{Q}\delta_{ij}$$

$$\mathrm{E}\{\boldsymbol{v}_k\} = 0, \mathrm{E}\{\boldsymbol{v}_i\boldsymbol{v}_j^{\mathrm{T}}\} = \boldsymbol{R}\delta_{ij}$$

Here $\boldsymbol{Q} \geqslant 0$, $\boldsymbol{R} \geqslant 0$ are covariance of dynamics model and measurement noise respectively, δ_{ij} is the Kronecker function.

3 The forward estimator for variable acceleration

3.1 The extended Kalman filter for acceleration estimation

For continuous state-discrete observations system, the state estimated and its error covariance are defined respectively as:

$$\hat{a} = \mathrm{E}\{a\}, P = \mathrm{E}\{(a - \hat{a})(a - \hat{a})^{\mathrm{T}}\}$$

The notation $(\ )_k^-$ denotes quantities of propagation before the k^{th} measurement, and $(\ )_k^+$ denotes the quantities of filter after the k^{th} measurement respectively. Furthermore, It is assumed that the initial state estimated and the initial error covariance are known respectively. The EKF is described in follow three steps:

(1) The state and error covariance propagation

The EKF propagates the state estimate $\hat{a}_{k-1}^+$ and its error covariance $\boldsymbol{P}_{k-1}^+$ from t_{k-1} to t_k by integrating the differential equation (5) and covariance Riccati equation respectively

$$\hat{a}_k^- = \frac{I_{\mathrm{sp}}\hat{a}_{k-1}^+}{I_{\mathrm{sp}} - \hat{a}_{k-1}^+(t_k - t_{k-1})} \tag{7}$$

$$\boldsymbol{P}_k^- = \boldsymbol{\Phi}(t_k, t_{k-1})\boldsymbol{P}_{k-1}^+\boldsymbol{\Phi}^{\mathrm{T}}(t_k, t_{k-1}) + \boldsymbol{Q} \tag{8}$$

Where $\boldsymbol{\Phi}(t_k, t_{k-1})$ is the system transfer matrix, which approximates with

$$\boldsymbol{\Phi}(t_k, t_{k-1}) \approx \boldsymbol{I} + 2\left(\frac{\hat{a}_{k-1}^+}{I_{\mathrm{sp}}}\right)(t_k - t_{k-1}) \tag{9}$$

(2) The optimal Kalman gain

$$\boldsymbol{K} = \boldsymbol{P}_k^-[\boldsymbol{H}_k^-]^{\mathrm{T}}\{\boldsymbol{H}_k^-\boldsymbol{P}_k^-[\boldsymbol{H}_k^-]^{\mathrm{T}} + \boldsymbol{R}\}^{-1} \tag{10}$$

Where $\boldsymbol{H}_k^-$ is the measurement matrix, which is the Jacobian matrix about measurement with the state variables.

$$\boldsymbol{H}_k^- = \left(\frac{\partial z}{\partial a}\right)_{a=\hat{a}_k^-} \tag{11}$$

(3) The state and covariance updating

$$\hat{a}_k^+ = \hat{a}_k^- + \boldsymbol{K}[z_k - z(t_k, \hat{a}_k^-)]$$

$$\boldsymbol{P}_k^+ = [\boldsymbol{I} - \boldsymbol{K}\boldsymbol{H}_k^-]\boldsymbol{P}_k^- \tag{12}$$

In the computation of partial derivations that describe the dependence of measurement on the acceleration, because each radar measurement, either the angle or distance, may be expressed as the functions of instantaneous position and velocity vector of satellite, and the instantaneous state of satellite, or as the functions of unknown acceleration, see formula (3). So the Jacobian matrix about measurement with the state variables can be expressed with step partial derivations

$$\boldsymbol{H}_k^- = \frac{\partial z}{\partial a} = \left(\frac{\partial z}{\partial \boldsymbol{r}}\right)\left(\frac{\partial \boldsymbol{r}}{\partial a}\right) + \left(\frac{\partial z}{\partial \dot{\boldsymbol{r}}}\right)\left(\frac{\partial \dot{\boldsymbol{r}}}{\partial a}\right) \tag{13}$$

3.2 The variation equations

In view of the full partial derivation (13), the terms $(\partial z/\partial \boldsymbol{r})$, $(\partial z/\partial \dot{\boldsymbol{r}})$ are the Jacobian matrixes about measurement with the instantaneous position and velocity of satellite respectively, one may obtain an analytical solution in general papers. The terms $(\partial \boldsymbol{r}/\partial a)$, $(\partial \dot{\boldsymbol{r}}/\partial a)$ are the Jacobian matrixes about the instantaneous state of satellite with the unknown acceleration. With the treatment of the satellite motion equation (1), one may not obtain any analytical expressions anymore, however, one may solve a special set of differential equations, which describes the dependence of instantaneous position and velocity transfer on the thrust acceleration. The variational equations offer the mathematical background to solve this problem.

In order to develop the variational equations for the Jacobian matrix about the instantaneous state of satellite with the unknown acceleration, omit the second harmonic effects J_2, and employ partial derivatives for the acceleration to two side of equation (1), yield

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \boldsymbol{r}}{\partial a}\right) = \frac{\partial \dot{\boldsymbol{r}}}{\partial a} \tag{14}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{\boldsymbol{r}}}{\partial a}\right) &= \frac{\partial}{\partial a}\left(\frac{\mathrm{d}\dot{\boldsymbol{r}}}{\mathrm{d}t}\right) \\ &= 3\frac{\mu}{r^4}\boldsymbol{r}\left(\frac{\partial r}{\partial a}\right) - \frac{\mu}{r^3}\left(\frac{\partial \boldsymbol{r}}{\partial a}\right) + \\ &\quad \boldsymbol{p} + a\left(\frac{\partial \boldsymbol{p}}{\partial a}\right)\end{aligned} \tag{15}$$

The thrust direction and its Jacobian matrix about acceleration depend on dedicated coordinate frame upon which the thrust defined[15-16]. One may unite the motion equation (1), which describes the instantaneous state with time, with the variational equations (14 - 15), which describe the changes of instantaneous state with the estimated acceleration

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{r} = \dot{\boldsymbol{r}} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\dot{\boldsymbol{r}} = g_r\boldsymbol{r} + g_w\boldsymbol{w}_e + a\boldsymbol{p} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\partial \boldsymbol{r}}{\partial a}\right) = \dfrac{\partial \dot{\boldsymbol{r}}}{\partial a} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\partial \dot{\boldsymbol{r}}}{\partial a}\right) = \left(\dfrac{3\mu}{r^5}\boldsymbol{r}\boldsymbol{r}^{\mathrm{T}} - \dfrac{\mu}{r^3}\right)\dfrac{\partial \boldsymbol{r}}{\partial a} + \boldsymbol{p} + a\dfrac{\partial \boldsymbol{p}}{\partial a} \end{cases} \tag{16}$$

With the combined equations (16), if the instantaneous states of satellite at sampling epoch t_{k-1} are known as $\boldsymbol{r}_{k-1}^{+}, \dot{\boldsymbol{r}}_{k-1}^{+}$, the variation matrixes are known as $(\partial \boldsymbol{r}/\partial a)\mid_{k-1}$, $(\partial \dot{\boldsymbol{r}}/\partial a)\mid_{k-1}$, and the optimal estimated acceleration $\hat{a}_k^{+}$. Integrate the equation (16) from t_{k-1} to t_k as a general initial differential problem. One may obtain the instantaneous state $\boldsymbol{r}_k^{-}$, $\dot{\boldsymbol{r}}_k^{-}$ at sampling epoch t_k, and the variation matrixes $(\partial \boldsymbol{r}/\partial a)\mid_{k-1}$, $(\partial \dot{\boldsymbol{r}}/\partial a)\mid_k$ at the same time, then the trustful measurement data could be predicted, and the Jacobian matrixes about measurement with the estimated acceleration could be obtained. With the Kalman gain equation (10), the improved acceleration at the measurement epoch t_k together with the associated covariance could be updated by equation (12). The observability of system (6) is totally determined by measurement matrix $\partial z/\partial a$, one may consider that any changes of acceleration would cause the different bias of instantaneous states absolutely, then the variation matrixes $(\partial \boldsymbol{r}/\partial a)$, $(\partial \dot{\boldsymbol{r}}/\partial a)$ are no chance to be zero from equation (16). So if the motion state of satellite is observable, whose observability has been discussed by many papers[10-14], the unknown acceleration as a state variant of dynamics system (6) can be estimated accurately.

4 The real flight mission scenery

To ascertain the forward estimator, the real flight mission scenery is summarized here. The satellite X was transferring to GEO orbit by firing the apogee motor. The first burn was scheduled on epoch 16: 14: 08, April 15th, 2009, the burn duration was 1835 seconds, and ended on epoch 16: 44: 44, the thrust direction fixed at local coordinate frame in the satellite centre. The measurement were carried out by a traditional ground station. The outputs were slant range and the pointing angles, the sampling frequency was one second, so the measurement equation was

$$z = (\rho_k, A_k, E_k, \dot{\rho}_k), k = 1, \cdots, N \tag{17}$$

And measurement data were corrupted with white Gaussian noise, its error covariance is

$$\boldsymbol{R} = \mathrm{diag}\left(10^2, \left(0.02\frac{\pi}{180}\right)^2, \left(0.02\frac{\pi}{180}\right)^2, 1^2\right)$$

With the initial conditions:

$$\hat{a}_0^{+} = 0.0,\ P_0^{+} = 2^2$$

and the instantaneous states of satellite and variation matrixes at initial epoch

$$\boldsymbol{r}_0^{+} = \boldsymbol{r}(t_0), \dot{\boldsymbol{r}}_0^{+} = \dot{\boldsymbol{r}}(t_0),$$

$$\left(\frac{\partial \boldsymbol{r}}{\partial a}\right)_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \left(\frac{\partial \dot{\boldsymbol{r}}}{\partial a}\right)_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By conducting the measurement data in real-time, the acceleration was estimated sequentially and the dynamics model was compensated online. Fig. 2 shows the estimated acceleration in the burn duration. Fig. 3 illustrates the detail information by zooming in the burn duration, the semi major axis and inclination estimated online are presented in fig. 4 and 5.

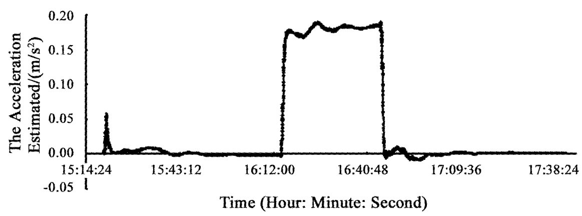

Fig. 2　The estimated acceleration versus time

Table 1 summaries the performances of the arithmetic presented here. If comparing with the post-maneuver orbit, the error in semi major axis is about 2 kilometers, relative to the semi major axis ' s incre-

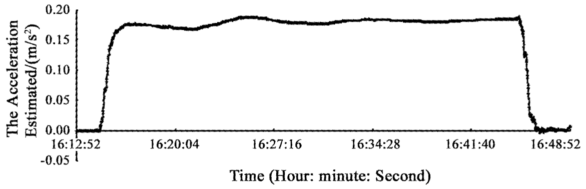

Fig. 3 The acceleration estimated during burn (Zoom in)

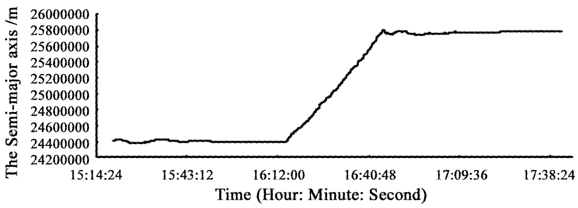

Fig. 4 The estimated Semi-major axis versus time

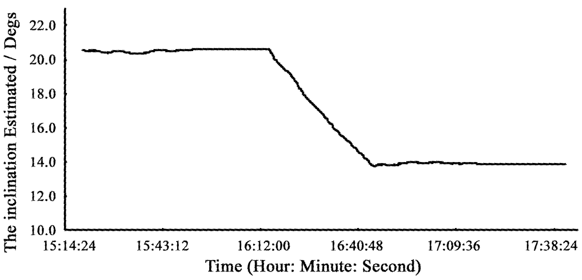

Fig. 5 The estimated inclination versus time

ments, the relative error is 0.5%, The eccentricity is about 10^{-2}, the relative error is 5%, and the inclination error is 10^{-1}, relative error is about 1%. The results totally satisfy the requirements of the flight control mission.

Table 1 The accuracy of the arithmetic

Items	Semi-major axis/km	Eccentricity	Inclination /Degs
Expected	25724.517	0.641551	14.010
Post orbit	25771.885	0.63846	13.884
Estimated online	25769.160	0.63863	13.851

The residuals are presented in Fig.6, which ascertain that the algorithm is of great convergent tendency.

5 Conclusions

This paper focuses on building on-line filter to estimate the continuous thrust acceleration to compensate the uncertainty of the dynamics model during the maneuver process. An elegant differential equation has been derived for variable acceleration, which was chosen as a state variable. The measurement equations have been built to reflect the relationships between the state space and measurement sampling space. An extended Kalman filter (EKF) has been developed for dynamic system to estimate forwardly the variable acceleration in real time, also the variational equation for the measurement vector to acceleration has been put forward in realizing the linearization of the discrete-time measurement equations. The real flight mission scenery has been introduced to ascertain that the algorithm developed here can estimate the acceleration precisely during the continuous thrust maneuver process.

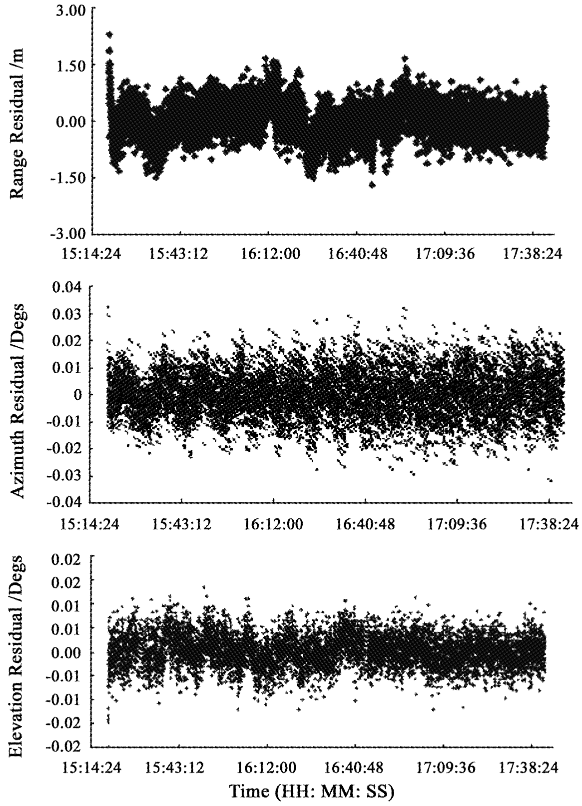

Fig. 6 The residuals (O-C) versus time

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