

Error Analysis for SINS Alignment Using Gravity Integration in Inertial Reference Frame*

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Abstract: Sensitivity analysis for Strap-down Inertial Navigation System (SINS) alignment using gravity integration in inertial frame is carried out with respect to IMU sensor errors and linear vibration. And the explicit analytical error formulations are derived. By inspecting the error formulations, it can be found that the proposed alignment algorithm is more sensitive to linear vibration than sensor errors and gyro biases can be estimated by the fitting slope of the calculated initial Euler angles. The analysis is well validated by simulation and thus it's helpful in design alignment process for Strap-down inertial navigation system.

Key words: error analysis; alignment; inertial reference frame; sensor errors; linear vibration

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基于重力矢量积分的 SINS 对准算法误差分析*

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摘 要: 针对基于重力矢量积分的捷联惯导初始对准算法, 分析了惯性器件误差和线运动干扰对对准精度的影响。推导了惯性器件误差与对准失准角之间的解析表达关系, 指出可从初始欧拉角估计值中提取陀螺漂移信息, 实现陀螺的在线标定。理论分析表明, 该算法将传统解析对准算法对角运动的敏感转化为对线运动的敏感, 线运动干扰成为影响算法对准精度的主要因素。仿真试验验证了上述分析的合理性。

关键词: 误差分析; 对准算法; 惯性参考系; 惯性器件误差; 线运动

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In a strapdown inertial navigation system, the purpose of alignment is to determine the transformation matrix which relates the instrumented body frame to the computational navigation frame^[1-2]. The poor initial alignment accuracy will end up with poor navigation performance. Since inertial navigation systems are entirely self-contained, the Direction Cosine Matrix (DCM) can be directly computed using the measurements knowledge of accelerometers and gyros, that is, the gravity and earth rotation vectors^[3], in the two frames. That is analytic alignment, or conventional analytic alignment (CAA), or direct analytic alignment (DAA)^[1]. Theoretically, an analytic alignment method for strapdown systems is functionally equal to the physical gyrocompassing in gimbaled systems. The error

characteristics are provided in detail in papers^[1,4-6]. The authors had investigated the impact of gentle swaying disturbance on CAA of SINS, and then the applicable conditions of CAA on swaying disturbance base are discussed in paper^[7].

The CAA method is suitable for alignment in a quasi-stationary base environment, which accuracy estimation is bounded by the angular vibration profile. To overcome the shortcoming above, considering that the observation of the gravity slow drift in the inertial space (the projection of the gravity in the inertial frame defines a cone) allows one to determine the Earth axis, thus the North direction, based on that, Lenonid^[8], Gaiffe^[9], Napolitano^[10] and Prof. Qin^[11] presented other approaches. Lenonid utilized gyro outputs for

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continuously tracking attitude system and accelerometers for attitude estimation which called gyrocompassing. Meanwhile, the error analysis of gyrocompassing had been done then the paper pointed out that azimuth estimation is based on the estimation of a very slow trend (proportional to the earth angular rate) which makes it very sensitive to linear vibrations. Noticing that the gravity vector is proportional to the up axis and the derivative of gravity vector is proportional to the east axis, Gaiffe and Napolitano presented a novel clew to deal with the alignment by virtue of an inertial frame and a low-pass filter. Lian^[12] researched on the method in detail and discussed the error characteristics. Motivated by Gaiffe and Napolitano and based on the conventional analytic alignment framework, Prof. Qin^[11] used the integrations of the gravity of two different period of time in the inertial frame as the reference vectors and this method is called indirect analytic alignment (IAA) here (compared with DAA). And Gu^[13] validated the performance of IAA by two-axis turntable test.

Compared with CAA, the reference vectors of IAA^[14-16] are more complex and the error characteristics of which are only researched by simulations by far. It is difficult to deduce the error analysis formulas, but it is important for us to comprehend and make use of the IAA. So we investigate the error analysis formulas of IAA aroused by inertial sensor errors and linear vibrations.

The coordinate frames used in this paper are defined in Section 2. In Section 3, the algorithmic principle for the IAA is presented. The error analysis formulas are described in Section 4. The simulation test results are illustrated in Section 5. Finally, the conclusion is presented.

1 Coordinate Frame Definitions

The coordinate frames used in this paper are defined as follows:

b Frame = “body” coordinate frame parallel to strap-down inertial sensor axes.

n Frame = “navigation” coordinate frame having z axis parallel to the upward vertical at the local position location. A “geographic” n Frame would have the x, y axes rotated around z to maintain the y axis parallel to local true north.

e Frame = “earth” referenced coordinate frame with

fixed angular geometry relative to the rotating earth.

i Frame = “inertial” non-rotating coordinate frame.

n_0 Frame = “inertial” non-rotating coordinate frame. It is formed by fixing the n frame at the beginning of the alignment in the inertial space.

b_0 Frame = “inertial” non-rotating coordinate frame. It is formed by fixing the b frame at the beginning of the alignment in the inertial space.

2 Algorithmic Principle for IAA

The Direction Cosine Matrix (DCM), which relates the body frame to the computational navigation frame, could be described as follows:

$$\mathbf{C}_b^n(t) = \mathbf{C}_{n_0}^n(t) \mathbf{C}_{b_0}^{n_0}(t) \quad (1)$$

Where $\mathbf{C}_b^{b_0}(t)$ is the rotation matrix of the body frame b relative to the reference frame b_0 and can be calculated using the gyro outputs as follows^[11]:

$$\dot{\mathbf{C}}_b^{b_0}(t) = \mathbf{C}_b^{b_0}(t) (\boldsymbol{\omega}_{ib}^b \times) \quad (2)$$

$\mathbf{C}_{n_0}^n(t)$ is as follows:

$$\mathbf{C}_{n_0}^n(t) = \begin{bmatrix} c\omega t & sLs\omega t & -cLs\omega t \\ -sLs\omega t & s^2Lc\omega t + c^2L & -sLcL(c\omega t - 1) \\ cLs\omega t & -sLcL(c\omega t - 1) & c^2Lc\omega t + s^2L \end{bmatrix} \quad (3)$$

Where $c\omega t = \cos(\omega_{ie}t)$, $s\omega t = \sin(\omega_{ie}t)$, $cL = \cos(L)$, $sL = \sin(L)$.

$\mathbf{C}_{b_0}^{n_0}$ could be calculated by the following equation:

$$\mathbf{C}_{b_0}^{n_0} = \begin{bmatrix} (\mathbf{v}_{t_1}^{n_0})^T \\ (\mathbf{v}_{t_1}^{n_0} \times \mathbf{v}_{t_2}^{n_0})^T \\ (\mathbf{v}_{t_1}^{n_0} \times \mathbf{v}_{t_2}^{n_0} \times \mathbf{v}_{t_1}^{n_0})^T \end{bmatrix}^{-1} \begin{bmatrix} (\mathbf{v}_{t_1}^{b_0})^T \\ (\mathbf{v}_{t_1}^{b_0} \times \mathbf{v}_{t_2}^{b_0})^T \\ (\mathbf{v}_{t_1}^{b_0} \times \mathbf{v}_{t_2}^{b_0} \times \mathbf{v}_{t_1}^{b_0})^T \end{bmatrix} \quad (4)$$

Where t_2 is the end time of alignment, and $0 < t_1 < t_2$; $\mathbf{v}_t^{n_0}$ and $\mathbf{v}_t^{b_0}$ are as follows:

$$\mathbf{v}_t^{n_0} = \int_0^t \mathbf{C}_{n_0}^{n_0}(t) \mathbf{g}^n dt \quad (5)$$

$$\mathbf{v}_t^{b_0} = - \int_0^t \mathbf{C}_b^{b_0}(t) \mathbf{f}^b dt \quad (6)$$

3 Error Analysis

In this section, the error characteristics of IAA are investigated. According to Eq. 1, the alignment error $\mathbf{C}_b^n(t)$ can be described as

$$\delta \mathbf{C}_b^n(t) = \delta \mathbf{C}_{n_0}^n(t) \mathbf{C}_{b_0}^{n_0}(t) + \mathbf{C}_{n_0}^n(t) \delta \mathbf{C}_{b_0}^{n_0}(t) + \mathbf{C}_{n_0}^n(t) \mathbf{C}_{b_0}^{n_0} \delta \mathbf{C}_b^{b_0}(t) \quad (7)$$

Eq. 7 contains three components on the right side. The first component is the attitude error of relation between

the current navigation frame and the n_0 frame caused by the time precision, while the time can be obtained precisely, the time error can be ignored rationally. The third term, due to the drift biases of gyros and the attitude algorithm, is downplayed because of the short duration of alignment using navigational grade gyros. The main error of IAA comes from the second term, which is the initial Euler misalignment between the b frame and n frame at the initial time. This component is derived from two aspects: inertial instruments errors and base vibration. So we pay more attention to the second error.

The equations for $C_{b_0}^{n_0}$ can be written in the form:

$$\hat{C}_{b_0}^{n_0} = M\hat{Q} \quad (8)$$

Where,

$$M = \begin{bmatrix} (\mathbf{v}_{t_1}^{n_0})^T \\ (\mathbf{v}_{t_1}^{n_0} \times \mathbf{v}_{t_2}^{n_0})^T \\ (\mathbf{v}_{t_1}^{n_0} \times \mathbf{v}_{t_2}^{n_0} \times \mathbf{v}_{t_1}^{n_0})^T \end{bmatrix}^{-1}, \hat{Q} = \begin{bmatrix} (\hat{\mathbf{v}}_{t_1}^{b_0})^T \\ (\hat{\mathbf{v}}_{t_1}^{b_0} \times \hat{\mathbf{v}}_{t_2}^{b_0})^T \\ (\hat{\mathbf{v}}_{t_1}^{b_0} \times \hat{\mathbf{v}}_{t_2}^{b_0} \times \hat{\mathbf{v}}_{t_1}^{b_0})^T \end{bmatrix}.$$

The elements of M are constant at any latitude when the t_1 and t_2 are fixed on, but \hat{Q} contains measurement uncertainties. The equation above can therefore be written as

$$\hat{C}_{b_0}^{n_0} = M(Q + \delta Q) = (I_3 + M\delta Q)C_{b_0}^{n_0} \quad (9)$$

where

$$\delta Q = \begin{bmatrix} (\delta \mathbf{v}_{t_1}^{b_0})^T \\ (\delta \mathbf{v}_{t_1}^{b_0} \times \mathbf{v}_{t_2}^{b_0} + \mathbf{v}_{t_1}^{b_0} \times \delta \mathbf{v}_{t_2}^{b_0})^T \\ (\delta \mathbf{v}_{t_1}^{b_0} \times \mathbf{v}_{t_2}^{b_0} \times \mathbf{v}_{t_1}^{b_0} + \mathbf{v}_{t_1}^{b_0} \times \delta \mathbf{v}_{t_2}^{b_0} \times \mathbf{v}_{t_1}^{b_0} + \mathbf{v}_{t_1}^{b_0} \times \mathbf{v}_{t_2}^{b_0} \times \delta \mathbf{v}_{t_1}^{b_0})^T \end{bmatrix}$$

$\hat{C}_{b_0}^{n_0}$ can be orthogonalized using the formula^[1]:

$$(\hat{C}_{b_0}^{n_0})_o = \hat{C}_{b_0}^{n_0} [(\hat{C}_{b_0}^{n_0})^T \hat{C}_{b_0}^{n_0}]^{-1/2} \approx [I_3 + (M\delta Q C_{b_0}^{n_0} - C_{b_0}^{n_0} \delta Q^T M^T) / 2] C_{b_0}^{n_0} \quad (10)$$

Let us assume that the initial Euler misalignment angles are ϕ_0 , then

$$(\phi_0 \times) = \frac{1}{2} [(C_{b_0}^{n_0} \delta Q^T M^T) - (C_{b_0}^{n_0} \delta Q^T M^T)^T] \quad (11)$$

Where

$$C_{b_0}^{n_0} \delta Q^T = \begin{bmatrix} (\delta \mathbf{v}_{t_1}^{n_0})^T \\ (\delta \mathbf{v}_{t_1}^{n_0} \times \mathbf{v}_{t_2}^{n_0} + \mathbf{v}_{t_1}^{n_0} \times \delta \mathbf{v}_{t_2}^{n_0})^T \\ (\delta \mathbf{v}_{t_1}^{n_0} \times \mathbf{v}_{t_2}^{n_0} \times \mathbf{v}_{t_1}^{n_0} + \mathbf{v}_{t_1}^{n_0} \times \delta \mathbf{v}_{t_2}^{n_0} \times \mathbf{v}_{t_1}^{n_0} + \mathbf{v}_{t_1}^{n_0} \times \mathbf{v}_{t_2}^{n_0} \times \delta \mathbf{v}_{t_1}^{n_0})^T \end{bmatrix}^T \quad (12)$$

3.1 Error Aroused by Sensor Errors

Eq. 2 and Eq. 6 are developed for the attitude and velocity updating. How the sensor errors affect the attitude and velocity are described by the error dynamics

equations. We derive these equations by applying a differential operator to Eq. 2 and Eq. 6. It is convenient to represent the attitude error angles of $C_{b_0}^{n_0}$ and velocity error by $\boldsymbol{\eta}^{b_0}$ and $\delta \mathbf{v}^{b_0}$ due to gyros drift $\boldsymbol{\varepsilon}^b$ and accelerometers bias ∇^b :

$$\dot{\boldsymbol{\eta}}^{b_0} = -C_{b_0}^{b_0}(t) \boldsymbol{\varepsilon}^b \quad (13)$$

$$\delta \dot{\mathbf{v}}^{b_0}(t) = \boldsymbol{\eta}^{b_0} \times \mathbf{f}^{b_0}(t) - C_{b_0}^{b_0}(t) \nabla^b \quad (14)$$

Consider a stationary alignment for a SINS and assume the body frame coincides with the navigation frame. Then Eq. 13 and Eq. 14 can be written:

$$\dot{\boldsymbol{\eta}}^{n_0} = -C_n^{n_0}(t) \boldsymbol{\varepsilon}^n \quad (15)$$

$$\delta \dot{\mathbf{v}}^{n_0}(t) = \mathbf{g}^{n_0} \times \boldsymbol{\eta}^{n_0} - C_n^{n_0}(t) \nabla^n \quad (16)$$

Considering the use of Laplace transformation and inverse Laplace transformation to above equations, the time-response approximate solution of $\delta \mathbf{v}^{n_0}(t)$ are:

$$\delta \mathbf{v}_x^{n_0}(t) = -(\nabla_E t + \frac{1}{2} g \boldsymbol{\varepsilon}_N t^2) \quad (17)$$

$$\delta \mathbf{v}_y^{n_0}(t) = -(\nabla_N t - \frac{1}{2} g \boldsymbol{\varepsilon}_E t^2 + \frac{1}{6} \cos L g \omega_{ie} \boldsymbol{\varepsilon}_U t^3) \quad (18)$$

$$\delta \mathbf{v}_z^{n_0}(t) = -(-\nabla_U t + \frac{1}{2} \cos L g \omega_{ie} \nabla_E t^2 + \frac{1}{3} \cos L g \omega_{ie} \boldsymbol{\varepsilon}_N t^3) \quad (19)$$

Considering $2t_1 = t_2 = t$ in Eq. 12, then substituting Eq. (17) ~ Eq. (19) into Eq. (12) and Eq. (11), the initial Euler error angles for the initial body attitude matrix $C_{b_0}^{n_0}$ are given up to one order of time by

$$\phi_{0E} = -\frac{\nabla_N}{g} \quad (20)$$

$$\phi_{0N} = -\frac{\nabla_E + \boldsymbol{\varepsilon}_N t}{g} \frac{1}{4} \quad (21)$$

$$\phi_{0U} = -\frac{\boldsymbol{\varepsilon}_E}{\omega_{ie} \cos L} + \frac{\nabla_E \tan L}{g} + \frac{\boldsymbol{\varepsilon}_U t}{2} \quad (22)$$

3.2 Error Aroused by Linear Vibration

To predigest analysis, assume the body frame coincides with the navigation frame and the linear vibration profile is $\delta \mathbf{f}(t)$, then the linear vibration velocity are $\delta \mathbf{v}_{t_1}^{n_0} = \int_0^{t_1} \delta \mathbf{f}(\tau) d\tau$ at time t_1 and $\delta \mathbf{v}_{t_2}^{n_0} = \int_0^{t_2} \delta \mathbf{f}(\tau) d\tau$ time t_2 . Substituting $\delta \mathbf{v}_{t_1}^{n_0}$ and $\delta \mathbf{v}_{t_2}^{n_0}$ into Eq. 12 and Eq. 11, then initial Euler error angles aroused by linear vibration are as follows:

$$\phi_{0E} = -\frac{4 \int_0^{t/2} \delta f_N(\tau) d\tau + \int_0^t \delta f_N(\tau) d\tau}{gt} \quad (23)$$

$$\phi_{0N} = \frac{2 \int_0^{t/2} \delta f_E(\tau) d\tau}{gt} \quad (24)$$

$$\phi_{0U} = \frac{4 \left[2 \int_0^{t/2} \delta f_N(\tau) d\tau - \int_0^t \delta f_N(\tau) d\tau \right]}{g\omega_{ie} \cos Lt^2} \quad (25)$$

4 Simulation

In this section, we carry out simulation to evaluate the error characteristics of the IAA method described above. The simulation mimicking typical alignment conditions are as follows; the first for a stationary SINS, the other one for a swaying SINS without linear vibration and another one for a swaying SINS with linear vibration. The SINS is assumed to be located at latitude 34°, longitude 108° and 440 m height. It has gyros with drift rate 0.01°/h and accelerometers with bias 5×10⁻⁵ g. The sampling rate is 200 Hz. The random walk coefficient are 0.003 °/√h and 8 μg/√Hz for gyros and accelerometers respectively. The initial Euler angles are 0° (pitch), 0° (roll) and 0° (yaw).

Firstly, consider the SINS to be completely stationary. Fig. 1 shows the initial Euler error angles obtained by IAA, as well as curve for our parameter setting in Eq. 20 ~ Eq. 22.

$$\begin{bmatrix} \phi_{0E} \\ \phi_{0N} \\ \phi_{0U} \end{bmatrix} = \begin{bmatrix} -0.1719 \\ 0.1719 \\ -2.6485 \end{bmatrix} + \begin{bmatrix} 0 \\ 4.1667 \times 10^{-5} \\ 8.3333 \times 10^{-5} \end{bmatrix} t \quad (26)$$

Nice agreement between IAA alignment results and analytic results is observed in Fig. 1. The initial alignment Euler angles obtained by IAA are plotted in Fig. 2.

The fitted slope of initial roll and yaw angles in Fig. 2 are -3.9933×10⁻⁵/s and 8.8531×10⁻⁵/s, or equivalently ε_N=0.0096 °/h and ε_U=0.0106 °/h respectively, which implies that we might instead estimate

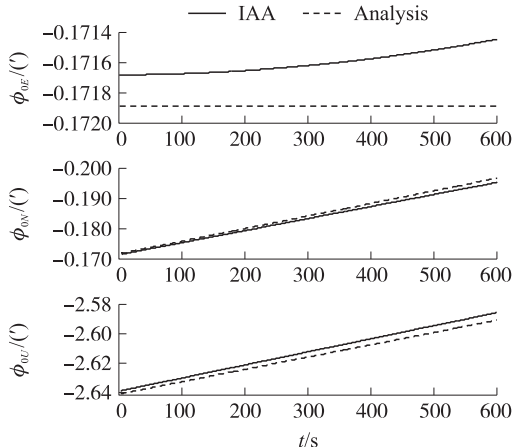


Fig.1 Initial Euler Error Angles on Static Base

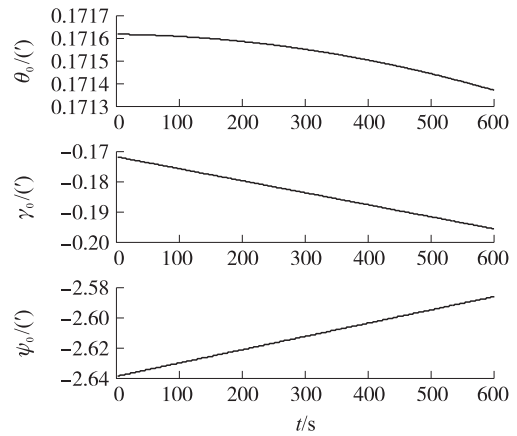


Fig.2 Initial Alignment Euler Angles

ε_N and ε_U using the actual slopes of initial alignment roll and yaw angles.

Secondly, we consider the SINS angularly swaying without linear vibration. Keep the initial angles unchanged and the three angle rate to 0.2π×sin(0.1πt)°/s, 0.18π×sin(0.06πt)°/s and 0.16π×sin(0.08πt)°/s. The angles estimates along with the true value are plotted in Fig. 3. Estimates of the constant initial Euler error angles by IAA are plotted in Fig. 4. Similar characteristic with those in the stationary case can be identified for IAA. It indicates that the IAA method could alternatively serve as an evaluation tool of SINS calibration in that the existence of significant biases would lead to apparent climbing trends in the estimates of initial Euler angles.

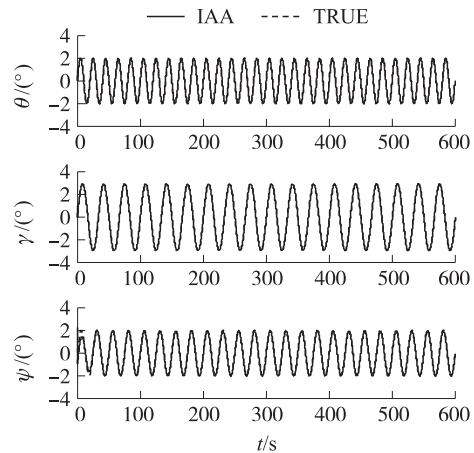


Fig.3 Alignment Result without Vibration

Thirdly, the SINS is swaying with both angular and linear vibrations, then keep the above angular motions unchanging and additionally add linear vibrations. We set zero initial ground velocity and the three ground velocity rates to 0.05sin(0.2πt) m/s, 0.05sin(0.3πt) m/s and 0.05sin(0.4πt) m/s. The alignment results

along with the true value are given in Fig. 5. The initial Euler angles by IAA are given in Fig. 6.

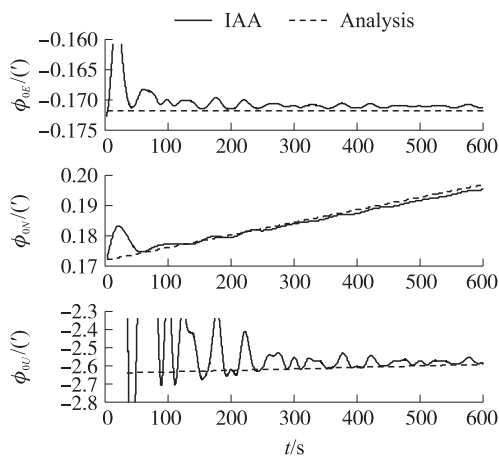


Fig. 4 Initial Euler Error Angles on Sway Base

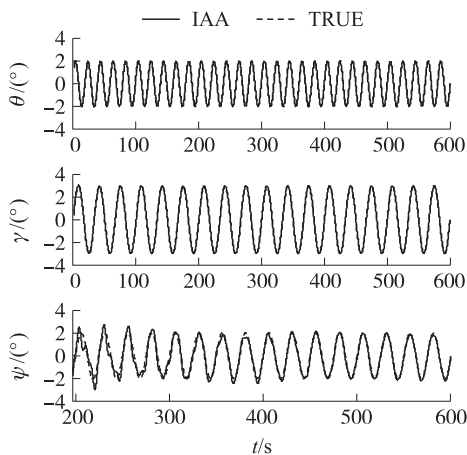


Fig. 5 Alignment Result with Vibration

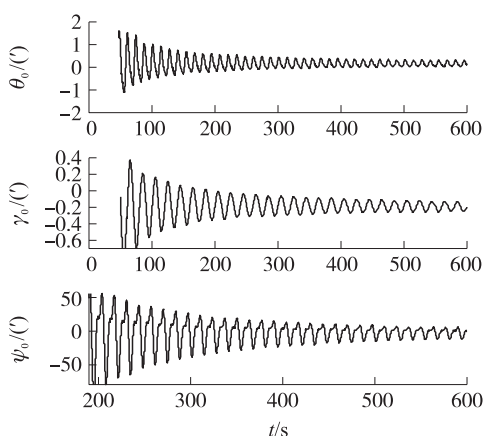


Fig. 6 Initial Euler Angles Estimates with Vibration

Both Figures imply that the liner vibration imposes considerable effect on the IAA method. The climbing trends of initial attitude angles are submerged in and cannot be distinguished from the misalignment angles. Compared with sensors errors, the IAA method is more sensitive to linear vibrations.

5 Summary and Conclusion

This paper introduces the indirect analytic alignment (IAA) method for strapdown inertial navigation system and the corresponding error characteristics. Sensitivity analysis of the IAA method with respect to sensor biased and linear vibration has been made and explicit error analytic formulas are derived for a stationary case. By fitting slope of initial Euler angles, the significant sensor biases can be estimated, but it is not available when serious linear vibration. The analysis is well validated by simulation. These results are helpful for investigating the indirect analytic alignment of strapdown inertial navigation system.

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