

FUZZY CLASSIFICATION WITH A GIS AS AN AID TO DECISION MAKING

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ABSTRACT

In this paper, we discuss various aspects of using fuzzy classification with a GIS. In particular, we show how fuzzy membership functions to particular classes can be computed for composite regions composed of lots of smaller regions belonging to different classes and how variables taking values in ranges with different boundary conditions can be handled in a mathematically rigorous way. We demonstrate our methodology for the problem of assessing the risk of desertification of burned forest areas in the Mediterranean region.

1. INTRODUCTION

The objective of the present work is to assess the degree of risk of desertification of burned forest areas using a fuzzy classification technique. It is important to estimate the risk of desertification in order to take proper measures for its prevention. Since the parameters involved in the study are fuzzy in nature and have to be classified by using fuzzy labels like low, medium, high etc., it is felt that it could be more appropriate to use fuzzy logic. Moreover, the use of remote sensing techniques and GIS along with fuzzy logic to evaluate the degree of risk would help an expert in a very efficient planning of resource allocation and decision making.

Work that has already been done on forest fire includes mapping and monitoring of forest fire areas (Prevedel, 1995), assessment of vegetation change (Jakubauskas *et al.*, 1990) and restoration of burned areas (Greer, 1994). Though there have been published work on assessment of areas affected by forest fire (Jakubauskas *et al.*, 1990), the concept of vagueness has never been considered. Attempts have been made to include uncertainty in the data (Stassopoulou), but only in terms of probability functions and not partial membership functions. There have been attempts to use GIS for the classification (Rokos *et al.*, 1993), but as on today no GIS package offers a facility to handle vague definitions.

The crux of any fuzzy logic problem lies in deriving the membership functions. In most of the fuzzy control systems, membership functions are chosen arbitrarily by the users based on their experience and

perspectives (Mandel, 1995). Hence the membership functions given by two users could be quite different. More recently, membership functions have been designed using optimisation procedures (Mandel, 1995) and fuzzy B-splines (Wang *et al.*, 1995). In image analysis and pattern recognition problems, the derivation of membership functions is still an issue, but attempts have been made to analyse the flexibility and uncertainty in membership function evaluation using bound functions and spectral fuzzy sets (Kaufmann, 1975). The most commonly used shapes for membership functions are triangular, trapezoidal and Gaussian.

In the present work, the membership functions have been derived by assuming Gaussian error distributions and extra experiments have also been performed with uniform error distributions. Arc/Info GIS has been used to store data and also to derive necessary secondary data and then the rules given by the experts have been implemented by using simple fuzzy operators.

2. FUZZY MEMBERSHIP FUNCTIONS

2.1. Study Parameters

The data that are used for the study pertain to a few sites in Attica, Greece. The variables that influence the degree of desertification were defined by the experts as Soil Erosion and Regeneration Potential. While the soil erosion is influenced by Ground Slope, Rock Permeability and Soil Depth, the Regeneration Potential is influenced by Ground Aspect and Soil Depth. Some of the data regarding slope and aspect could be derived from Digital Elevation Models using the GRID module of Arc/Info GIS package.

2.2. Membership Functions

Let us assume that the class membership of a fuzzy variable is determined by a measurement concerning the variable performed with a given accuracy expressed by the standard error in the measuring process. In other words, let us say that the value of a given variable t is measured to be μ and the error in this measurement is assumed to be Gaussian with zero mean and standard deviation σ . Our objective is to derive the membership functions of classes defined for the variable t as ranges of its values. It is obvious, for example, that if t is assigned to a certain class c if its value ranges between t_1 and t_2 , then the probability of t belonging to this class is given by

$$f(\mu) = \frac{1}{A} \int_{t_1}^{t_2} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad (1)$$

where A is given by

$$A = \int_{t_{min}}^{t_{max}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad (2)$$

where t_{min} and t_{max} are the minimum and maximum values that t could take.

Thus, the probability of the variable t belonging to class c if its value was measured to be μ with standard error σ , is given by

$$f(\mu; t_1, t_2) = \frac{\operatorname{erf}\left(\frac{t_2-\mu}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{t_1-\mu}{\sqrt{2}\sigma}\right)}{\operatorname{erf}\left(\frac{t_{max}-\mu}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{t_{min}-\mu}{\sqrt{2}\sigma}\right)} \quad (3)$$

where

$$\operatorname{erf}(z) \equiv \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt \quad (4)$$

To evaluate the error functions, the following rational approximation is used [1]: For $0 \leq x < \infty$ $\operatorname{erf}(x) = 1 - (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5) e^{-x^2}$

where

$$t = \frac{1}{1 + px}$$

$$p = 0.3275911 \quad a_1 = 0.254829592$$

$$a_2 = -0.284496736 \quad a_3 = 1.421413741$$

$$a_4 = -1.453152027 \quad a_5 = 1.061405429$$

The error of this approximation is less than 1.5×10^{-7} .

A membership function of a certain class to be used within the framework of fuzzy logic is a function which when given as input a certain measurement, returns the probability with which the variable can be assigned to the particular class. Thus, we have to define a membership function for each class we have and each of these functions should be a function of the measurement value. It should also depend parametrically on the limiting values that define the class and the error in the measurement. It is obvious from the above that the membership function of class c is given by equation (3) when plotted as a function of μ . Also, it is clear from the definitions that the values of the functions sum up to 1. Different membership functions could be used for the different variables if extra information was available. Since the fuzzy variables we have in our problem have their own peculiarities when it comes to defining class boundaries, we shall discuss each variable separately.

2.2.1. Slope

Slope has been classified into the following 4 classes based on the degree to which they influence soil erosion. It is obvious that the steeper the slope, the greater is the soil erosion.

Gentle: 0 - 20%
 Moderate: 21 - 40%
 Moderately steep: 41 - 70%
 Steep: > 70%

The membership function for each class of slope can be derived with the help of equation (3) for various values of t within the class interval $[t_1, t_2]$. Now, the probability of slope belonging to any particular class for a given value of μ can be evaluated from the membership function. The slope can be expressed in degrees or percent. When expressed as a percentage, the slope is 100% when the angle is 45° and approaches infinity as the angle approaches the vertical which is 90° . From the mathematical point of view, for every direction there is a twofold ambiguity in estimating a slope as the ground may slope

upwards or downwards. If we assume that one of these directions is positive slope, the other can be thought of as the negative slope. However, for the purpose of evaluating the risk of soil erosion, positive or negative slope does not matter. Thus, we do not need to consider negative values of the measurement μ as this is always going to be given to us as a positive number and the negative value case is the mirror image of the positive value case. What matters is how we treat the error distribution when class boundaries are crossed. The choice of Gaussian error probability density function implies that we have infinite tails which must influence all membership functions. In practice, if $G(\mu)$, $M_1(\mu)$, $M_2(\mu)$ and $S(\mu)$ indicate the membership functions for the classes gentle, moderate, moderately steep and steep respectively, we have:

$$\begin{aligned}
 G(\mu) &= f(\mu; -20, 20) \\
 M_1(\mu) &= f(\mu; 20, 40) + f(\mu; -40, -20) \\
 M_2(\mu) &= f(\mu; 40, 70) + f(\mu; -70, -40) \\
 S(\mu) &= f(\mu; 70, \infty) + f(\mu; -\infty, -70)
 \end{aligned}$$

where the function $f(\mu; t_1, t_2)$ is defined by equation (3).

These functions are plotted in Figure-1 for $\sigma = 4.5$. Note that for any particular value of the slope, the values of the membership functions sum up to 1.

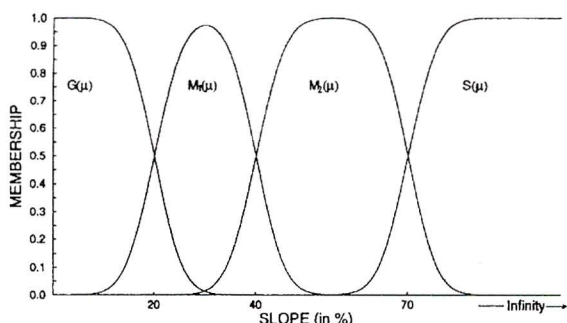


Figure 1 - Membership Functions for Slope

2.2.2 Soil Depth

This is classified into 3 classes.

1. Bare: < 5 cm
2. Shallow: 5 - 30 cm
3. Deep: > 30 cm

The Gaussian distribution of the error in measuring soil depth is truncated at $x = 0$ as soil depth cannot have negative values. Thus, if $B(\mu)$, $S(\mu)$ and $D(\mu)$

are the membership functions for the classes Bare, Shallow and Deep respectively, we have:

$$\begin{aligned}
 B(\mu) &= f(\mu; 0, 5) \\
 S(\mu) &= f(\mu; 5, 30) \\
 D(\mu) &= f(\mu; 30, \alpha)
 \end{aligned}$$

where $f(\mu; t_1, t_2)$ is given by equation (3) with $t_{\min} = 0$ and $t_{\max} = \infty$. These functions are plotted in Figure-2 for $\sigma = 2.5$

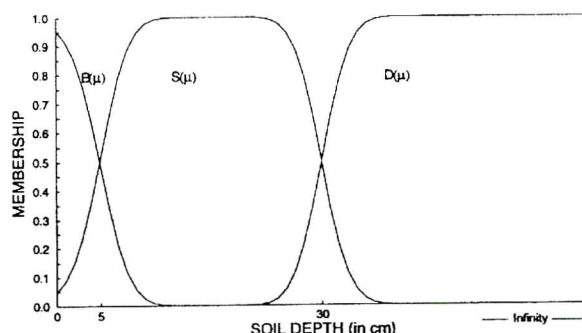


Figure 2 - Membership Functions for Soil Depth

2.2.3 Aspect

The aspect or orientation of a ridge can be expressed as the angle the normal to the ridge forms with the north direction. This angle could take a value from 0° to 360° and it could belong to any of the following classes :

1. North: $0 - 4^\circ, 315 - 360^\circ$
2. East: $45 - 135^\circ$
3. South: $135 - 225^\circ$
4. West: $225 - 315^\circ$

The aspect takes a range of possible values with cylindrical boundaries. The implication of this is that theoretically, since the tails of the Gaussian distribution are infinitely long, each class membership function would be the sum of an infinite number of contributions from segments of these tails that are 360° apart i.e., an infinite sum of evaluations of function (3) between limits that differ by 360° . In practice, of course, the contribution from these tails is insignificant from the mathematical point of view and meaningless from the point of view of the particular application that we are considering here. Thus the membership functions $N(\mu)$, $E(\mu)$, $S(\mu)$ and $W(\mu)$ for the four classes North, East, South and West respectively are :

$$N(\mu) = f(\mu; 0, 45^\circ) + f(\mu; 315, 360^\circ) + f(\mu; 360, 405^\circ) + \dots$$

$$E(\mu) = f(\mu; 45, 135^\circ) + f(\mu; 405, 495^\circ) + \dots$$

$$S(\mu) = f(\mu; 135, 225^\circ) + f(\mu; 495, 585^\circ) + \dots$$

$$W(\mu) = f(\mu; 225, 315^\circ) + f(\mu; 585, 675^\circ) + \dots$$

with $t_{\min} = -\infty$ and $t_{\max} = \infty$.

Figure 3 shows these membership functions for $\sigma = 18$.

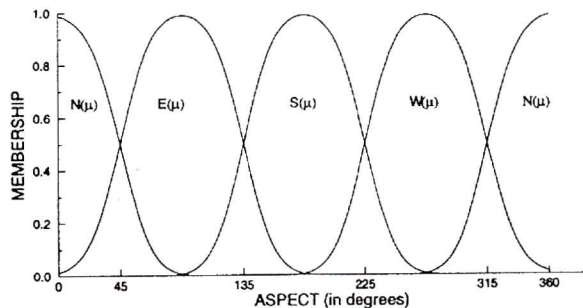


Figure 3 - Membership Functions for Aspect

2.2.4 Rock Permeability

Rock permeability refers to the ease with which water may run through the rock. The higher the rock permeability, the lower is the risk of soil erosion. The different types of rocks found in the study area are Hard Limestone, Schists, Metamorphic, Calcareous tertiary deposits, Siliceous tertiary deposits and Colluvium. While the metamorphic rocks and schists (which is an advanced grade of metamorphic rock) are impermeable, the rest are permeable. In the data that is available, rock permeability is defined for a sample site as a whole. Since the information given is only whether a sample site consists of either permeable rocks or impermeable rocks, rock permeability is considered as a non-fuzzy variable, even though it need not necessarily be. We shall see later that, in cases where we are concerned with the classification of a composite site, i.e., a site that consists of several patches each one having its own geology, the membership of the composite region into each one of the classes represented by the subregions is calculated as the proportional area each class of the subregions occupies within the composite region.

3. FUZZY CLASSIFICATION WITH GIS

3.1. The Role of GIS

The primary data to be used in the study to assess the degree of risk of desertification were provided on the Arc/Info GIS. Some secondary data were derived from the primary data using the potentialities of

Arc/Info. This GIS is better described in (Rokos *et al.*, 1993). Data included four test areas of different sizes chosen based on the availability of relevant satellite data. From these test areas, 53 sample sites had been chosen in such a way that they would represent maximum site variability. The various GIS layers were of rock permeability, soil depth and a Digital Elevation Model. The GIS data consisted of both vector and raster data types. Table 1 shows the different GIS layers used in the study.

Table 1 - GIS DATA and DATA TYPES

PRIMARY DATA	
GIS LAYERS	DATA TYPE
Sample site boundaries	vector
Soil depth	vector
Rock permeability	vector
DEM	raster

DERIVED DATA	
GIS LAYERS	DATA TYPE
Slope	raster
Aspect	raster

Since the data regarding rock permeability, soil depth and DEM were provided for the entire study area, the required data were extracted by clipping with the sample site boundaries. Pixel-wise slope and aspect values were obtained from the DEM using the GRID module of Arc/Info. GRID was used to derive the slope and aspect values as it can accurately portray continuous surfaces. GRID is a raster based geo-processing system integrated with Arc/Info. A grid in Arc/Info represents a single theme and is made up of cells of a particular size representing the resolution of the data and the cell values representing the class within the theme to which it belongs. Each integer grid would have an associated Value Attribute Table which stores the cell values.

Slope is evaluated as the maximum rate of change in value from each cell to its neighbours and an output slope grid could have slope values in degrees or percent. Aspect is evaluated as the direction of slope. The pixel based slope values were generalised to each sample site by averaging the slope values of all pixels in the sample site. Since the aspect has cylindrical boundaries, evaluating the mean aspect value of all the pixels in a site could result in the aspect falling into a completely wrong class. For example, if a site contained aspect values belonging to North i.e., between 0 to 45° and 315 to 360°, then evaluating the aspect value of the site as the mean of all pixel values could classify it even into the class 'South'. In order

to eradicate this problem, the following methodology has been adopted.

1. All N pixel values of a site were sorted in ascending order of aspect value.
2. A new sequence of N numbers was created by subtracting 360° from each pixel value.
3. The old and the new sequences were concatenated, thus creating a single sequence of 2N numbers i.e., twice as long as the previous one, the first half of which is the same as the second half shifted by -360°.
4. Mean and variance were then calculated in a sliding window of length N.
5. The mean corresponding to the minimum variance was chosen as the mean aspect value of the site.

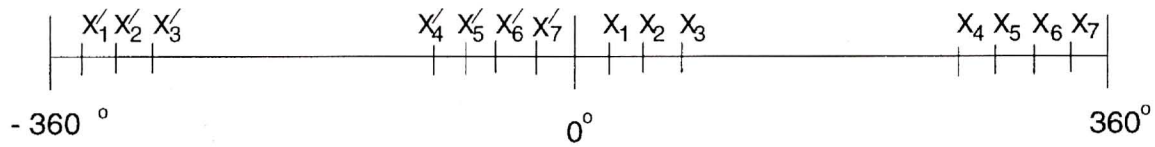


Figure 4 - Inducing Continuity in Aspect

Figure 4 gives an example of how this trick solves the problem of discontinuity at 360°/0°. Suppose that N = 7 and the values X₁, ..., X₇ are placed as shown along the positive real axis of Figure 4. Clearly, the average of these aspects should be either near 0° or 360°. However, if we compute it by straight averaging, we shall find a number near 180°. By shifting the sequence 360° to the left, we create the ghost members of the sequence X'₁, ..., X'₇. We then consider every 7 successive members of this extended sequence and compute their average and their variance. The variance will be minimum when the sliding window of length 7 contains numbers X'₄, X'₅, X'₆, X'₇, X₁, X₂, X₃. The average of these numbers will be around 0 which is the correct value.

3.2 Fuzzy Classification

The domain expert's knowledge was implemented over the framework of GIS and then, the fuzzy classification technique was used for decision making. The domain expert's knowledge is expressed by two sets of rules, one for natural regeneration potential and the other for risk of soil erosion. Both the antecedents and the consequents in the rules are fuzzy. Rock permeability, soil depth, slope and aspect were the fuzzy variables involved in the rules. The rules are shown in the following Table 2 and Table 3.

Let x₁ be the value of slope in a site. Then

$$\{x_1, \mu_S(G), \mu_S(M), \mu_S(S)\}$$

would represent the membership grades of x₁ to the classes gentle, medium and steep of the fuzzy variable Slope (S). Let x₂ be the aspect of a site in degrees, Then,

$$\{x_2, \mu_A(N), \mu_A(E), \mu_A(W), \mu_A(S)\}$$

would represent the membership grades of x₂ to the classes North, East, West and South of the fuzzy variable Aspect (A). If x₃ is the value of Soil Depth, then

$$\{x_3, \mu_{SD}(B), \mu_{SD}(S), \mu_{SD}(D)\}$$

represents the membership grades of x₃ to the classes bare, shallow and deep of the fuzzy variable Soil Depth (SD). If x₄ is the permeability of rock in the site, then

$$\{x_4, \mu_R(P), \mu_R(I)\}$$

would represent the membership grades of x₄ to the classes permeable and impermeable of the variable Rock Permeability (R).

The actual membership grades were evaluated from equation (3) of section 2.

Once the membership grades to the fuzzy variables are evaluated, the membership grades to the natural regeneration potential and risk of soil erosion are obtained from the fuzzy relations given in Table 2 and Table 3 using the fuzzy equivalents of Logical AND and OR namely, Max and Min. Hence, the

Table 2 - Tabulated rules for natural regeneration potential

		SOIL DEPTH (SD)		
		BARE	SHALLOW	DEEP
A S P E C T (A)	NORTH	SG	SL	NL
	EAST	SG	SL	NL
	WEST	SE	ML	SL
	SOUTH	SE	ML	SL

NL - No limitation
 SL - Slight Limitation
 ML - Moderate Limitation

SG - Strong Limitation
 SE - Severe Limitation

Table 3 - Tabulated rules for risk of soil erosion

		PERMEABILITY (R) & SOIL DEPTH (SD)					
		PERMEABLE			IMPERMEABLE		
S L O P E (S)	GENTLE	BARE	SHALLOW	DEEP	BARE	SHALLOW	DEEP
		*	SR	NSR	*	HR	SR
	MEDIUM	*	MR	SR	*	VHR	MR
	STEEP	*	MR	SR	*	VHR	HR

* The land with bare soil is already eroded. No further erosion can occur.

NSR - No to slight risk
 SR - Slight risk
 MR - Moderate risk

HR - High risk
 VHR - Very high risk

STEEP includes MODERATELY STEEP

membership grades for natural regeneration potential could be defined as

$$\begin{aligned} \mu_{RP}(NL) &= [\mu_A(N) \wedge \mu_{SD}(D)] \vee [\mu_A(E) \wedge \mu_{SD}(D)] \\ \mu_{RP}(SL) &= [\mu_A(S) \wedge \mu_{SD}(D)] \vee [\mu_A(N) \wedge \mu_{SD}(S)] \vee \\ &[\mu_A(W) \wedge \mu_{SD}(D)] \vee [\mu_A(E) \wedge \mu_{SD}(S)] \\ \mu_{RP}(ML) &= [\mu_A(S) \wedge \mu_{SD}(S)] \vee [\mu_A(W) \wedge \mu_{SD}(S)] \\ \mu_{RP}(SG) &= [\mu_A(N) \wedge \mu_{SD}(B)] \vee [\mu_A(E) \wedge \mu_{SD}(B)] \\ \mu_{RP}(SE) &= [\mu_A(S) \wedge \mu_{SD}(B)] \vee \mu_A(W) \wedge \mu_{SD}(B) \end{aligned}$$

where RP represents the 'Regeneration Potential', \wedge and \vee represent the Minimum and Maximum operators respectively. While the Minimum operation would give the largest fuzzy subset contained in the sets, the Maximum operation would give the smallest fuzzy subset contained in the sets. In other words, any chain connected in a series position is associated with

\wedge and a chain connected in a parallel position is associated with \vee .

The membership grades to the risk of soil erosion could be derived from the following operations.

$$\begin{aligned} \mu_{SE}(NSR) &= [\mu_S(G) \wedge \mu_{SD}(D) \wedge \mu_R(P)] \\ \mu_{SE}(SR) &= [\mu_S(M) \wedge \mu_{SD}(D) \wedge \mu_R(P)] \\ &\vee [\mu_S(S) \wedge \mu_{SD}(D) \wedge \mu_R(P)] \vee [\mu_S(G) \wedge \mu_{SD}(S) \wedge \mu_R(P)] \vee \\ &[\mu_S(G) \wedge \mu_{SD}(D) \wedge \mu_R(I)] \\ \mu_{SE}(MR) &= \\ &[\mu_S(M) \wedge \mu_{SD}(S) \wedge \mu_R(P)] \vee [\mu_S(S) \wedge \mu_{SD}(S) \wedge \mu_R(P)] \vee [\mu_S(M) \\ & \wedge \mu_{SD}(D) \wedge \mu_R(I)] \\ \mu_{SE}(HR) &= [\mu_S(S) \wedge \mu_{SD}(D) \wedge \mu_R(I)] \\ &\vee [\mu_S(G) \wedge \mu_{SD}(S) \wedge \mu_R(I)] \\ \mu_{SE}(VHR) &= [\mu_S(M) \wedge \mu_{SD}(S) \wedge \mu_R(I)] \\ &\vee [\mu_S(S) \wedge \mu_{SD}(S) \wedge \mu_R(I)] \end{aligned}$$

where SE represents the 'Risk of Soil Erosion'.

While evaluating the membership grades of risk of soil erosion, slope has been classified into 3 classes only, as for all practical purposes, slopes > 40% are considered steep. Since only linguistic data were available for soil depth and rock permeability and also since a sample site could consist of more than one type of rock permeability and more than one type

of soil depth, the membership grade to a particular class of these variables was evaluated as the proportion of a sample site belonging to that class.

Finally, to obtain the degree of risk of desertification based on the natural regeneration potential and risk of soil erosion, the fuzzy relations given in Table 4 were used.

Table 4 - Tabulated rules for risk of soil erosion

		REGENERATION POTENTIAL (RP)				
		NL	SL	ML	SG	SE
E R O S I O N (SE)	NSR	NR	LR	LR	MR	MR
	SR	LR	LR	MR	MR	HR
	MR	LR	MR	MR	HR	HR
	HR	MR	MR	HR	HR	VHR
	VHR	MR	HR	HR	VHR	VHR

NR - No risk

LR - Low risk

MR - Moderate risk

HR - High risk

VHR - Very high risk

The membership grades to the risk of desertification were evaluated from the following equations.

$$\mu_D(\text{NR}) = \mu_{\text{RP}}(\text{NL}) \wedge \mu_{\text{SE}}(\text{NSR})$$

$$\mu_D(\text{LR}) = [\mu_{\text{RP}}(\text{SL}) \wedge \mu_{\text{SE}}(\text{NSR})] \vee [\mu_{\text{RP}}(\text{ML}) \wedge$$

$$\mu_{\text{SE}}(\text{NSR})] \vee [\mu_{\text{RP}}(\text{NL}) \wedge \mu_{\text{SE}}(\text{SR})]$$

$$\vee [\mu_{\text{RP}}(\text{SL}) \wedge \mu_{\text{SE}}(\text{SR})] \vee [\mu_{\text{RP}}(\text{SL}) \wedge \mu_{\text{SE}}(\text{MR})]$$

$$\mu_D(\text{MR}) = [\mu_{\text{RP}}(\text{SG}) \wedge \mu_{\text{SE}}(\text{NSR})] \vee [\mu_{\text{RP}}(\text{SE}) \wedge \mu_{\text{SE}}$$

$$(\text{NSR})] \vee [\mu_{\text{RP}}(\text{ML}) \wedge \mu_{\text{SE}}(\text{SR})] \vee [\mu_{\text{RP}}(\text{SG}) \wedge \mu_{\text{SE}}(\text{SR})]$$

$$\vee [\mu_{\text{RP}}(\text{SL}) \wedge \mu_{\text{SE}}(\text{MR})] \vee [\mu_{\text{RP}}(\text{ML}) \wedge \mu_{\text{SE}}(\text{MR})]$$

$$\vee [\mu_{\text{RP}}(\text{NL}) \wedge \mu_{\text{SE}}(\text{HR})] \vee [\mu_{\text{RP}}(\text{SL})$$

$$\wedge \mu_{\text{SE}}(\text{HR})] \vee [\mu_{\text{RP}}(\text{NL}) \wedge \mu_{\text{SE}}(\text{VHR})]$$

$$\mu_D(\text{HR}) = [\mu_{\text{RP}}(\text{SE}) \wedge \mu_{\text{SE}}(\text{SR})] \vee [\mu_{\text{RP}}(\text{SG}) \wedge \mu_{\text{SE}}(\text{MR})]$$

$$\vee [\mu_{\text{RP}}(\text{SE}) \wedge \mu_{\text{SE}}(\text{MR})] \vee$$

$$[\mu_{\text{RP}}(\text{ML}) \wedge \mu_{\text{SE}}(\text{HR})] \vee [\mu_{\text{RP}}(\text{SG}) \wedge \mu_{\text{SE}}(\text{HR})] \vee$$

$$[\mu_{\text{RP}}(\text{SL}) \wedge \mu_{\text{SE}}(\text{VHR})] \vee$$

$$[\mu_{\text{RP}}(\text{ML}) \wedge \mu_{\text{SE}}(\text{VHR})]$$

$$\mu_D(\text{VHR}) = [\mu_{\text{RP}}(\text{SE}) \wedge \mu_{\text{SE}}(\text{HR})] \vee [\mu_{\text{RP}}(\text{SG}) \wedge$$

$$\mu_{\text{SE}}(\text{VHR})] \vee [\mu_{\text{RP}}(\text{SE}) \wedge \mu_{\text{SE}}(\text{VHR})]$$

The results obtained by using input values from the GIS were compared with the expert's classification. These results are given in Table 5. The values in bold

represent the category into which the expert had classified a site.

4 DISCUSSION AND CONCLUSIONS

From the above table it can be seen that the fuzzy classification system agrees with the expert in 21 out of the 53 sites when the input data are read from the GIS. If we allow up to one neighbouring class disagreement, the fuzzy classification system agrees with the expert's classification in 45 out of the 53 sites. The fuzzy classification system becomes a simple rule-based hard classifier when the input data are the field data with which no uncertainty value can be associated. In fact, the classification obtained by the field data agrees everywhere with the expert (an indication that the rules provided by the expert have been correctly implemented). The disagreement we observe between the GIS classification and the other two, may stem from one of the following reasons:

- The regions that are totally wrongly classified are the regions for which the GIS data are in complete disagreement with the field data. Clearly the GIS data are much more unreliable than the field data, mainly

SAMPLE SITE		RISK OF DESERTIFICATION				
SITE No.	SITE NAME	NR	LR	MR	HR	VHR
1	B-2	0.000	0.000	0.038	0.567	0.130
2	B-6	0.000	0.000	0.000	0.000	0.875
3	B-5	0.000	0.000	0.037	0.624	0.000
4	B-3	0.000	0.048	0.154	0.627	0.000
5	B-1	0.000	0.000	0.049	0.470	0.200
6	B-4	0.000	0.000	0.000	0.000	1.000
7	P-11	0.000	0.000	0.998	0.000	0.000
8	P-9	0.000	0.000	0.520	0.480	0.000
9	P-12	0.000	0.000	0.711	0.000	0.000
10	PD2-4	0.000	0.000	0.000	0.000	1.000
11	PD2-3	0.000	0.000	0.000	0.000	1.000
12	P-10	0.000	0.253	0.531	0.000	0.000
13	PD2-2	0.000	0.000	0.027	0.090	0.803
14	PD2-1	0.000	0.000	0.000	0.000	1.000
15	P-8	0.000	0.830	0.170	0.000	0.000
16	P-7	0.160	0.284	0.631	0.352	0.080
17	P-1	0.001	0.740	0.000	0.000	0.000
18	P-6	0.000	0.937	0.000	0.000	0.000
19	PD1-7	0.000	0.000	0.021	0.752	0.000
20	PD1-6	0.000	0.000	0.144	0.617	0.000
21	P-3	0.000	0.991	0.000	0.000	0.000
22	P-14	0.000	0.790	0.000	0.000	0.000
23	P-13	0.000	0.914	0.000	0.000	0.000
24	PD1-5	0.000	0.000	0.094	0.648	0.030
25	PD1-4	0.000	0.000	0.098	0.581	0.020
26	P-5	0.000	0.992	0.000	0.000	0.000
27	PD1-3	0.000	0.000	0.012	0.515	0.00
28	P-4	0.284	0.710	0.000	0.000	0.000
29	PD1-2	0.000	0.000	0.000	0.000	1.000
30	P-16	0.042	0.582	0.000	0.000	0.000
31	PD1-1	0.000	0.000	0.079	0.764	0.000
32	P-15	0.091	0.639	0.000	0.000	0.000
33	P-2	0.001	0.670	0.000	0.000	0.000
34	L-1	0.000	0.000	0.558	0.436	0.000
35	L-2	0.000	0.000	0.061	0.525	0.000
36	L-3	0.000	0.000	0.008	0.709	0.000
37	L-5	0.000	0.455	0.545	0.010	0.000
38	L-4	0.240	0.050	0.240	0.004	0.000
39	L-6	0.000	0.000	0.503	0.175	0.000
40	TB-1	0.000	0.000	0.146	0.496	0.360
41	TB-2	0.000	0.000	0.123	0.458	0.000
42	TP-2	0.000	0.022	0.040	0.040	0.000
43	TP-1	0.000	0.000	0.000	0.000	1.000
44	TP-3	0.000	0.071	0.570	0.000	0.000
45	TPD2-1	0.000	0.000	0.000	0.000	1.000
46	TPD1-3	0.000	0.000	0.002	0.614	0.000
47	TP-4	0.187	0.813	0.000	0.000	0.000
48	TPD1-2	0.000	0.000	0.000	0.872	0.000
49	TP-6	0.000	0.798	0.027	0.190	0.000
50	TPD1-1	0.000	0.000	0.000	0.591	0.000
51	TP-5	0.113	0.490	0.000	0.000	0.000
52	TL-1	0.000	0.366	0.572	0.330	0.000
53	TL-2	0.000	0.000	0.186	0.381	0.000

Table 5: RESULTS

due to the difference in scale. (The test sites were only of size $250 \times 250 \text{ m}^2$.) Our approach is aimed exactly at modelling this uncertainty, but the approximation of the error distributions by Gaussians may not be the best one. However, when we repeated the calculations assuming uniform distributions, i.e., triangular or trapezoidal membership functions which are commonly used, the results became much worse. The correct modelling of this uncertainty is part of our future work.

- We believe that by far the most significant reason of disagreement between the fuzzy classification and the expert's assessment is the expert's assessment itself. This was not done using some sort of accumulated experience and background knowledge which should have been elicited by some Knowledge Engineering techniques. It was rather done using a linear superposition rule of class labels, which is the very type of rule which we argue should be replaced by fuzzy classification! Thus, there is no guarantee that the expert's classification is more correct than the fuzzy classification. Only the study of historical data retrospectively could determine the correct classification method, but that is beyond the scope of this project.

In summary, we have shown in this paper how the fuzzy membership functions can be derived from the error distributions in the measurement data and in the information provided by the GIS layers. In particular, we dealt with the case of free boundary conditions, as is the case of measuring the soil depth, mirror image boundary conditions, as is the case of ground slope, and cylindrical boundary conditions, which is the case in aspect. It must be emphasised that although in the work presented here, we assumed Gaussian distributions of errors, the approach could be used with any type of error probability density function.

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