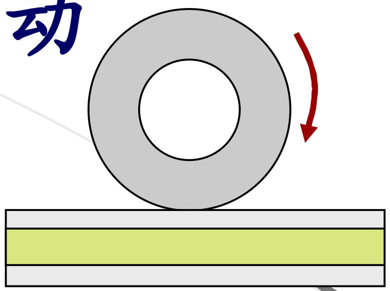
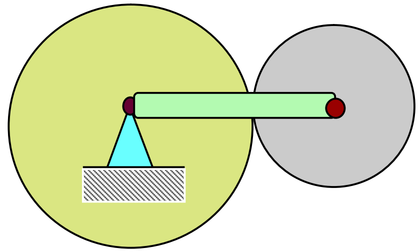
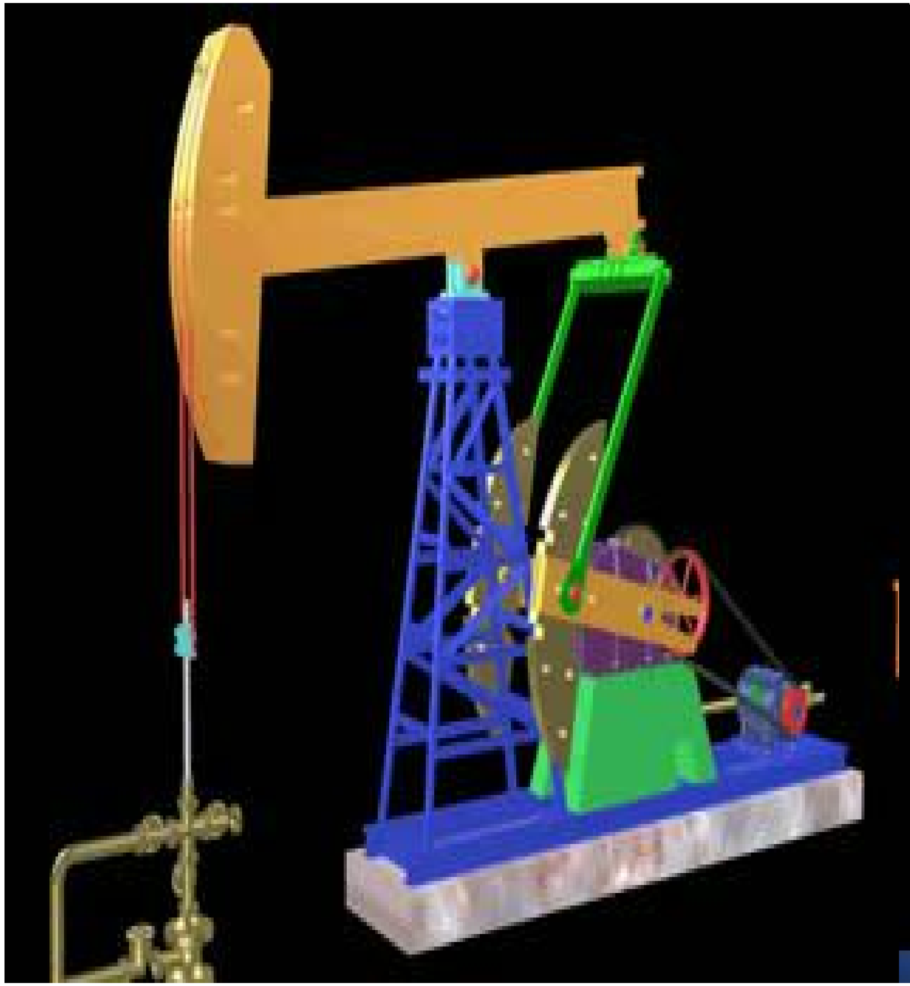

理论力学
运动学

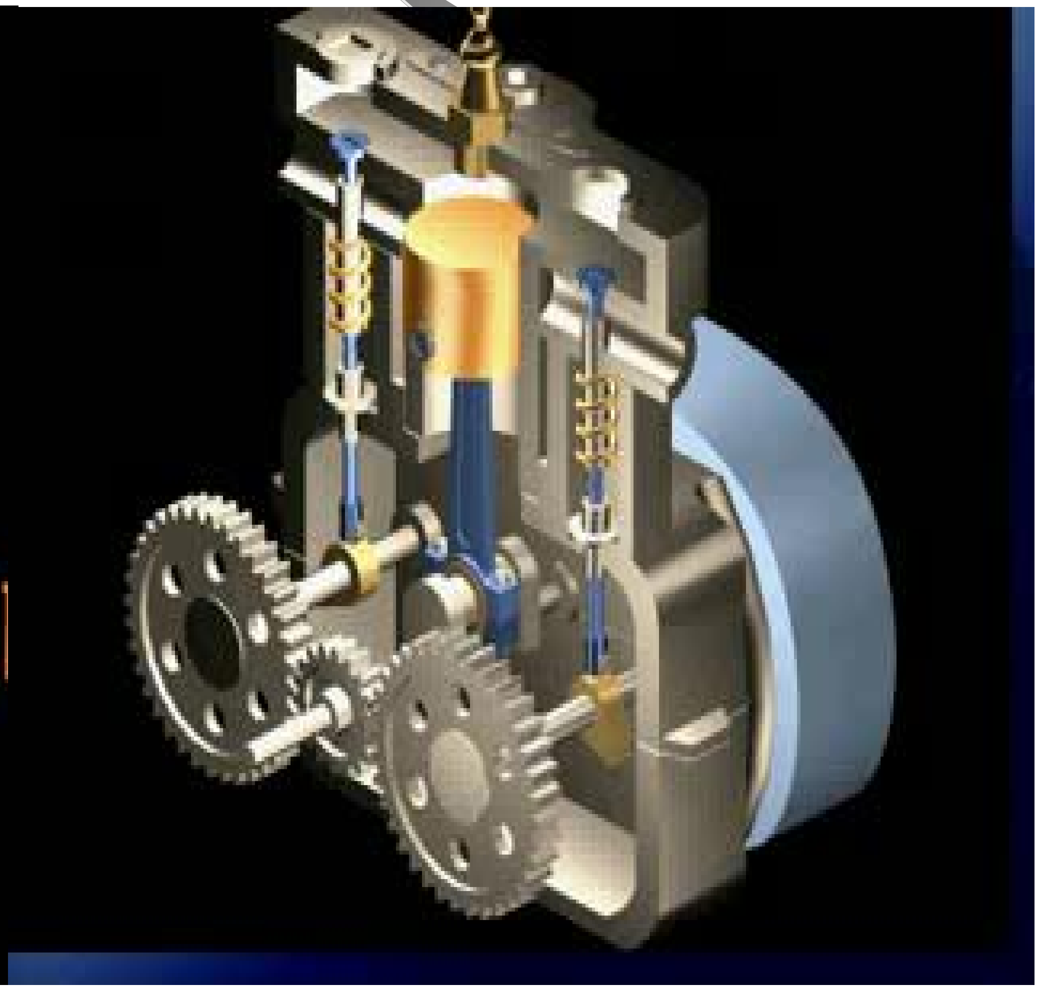
第七章 刚体的平面运动

平行于固定平面的运动。









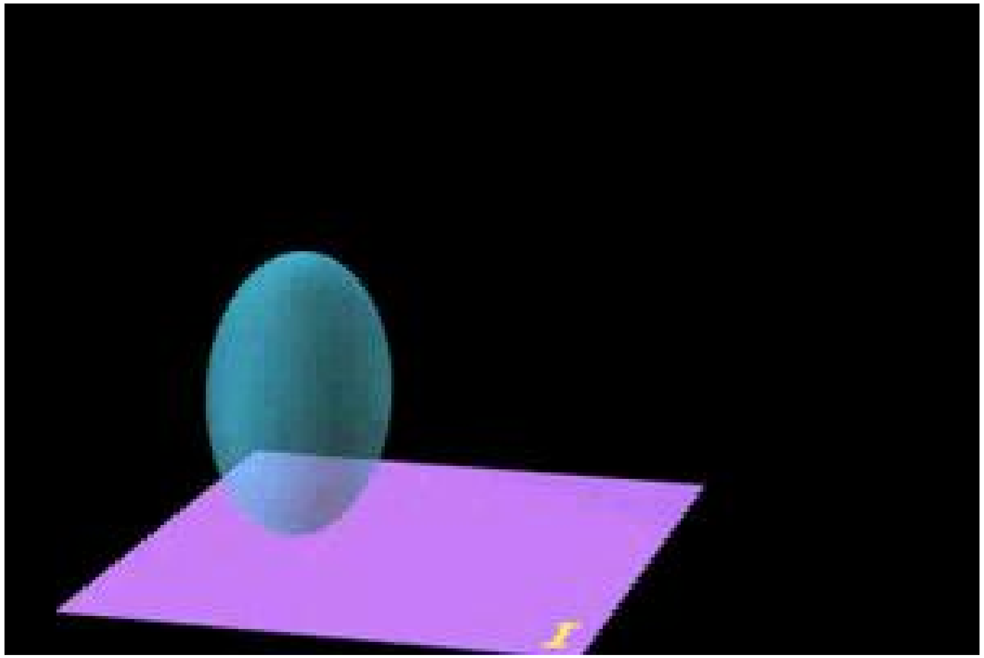
§ 7-1 刚体的平面运动方程

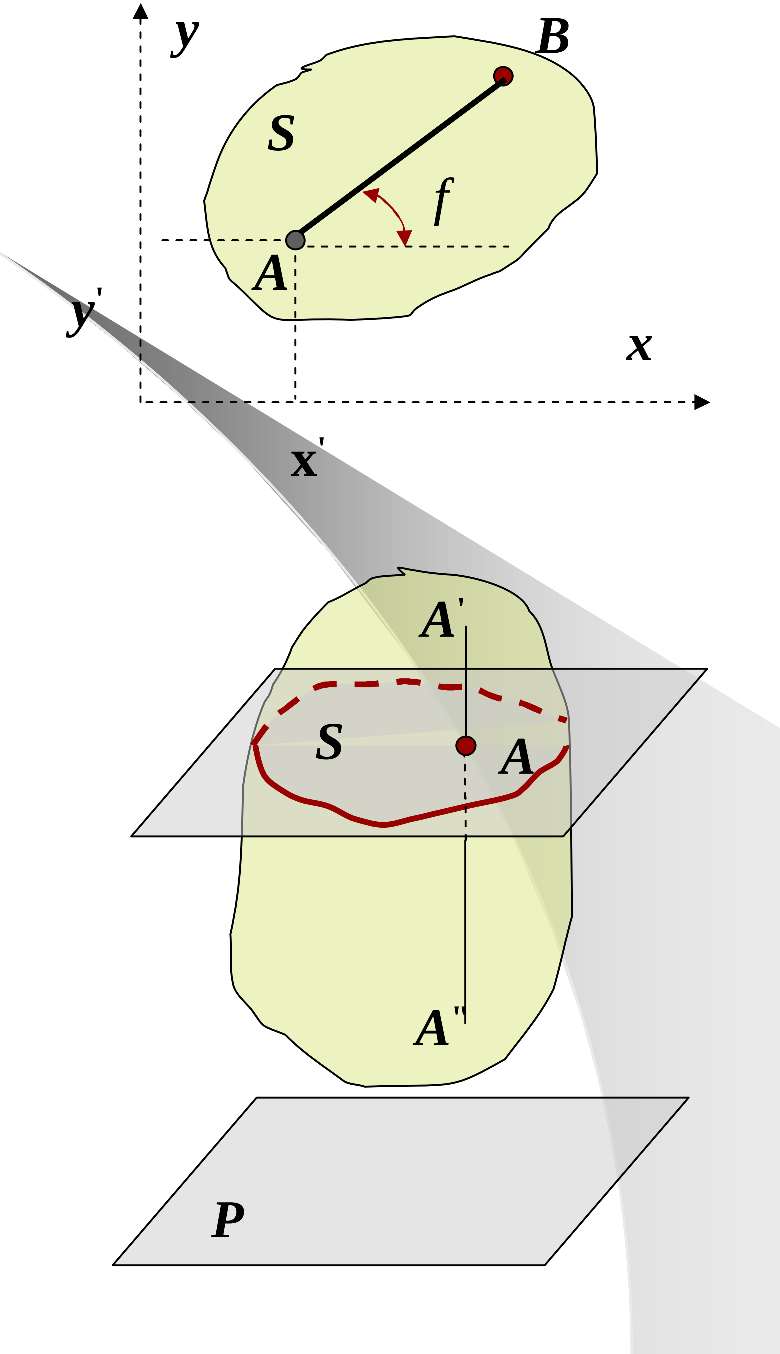
A：基点　　平面运动方程

$x' = f_1(t)$；

$y' = f_2(t)$；

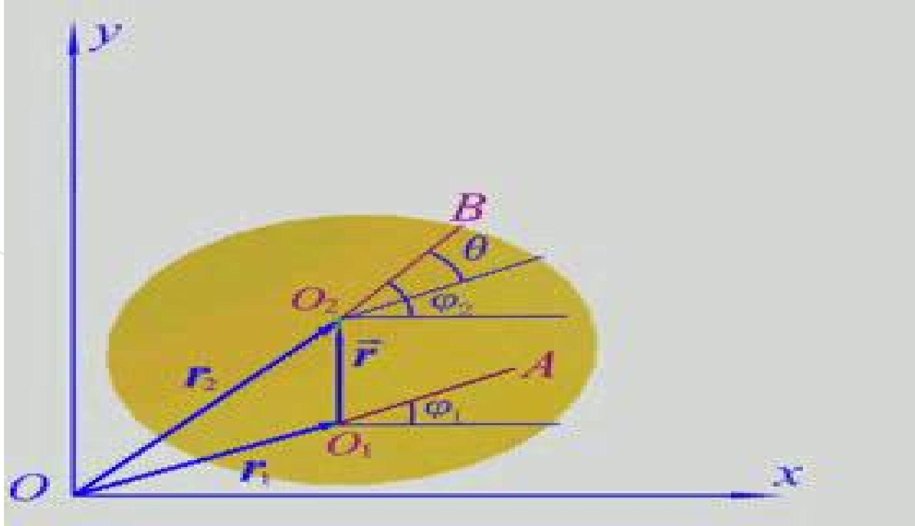
$f = f_3(t)$；





平面运动方程:

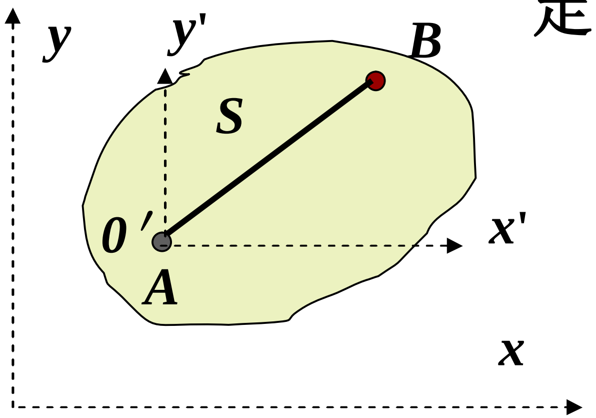
$x' = f_1(t)$; $y' = f_2(t)$; $f = f_3(t)$;



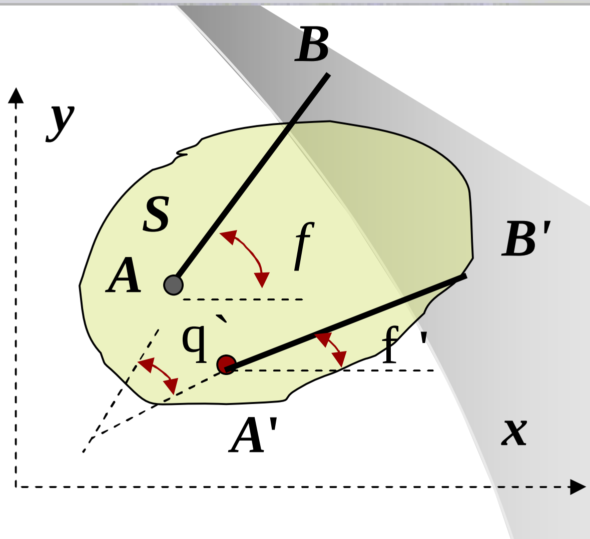
讨论:

1. $f = c$; $x' = f_1(t)$; $y' = f_2(t)$; 平动。

2. $x' = c_1$; $y' = c_2$; $f = f_3(t)$; 定轴转动。



$f - f' = q$



$$v_{0'x} = \frac{dx_{0'}}{dt} = f'_1(t);$$

$$v_{0'y} = \frac{dy_{0'}}{dt} = f_2'(t);$$

平动。

$$w = \frac{df}{dt} = \frac{df'}{dt} = f'_3(t); \quad a = f''_3(t).$$

v与基点有关。

w, a与基点无关。

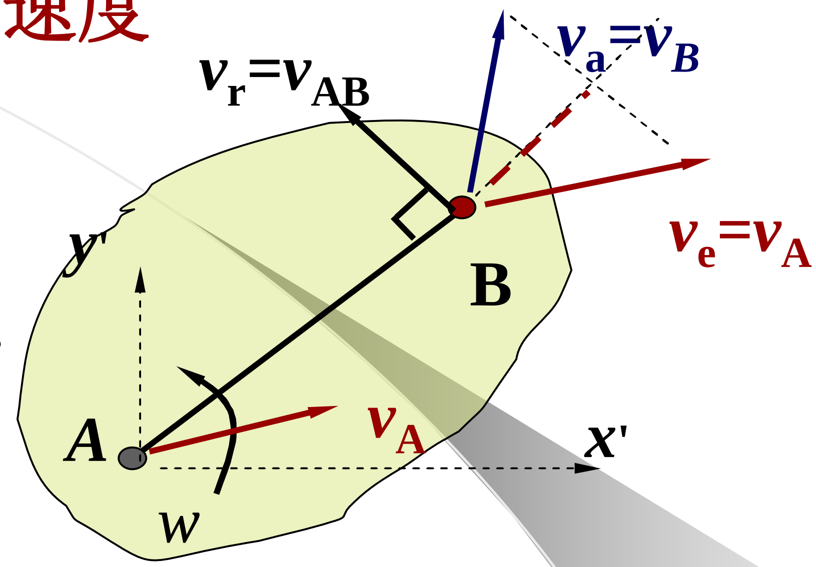
§ 7-2 平面图形上各点的速度

一、基点法(合成法)

$v_{AB}=w*AB$;

$\vec{v}_B = \vec{v}_A + \vec{v}_{AB}$;

$(\vec{v}_a = \vec{v}_e + \vec{v}_r)$;



1. v_A平动速度;

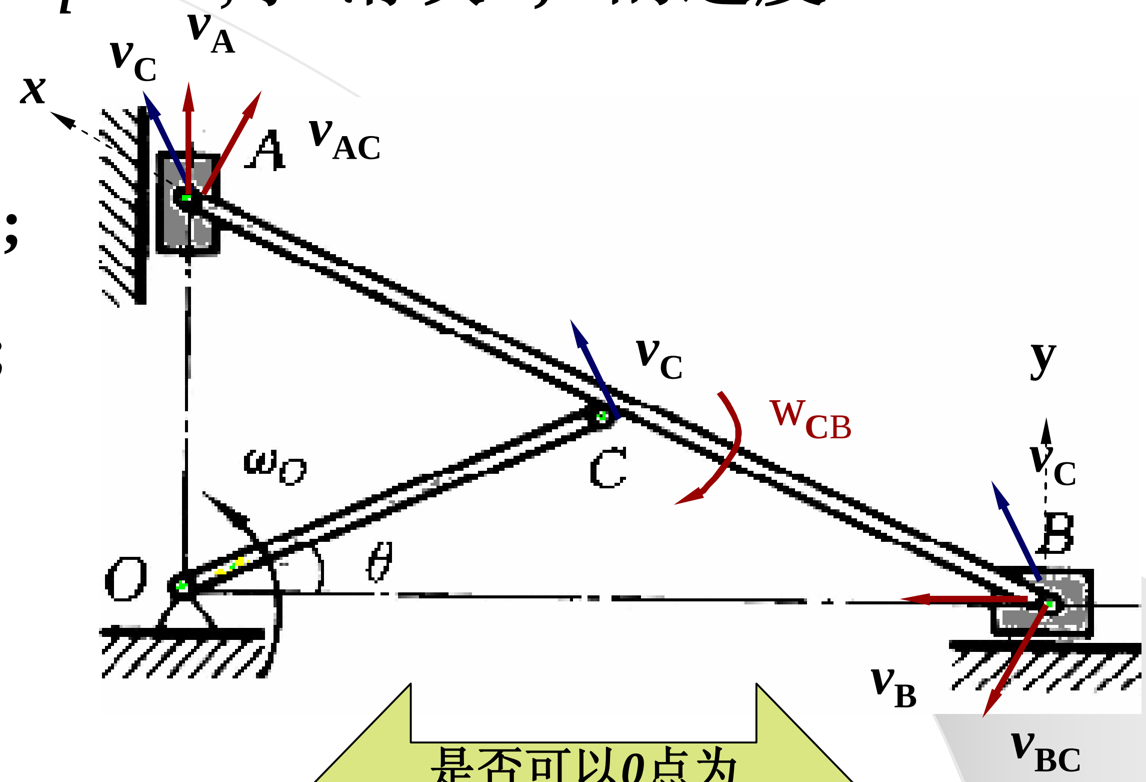
2. v_{AB}相对转动速度;

二、投影法

$[v_B]_{AB}=[v_A]_{AB}$;

1.瞬时状态;
2.可解二个未知量(大小，方向)。

例7—1：椭园规机构如图示，杆0C绕0作匀角速度转动，已知:$OC=AC=AB=R$，$q=30^0$,求:滑块A,B的速度。



解:

[A] 基点法 $v_C=Rw_0$;

x: $v_A\cos60^0=v_C\cos30^0$;

$$v_A=\sqrt{3}Rw_0.$$

[B] 基点法

x: $v_B\cos30^0=v_C\cos30^0$;

$v_B=v_C=Rw_0$;

y: $v_{BC}\cos30^0=v_C\cos30^0$;

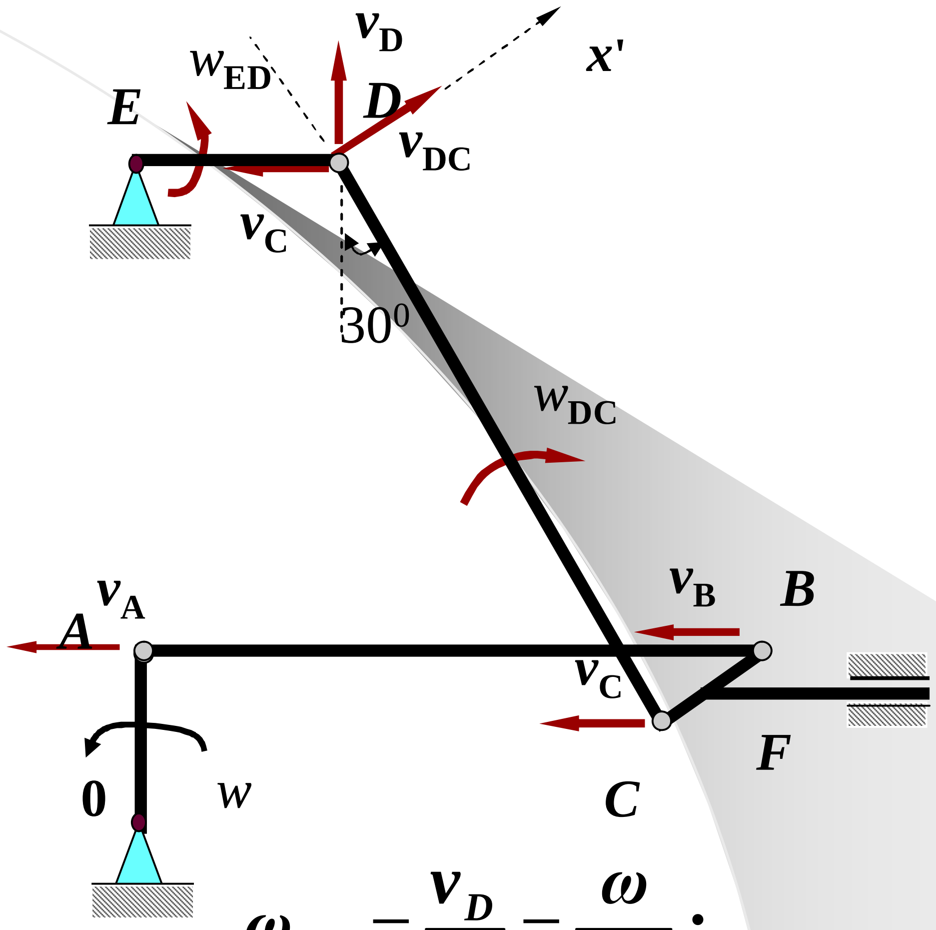
$v_{BC}=v_C=Rw_{CB}$;

$w_0=w_{CB}=w_{AB}$;

是否可以0点为基点求B点速度.

同样可以A点为基点求B点速度.

例7—2：机构如图示，杆0A绕0作匀角速度转动，已知: $DC=6r$，$OA=ED=r$, 求:滑块F的速度和杆ED的角速度。



解:

[A] 基点法 $v_A=rw$;

AB作瞬时平动: $v_A=v_B$;

BC作平动: $v_B=v_C=v_F$;

[C] 基点法

CD: $v_C\cos60^0=v_D\cos30^0$;

$$v_D=\frac{r\omega}{\sqrt{3}};$$

x': $v_D\cos60^0=v_{DC}-v_C\cos30^0$;

$$\omega_{ED}=\frac{v_D}{r}=\frac{\omega}{\sqrt{3}};$$

$$v_{DC}=2\frac{\sqrt{3}}{3}rw;\qquad w_{DC}=\frac{v_{DC}}{6r}=\frac{\sqrt{3}}{9}w;$$

三、瞬时速度中心法

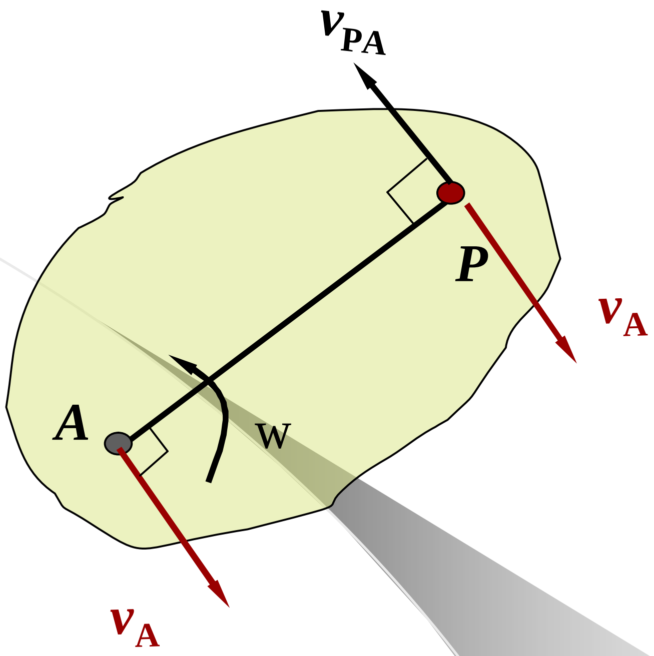
$$\vec{v}_P = \vec{v}_A + \vec{v}_{AP} = 0;$$

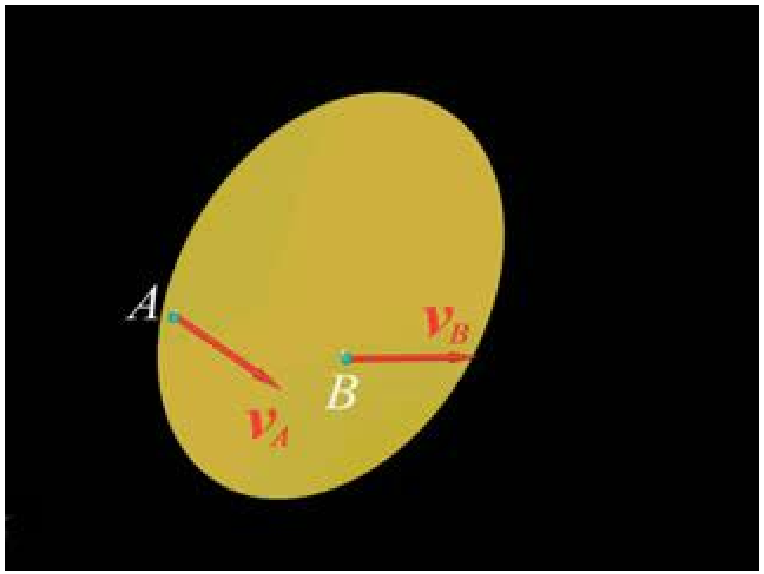
速度瞬心:P;

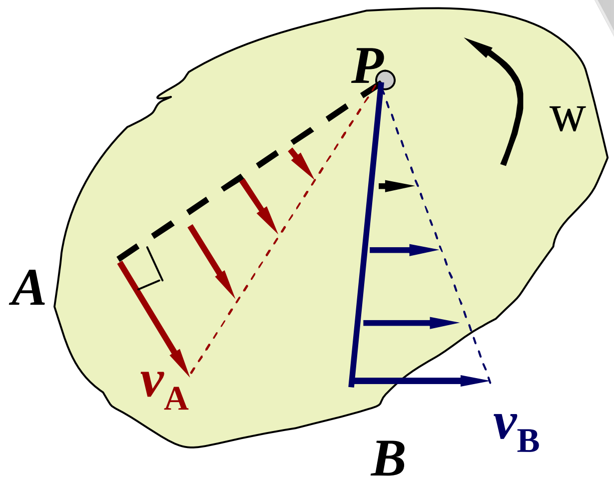
$PA = v_A / w$;

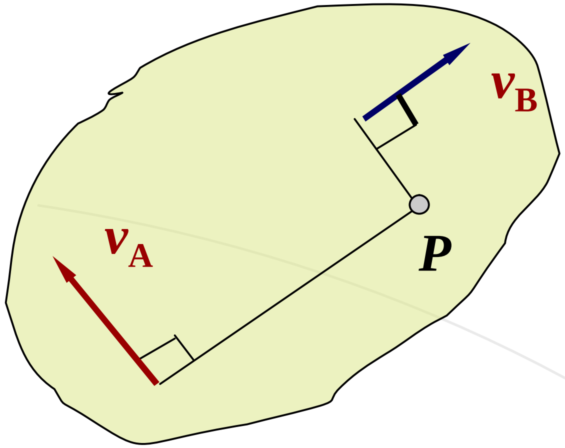
$v_P = v_A - v_{AP} = 0$;

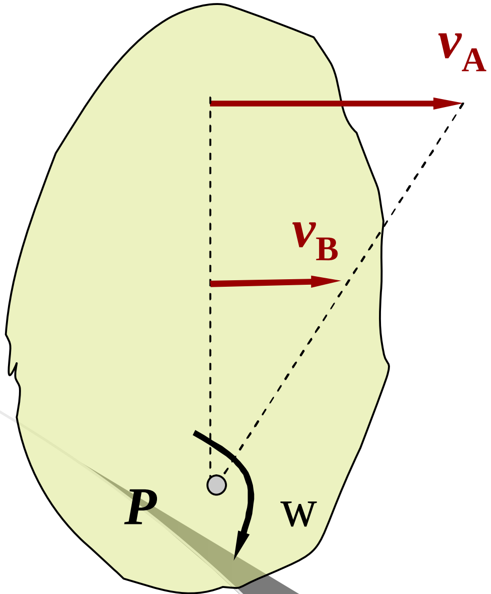
不同瞬时，不同瞬心;

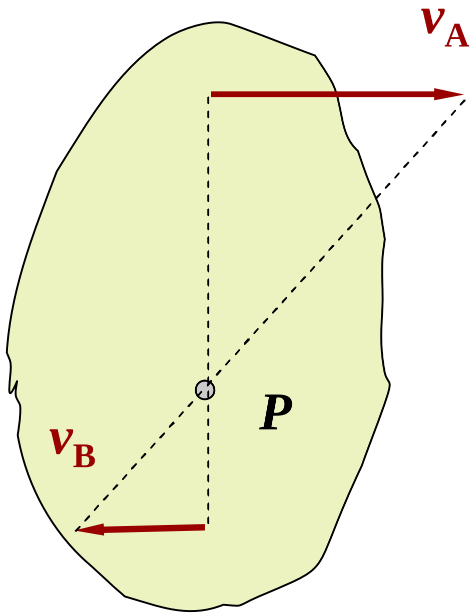


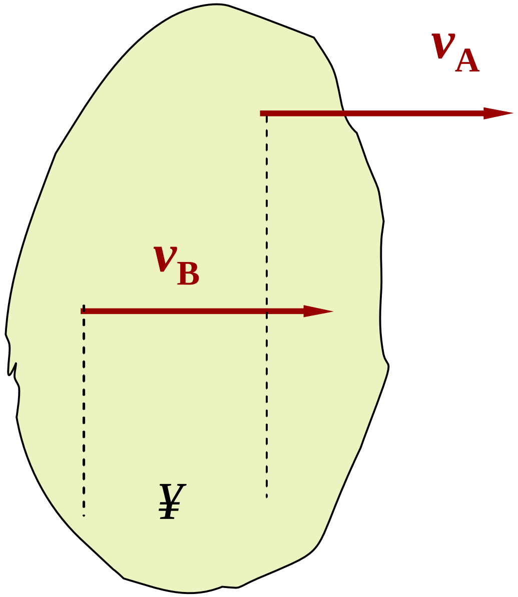


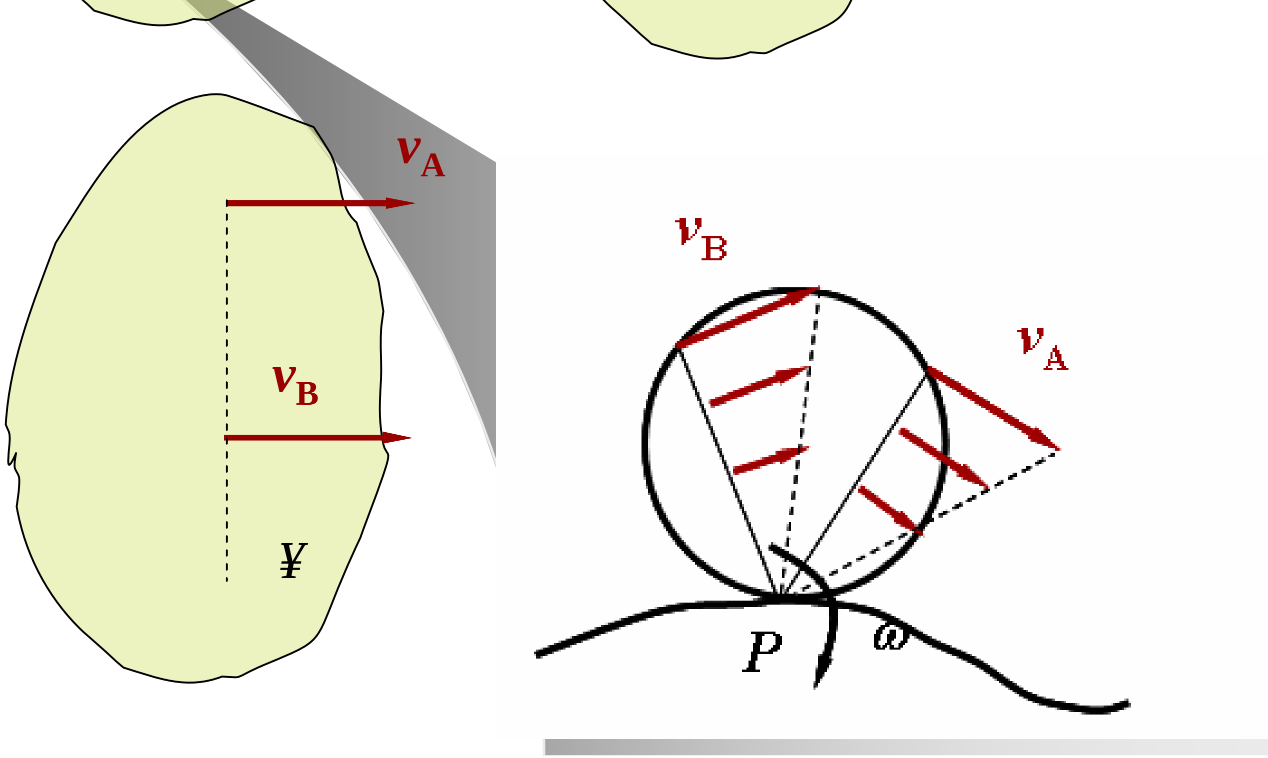












瞬心位置确定

例7—1A：椭园规机构如图示，杆0C绕0作匀角速度转动，已知:$OC=AC=AB=R$，$q=30^0$,求:滑块A,B的速度。

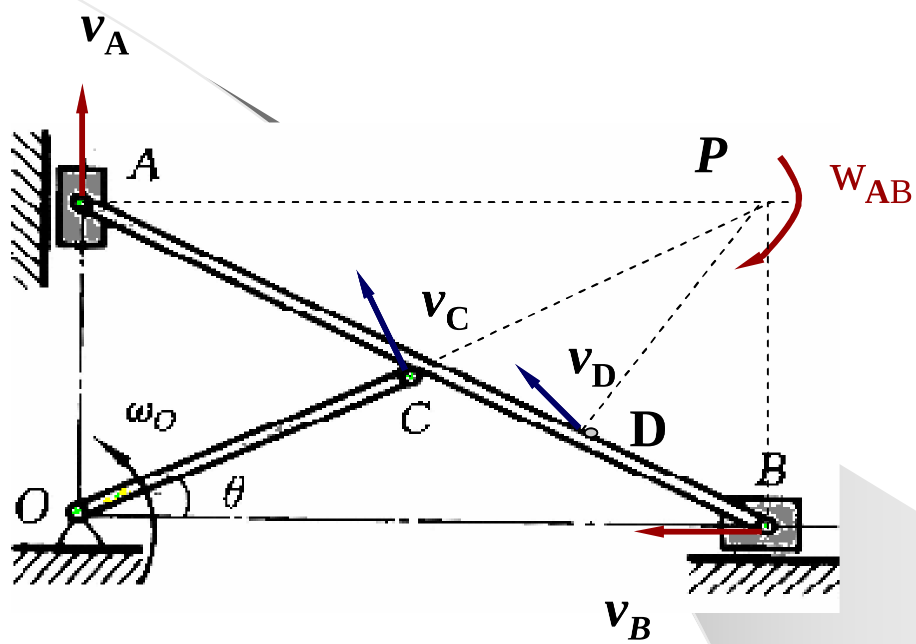
解:

瞬心法　　$v_c=Rw_0$;

$$\omega_{AB}=\frac{v_C}{R}=\omega_0\,;$$

$$v_A=2R\cos\theta\; w_{AB}=\sqrt{3}Rw_0.$$

$$v_B=2R\sin\theta\; w_{AB}=Rw_0.$$

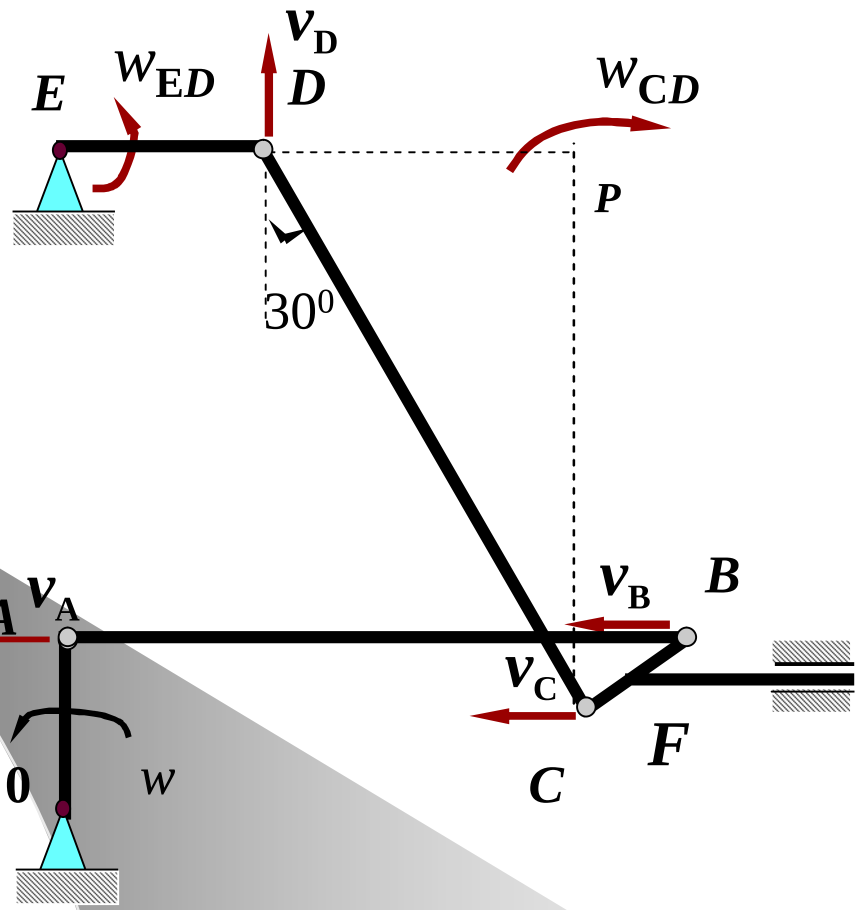


$v_D=PD\,w_{AB}$;

***P*点为**
基点求D点速度.

例7—2A： 机构如图示，杆0*A*绕0作匀角速度转动，已知:0*A*=*r*, *DC*=6*r*，,求:滑块*F*的速度和杆*ED*的角速度。

解: $v_A = rw$;



*AB*作瞬时平动: $v_A = v_B$;

*BC*作平动: $v_B = v_C$;

$v_C = PC\, w_{CD} = 6r\cos 30^0\, w_{CD}$;

$$w_{CD} = \frac{\sqrt{3}}{9} w;$$

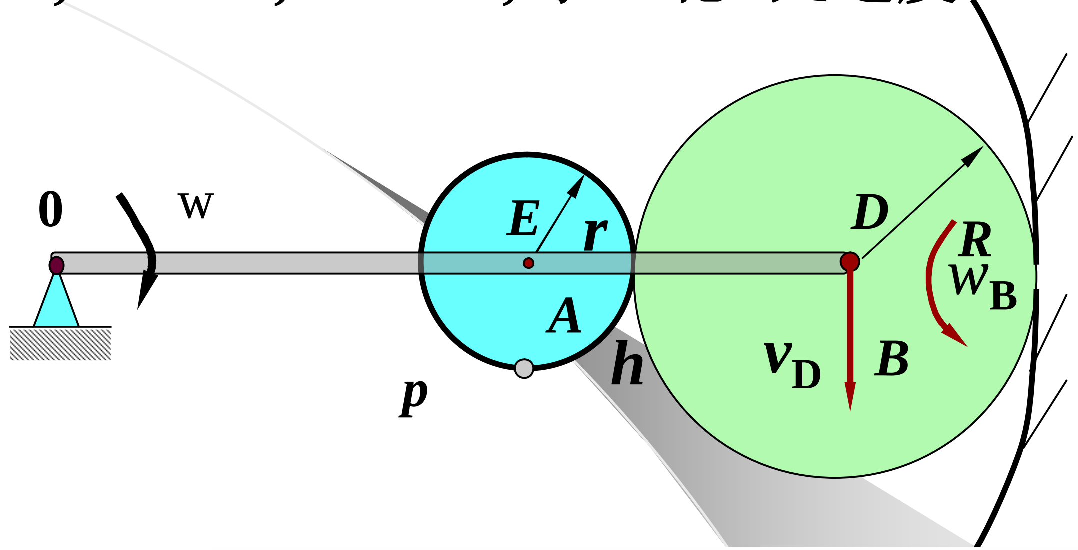
$v_D = 6r\cos 60^0\, w_{CD}$;

$$v_D = \frac{r\omega}{\sqrt{3}}; \qquad \omega_{ED} = \frac{\omega}{\sqrt{3}};$$

例7—3： *0AB*杆做匀速转动带动A、B摩檫轮,B摩檫轮与外曲面做纯滚动，已知:w=3 1/s,*0E*=8cm, *r* =4cm, *R*=9cm, 求:A轮P处速度。

解:

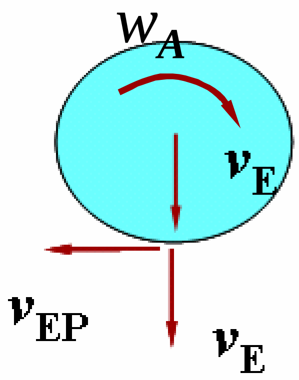
$v_D = w(0E + r + R) = 63;$



$w_B = \dfrac{63}{R}$ $v_h = w_B \times 2R = 126;$

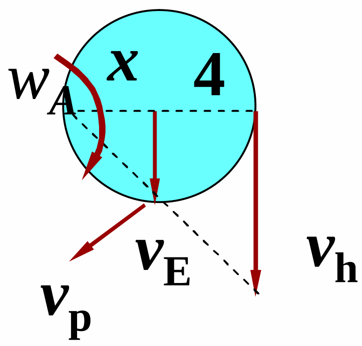
$v_E = 0E \cdot w = 24;$

[A] 瞬心法 $x = 0.9240;$



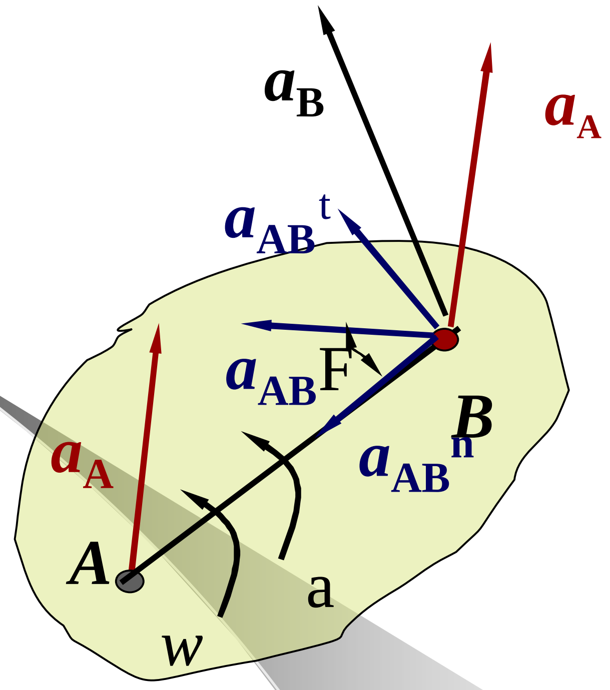
$$\frac{v_h}{4 + x} = \frac{v_E}{x} = w_A;\quad w_A = 25.5$$

$$v_p = \sqrt{(w_A r)^2 + v_E^2} = 104.8cm/s;$$



$w_A r = w_B 2R$ ？

§ 7–3 平面图形上各点的加速度



基点法(合成法)

$a_{AB}^{t}=a*AB$;　　　垂直AB;

$a_{AB}^{n}=w^{2}*AB$;　　　指向A;

$$a_{AB}=\sqrt{(a_{AB}^{n})^{2}+(a_{AB}^{\tau})^{2}}=AB\sqrt{\omega^{4}+\alpha^{2}}$$

$$\tan f=\frac{a_{AB}^{\tau}}{a_{AB}^{n}}=\frac{a}{w^{2}}$$

$a_{AB}=a_r$;　$a_A=a_e$;　$a_B=a_a$;

$$\vec{a}_B=\vec{a}_A+\vec{a}_{AB};$$

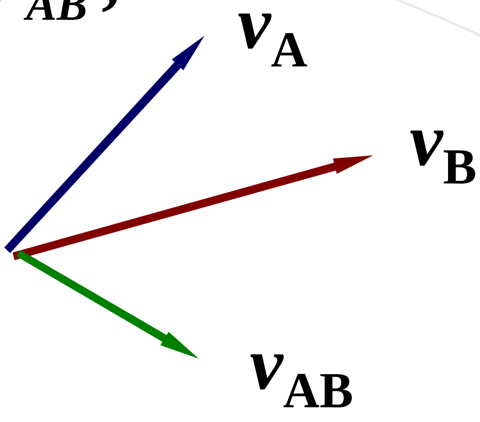
$$\vec{a}_B^{t}+\vec{a}_B^{n}=\vec{a}_A^{t}+\vec{a}_A^{n}+\vec{a}_{AB}^{t}+\vec{a}_{AB}^{n};$$

1.瞬时状态;
2.可解二个未知量
(大小，方向。

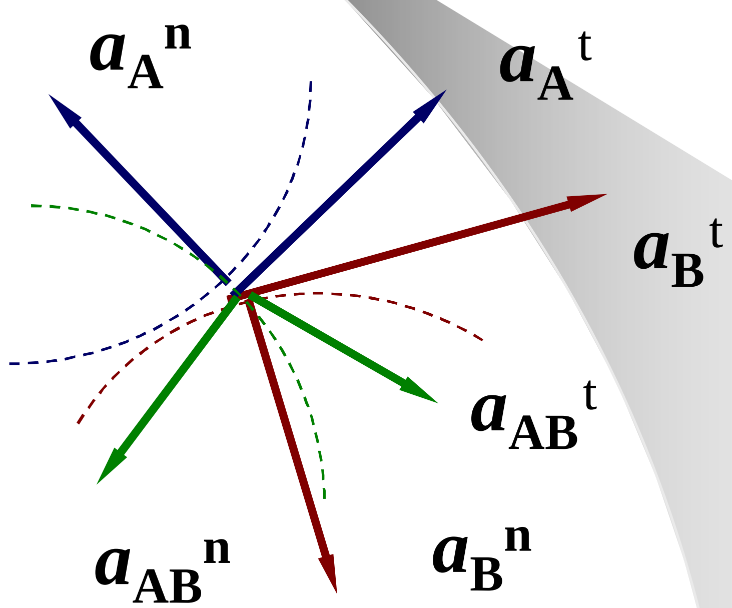
提问：为什么没有哥氏加速度?

是否有加速度投影法

$$\vec{v}_B = \vec{v}_A + \vec{v}_{AB};$$



[速度矢量图]



[加速度矢量图]

$$\vec{a}_B^t + \vec{a}_B^n = \vec{a}_A^t + \vec{a}_A^n + \vec{a}_{AB}^t + \vec{a}_{AB}^n;$$

例7—4： 往复机构如图示，杆0A绕0作匀角速度转动，已知:0A=r, q =60^0，求:滑块B的速度，加速度。

解：

$v_A = rw \qquad AB = l = \sqrt{3}r;$

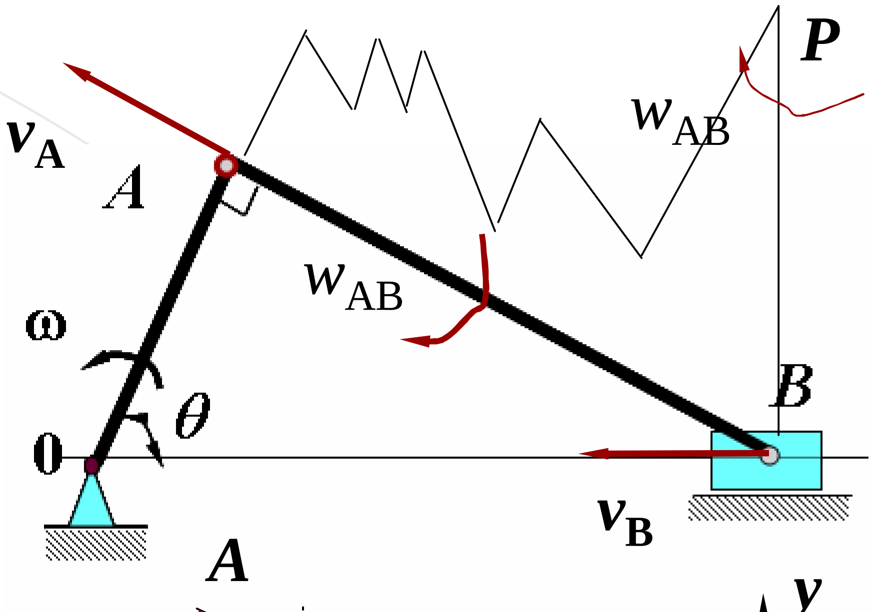
$AP = \sqrt{3}l = 3r; \qquad BP = 2l;$

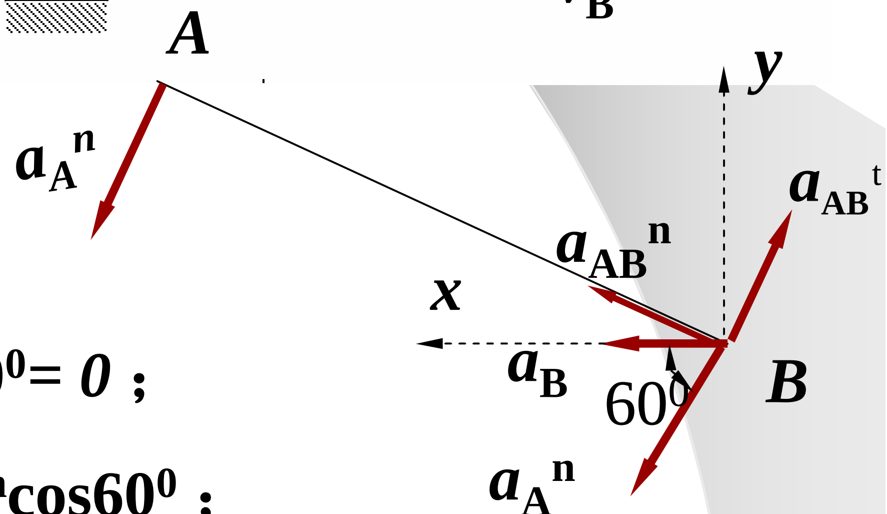
$\omega_{AB} = \frac{v_A}{AP} = \frac{\omega}{3}; \qquad v_B = \frac{2\sqrt{3}w}{3}r;$

$a_A^t = \alpha r = 0; \qquad a_A^n = w^2 r;$

$a_{AB}^n = w_{AB}^2 \times BA = 2\frac{\sqrt{3}r}{9}w^2;$

$a_{AB}^\tau = \frac{8}{9}r\omega^2; \qquad a_B = -\frac{2}{9}r\omega^2;$





y: $a_{AB}^t \cos 30^0 + a_{AB}^n \cos 60^0 - a_A^n \cos 30^0 = 0$;

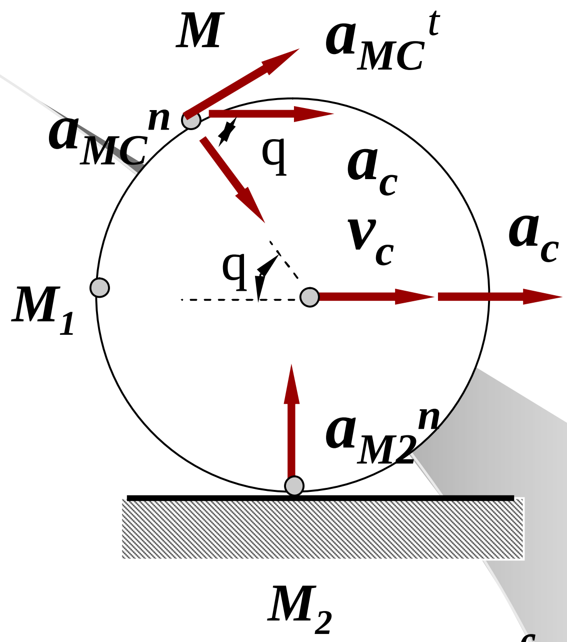
x: $a_B = -a_{AB}^t \cos 60^0 + a_{AB}^n \cos 30^0 + a_A^n \cos 60^0$;

$\vec{a}_B^t + \vec{a}_B^n = \vec{a}_A^t + \vec{a}_A^n + \vec{a}_{AB}^t + \vec{a}_{AB}^n;$

如以AB轴投影方程

例7—5：园轮做纯滚动，已知:半径=r,轮心速度，加速度,求:园轮边缘任意点M的加速度。

解： 纯滚动条件：



$F\ r=x;\qquad w\ r=v_c;\qquad a\ r=a_c;$

$a_{MC}^{n}=w^2 r=\frac{v_C^2}{r};\ a_{MC}^{t}=a\ r=a_c;$

$a_M=\sqrt{a_{Mx}^2+a_{My}^2}$

x: $a_{Mx}=a_C+a_{MC}^{t}\sin q+a_{MC}^{n}\cos q$;

y: $a_{My}=a_{MC}^{t}\cos q-a_{MC}^{n}\sin q$;

讨论：1.求M_1的加速度; $q=0$;

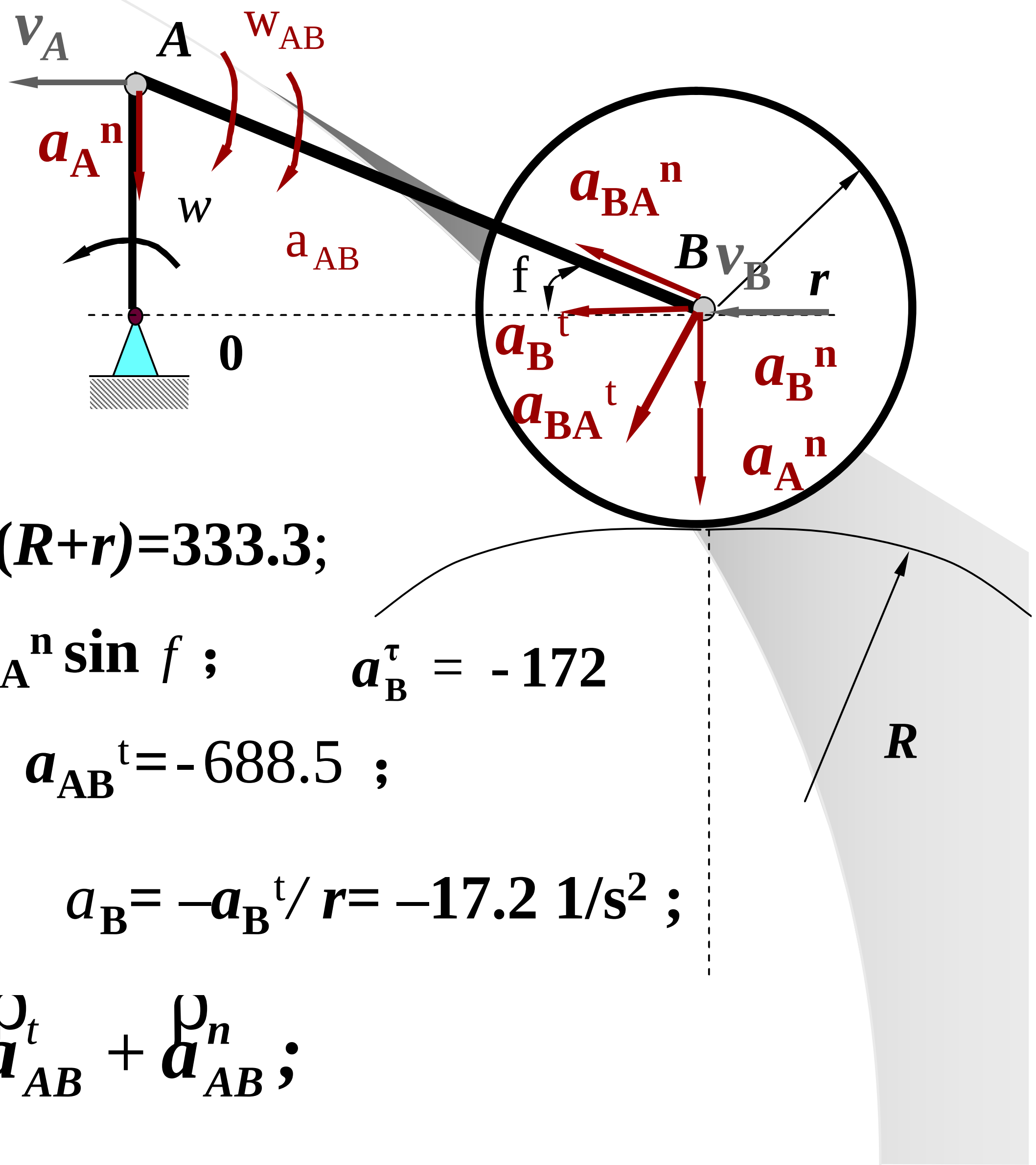
$$a=\sqrt{(a_C+\frac{v_C^2}{r})^2+a_C^2};\qquad \tan f=\frac{a_C}{a_C+\frac{v_0^2}{r}};$$

2.求M_2的加速度;$q=-p/2$;　$a_x=a_C-a_C=0$;　$a_y=v_0^2/r$;

M_2是速度瞬心，但加速度不等于零。

例7—6： 园轮在曲面做纯滚动，*0A*杆做匀速转动，已知:w=10 1/s, *0A*=*r*=10cm,*AB*=*l*=40cm, *R*=20cm,求:园轮,杆AB的角加速度。

解： $v_A = v_B = 10w;$



$w_B = v_B / r = w; \quad w_{AB} = 0;$

$$\cos f = \frac{\sqrt{40^2 - 10^2}}{40} = 0.968;$$

$$\sin f = \frac{1}{4} = 0.25; \qquad a_{BA}^n = 0;$$

$a_A^n = w^2 r = 1000; \quad a_B^n = v_B^2/(R+r) = 333.3;$

$AB:\ a_B^t \cos f - a_B^n \sin f = -a_A^n \sin f;$ $\quad a_B^\tau = -172$

$y:\ a_B^n = a_{AB}^t \cos f + a_A^n;$ $\quad a_{AB}^t = -688.5;$

$a_{AB} = -a_{AB}^t / l = -17.2\ 1/s^2;$

$a_B = -a_B^t / r = -17.2\ 1/s^2;$

$$\vec{a}_B^t + \vec{a}_B^n = \vec{a}_A^t + \vec{a}_A^n + \vec{a}_{AB}^t + \vec{a}_{AB}^n;$$

例7—7： 图示连杆机构，0_1A以匀角速度w=2**rad/s**转动,并带动滑块运动,已知: $0_1A = 0_2B = CD = 20cm$, $AB = 0_10_2 = 40cm$, 求:*CD*杆中点E速度,加速度。

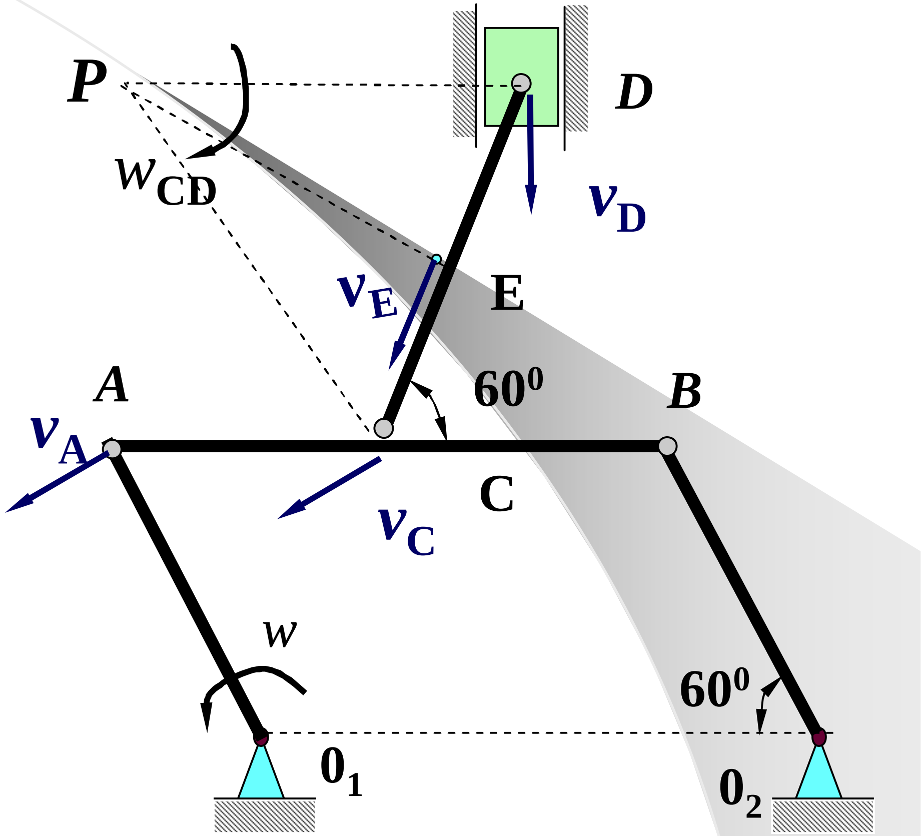
解： 1:求速度

$v_A = 20w = 40;$

$v_A = v_C = v_B;$ $v_C = v_D;$

$v_E = v_C \cos 30^0 = 20\sqrt{3}\ cm/s;$

$w_{CD} = v_C / CP = 2\ rad/s;$



例7—7A： w=2rad/s, $0_1A = 0_2B=CD$=20cm ,$AB=0_10_2$ =40cm, 求:a_E。

解： 2:求加速度

$a_A^n = a_C^n$ =80cm/s²;

a_{EC}^n= 40cm/s²; a_{DC}^n= 80cm/s²;

求E加速度有三个未知量a_{EC}^t，a_E大小,方向。

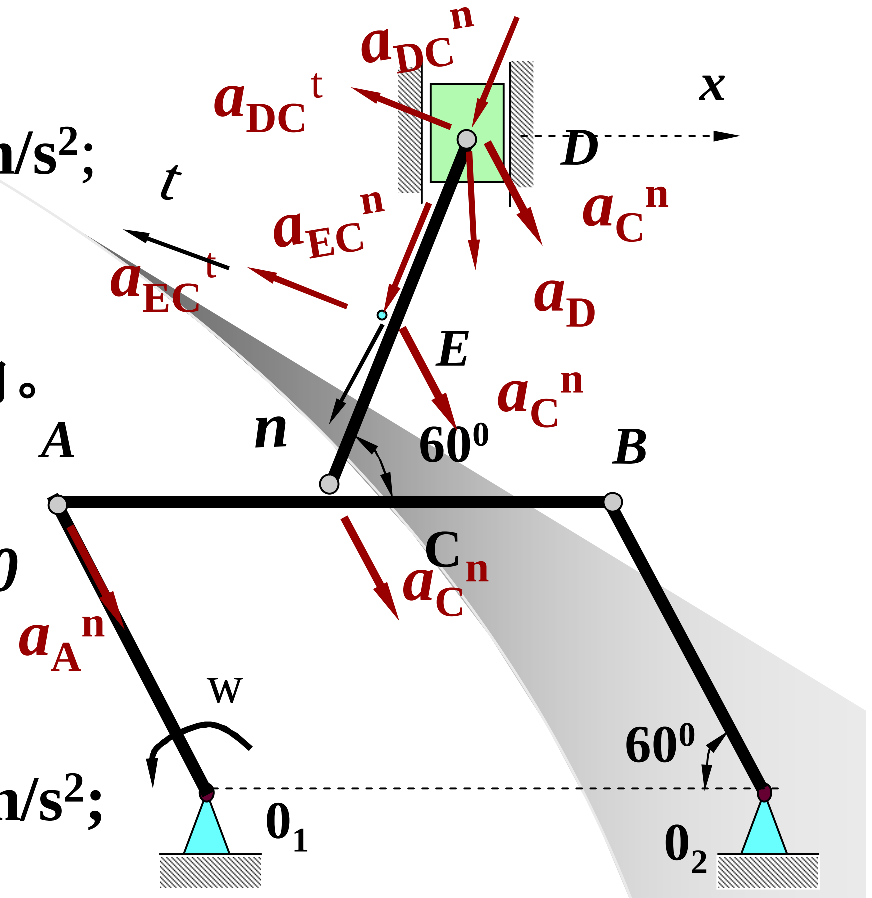
先求D点； D点:

x: $a_C^n \cos 60^0 - a_{DC}^n \cos 60^0 - a_{DC}^t \cos 30^0 = 0$

$a_{DC}^t = 0$; $a_{DC} = 0$; $\vec{a}_E = \vec{a}_E^t + \vec{a}_E^n$;

E点： n: $a_E^n = a_C^n \cos 60^0 + a_{EC}^n$ = 80cm/s²;

t: $a_E^t = -a_C^n \cos 30^0 + a_{EC}^t$ =69.2cm/s²;

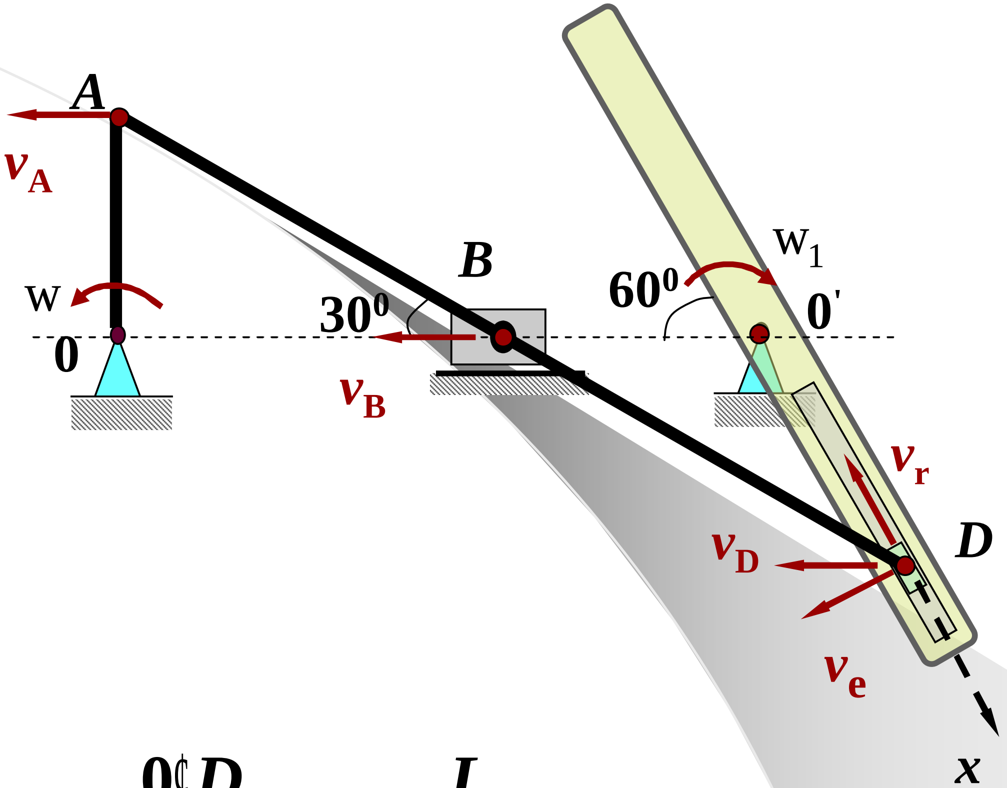


$\tan b = \dfrac{a_E^t}{a_E^n}$; $b = 40.8^0$。

$$a_E = \sqrt{(a_E^n)^2 + (a_E^\tau)^2} = 105.8\text{cm}/\text{s}^2;$$

是否可从D点求E点的加速度?

例7—8：曲柄绕0匀速转动，已知:w, $AB=BD=L, 0A=r$, 求:w_1。



解： [速度:*B*]　$v_A = v_B = v_D = rw$，

$$\vec{v}_a = \vec{v}_e + \vec{v}_r = v_D\,;$$

$$v_e = v_D\sin60^0 = rw\sin60^0,$$

$$v_r = v_D\sin30^0 = rw\sin30^0,$$

$$w_1 = v_e / D0', \qquad \frac{0¢D}{\sin30^0} = \frac{L}{\sin120^0}$$

$$0¢D = \frac{L}{\sin60^0}\sin30^0 = \frac{r}{\sin60^0} \qquad w_1 = \frac{3}{4}w$$

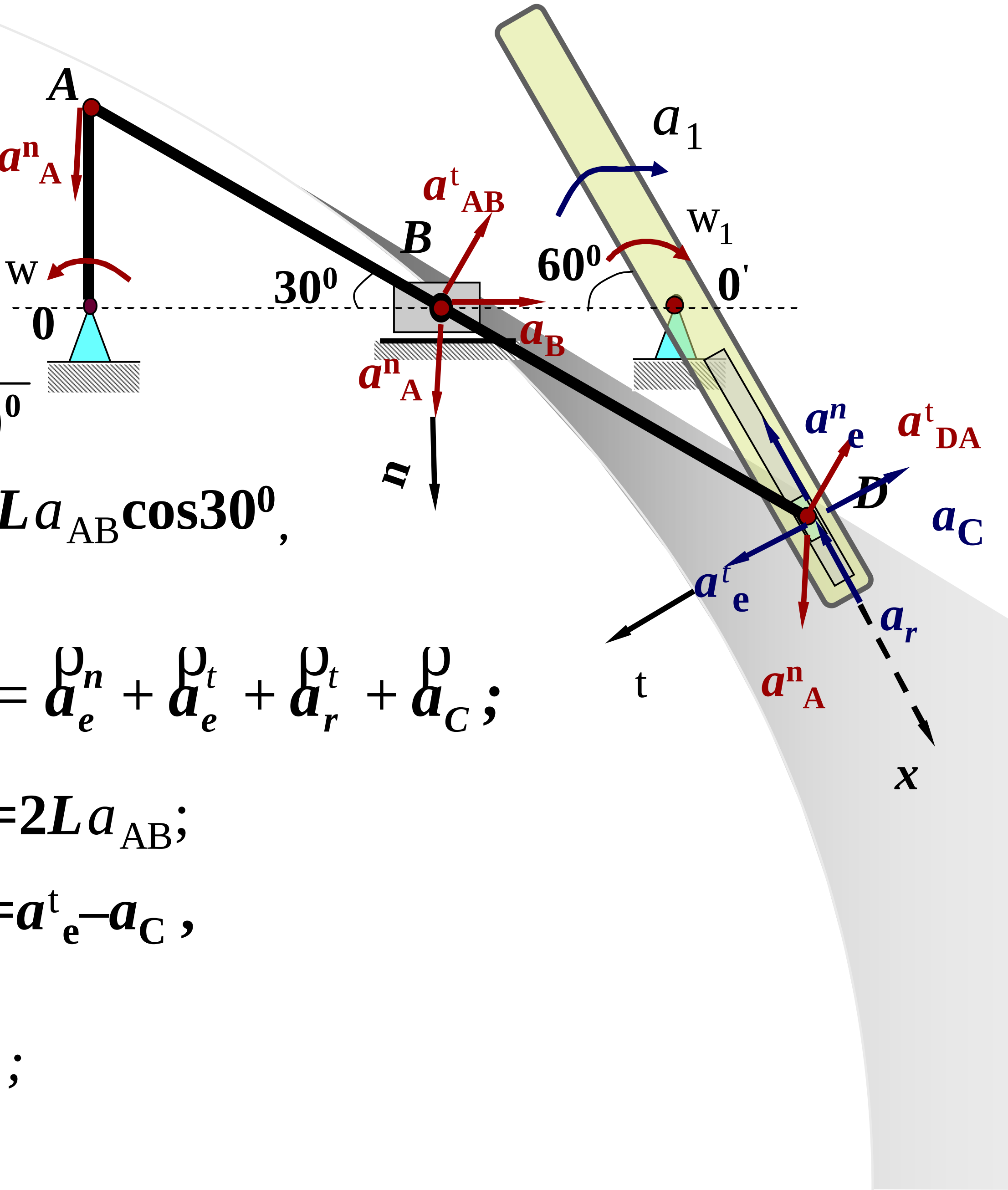
例7—8A：曲柄绕0匀速转动，已知:w, $AB=BD=L, 0A=r$, 求:a_1。

解： $a^n_A = w^2 r$; $a^n_{AB} = 0$;

$$\vec{a}_B = \vec{a}_A + \vec{a}_{AB} =$$

$$\vec{a}^n_A + \vec{a}^t_{AB} + \vec{a}^n_{AB};$$

$$a_{AB} = \frac{a^t_{AB}}{L} = \frac{rw^2}{L\cos 30^0}$$



n: $0 = a^n_A - a^t_{BA}\cos 30^0 = rw^2 - L a_{AB}\cos 30^0$,

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$$\vec{a}_D = \vec{a}^n_A + \vec{a}^t_{DA} + \vec{a}^n_{DA}; \quad \vec{a}_D = \vec{a}^n_e + \vec{a}^t_e + \vec{a}^t_r + \vec{a}_C;$$

$$a_C = 2v_r w_1; \qquad a^t_{DA} = 2L a_{AB};$$

t: $a^n_A \sin 30^0 - a^t_{DA}\cos 30^0 = a^t_e - a_C$,

$$a^\tau_e = -\frac{\omega^2 r}{2}; \quad \alpha_1 = -\frac{\omega^2 r}{2L};$$


本章结束