理论力学

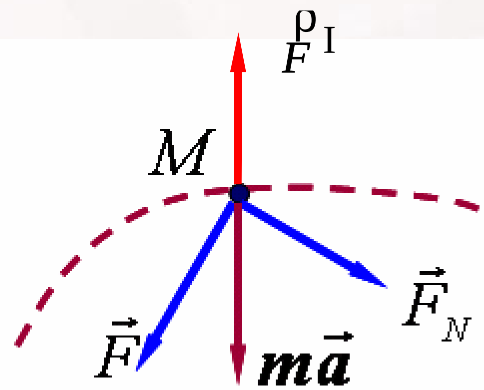
动力学

第十二章 达朗伯原理(动静法)

§12-1 惯性力 质点系的达朗伯原理

$$\vec{F} + \vec{F}_N = m\vec{a} \qquad \vec{F} + \vec{F}_N - m\vec{a} = 0$$

$$\vec{F}_I = -m\vec{a} \qquad \vec{F} + \vec{F}_N + \vec{F}_I = 0$$



一、惯性力的大小与方向

$$\vec{F}_I = -m\vec{a}$$

二、惯性力的作用物体

惯性力作用在施力物体上

一、质点的达朗伯原理

$$\vec{F}+\vec{F}_{\mathrm{N}}+\vec{F}_{\mathrm{I}}=0$$

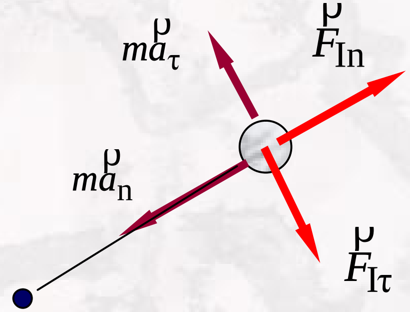
作用在质点上的主动力、约束力与惯性力构成一平衡力系。

主动力+约束力+惯性力=平衡力系

直角坐标系：

$$F_x^{\mathrm{I}}=-ma_x=-m\frac{\mathrm{d}x^2}{\mathrm{d}t^2}$$

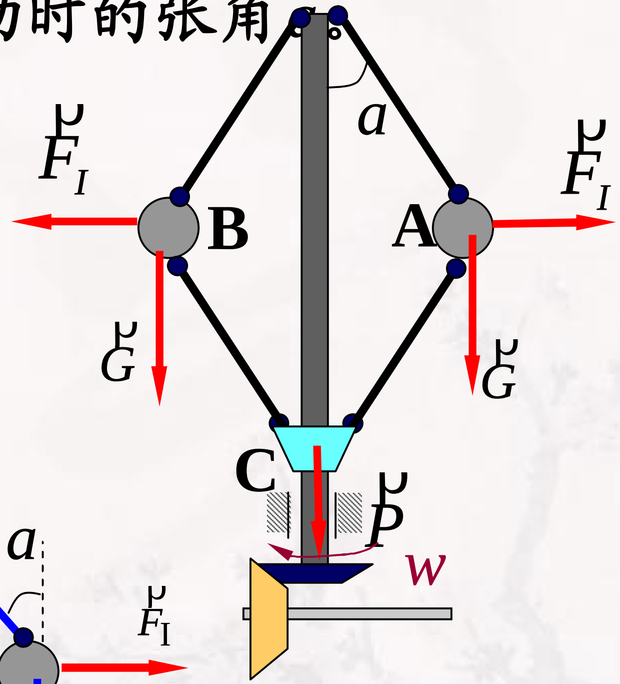
$$F_y^{\mathrm{I}}=-ma_y=-m\frac{\mathrm{d}y^2}{\mathrm{d}t^2} \qquad F_z^{\mathrm{I}}=-ma_z=-m\frac{\mathrm{d}z^2}{\mathrm{d}t^2}$$



自然坐标系：

$$F_{\mathrm{n}}^{\mathrm{I}}=-ma_{\mathrm{n}}=-m\frac{v^2}{\rho} \qquad F_\tau^{\mathrm{I}}=-ma_\tau=-m\frac{\mathrm{d}v}{\mathrm{d}t}$$

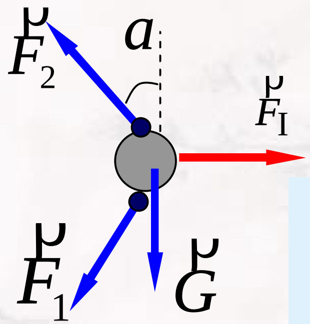
例12-1：飞球调速器以等角速度ω转动，已知：重锤重P，飞球A、B均重G，各联杆长为b，试求：A、B在转动时的张角α。



解： 惯性力： $F_I=\dfrac{G}{g}b\omega^2\sin\alpha$

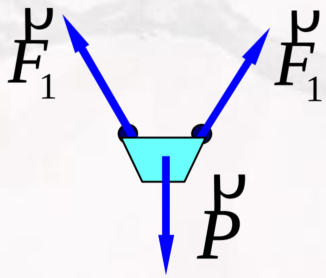
[A]: $\sum F_{ix}=0$ $\dfrac{G}{g}b\omega^2\sin\alpha-(F_1+F_2)\sin\alpha=0$

$\sum F_{iy}=0$ $G+(F_1-F_2)\cos\alpha=0$

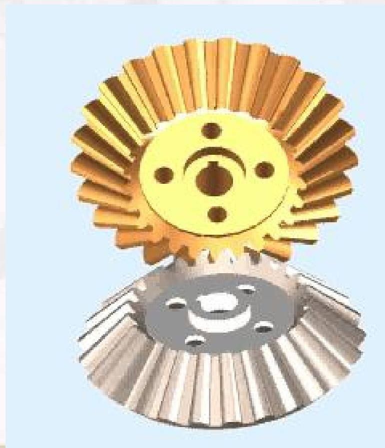


得： $F_1=\dfrac{G}{2g}b\omega^2-\dfrac{G}{2\cos\alpha}$

[C]: $F_1=\dfrac{P}{2\cos\alpha}$



得： $\cos\alpha=\dfrac{G+P}{Gb\omega^2}g$



二、 质点系达朗伯原理

对M_i质点：$\vec{F}_{Ii} = -m_i \vec{a}_i$

$$\vec{F}_i^e + \vec{F}_i^i + \vec{F}_{Ii} = 0$$

$$\sum \vec{F}_i^i \equiv 0、\sum \vec{M}_O(\vec{F}_i^i) \equiv 0$$

对质点系：

$$\sum \vec{F}^e + \sum \vec{F}_I = 0 \quad （主矢）$$

$$\begin{cases} \sum F_x^e + \sum F_{Ix} = 0 \\ \sum F_y^e + \sum F_{Iy} = 0 \\ \sum F_z^e + \sum F_{Iz} = 0 \end{cases}$$

质点系运动的每一瞬时，每个质点的惯性力与作用于该质点系的外力组成平衡力系。

$$\sum \vec{M}_0(\vec{F}_i^e) + \sum \vec{M}_0(\vec{F}_{Ni}) + \sum \vec{M}_0(\vec{F}_{Ii}) = 0 \quad （主矩）$$

$$\begin{cases} \sum M_x(\vec{F}_i^e) + \sum M_x(\vec{F}_{Ni}) + \sum M_x(\vec{F}_{Ii}) = 0 \\ \sum M_y(\vec{F}_i^e) + \sum M_y(\vec{F}_{Ni}) + \sum M_y(\vec{F}_{Ii}) = 0 \\ \sum M_z(\vec{F}_i^e) + \sum M_z(\vec{F}_{Ni}) + \sum M_z(\vec{F}_{Ii}) = 0 \end{cases}$$

例12-2：在滑轮机构中，物块A重P_1=1kN，物块B重P_2=0.5kN，试求：轴承处的约束力。

解：因：$r_1 = 2r_2$　有：$a_1 = 2a_2$

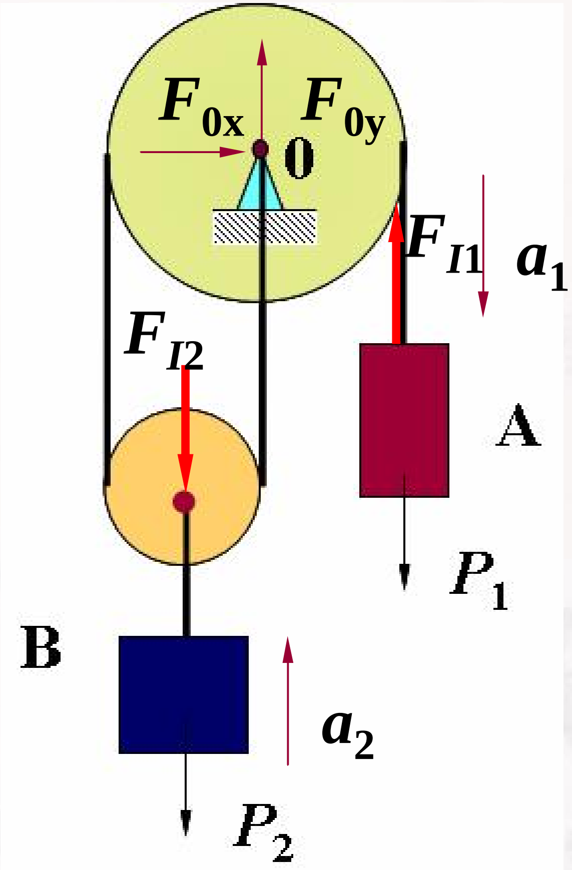
惯性力：$F_{I1} = \frac{P_1}{g} a_1$　　$F_{I2} = \frac{P_2}{g} a_2$

$$\sum M_0 = 0 \quad (F_{I1} - P_1) r_1 + (P_2 + F_{I2}) \frac{r_1}{2} = 0$$

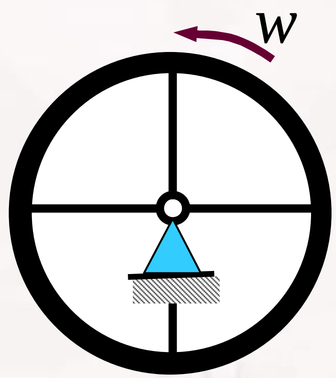
有：$$a_1 = \frac{P_1 - \frac{P_2}{2}}{P_1 + \frac{P_2}{4}} g$$

$$\sum F_x = 0 \qquad F_{0x} = 0$$

$$\sum F_y = 0 \qquad P_1 + P_2 + \frac{P_2}{2g} a_1 - \frac{P_1}{g} a_1 - F_{0y} = 0 \qquad F_{Oy} = 1\text{kN}$$



例12-3：飞轮重P，半径为R，以w匀角速度转动，轮辐质量不计。试求:轮缘横截面的张力。



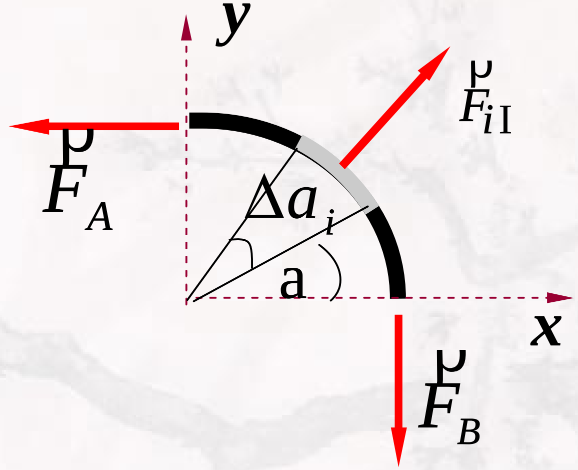
解：

$$F_{i\mathrm{I}} = \frac{P}{2pRg} R\Delta a_i R w^2$$

$$å\, F_{ix} = 0 \qquad \sum F_{i\mathrm{I}} \cos a - F_A = 0$$

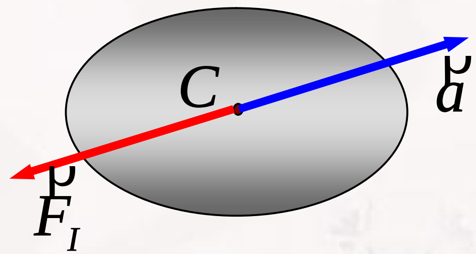
$$F_A = \int_0^{\frac{p}{2}} \frac{P}{2pg} R w^2 \cos a \,\mathrm{d}a = \frac{PR\omega^2}{2\pi g}$$

$$F_A = F_B$$



§12-2　刚体的惯性力系简化

1、移动刚体： $\vec{F}_{\mathrm{I}} = -m\vec{a}_C$

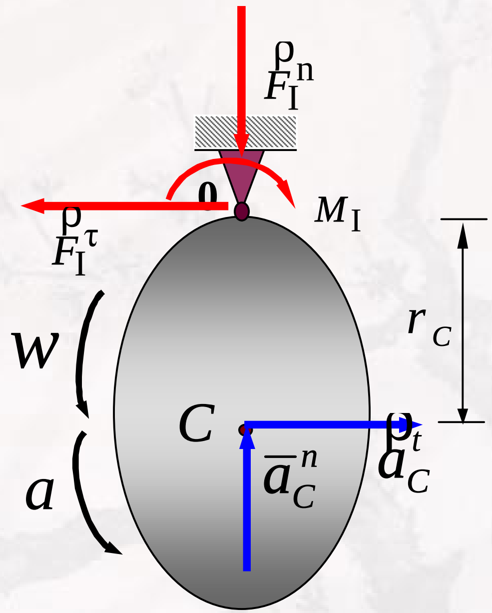


$$\vec{M}_C(\vec{F}_{\mathrm{I}}) = -\frac{\mathrm{d}\vec{L}_C}{\mathrm{d}t} = -J_C\vec{\alpha}$$

$$\because \vec{\alpha} = \mathbf{0} \qquad \therefore \vec{M}_C(\vec{F}_{\mathrm{I}}) = 0$$

刚体移动时，惯性力系向质心C简化，得到作用在质心上的一个合惯性力。

2、定轴转动刚体： （向轴心O简化 ）



$$\vec{F}_{\mathrm{I}} = -m\vec{a}_C \qquad \vec{M}_0(\vec{F}_{\mathrm{I}}) = -\frac{\mathrm{d}L_0}{\mathrm{d}t} = -J_0\alpha$$

$$M_{\mathrm{I}o} = -J_o\alpha \qquad F_{\mathrm{I}}^{\mathrm{n}} = mr_C w^2$$

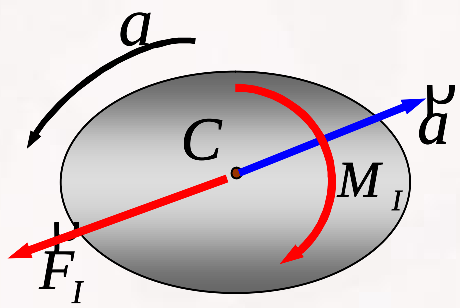
$$F_{\mathrm{I}}^{\tau} = mr_C\alpha \qquad M_{\mathrm{I}} = J_O\alpha$$

3、平面运动刚体

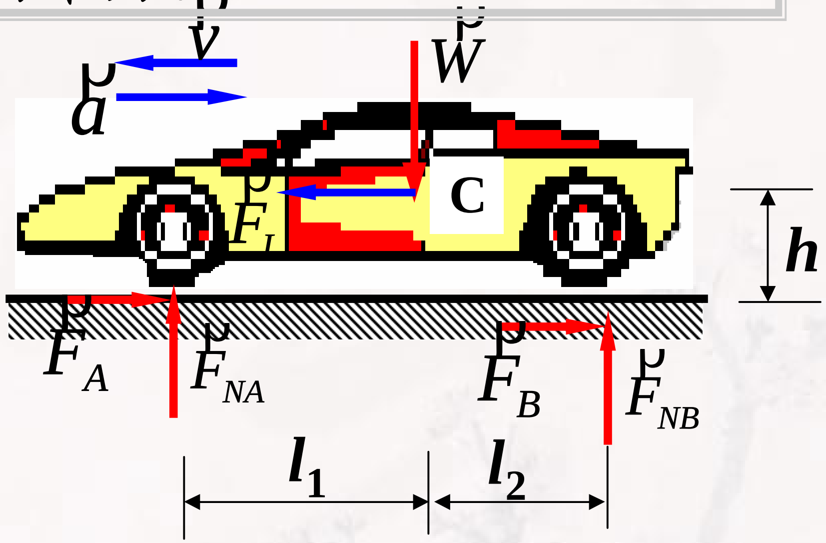
$$\vec{F}_{\mathrm{I}} = -m\vec{a}_C$$

$$M_C(\vec{F}_{\mathrm{I}}) = -\frac{\mathrm{d}L_C}{\mathrm{d}t} = -J_C a$$

$$M_{\mathrm{I}C} = -J_C a$$



例12-4：重W的轿车，以速度v_0行驶，因刹车制动，车滑行一段S才停车。试求：前、后轮的法向约束力。



解：　作减速运动：

$$a = \frac{v_S^2 - v^2}{2S} = \frac{v_0^2}{2S} \qquad F_I = \frac{W}{g}a$$

$$\sum M_B = 0 \quad F_{NA}(l_1 + l_2) - Wl_2 - F_I h = 0$$

$$F_{NA} = \frac{W}{l_1 + l_2}(l_2 + \frac{a}{g}h)$$

$$\sum F_y = 0 \quad F_{NA} + F_{NB} - W = 0$$

故刹车时 $F_{NA}>F_{NB}$，　车头下沉

$$F_{NB} = \frac{W}{l_1 + l_2}(l_1 - \frac{a}{g}h)$$

当 $a=0$

$$F'_{NA} = \frac{W}{l_1 + l_2}l_2 \qquad F'_{NB} = \frac{l_1}{l_1 + l_2}W$$

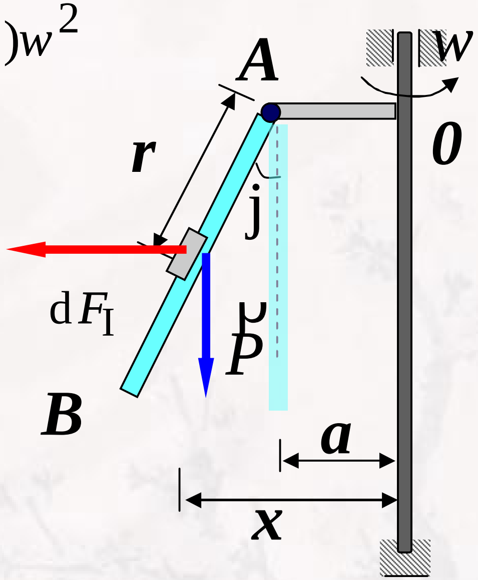
例12-5：铅直轴以角速度转动，水平杆OA固定在轴上，在A点绞连匀质杆AB。设OA=0,AB=L，试求图示情况下的角速度w值。

解： 方法一：积分

$$\mathrm{d}F_{\mathrm{I}} = \mathrm{d}m x w^2 = \frac{P}{gl}\mathrm{d}r(a + r\sin j)w^2$$

$$x = a + r\sin j$$

$$\sum M_A = 0 \qquad P\frac{l}{2}\sin j - \int_0^l \mathrm{d}F_{\mathrm{I}} r\cos j = 0$$



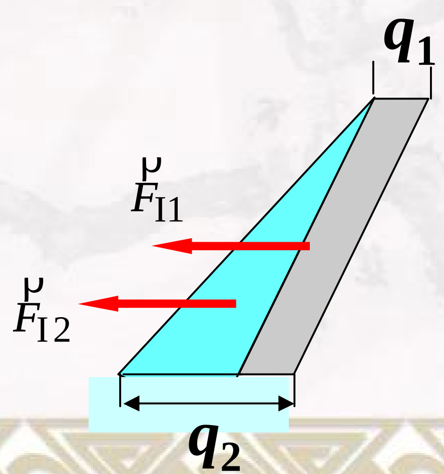
方法二：直接法

$$q_1 = \frac{P}{g}\frac{w^2 a}{l} \qquad q_2 = \frac{P}{g}\frac{w^2(a + l\sin j)}{l} = q_1 + \frac{P}{g}\frac{w^2 l\sin j}{l}$$

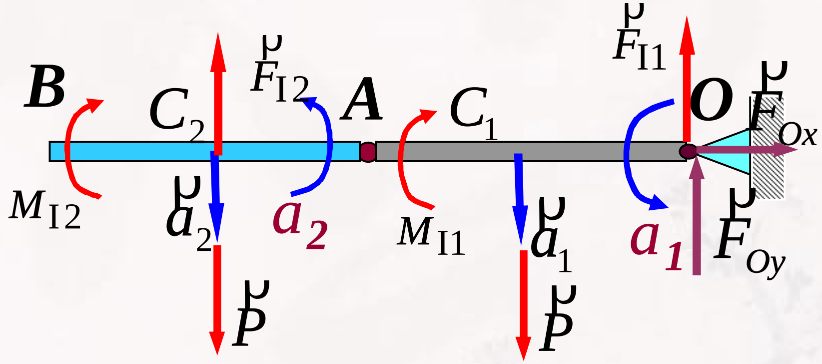
$$F_{\mathrm{I}1} = q_1 \cdot l \qquad F_{\mathrm{I}2} = \frac{p}{g}w^2\sin j\,\frac{1}{2}l$$

$$\sum M_A = 0, \qquad P\cdot\frac{l}{2}\sin j = F_{\mathrm{I}1}\frac{l}{2}\cos j + F_{\mathrm{I}2}\frac{2l}{3}\cos j$$

$$w = \sqrt{\frac{3g\sin j}{3a\cos j + l\sin 2j}}$$



例12-6: 杆长均为b、重力均为P 的均匀细杆从水平位置无初速开始运动，试求两杆在该瞬时的角加速度。



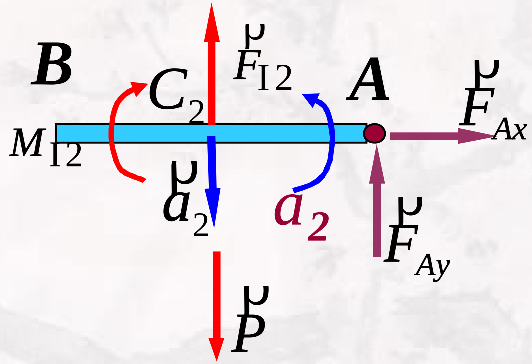
解： $F_{I1}=\dfrac{P}{g}a_1$ $F_{I2}=\dfrac{P}{g}a_2$

$M_{I1}=J_O\alpha_1$ $M_{I2}=J_{C2}\alpha_2$

$a_2=b\alpha_1+\dfrac{b}{2}\alpha_2$ $a_1=\dfrac{b}{2}\alpha_1$

[整体] $\sum M_O=0$

$$(P-F_{I2})\frac{3}{2}b+P\frac{b}{2}-\frac{P}{3g}b^2\alpha_1-\frac{P}{12g}b^2\alpha_2=0$$



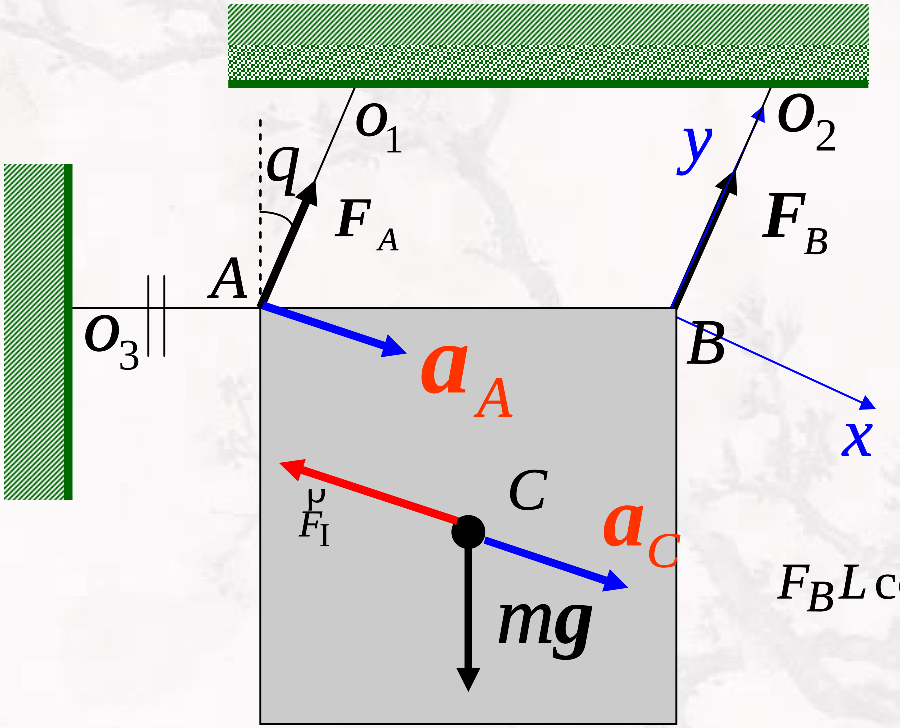
[AB] $\sum M_A=0$

$$(P-F_{I2})\frac{b}{2}-\frac{P}{12g}b^2\alpha_2=0$$

解得： $11\alpha_1+5\alpha_2=\dfrac{12g}{b}$ $3\alpha_1+2\alpha_2=\dfrac{3g}{b}$

有： $\alpha_1=\dfrac{9g}{7b}$， $\alpha_2=-\dfrac{3g}{7b}$

例12-7：已知：$m, q, AO_1 \underline{\underline{//}} BO_2, O_1O_2 \underline{\underline{//}} AB$，试求水平绳切断后的瞬时，板质心加速度和两个绳索的拉力。



解：受力分析与运动分析

$$F_I = ma_c$$

建立"平衡方程"，并求解

$$\sum F_x = 0 \quad mg\sin q - F_I = 0$$

$$a_C = g\sin q$$

$$\sum M_A = 0$$

$$F_B L\cos q - mg\frac{L}{2} - F_I\frac{L}{2}\cos q + F_I\frac{L}{2}\sin q = 0$$

$$F_B = \frac{mg}{2}(\sin q + \cos q)$$

$$\sum F_y = 0 \quad F_B + F_A - mg\cos q = 0$$

$$F_A = \frac{mg}{2}(\cos q - \sin q)$$

例12-8：匀质转轴重G，质心C到转轴的距离是e，转轴以匀速度w绕水平轴转动。试求当质心C 转动最低位置时轴承所受压力。

解：

$$a = 0,\ M_I = 0 \qquad F_I = mew^2$$

$$\sum M_B = 0$$

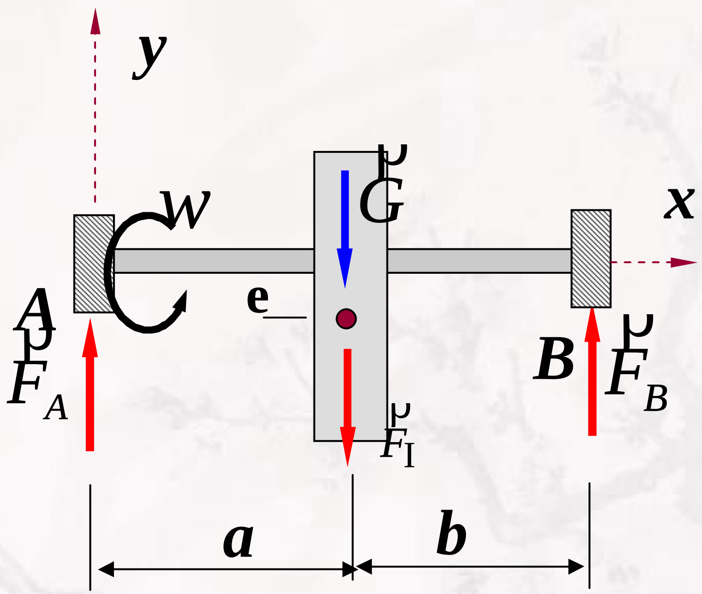
$$(F_I + G)b - F_A(a+b) = 0$$

$$\sum M_A = 0$$

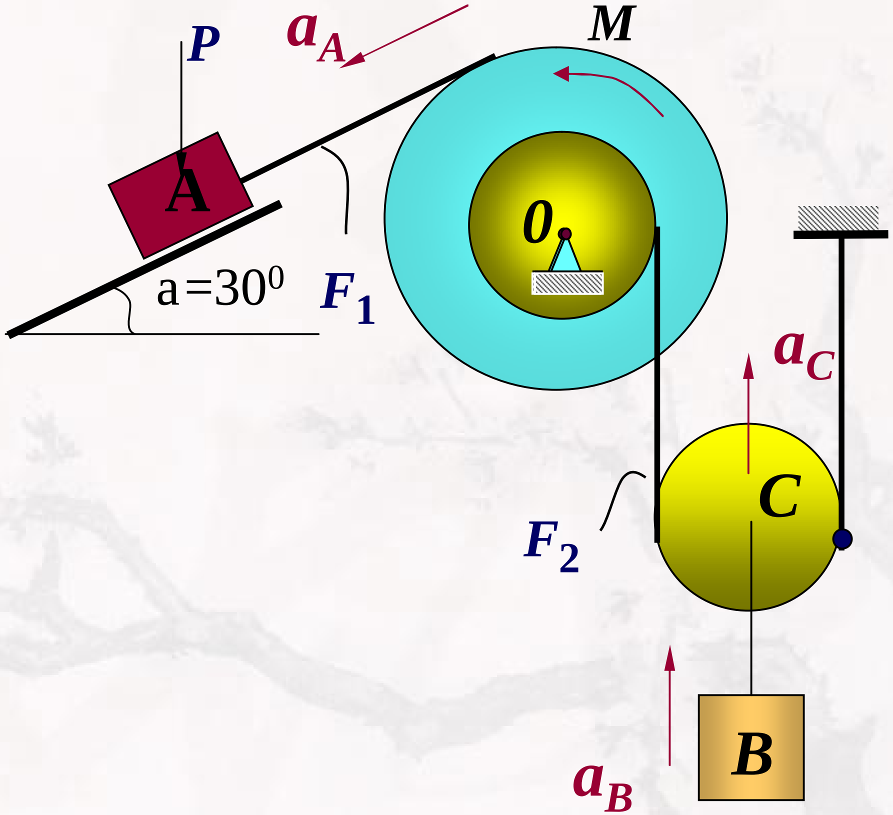
$$F_B(a+b) - (F_I + G)a = 0$$

$$F_A = \frac{b}{a+b}(F_I + G) = \frac{b}{a+b}\left(\frac{ew^2}{g} + 1\right)G$$

$$F_B = \frac{a}{a+b}(F_I + G) = \frac{a}{a+b}\left(\frac{ew^2}{g} + 1\right)G$$



例11-10A 运动机构如图示，已知：$m_A=m, m_B=m/2, m_C=m/3$,鼓轮的迴转半径为r，质量为m,鼓轮小半径为r，大半径为R， 轮C的半径为r，物体A接触的摩擦因数为f，试求物体A下落时的绳F_1, F_2张力(用a_A 表示)。



解：已知：

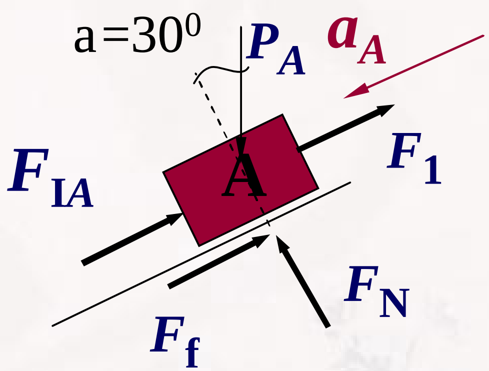
$$v_A^2=\frac{\left[\frac{2M}{Rm}+g(1-\sqrt{3}f-\frac{5r}{6R})\right]}{1+\frac{r^2}{R^2}+\frac{r^2}{4R^2}}x_A,$$

$$a_A=\frac{\left[\frac{2M}{Rm}+g(1-\sqrt{3}f-\frac{5r}{6R})\right]}{2(1+\frac{r^2}{R^2}+\frac{r^2}{4R^2})},$$

[A]:

$$F_1 - \frac{m}{g}\sin\alpha + ma_A + \frac{m}{g}\cos\alpha \quad f = 0,$$

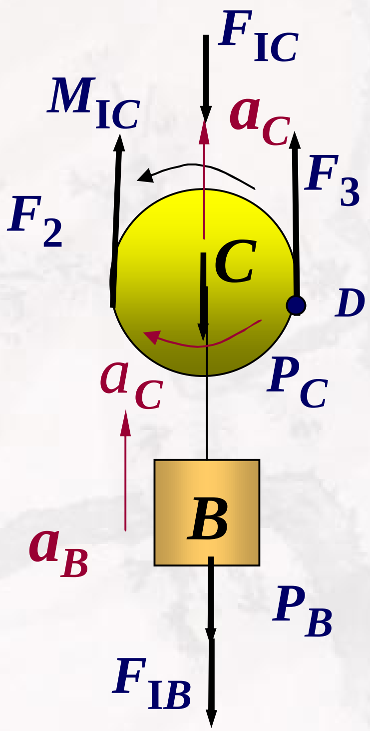
$$F_f = P_A\cos\alpha \quad f, \qquad F_1 = \frac{m}{2g} - ma_A - \frac{m}{g}\frac{\sqrt{3}}{2}f,$$



[C]:

$$\sum M_D = 0;\ -F_2 2r + (P_B + F_{IC} + P_C + F_{IB})r + \frac{mr^2}{2\cdot 3}a_c = 0,$$

$$F_2 = \frac{5}{12}mg + \frac{m}{4}a_A, \qquad \ddot{\varphi}_C r = \ddot{x}_B = \ddot{x}_C, \qquad 2\ddot{\varphi}_C r = \ddot{x}_A,$$

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