Chapter 9 Graphs

9.8 Graph Coloring

1. Introduction

- Introduction
  - The Coloring of Map
    - When a map is colored, two regions with a common border are customarily assigned different colors.

      ---- try to find a small number of colors.
    - Example 0:
      - For the map shown on the left of Figure 1, four colors are enough, but three are not enough

1. Introduction

- The Dual Graph of a Map
  - Each map in the plane can be represented by a graph. To set up this correspondence, each region of the map is represented by a vertex. Edges connected two vertices if the regions represented by these vertices have a common border.
  - The resulting graph is called the dual graph (对偶图) of the map.
  - For example, see Figure 1 and 2 (page 614).

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- Definition 1 (page 667)
  - A coloring of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.
- Definition 2 (page 667)
  - The chromatic number (色数) of a graph is the least number of colors needed for a coloring of this graph.

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- Theorem 1 (page 604)
  - The Four Color Theorem (四色定理)
  - The chromatic number of a planar graph is no greater than four.
  - Remark
    - For the story of the proof, please see page 668.
    1. It was posed as a conjecture (猜想) in 1850.
    2. It was finally proved by two American mathematicians in 1976.Their proof relies on a careful case-by-case analysis carried by computer.

1. Introduction

- Theorem 1 (page 604)
  - Remark(cont.)
    3. Prior to 1976, many incorrect proofs were published, often with hard-to-find-errors.
  - Note
    1. The Four Color Theorem applies only to planar graphs.
    2. Nonplanar graphs can have arbitrary large chromatic numbers.

1. Introduction

- Example 1 (page 668)
  - What are the chromatic numbers of the graphs G and H shown in Figure 3?
  - Solution:
    - G ----- The chromatic number of G is 3.
    - H ----- The chromatic number of H is 4.
    - Why?

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- Example 2 (page 669)
  - What is the chromatic number of Kn?
  - Solution:
    - A coloring of Kn can be constructed using n colors by assigning a different color to each vertex.
    - There are no fewer colors to color the graph.
    - Therefore, the chromatic number of Kn is n.
  - Remark (备注)
    - Recall that Kn is not planar when n≥5, so this result does not contradict the Four Color Theorem.

1. Introduction

- Example 3 (page 670)
  - What is the chromatic number of the complete bipartite graph $K_{m,n}$, where m and n are positive integers?
  - Solution:
    - Only two colors are needed.
    - A coloring of $K_{3,4}$ with two colors is displayed in Figure 6.

1. Introduction

- Example 4 (page 617)
  - What is the chromatic number of the graph Cn? (Recall that Cn is cycle with n verteices.)
  - Solution
    - When n is even, the chromatic number is 2.
    - When n is odd and n>1, the chromatic number is 3.
    - For the direct understanding , please see Figure 7 (page 670).

2. Application of Graph Coloring

- Example 5 (Scheduling of Final Exams, page 671)
  - How can the final exams at a University be scheduled so that no student has two exams the same time?
  - Solution: The scheduling problem can be solved using a graph model.
    - Vertices represented courses.
    - If there is a common student in the courses, an edge between the two corresponding vertices exist.

2. Application of Graph Coloring

- Example 5 (Scheduling of Final Exams, page 671)
  - Solution:
    - Now we want to color the graph. Each time slot for a final exam is represented by a different color.
    - The scheduling of the exams corresponds to a coloring of the associated graph.

2. Application of Graph Coloring

- Example 5 (Scheduling of Final Exams, page 671)
  - Solution:
    - Now we want to color the graph. Each time slot for a final exam is represented by a different color. The scheduling of the exams corresponds to a coloring of the associated graph.
    - For instance, suppose there are seven finals to be scheduled. Suppose the courses are numbered 1 through 7. Suppose that the following pairs of courses have common students: 1 and 2, 1 and 3, 1 and 4, 1 and 7, 2 and 3, 2 and 4, 2 and 5, 2 and 7, 3 and 4, 3 and 6, 3 and 7, 4 and 5, 4 and 6, 5 and 6, 5 and 7, and 6 and 7.

2. Application of Graph Coloring

- Example 5 (Scheduling of Final Exams, page 671)
  - Solution:
    - Since the chromatic number of this graph is 4, four time slots are needed.
    - For the direct understanding, please see Figure 8 and 9 (page 672).

2. Application of Graph Coloring

- Example 6 (Frequency Assignments, page 672)
  - Television channel 2 through 13 are assigned to stations in North American so that no two stations within 150 miles can operate on the same channel. How can the assignment of channels be modeled by graph coloring?

2. Application of Graph Coloring

- Example 6 (Frequency Assignments, page 672)
  - Solution:
    - Construct a graph
    - A vertex represents a station
    - Two vertices are constructed by an edge if they are located within 150 miles of each other.
    - An assignment of channels corresponds to a coloring of the graph, where each color represents a different channel

Homework

- Page 672~675
  - 5~11, 13(read), 15(read), 16