PLANNING THE TIMING AND SCALE OF LUMPY INVESTMENTS

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Development planners frequently encounter and evaluate single investment projects that are large enough to dramatically alter the structure of a less developed economy -- the Tarbella Dam in Pakistan, the Volta River Project in Ghana, and the Aswan High Dam in Egypt are notable instances. But it is misleading to suggest that such investments are always linked to agriculture or overhead services. In fact, individual projects in manufacturing can also be lumpy -- requiring a substantial fraction of the total investment foreseen over a medium term plan.[1] For example, one need only think of projects such as steel mills, large engineering shops, or petrochemical complexes in African or small Asian nations.

Lumpy investment projects yield major external economies (or diseconomies) to other sectors -- it is this that ultimately characterizes them as "lumpy." There is little doubt that too much is written about external economies and too little done to incorporate formally their existence into our project evaluation

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[1] Theoretical and practical planners have not settled on a lower bound for the ratio of project to total investment requirements that qualifies a project as "lumpy." Where the project is a major user of several specific resources, one of these, rather than investment financing per se, may be crucial in establishing "lumpiness." We shall have more to say on this as we proceed.

criteria. Economists have catalogued a large number of specific cases of interdependence between sectors in which the market mechanism fails to provide the information necessary for optimal decentralized decision making; benefit-cost analysis has been modified apace to include these linkages. But the attempt to modify benefit-cost analysis to incorporate essential features of the general equilibrium system within which decisions are made has largely been a failure. With the tools now at hand, the fruitless addition of epicycles can and should be brought to a halt.

This paper presents a general equilibrium model used to evaluate lumpy investments in the manufacturing sector of South Korea. The model is based on the dynamic input-output model. The choices between imports and exports in the various sectors and between investment and consumption in each period are made to maximize a utility function based on aggregate consumption.

In its general formulation, this type of model is well suited to handle three types of interdependence which give rise to external economies: input-output linkages, competition between sectors for scarce resources, and changing comparative costs associated with changes in resource endowments. The model formulated here, however, also includes another prevalent source of external economies, decreasing unit costs within inter-related sectors.[2] The model is a

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[2] On the failure of the market mechanism as an investment allocation device in the presence of economies of scale see Chenery /2/. Excellent surveys of the empirical work on the prevalence of decreasing costs are contained in Haldi and Whitcomb /6/ and Moore /8/.

linear programming model but for the fact that some variables are restricted to be either zero or one. These variables are used to specify increasing returns to scale in two sectors -- petrochemicals and iron and steel.

This model does not include all the respects in which lumpy investment projects in manufacturing represent a structural break. Learning effects, induced technological change, and income distribution are among the elements neglected. In principal there is no reason for excluding such elements, short of the lack of empirical knowledge needed to include them. While an inter-temporal, multi-sectoral optimizing model such as that used here may over-simplify the characteristics and dynamics of production in some sectors and neglect some of the more important means whereby economic relationships and parameters are changed, it does recognize that an economy's comparative advantage changes over time as a result of past and present production activity. Furthermore, such a model is one of the few available means for evaluating projects within a general equilibrium framework.

In the sections that follow we will develop the properties of a general equilibrium model in which economies of scale are present, outline the Korean model's structure, and analyze some solutions to the Korean model for insights into the nature of lumpy investment projects in manufacturing.

Economies of Scale in Inter-related Sectors

Economies of scale are empirically significant in a number of major industries. Typically, decreasing costs occur in the construction of capacity and in the use of manpower in plants manufacturing petroleum, chemicals, petrochemicals, steel, cement, and aluminum, to mention but a few.[3] Empirical studies of plant construction costs have found that in these cases the elasticity of total cost with respect to plant size is constant over a wide range and frequently is between 0.6 and 0.8.[4] Thus doubling plant size increases costs by between sixty and eighty per cent. These figures refer to the construction of a balanced plant, not to the expansion of plant capacity by breaking one or several limiting capacities within the plant.

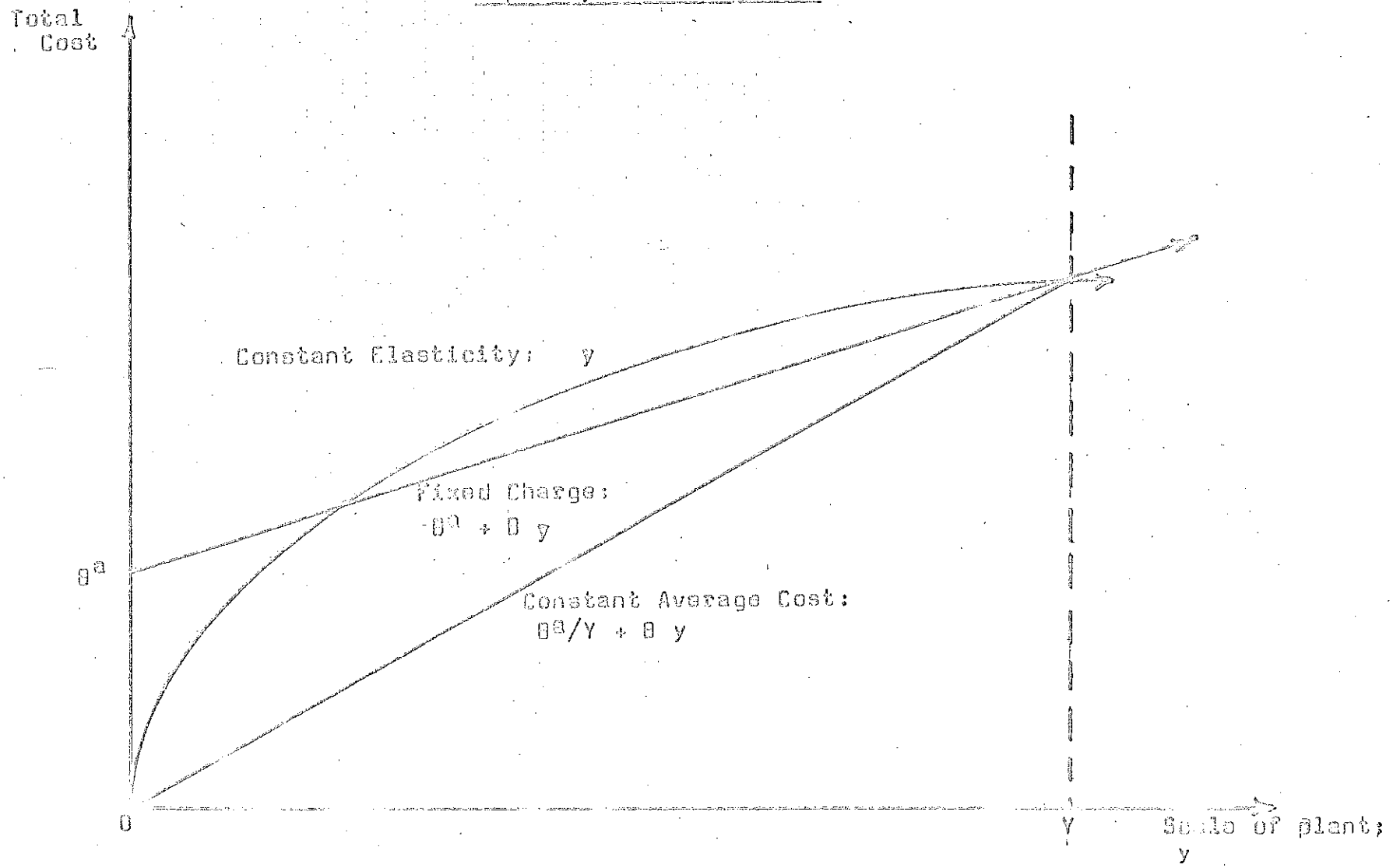
Figure I exhibits a constant elasticity capacity cost function along with a "fixed charge" cost function which is used here to approximate the former. The latter is preferred since it can be used in numerically solvable models while the former cannot. The approximation is of the form: Total cost = $B^a\delta + B\,y$, where "B^a" and "B" are respectively the fixed and variable charges associated with capacity construction, "y" is the size of plant constructed, and "δ" is a zero-one variable equal to zero if **no** plant is built and equal to one if a plant, regardless of size, is built. The average cost of capacity is $B + B^a/y$, and it declines asymptotically to "B", the variable cost of capacity.

[3] In addition, decreasing costs exist where the set-up time between production runs is important, as in the metal working industries.

[4] Haldi and Whitcomb /6/ .

FIGURE I

Capacity Cost Functions



To illuminate the workings of a model incorporating economies of scale in inter-related sectors, we now consider a simple model which can be approached through both graphs and algebra. The structure of this model is identical in its major respects to that of the model used to evaluate Korean investment in a petrochemical complex and an integrated steel mill. The parameters and variables used in the model are defined in Table I. The first subscript ("i" or "j") associated with each variable denotes a sector, the second ("t") denotes the time period. The equations that hold in each period in the model follow:

Sector output equal to demand less imports --

$$(1)\quad x_{i,t} = \sum_{j=1}^{2} A_{ij}\, x_{j,t} + \sum_{j=1}^{2} B_{ij}\, y_{j,t} + \sum_{j=1}^{2} B^{a}_{ij}\, \delta_{j,t} + S^{a}_{i} + S_{i}\, cn_{t} + e_{i,t} - m_{i,t} \qquad i = 1,2$$

Production no greater than capacity --

$$(2)\quad x_{i,t} \leq k_{i,t} \qquad i = 1,2$$

Capacity updated by investment --

$$(3)\quad k_{i,t} = k_{i,t-1} + y_{i,t} \qquad i = 1,2$$

Foreign exchange use no greater than supply --

$$(4) \quad \sum_{i=1}^{2} N_i x_{i,t} + \sum_{i=1}^{2} M_i y_{i,t} + \sum_{i=1}^{2} M^a \delta_{i,t} + \sum_{i=1}^{2} P_i m_{i,t} - \sum_{i=1}^{2} e_{i,t} \leq f_t$$

Obtaining fixed charge capacity cost functions --

$$(5a) \quad y_{i,t} \leq Y_i \delta_{i,t} \qquad i = 1,2$$

$$(5b) \quad \delta_{i,t} = 0 \text{ or } 1 \qquad i = 1,2$$

Interdependence between sectors is found on three levels in the model: user - supplier relations on both current and capital account, competition for investment resources, and competition for scarce foreign exchange. Production and investment require non-domestically produced inputs. In addition, trade is important since imports are an alternative supply source to production. Domestic prices are assumed equal to export prices for commodities which are internationally traded, as both are in this simple model. The last two assumptions can be changed without altering the behavior of the model. For convenience, as well as the lack of data, labor is not included in the model.

TABLE I: Symbols Used in the Model

Parameters --

A_{ij}	input from sector "j" required per unit gross output in sector "i"
N_i	non-competitive imports required per unit gross output in sector "i"
B_{ij}	variable requirement of sector "j's" output per unit of capacity constructed in sector "i"
B^a_{ij}	fixed requirement of sector "j's" output per unit of capacity constructed in sector "i"
M_i	variable requirement of non-competitive imports per unit of capacity constructed in sector "i"
M^a_i	fixed requirement of non-competitive imports per unit of capacity constructed in sector "i"
S^a_i, S_i	parameters in the commodity consumption functions, consumption of the "i'th" good equals $S^a_i + S_i\ cn_t$
P_i	the c.i.f. import price of the "i'th" commodity
Y_i	size of the largest plant that can be built in sector "i"

Variables --

$x_{i,t}$	gross production of commodity "i" in period "t"
$k_{i,t}$	capacity in sector "i" in period "t"
$y_{i,t}$	size of plant constructed in sector "i" in period "t"
$\delta_{i,t}$	zero-one variable for capacity construction in sector "i" in period "t"
$m_{i,t}$	imports of the "i'th" commodity in period "t"
$e_{i,t}$	exports of the "i'th" commodity in period "t"
cn_t	aggregate consumption in period "t"
f_t	net foreign capital inflow in period "t" (exogenous)

The fixed and variable charge activities associated with capacity provision in each sector are linked by equation (5a) which requires that at least the proportion $y_{i,t}/Y_i$ of the fixed charge be incurred when a plant of size "$y_{i,t}$" is constructed in the "i'th" sector. Equation (5b) insures that if "$\delta_{i,t}$" is greater than zero, then it must be one. Equation (5a) alone, without the addition of (5b), gives a capacity cost function with constant average (equal to marginal) cost: Total Cost = $(B^a_{ij}/Y_i + B_{ij})\, y_{i,t}$. Thus, if it is expected that a plant of size "x" will be constructed in a given period, Y_i may be set to "x" for that period and the existence of economies of scale could thereby be formally neglected. That this approach is dangerous will become evident.

Figure II presents a tableau of the model for the two period case. Illustrative parameter values have been used so that numerical solutions can be obtained. Purely for clarity and so that the reader can follow the solution procedure with minimal difficulty, inter-dependence between sectors has been neglected in the input - output matrix and in the variable capacity cost coefficients, and non-competitive imports have been suppressed. Positive numbers denote outputs, negative numbers, inputs. The objective function is maximize cn_2.

Without a stipulation about investment in the terminal period (period two here), the maximum is achieved by investing nothing in that period. Terminal investment levels could be pre-set, or investment could be endogenously determined to achieve a given post-terminal

FIGURE II

Tableau of Two Period Model

	Constraints	t = 1											t = 2											Right Hand Side
		x_1	x_2	y_1	y_2	δ_1	δ_2	e_1	e_2	m_1	m_2	cn_1	x_1	x_2	y_1	y_2	δ_1	δ_2	e_1	e_2	m_1	m_2	cn_2	
t=1	Product balance	1	0	-1	0	-10	-15	-1		1		-.5												= 0
		0	1	0	-1	-20	-20		-1		1	-.5												= 0
	Capacity cost			-1		100																		$\geq$ 0
					-1		100																	$\geq$ 0
	Zero-one reg't.					1																		= 0 or 1
							1																	= 0 or 1
	Capacity limit	-1																						$\geq$ -250.0
			-1																					$\geq$ -273.4
	Foreign exchange	0	0	0	0	0	0	1	1	-1	-1.5													$\geq$ 0
t=2	Product balance												1	0	-1	0	-10	-15	-1		1		-.5	= 0
													0	1	0	-1	-20	-20		-1		1	-.5	= 0
	Capacity cost														-1		100							$\geq$ 0
																-1		100						$\geq$ 0
	Zero-one reg't.																1							= 0
																		1						= 0
	Capacity limit			1									-1											= 0 or 1
					1									-1										= 0 or 1
	Foreign exchange												0	0	0	0	0	0	1	1	-1	-1.5		$\geq$ 0
	Terminal invest.												-.1			1								$\geq$ 25
														-.1				1						$\geq$ 40

Note: Time subscripts have been suppressed.

growth rate. The latter course has been chosen and the post-terminal growth rate is ten per cent.[5]

Matters are simplified considerably if we assume that $e_{i,1}$, $m_{i,1}$ $(i = 1,2)$ f_1, and f_2 equal zero. As initial conditions we take $y_{1,1} = 250.0$, $y_{2,1} = 273.4$. With these simplifications the problem is easily shown to reduce to

maximize Δ, subject to

$$(6) \quad 1\, y_{1,1} + 10\, \delta_{1,1} + 15\, \delta_{2,1} \leq 50.0$$

$$(7) \quad 2\, y_{2,1} + 20\, \delta_{1,1} + 20\, \delta_{2,1} \leq 73.4$$

$$(8) \quad -1\, y_{1,1} - 1.11\, m_1 + 1.11\, e_1 + .55 \leq 2.8$$

$$(9) \quad -1\, y_{2,1} - 1.25\, m_2 + 1.25\, e_2 + .63 \leq -16.6$$

$$(10) \quad e_1 + e_2 - m_1 - 1.5\, m_2 = 0, \text{ where } \Delta = cn_2 - cn_1 \quad .$$

The time subscripts have been dropped from the trade variables, and equations (5a) and (5b) for investment in period one have not been reproduced here. Equations (6) and (7) are the capacity constraints for period one; (8) and (9) are the second period's capacity constraints. Equation (10) is the foreign exchange constraint for the last period. The problem in this reduced form has nine equations and seven variables.

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[5] The terminal conditions are that $y_{i,2} = .10\,(x_{i,2} - \sum_{i=1}^{2} B^a_{ij})$. The fixed charges are subtracted from production since the capacity needed to meet these demands already exists and need not grow.

There are four possible patterns of investment in the two sectors considered together, depending upon which sectors' capacities are increased in the first period. Each pattern is associated with a different combination of values for the two zero-one variables $\delta_{1,1}$ and $\delta_{2,1}$. One method of solving this mixed integer - continuous variable programming problem is to solve each of the linear programming problems obtained for each combination of the zero-one variables after the fixed charges incurred for that pattern of investment have been subtracted from the right hand sides and the scale activities for projects not undertaken have been removed. of equations (6) and (7). The objective values associated with the optimal solutions for the alternative patterns can then be compared, and the pattern with the highest optimal objective chosen as best.

The first four columns of Table II give the optimal solutions for each pattern of investment. The optimal solution is characterized by investment in both sectors in the initial period; consumption growth is twenty between the first and second periods. The pattern $\delta_{1,1} = 0$, $\delta_{2,1} = 0$ results in a decline of consumption since there is not enough of the first commodity in the second period to trade for the required amount of the second commodity even if consumption growth is zero.

The method of solution used above is too cumbersome if there are a large number of zero-one variables (i.e. patterns of investment over time).[6] However, its economic implications are of the utmost

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[6] If there are "n" zero-one variables, the number of linear programming problems to be solved is 2^n.

TABLE II

Solutions to Two Period Model

Solution	A	B	C	D	E	F	G
Investment Pattern:							
$\delta_{1,1}$	0	1	0	1	.41	1	0
$\delta_{2,1}$	0	0	1	1	.29	.24	.33
Objective Value:							
cn_2	386.1	414.7	410.5	420.0	444.2	435.4	417.2
Activity Levels:							
$y_{1,1}$	.0	40.0	.0	25.0	41.5	36.4	.0
$y_{2,1}$	.0	.0	26.7	16.7	29.5	24.3	33.4
m_1	.0	.0	8.0	.0	.0	.0	6.0
m_2	6.3	20.8	.0	10.0	11.9	11.8	.0
Shadow Prices:							
First Period Capacity							
Sector One	.00	.72	.00	.72	.58	.72	.00
Sector Two	.00	.00	.40	.48	.40	.40	.04
Second Period Capacity							
Sector One	.72	.72	.90	.72	.72	.72	.90
Sector Two	.96	.96	.80	.96	.96	.96	.80
Profit on $y_{i,1}$							
Sector One	.72	.00	.90	.00	.14*	.00	.90
Sector Two	.96	.96	.00	.00	.17*	.19*	.73*
Profit on $\delta_{i,1}$							
Sector One	.00	-7.18	-4.00	-16.78	.00°	-15.20°	-97.77°
Sector Two	.00	-10.79	-6.00	-20.39	.00°	.00°	.00°
Investment Penalty[1)]							
Sector One							
$\delta_{1,1} = 1$	-	-	-	-	8.8	-	-
$\delta_{1,1} = 0$	-	-	-	-	25.1	-	-
Sector Two							
$\delta_{2,1} = 1$	-	-	-	-	11.1	14.3	5.4
$\delta_{2,1} = 0$	-	-	-	-	25.7	21.4	9.0

Notes: *: Shadow price on associated (5a) constraint.
o: Shadow price on associated (5b)/(5b') constraint.
1: Derivation explained in text.

significance. From micro-economic theory it is well known that, even in the presence of economies of scale, Pareto-optimality requires marginal cost pricing (disregarding the complications that arise in the theory of second best). Pricing the output of a project with economies of scale at marginal cost, however, results in a loss on the project. Thus the standard project selection criterion, that total benefits exceed total costs, is not satisfied. This does not mean that the project should be discarded, however, for the benefit-cost criterion is really not applicable when decreasing costs are present. The proper selection rule in this case is a two-fold one. First, the scale of the project must be set at the level where marginal cost equals marginal revenue (it is at the margin that the benefit-cost rule applies). Then, for the project of this scale, the sum of the producers' and consumers' surplus resulting from the project's being undertaken is computed. If this sum is positive, the project should be chosen for investment.[7]

Now consider our straightforward solution procedure. Each linear programming problem includes constraints (6) through (10) (and only these) and excludes the zero-one activities (as well as scale activities pre-set to zero) since the right hand side has been reduced by the fixed charges incurred. In each problem, therefore, the scale of the project, $y_{i,l}$, is determined so that, at the margin, benefits equal costs. This is a fundamental property of linear programming. The surplus due to a project is equal to the difference between the optimum objective

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[7] See Oort /10/.

value and the objective value for the alternative decision regarding investment in that project (all other project decisions remaining the same). The economic surplus from the first project is thus 9.5; from the second -- 5.3.[8]

These ideas are illustrated graphically in Figure III. Curves AB and CD are respectively the marginal and average cost curves for the firm; the demand curve is EF. In this (extreme) case there is no quantity for which demand price equals average cost so that even with average cost pricing the firm operates at a loss. Yet the firm's operation at scale "Y" is profitable to the economy as a whole since the surplus, roughly measured by AEG, exceeds the operating deficit of AGHI. The specification of project choices within a mixed integer programming model adds operational significance to the concepts of producer and consumer surplus which have, up to now, been only useful pedagogical devices.

Marginal rules are sometimes sufficient for project evaluation. The shadow prices for capacity in the two periods for a given pattern of investment can be used to determine whether a project not currently undertaken might profitably be selected for investment. The shadow price of a resource multiplied by the change in that resource's availability gives an upper bound to the change

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[8]The measure of surplus is really not unique since it depends on the pattern of investment elsewhere in the economy -- i.e. consumers' and producers' surplus are partial equilibrium concepts. An alternative measure of surplus for the first project is 28.6, for the second -- 24.4. The measure chosen in the text alone has the proper economic meaning, however.

FIGURE III

The Surplus Criterion

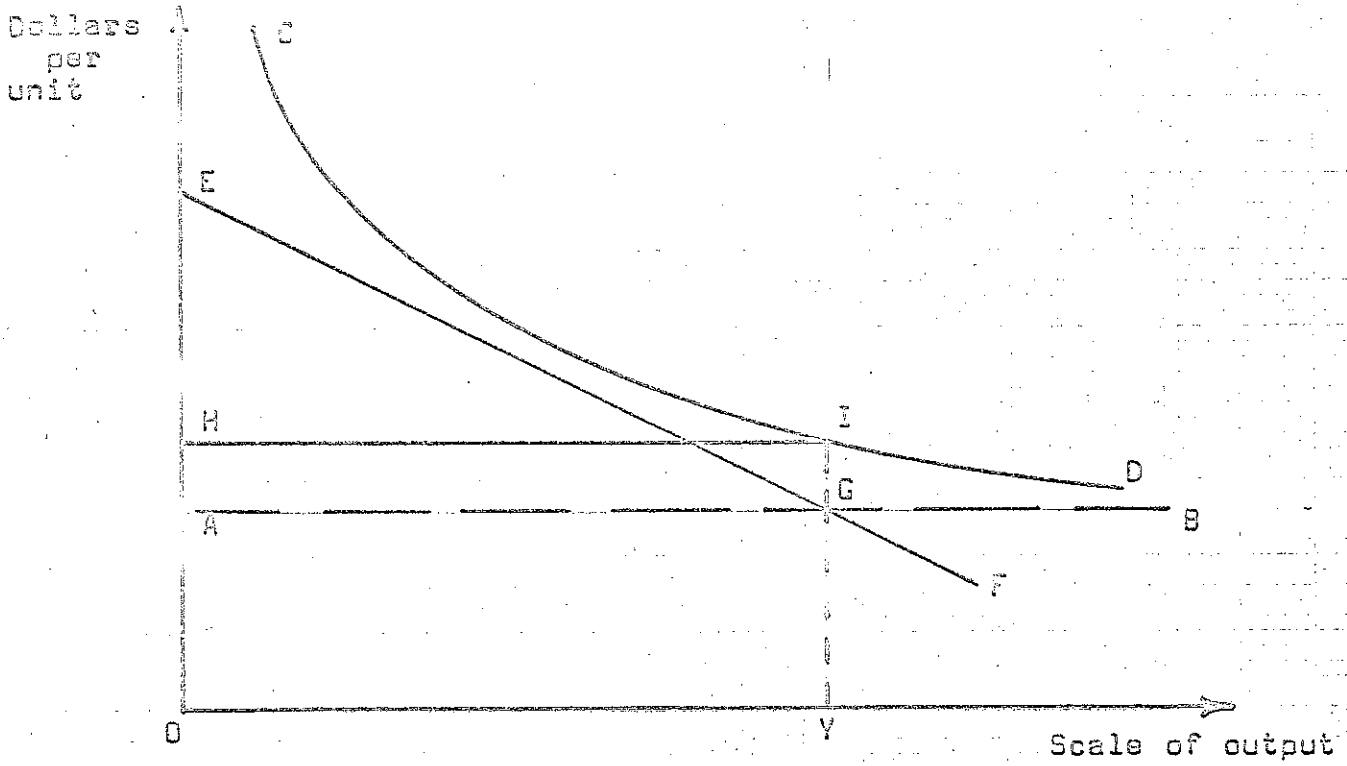
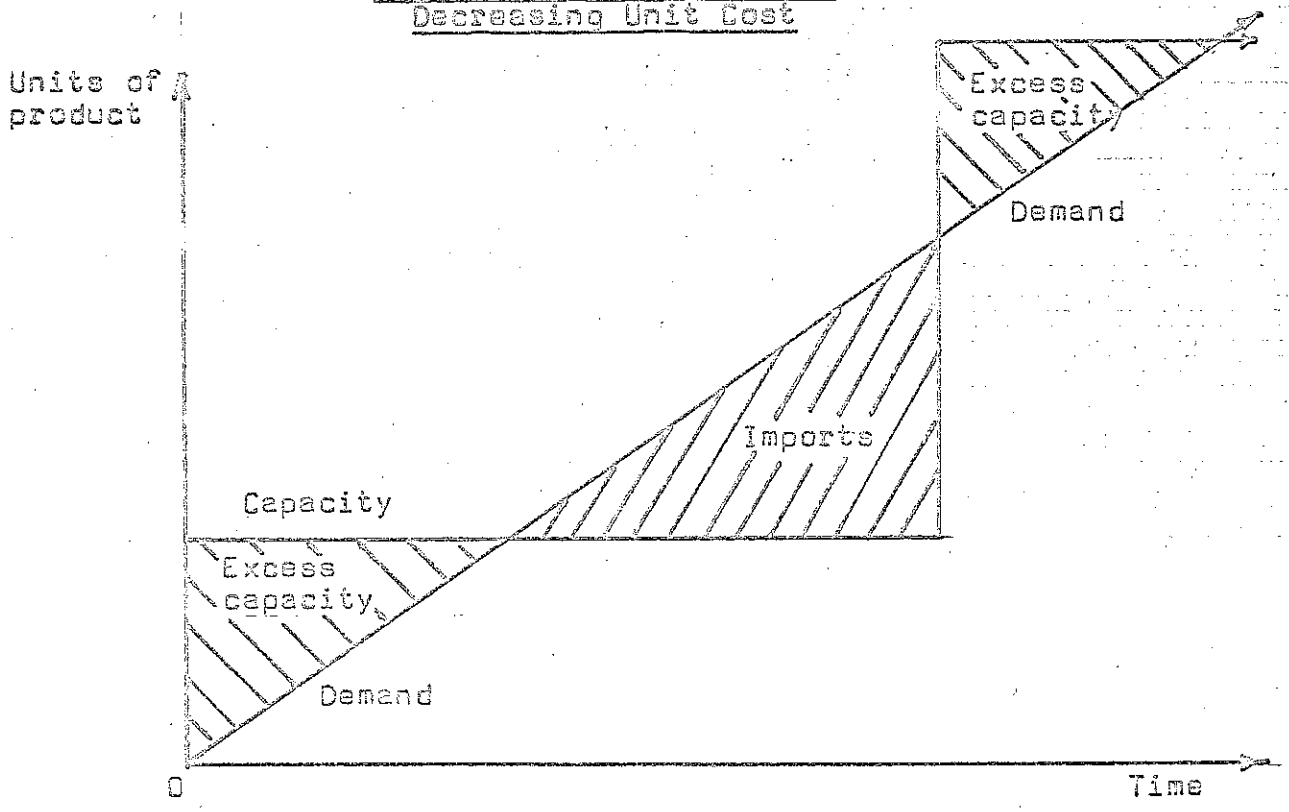


FIGURE V

Capacity Expansion Path: Decreasing Unit Cost



in the objective value that results from altering the supply of the resource. Thus if $\delta_{i,t}$ is zero for a given pattern, a sufficient condition that the project should not be undertaken given the pattern of investment elsewhere is that

$\sum_{j=1}^{2} B_{ji} \, sp_{j,t} - \sum_{s=t+1}^{\tau} sp_{i,s} \leq 0$, where "$\tau$" is the terminal period and "$sp_{i,s}$" is the shadow price of capacity "i" in period "s". That is, neglecting the fixed charge, the marginal cost of the first unit of capacity must exceed its marginal benefit.[9] The opposite condition is necessary, but not sufficient, for the project's selection. In Table II it can be seen that both projects pass the necessary conditions for selection in the patterns in which they are not built since the profit on the scale activities is positive. The profit on the zero-one activities is always negative and so cannot be used to obtain conditions for project selection.

The marginal conditions given above are not substitutes for determining project surplus. They are meaningful only for changes in a given pattern of investment intended to enlarge the set of undertaken projects. They are not sufficient to determine the optimal solution but are satisfied for rejected projects in that solution. In other words, only the positive surplus condition is a sufficient criterion.

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[9] A stronger, but somewhat meaningless, condition is that

$\sum_{i=1}^{2} B_{ji} \, sp_{j,t} \, y_{i,t} + \sum_{i=1}^{2} B^{a}_{ji} \, sp_{j,t} - \sum_{s=t+1}^{\tau} sp_{i,s} \, y_{i,t} \leq 0$, where $y_{i,t}$ is greater than or equal to the optimal scale of the project if undertaken.

There are several alternatives to complete enumeration as a solution procedure. Each requires the solution of a number of linear programming problems in which equations (5b) are replaced by $\delta_{i,t} \leq 1$ (5b') for some plant cost functions: this substitution replaced the fixed charge cost function by the constant average cost function shown in Figure I.[10] All begin by solving the "continuous" problem in which the average cost function is used for each project. This solution is Solution E in Table II. (Note that this is an infeasible solution to the problem when properly stated.)

To this point shadow prices have been computed using only the dual equations for equations (6) through (9); prices will now be computed using the full set of "reduced form" equations including (5a) and (5b), or (5b') where appropriate. We substitute average for marginal cost pricing where the fixed charge need not be fully incurred.

Plants of scale 41.5 and 29.5 are built in sectors one and two respectively when the output from both plants is priced at average cost. Using the information contained in this solution, "penalties" can be computed for not building a plant at all ($\delta_{i,t} = 0$) and for incurring the entire fixed charge when built ($\delta_{i,t} = 1$). For example, if the entire fixed charge in sector one is to be incurred, the slack activity on the corresponding

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[10] Methods of solving mixed integer - continuous variable programming problems are surveyed in Balinski /1 /.

equation (5b') must leave the basis and the slack for (5a) must enter. The profit on the latter slack in Solution E is $-[(10.0)(0.58) + (20.0)(0.40)]/100.0 = -.138$ (equal to minus the shadow price of constraint (5a)). From the simplex tableau (not shown here) it can be found that in the new basis the slack on (5a) will be used at level 63.6, so that the minimum loss from paying the entire fixed charge is $(63.6)(.138) = 8.78$. On the other hand, if the plant is not built in sector one, it will be necessary to export rather than import the second commodity in order to pay for imports of the first. In addition, there will be a surplus of commodity one in period one. The shadow price of foreign exchange in Solution E is 0.80 [$= (1.11)(0.72)$], and imports of the first commodity in the new basis will be 6.0, so that the minimum loss if the plant is not built is $4.80 + (0.58)(35.0) = 25.1$ (the second term in the sum is the loss on introducing the slack for commodity one in the first period). Penalities associated with the fixed charge for investment in sector two can be similarly obtained and are given in Table II.

Now, with penalties in hand, we proceed to solve another linear programming problem. This time we shall require that the entire fixed charge in sector one be paid. The solution is given as Solution F in Table II. The actual reduction in consumption when the full cost of the plant must be paid is 8.8 which is slightly more than the minimum estimated reduction (i.e. the penalty for forcing $\delta_{1,1}$ to zero). It is readily apparent that there is no

need to try the solution in which the plant in sector one is not constructed, for the actual reduction in consumption when the plant is built is less than the minimum reduction if it is not built. (Nonetheless, for the reader's convenience the solution in this case is provided, Solution G.) From the penalties for the fixed charge in sector two in Solution F we decide to try $\delta_{2,1}$ at level one, rather than zero. Solution D, the optimal solution, is thereby reached; it is known to be optimal without further solutions since the actual reduction in consumption between solutions F and D is less than the minimum reduction for not building a plant in sector two.

Using the solution process outlined above, the optimal solution was obtained in three solutions rather than four as was the case with complete enumeration. Its advantage is thus that fewer linear programming problems may have to be solved.[11] The procedure is also interesting in its use of average cost pricing to obtain upper bounds on succeeding solutions. But one should not be mislead into thinking that, as a consequence, average cost pricing is used to determine the shadow prices in the optimal solution. This is not the case unless the maximum plant size constraints (5a) are binding, which they generally are not in empirical applications to less developed countries.

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[11] The procedure used is an adaptation of the branch and bound solution technique presented in Davis, Kendrick and Weitzman /4 /. Here we have estimated bounds using the full set of basis changes; Davis, Kendrick and Weitzman use only the first basis change since it is not always so evident what the full set will be.

In Figure IV the isoquant for second period consumption equal to 420 is given in the initial capacity space as ABCDEFG. The isoquant does not have the typical shape associated with linear programming; it bends outward at B and F.[12] It assumes this shape because the fixed charges are incurred once and for all when the first unit of capacity is built rather than continuously. At point A only the plant in the second sector is constructed and the growth in demand for sector one's output is met through imports. Between points A and G there is substitution of production in the first sector for production in the second through trade. To build the first increment of capacity in sector one requires that the fixed charge be paid (point C); thereafter capacity can be expanded at constant marginal cost. At point E the fixed charge for the second plant has been paid but no capacity provided; at G only capacity in the first sector is increased. Again, the fact that the fixed charges are incurred only once gives the outward bending portions of the curve. In other words, it is not possible to combine points A and G as "activities" and thereby use AG as the isoquant (as would seem to be the case by analogy to linear programming.)[13]

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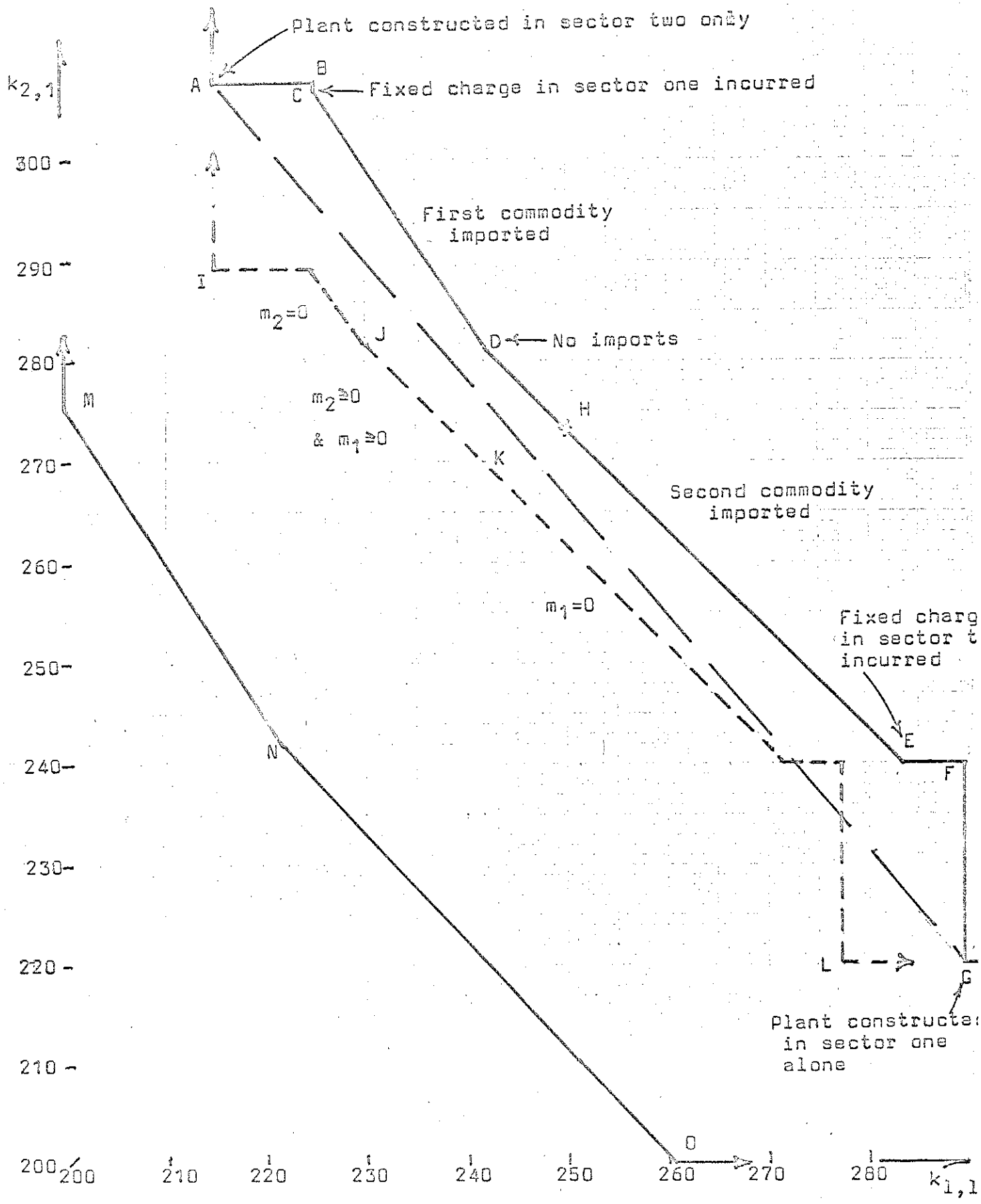
[12] Mathematically, the set of capacity points for which $cn_2 \geq 420$ is non-convex.

[13] In fact, the "process rays" on which A and G lie do not emanate from the origin.

FIGURE IV

Consumption Isoquants



The point corresponding to the initial capacity endowments in the problem just solved is point H. The relative prices of period one capacities are given by the slope of the line DE. With the solution to a maximization problem completed, in linear programming it is possible to restate the problem as one of minimizing the cost of achieving the same level of the maximand using the relative prices obtained in the original problem as the prices of the scarce resources. The duality property of linear programming insures that the resource requirement will be identical in both optimal solutions (this must be modified in the degeneracy case). That is not true here, for if we now seek to minimize the cost of obtaining 420 units of second period consumption, we find that the point G minimizes the initial capacity cost. Using the initial capacities given by G is cheaper than using those given at any point along DE. This is but an illustration of the general conclusion that in a model with decreasing costs there is not a simple correspondence between the set of resource prices and the set of resource endowments. This is further seen in the fact that the value of the primal is not equal to that of the dual; looking back at Table II, Solution D, maximum second period consumption is 420 but the value of the resources priced at the corresponding shadow prices is 754.[14]

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[14] The primal value is always exceeded by the dual value where there are economies of scale; just as with decreasing average cost in a firm, paying factors their marginal products more than exhausts the value of the firm's output. See Henderson and Quandt /7 /, p.64.

Much of what has been said in this section implies that the use of prices in a system of decentralized decision-making results in an inefficient allocation of resources if economies of scale are present. This is in fact the case. Not only do firms make a loss if they price at marginal cost, in addition there is no guarantee that an efficient allocation of resources will result.

Furthermore, as we have seen, the concept of average cost pricing is useful in computing solutions but cannot be applied to obtain efficient resource use in a decentralized decision-making context since, at the optimum, resources must be priced at marginal cost lest they be underutilized. The notion of economic surplus due to undertaking a project is instructive in this context. The surplus accrues to the economy as a whole, not merely to those who purchase the plant's xoutput. It is therefore somewhat illogical to force the purchasers to pay for something that ultimately benefits all.[15]

Figure IV also shows the isoquant for second period consumption equal to 420 when foreign capital in amount 22 is available in period two (IJKL). This is an interesting case since both commodities are imported between J and K. Also present in the figure is the isoquant MNO for the case in which equations (5b') replace (5b). Note that points along this isoquant yield consumption less than 420 in terms of the original problem.

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[15] On decentralized decision-making schemes in the presence of scale economies see Vietorisz/11/.

Characteristics of a More Complex Model

The model applied to planning the timing and scale of investment in petrochemicals and iron and steel in Korea differs from the simple model only in complexity and scope.[16] In the first place it includes all activity in the economy in its seventeen different commodities and nineteen production activities. Greater detail does not mean that the results of the model can be applied to decisions relating to sectors other than petrochemicals and iron and steel, however. It is necessary because decisions regarding investment in sectors connected to these two on either the supply or demand side influence the benefits and costs of the petrochemicals complex and integrated steel mill and vice versa. This two way influence can be captured only by disaggregating the using and supplying sectors. Activity in the sectors not closely related to petrochemicals or iron and steel must be present in the model since these sectors are indirectly related to them through their demand for investment resources, foreign exchange, and other resources in limited supply. An operational model must be focused on a limited number of choices; simply multiplying the number of sectors does not produce an operational model for investment decision purposes because these decisions regard projects, not sectors.

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[16] For a more complete discussion of the Korea model see Westphal /12 /.

The simple model could not be used to illustrate one consequence of decreasing costs because it was restricted to project decisions in a single period. In sectors where the cost of capacity exhibits increasing returns, the construction of plants is likely to be staggered. With economies of scale in capacity provision it pays to over-build capacity since unit capacity costs are thereby reduced. The opportunity cost of building over-capacity exists in the investment resources that are unutilized due to excess capacity during the period before the plant is operated fully. The trade-off between the cost of excess capacity and the saving from building larger plants will determine just how much over-investment is optimal when the plant is constructed.[17]

Where it is possible to import the commodity produced by the plant, it is usually profitable to delay the construction of a plant beyond the time when currently existing plants are fully utilized. This action again permits the construction of larger plants and results in lower average capacity costs. The penalty associated with imports is their higher cost. The typical pattern of capacity expansion in a sector with economies of scale is illustrated in Figure V.[18]

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[17] Chenery /3 / presents a one sector model in which these ideas are formally developed.

[18] This is taken from a book by Manne and associates /9./ in which the one sector model with trade and regional dimensions is developed.

When analyzing large projects within interdependent sectors it is necessary to answer the following questions for each project: what will happen to the level of total investment if it is undertaken; how is investment affected in the using and supplying industries related to the project and what will be the scale of the project and the degree of its initial utilization; and, how will the project influence the growth of supply in the rest of the economy? For small projects it may not be possible to identify these reactions to project construction; for lumpy projects it is both possible and desirable since the profitability of a lumpy project ultimately depends upon the adjustments that must be made in the rest of the economy to implement it.

The importance of the first question lies in the significance of the sacrifice that must be made in capacity expansion in other sectors and the extent of that sacrifice. This, after all, is the opportunity cost of a lumpy investment. With regard to the second question, it may be the case that investment in a lumpy project limits the investment that can be made in its suppliers and users. The utilization of the project will consequently be incomplete, at least initially, and this reduces its benefits. There is an obvious trade-off here between the scale of the project and its utilization through the extent of investment in related sectors. Finally, to the degree that the objective of economic activity is the enlarging of the supply of goods and services, the last question relates the project to the objectives of economic activity to determine its benefits.

The number of patterns of investment over time that can occur in a set of inter-related sectors is by no means limited: simultaneous investment in each sector with complete initial utilization of all projects' capacity must be weighed against larger projects with more excess capacity but lower unit capacity costs; imports of intermediate products for use in the lumpy project must be compared with domestic production; imports of the project's output should be analyzed as an alternative to increasing (or initiating) its production; and so on. One really needs a multi-sectoral model in which resources are simultaneously allocated among sectors to examine these alternatives. We now turn briefly to the results of such a model applied to Korea.

Aggregate Results for the Korea Model

The conclusion from the preceding argument is that the economy's accomodation to investment in a lumpy project determines its advisability. The Korean economy's probable accomodation to the construction of a petrochemicals complex and an integrated iron and steel mill is best seen by examining alternative solutions (corresponding to different investment patterns in the two projects) for a given set of parameter values within the model. The model specification employed here includes the following features:

1) Interdependence between sectors through an input-output matrix and an input-capacity matrix in which sectors simultaneously supply to and demand from one another.

2) Three two-year periods linked by capacity expansion activities with a one period gestation lag and by foreign exchange accumulation.

3) A requirement that the absolute change in consumption between the second and third periods be continued into the future at a compound growth rate of eight per cent through investment in the terminal period.

4) An upper limit of thirty per cent on the marginal propensity to save, and upper limits on competitive imports and exports which are endogenously determined.

5) A reasonable estimate of the foreign capital inflow available to Korea over the six-year period stated in the form of a limit to the accumulated debt plus interest charges (at ten per cent per anum) that can remain in the third period.

6) Most importantly, economies of scale in the cost of a petrochemicals complex and an integrated iron and steel mill.

Projects to build these plants are indeed lumpy. Together they require about six per cent of the total investment over the six years. Imported machinery accounts for about seventy per cent of the cost.

The aggregate characteristics of one set of solutions are presented here in Table III.[19] The objective function to be

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[19]There is not room here to present complete sectoral breakdowns. They may be obtained from the author on request.

TABLE III

Comparison of Summary Statistics -- Korea Model[1]

	Year	PC	IS	Welfare	GDP	C	I	E	M	M^1	M^2	M^3	M-E	EC
A.	1967-68	.4*	.2*	138.819	983	750	298	109	240	53	65	122	137	59
	1969-70	.1*	.1*		1199	891	356	148	272	70	96	105	132	58
	1971-72	.2*	.1*		1539	1108	464	298	414	122	148	144	134	10
B.	1967-68	1	1	138.622	984	750	301	109	246	54	65	127	144	58
	1969-70	1	1		1204	895	350	148	269	61	98	110	129	35
	1971-72	1	1		1526	1099	451	298	409	117	146	146	129	11
C.	1967-68	0	0	138.632	986	751	271	109	209	58	65	86	107	53
	1969-70	0	0		1145	850	337	143	260	78	84	98	125	238
	1971-72	0	0		1512	1114	485	298	468	191	136	140	188	0
D.	1967-68	1	0	138.701	985	750	300	109	243	56	65	122	141	55
	1969-70	0	0		1174	869	341	177	288	104	88	96	121	172
	1971-72	0	0		1526	1104	463	298	422	147	141	134	142	15
E.	1967-68	0	0	138.682	985	750	271	109	210	58	65	87	107	55
	1969-70	1	0		1164	862	374	169	320	100	88	133	161	185
	1971-72	0	0		1531	1112	462	298	424	150	141	133	144	15
F.	1967-68	0	1	138.655	985	750	288	109	227	56	65	107	125	55
	1969-70	0	0		1161	863	341	145	263	75	90	98	127	208
	1971-72	0	0		1517	1100	475	298	439	158	144	137	159	10
G.	1967-68	0	0	138.621	985	750	272	109	211	59	65	87	109	53
	1969-70	0	1		1156	858	362	147	285	85	86	115	147	218
	1971-72	0	0		1519	1100	476	298	439	157	145	137	159	0
H.	1967-68	1	1	138.699	984	750	322	109	265	57	65	143	163	57
	1969-70	0	0		1198	888	341	176	282	91	96	95	117	103
	1971-72	0	0		1532	1101	445	298	395	118	148	129	115	23
I.	1967-68	0	1	138.697	983	750	285	109	225	53	65	107	123	59
	1969-70	1	0		1188	882	382	159	315	87	95	134	165	74
	1971-72	0	0		1536	1106	445	298	396	121	148	127	116	29
J.	1967-68	0	0	138.665	983	750	268	109	208	55	65	88	106	58
	1969-70	1	1		1189	880	405	182	356	115	92	149	185	54
	1971-72	0	0		1540	1113	444	298	397	122	150	125	117	27
K.	1967-68	1	0	138.678	984	750	305	109	249	60	65	125	147	57
	1969-70	0	1		1202	888	364	187	312	108	93	111	137	80
	1971-72	0	0		1536	1107	443	298	395	120	149	126	115	29

Notes:
1: All National Income Account figures are yearly averages in billions of won
PC, IS: A "1" denotes plant built in that period
GDP: Gross domestic product
C: Consumption
I: Investment
E: Exports
M: Total imports
M^1: Competitive imports
M^2: Non-competitive intermediate imports
M^3: Non-competitive imports of capital goods
M-E: Foreign capital inflow
EC: Sum over all sectors of excess capacity. Note that there must be terminal excess capacity in petrochemicals and iron and steel if the projects are constructed to provide for post-terminal growth.
*: Amount of fixed charge incurred

maximized is the discounted sum of the welfare in each period during and after the plan. Welfare in each period is a piecewise linear approximation to non-linear function of per capita consumption so that the marginal rate of substitution between consumption in different periods is a variable.[20] The annex to this paper gives solution characteristics for the same model specification when the objective is simply the discounted sum of consumption over the infinite horizon starting in the first period of the model.

Not all of the possible patterns of investment are of interest.[21] In a large number of experiments with alternative specifications of key parameters in the model, those presented here have been found to be the most interesting. In each but the first two, no more than a single plant is built in either sector over the plan period. For comparative purposes Table III gives the continuous solution (A) and the solution obtained by forcing plants of the size constructed in that solution to be built along with full payment of the fixed

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[20] The objective is maximize $\sum_{t=1}^{\infty} (1.10)^{-t} P_t^{0.5} \log (cn_t/P_t)$, where the pure time discount is ten per cent and P_t denotes total population in the period. The capital stock in the post-terminal year is used as a proxy for post-terminal consumption over an infinite horizon on the assumption that consumption grows post-terminally at the compound yearly growth rate of eight per cent. For the properties of a welfare function with a constant elasticity of marginal utility such as the one employed here see Feldstein /5/.

[21] There are $2^6 = 64$.

charges (Solution B). A comparison of solution B with the other solutions indicates the cost associated with planning on the basis of average costs where there are economies of scale. Planning on the basis of the continuous solution would require the construction of small petrochemicals and steel complexes every two years.[22] The true cost of this investment pattern surpasses that of every other solution enumerated but one.

Only the construction of a petrochemicals plant in the first period has a positive surplus -- Solution C is the optimal one. The surplus is equal to .05 per cent of the optimal objective value. The fact that the objective values lie so close together in this case merely reflects the close similarity between alternative solutions. The compound growth rate of gross domestic product lies between 10.5 and 10.8 per cent in all solutions, that of consumption between 7.4 and 7.6.[23] With alternative objective functions (or export and terminal debt limits) which would associate a different resource allocation with each investment pattern, the differences in objective values could be greater.[24]

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[22] Note that, in fact, the continuous solution produces an inconsistent plan in which resources are over-utilized.

[23] Over the last four years gross domestic product in Korea has grown at a compound growth rate above eight per cent even though the last two years have seen bad rice harvests.

[24] This is especially true if the post-terminal capital stock does not enter the objective function, see Westphal /12/ where the surplus on each project is more than one per cent of optimal welfare value.

The gain of one twentieth of a per cent in welfare over an infinite horizon associated with shifting resources to the petrochemicals sector should not be minimized; however, the estimate is well within the model's margin of error due to inaccurate data. But, though the solutions lie close together in welfare value, the resource allocation associated with each is markedly distinct. The model clearly implies that undertaking either project has definite implications for resource allocation throughout the economy in the plan period.

The gap between imports and exports and the amount of excess capacity throughout the economy display the greatest sensitivity to the pattern of investment in petrochemicals and iron and steel. Of the determinants of the import - export gap, competitive imports and non-competitive imports of capital goods are the most sensitive. For the plant scales observed in these solutions, the petrochemicals complex costs about eighty billion won ($296 million) and the steel mill about forty five billion won ($167 million).[25] The construction of the former will result in a savings of foreign exchange by reducing imports of petrochemicals thirty-eight billion won by 1972. For the latter the savings will be thirty billion won.

By comparing the solutions it can be seen that when a major project is undertaken the total non-competitive imports of capital

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[25]The capacity of the petrochemicals complex is about 55,000 tons of ethylene (the major product stream) and that of the steel mill is about three quarters of a million tons annual output.

goods rise dramatically (by an amount almost equal to the foreign exchange component of the project). In subsequent periods competitive imports tend to be less as borrowing is reduced to offset the greater borrowing needed to finance the project in an earlier period. This is especially clear if one compares solutions C and D or C and F. The timing of foreign capital inflows responds to the pattern of investment so that the projects are financed through foreign capital in the period of construction. Foreign capital is used to alter the timing of investment, but not the total amount of investment. (Total investment during the plan does not differ by more than one or two per cent between solutions.)

The influence of the timing of the projects on the time path of competitive imports is somewhat obscured by the presence of large amounts of excess capacity in the processed food and chemicals sectors in 1969-1970. In all solutions, it is profitable to build up capacity in these sectors in the first period though it will not be used until the third. The excess capacity in 1969-1970 is due to the combined shortages of foreign exchange to finance imports for use in production and of capacity in the overhead sectors. Patterns of investment in which capacity is constructed initially in petrochemicals and/or iron and steel result in sets of production levels which minimize the effect of these shortages since exports of these commodities are then possible.

With so much variability in the aggregate characteristics of the solutions, it is not surprising that investment and production

levels in sectors other than petrochemicals or iron and steel are quite sensitive to the timing of the projects. The timing of investment in each sector is more sensitive to project timing than is total investment in each over the six year period. As might be expected, the sectors most influenced are those related through supply and demand to petrochemicals or iron and steel. Table IV compares solutions C, D and F in this regard.

Conclusion

There can be no question that investments in petrochemicals and iron and steel in Korea are lumpy and will have a significant impact on resource allocation decisions throughout the economy. It is this fact that makes a general equilibrium approach to their evaluation desirable. But not all projects in less developed countries are characterized by lumpiness -- some, like the construction of a textiles plant or a small irrigation project, are indeed marginal in their influence on the structure of future activity and can be analyzed using partial equilibrium tools. The determinants of the dividing line between lumpy and marginal projects have yet to be adequately examined, but the characteristics of the model explored here suggest that projects characterized by indivisibility and high cost should be considered lumpy and thus analyzed within a general equilibrium model.

TABLE IV

Capacity Growth in Solutions C, D & F

Sector	Solution A				Solution B				Solution C			
	1	2	3	4	1	2	3	4	1	2	3	4
Agriculture,Forestry,Fishing	6	6	7	3	6	6	7	3	6	6	7	3
Mining (c)	8	8	13	5	8	8	13	5	8	8	13	5
Processed Food, Wood Products	65	-5	58	16	18	31	59	16	47	6	60	16
Fiber Spinning (b)	67	21	27	17	72	17	24	16	67	20	26	17
Textiles Fabrics (b)	26	52	47	19	29	48	44	18	27	50	46	19
Light Manufacturing (b)	-11	140	77	25	48	41	41	20	-11	139	81	25
Chemicals	68	-6	47	15	75	6	107	23	67	-6	46	15
Fertilizer	35	-8	-5	3	35	8	-5	3	35	-8	-5	3
Petroleum (a)	77	43	40	24	91	38	37	24	85	40	39	24
Construction, Cement	33	42	36	17	35	36	32	16	35	38	33	16
Iron & Steel	161	38	39	31	162	34	36	30	-9	99	121	26
Steel Products (e)	23	33	34	14	24	29	31	13	-10	83	58	17
Finished Metal Products (d)	55	53	47	23	59	46	43	22	59	50	46	23
Machinery,Transport Equip.(d)	79	62	51	28	76	61	50	27	82	57	49	27
Overhead & Services (a,c)	18	35	34	13	19	34	31	13	18	35	32	13
Petrochemicals					x							
Integrated Steel Mill									x			

Notes: 1, 2, 3: Per cent capacity growth in 1967-68, 1969-70, and 1971-72 respectively.
4: Compound growth rate of capacity, 1967 to 1973.
x: Plant constructed in this sector in this period.
a: Principal suppliers of petrochemicals complex.
b: Principal users of petrochemicals.
c: Principal suppliers of integrated steel mill.
d: Principal users of integrated mill's outpout
e: Traditional iron and steel products products produced in small, somewhat outdated plants. The integrated steel mill produces both these forms of ferrous metal products as well as formerly non-produced forms.

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