Income elasticity of demand within individual consumer groups and the level of income elasticity of the entire market demand

Příjmová pružnost poptávky v rámci jednotlivých spotřebitelských skupin a úroveň příjmové elasticity celé tržní poptávky

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Abstract: The paper is focused on the derivation of the mathematical relationship among the income-elasticity level of the entire market demand and the income-elasticity values of the demand functions of the consumers' groups buying on the defined market. The determination of the mathematical term was based on the linearity of the relevant demand functions. Under the linearity assumption, the income elasticity coefficient of the entire market demand equals the weighted sum of the income-demand elasticities of the differentiated consumer groups buying on the given market. The weights in the aggregation formula are defined as the related demand shares, i.e. as the proportions of the groups' demands to the entire market demand. The derived aggregation equation is quite held if no demand interactions (e.g. the snob or fashion effect) are recorded among differentiated consumers' groups. The derived formula was examined by using empirical data about the consumer behaviour of Czech households in the market of meat and meat products (Czech Statistical Office). However, the application potential of the achieved term for the income-elasticity aggregations is much broader within the consumer-behaviour analysis. In addition to the subject aggregations of the demand functions, we can also apply the derived formula for the analysis and estimations of the income elasticities within the demand-object aggregations, i.e. the multistage analysis of the income elasticity of consumer demand. Another possibility of the use of the aggregation equation is for the evaluations and estimations of the income elasticity of the region-demand functions in relation to the subregions' demands or reversely.

Key words: income elasticity, market demand, consumer groups, group's demand, income-elasticity aggregation

Abstrakt: Příspěvek se zaměřil na vymezení matematického vztahu mezi příjmovou elasticitou tržní poptávky a hodnotami příjmových elasticit u poptávkových funkcí jednotlivých spotřebitelských skupin, které se vyskytují na daném trhu. Určení tohoto vtahu bylo prováděno za předpokladu lineárních aproximací jednotlivých poptávkových funkcí. Odvozený vztah byl pak vyzkoušen na empirických datech z oblasti chování českých spotřebitelů na trhu s masem a masnými výrobky. Vedle odvození a aplikace zkoumaného vztahu jsou v tomto článku rovněž naznačeny některé další možnosti jeho využití při analýze spotřebitelského chování. Při prováděné analýze bylo zjištěno, že za předpokladu linearity příslušných poptávkových vztahů lze hodnotu koeficientu příjmové elasticity tržní poptávky určit z váženého součtu dílčích koeficientů příjmové elasticity poptávky za jednotlivé spotřebitelské skupiny, které se nachází na daném trhu. Váhy v daném součtu jsou definovány jako podíly příslušné úrovně dílčí poptávky na celkové tržní poptávce. Takto formulovaný vztah ovšem platí pouze v případě, že mezi poptávkami jednotlivých spotřebitelských skupin neexistují vzájemné interakce, typu módní nebo snobský efekt ap. Použití odvozeného vztahu je však v rámci analýzy spotřebitelského chování mnohem širší. Vedle agregace poptávkových vztahů ve smyslu spotřebitelských subjektů je totiž stejně možné získanou rovnici použít při hodnocení příjmové pružnosti při předmětové agregaci poptávkových vztahů, tedy vícestupňová analýza příjmové elasticity spotřebitelské poptávky. V rámci zavedených předpokladů lze odvozenou rovnici využít také při hodnocení příjmové elasticity spotřebitelské poptávky po určitém statku na úrovni určitého územního celku, který je tvořen menšími celky (regiony nebo subregiony).

Klíčová slova: příjmová elasticita, tržní poptávka, spotřebitelské skupiny, poptávka skupiny agregace příjmové elasticity

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Supported by the Ministry of Education, Youth and Sports of the Czech Republic (Grant No. MSM 6215648904 – thematic direction No. 4 The development tendency of agribusiness, forming of segmented markets within commodity chains and food networks in the process of integration, globalization and changes of agrarian policy).

INTRODUCTION AND AIM OF PAPER

The research of the income-demand elasticity gives a lot of useful information. For instance, we can use the given information for adjusting the economically effective level of the household-income taxation, see Banks et al (1996). First and foremost, the knowledge of the income-elasticity level of consumer demands is quite essential for the correct analyses and estimations of price elasticity of the relevant demand functions, it is obvious from the new approach to the construction of demand models, see Deaton, Muellbauer (1980) or Pollak, Wales (1992).

Income elasticity of demand reactions is measured by the means of the elasticity coefficients in percentage terms, thus without regard to the original units. Due to this property of the elasticity coefficients, it is possible to compare the income-demand reactions in the varied consumption fields or among different consumers, respectively among different consumers groups. The second possibility of the comparison is particularly effective in socio-economical researches. Within these researches, the coefficients of income elasticity could be used for the numerical description of the consumption preferences within studied consumer subjects. The quantitative analyses and the mutual comparisons of the preferences of consumer subjects are possible too, McDowell et al. (1997) or Syrovátka (2001).

For the evaluations and estimations of income elasticity of the consumer demands, the analysis of relationships between the income-elasticity values of the individual demand functions and the level of income elasticity in their aggregate is very useful as well. The given relationship may be researched under the aggregation by the consumption items or under the aggregation by the consumer subjects. The paper was focused on the determination of the mathematical term between the income elasticity level of the entire market demand and income elasticities of demand functions of differentiated consumer groups, purchasing on the given market. The formula was derived under the linearity assumptions of all the related demand functions. The defined formula was applied in the field of consumer behaviour of Czech households on the market for meat and meat products.

METHODOLOGY – DERIVATION OF STUDIED RELATIONSHIPS

Let us suppose that the linear model (1) simulates market demand for certain normal (non-inferior) goods:

$$Q = A - B \times p + C \times \overline{m} \tag{1}$$

Where, Q denotes the total market demand for the goods, p is their market price and $\overline{m}$ represents the average level of incomes of the consumers buying the goods on the target market. With respect to the linear definition of the market-demand model (1), the coefficient of income elasticity (η) is given as:

$$\eta = C \times \frac{\overline{m}}{Q} \tag{2}$$

Further, let us suppose that the demand side on the target market is compound from k consumers' groups. All the consumers' groups (1, 2, ... k) pay the same market price for the given goods (p), but the average income within these groups is different $\overline{m}_1, \overline{m}_2, \ldots \overline{m}_k$ as well as the quantities of the groups' demands ($q_1, q_2, \ldots q_k$). For the simulations of these groups' demands, the linear models (3-1), (3-2) ... (3-k) are sufficiently exact too:

$$q_1 = a_1 - b_1 \times p + c_1 \times \overline{m}_1 \tag{3-1}$$

$$q_2 = a_2 - b_2 \times p + c_2 \times \overline{m}_2 \tag{3-2}$$

.....

$$q_k = a_k - b_k \times p + c_k \times \overline{m}_k \tag{3-k}$$

Within the introduced system of the linear demand functions (3-1), (3-2) to (3-k), the coefficients of income elasticities are defined as follows:

$$\eta_1 = c_1 \times \frac{\overline{m}_1}{q_1} \tag{4-1}$$

$$\eta_2 = c_2 \times \frac{\overline{m}_2}{q_2} \tag{4-2}$$

.....

$$\eta_k = c_k \times \frac{\overline{m}_k}{q_k} \tag{4-k}$$

If the individual demand functions of consumers' groups are completely independent, i.e. there are not any mutual relationships among the groups' demands, we can simply determine the model of the entire market demand as follows:

$$Q(p,\overline{m}_1,\overline{m}_2,\ldots,\overline{m}_k) = q_1(p,\overline{m}_1) + q_2(p,\overline{m}_2) + \ldots + q_k(p,\overline{m}_k) \tag{5}$$

Associated with the introduced linear definition of the individual demand functions (3-1), (3-2) ... (3-k), it is possible to write the market-demand model (5) as:

$$Q = [a_1 - b_1 \times p + c_1 \times \overline{m}_1] + [a_2 - b_2 \times p + c_2 \times \overline{m}_2] + ... + [a_k - b_k \times p + c_k \times \overline{m}_k] =$$
$$= [a_1 + a_2 + ... + a_k] - [b_1 + b_2 + ... + b_k] \times p + [c_1 \times \overline{m}_1 + c_2 \times \overline{m}_2 + ... + c_k \times \overline{m}_k] \quad (6)$$

With respect to the aim of this article and the initial assumptions, the notation of the linear model (6) can be rearranged into the equation (7):

$$Q = A - B \times p + c_1 \times \overline{m}_1 + c_2 \times \overline{m}_2 + ... + c_k \times \overline{m}_k \quad (7)$$

where the partial intercepts (a_1, $a_{2,}$ ... a_k) as well as the partial price parameters (b_1, $b_{2,}$ ... b_k) were summed up. The market-demand model in the form (7) that reflects the different income-demand functions of the consumers' groups may also be obtained by the substitution of the following term:

$$C = \frac{\overline{m}_1}{\overline{m}} \times c_1 + \frac{\overline{m}_2}{\overline{m}} \times c_2 + ... + \frac{\overline{m}_k}{\overline{m}} \times c_k \quad (8)$$

into the market-demand model (1). Substituting of the found term (8) into the income elasticity coefficient (2), we achieve the decomposition formula for the value of this elasticity coefficient (η) into k elasticity components:

$$\eta = \eta(1) + \eta(2) + ... + \eta(k) \quad (9)$$

The first component in the decomposition term (9) defines the elasticity of the entire market demand in response to the changes in the average level of incomes of the 1st consumer group:

$$\eta(1) = c_1 \times \frac{\overline{m}_1}{m} \times \frac{m}{Q} = c_1 \times \frac{\overline{m}_1}{Q} \quad (10\text{-}1)$$

The second component of the term (9) then measures the elasticity of the entire market demand with respect to the changes in the average level of incomes of the 2nd consumer group:

$$\eta(2) = c_2 \times \frac{\overline{m}_2}{m} \times \frac{m}{Q} = c_2 \times \frac{\overline{m}_2}{Q} \quad (10\text{-}2)$$

Analogically, we can explicate the k^{th} component of the decomposition equation (9). Thus, the component k records the elasticity of the entire market demand in relation to the changes in the level of the average income of the k^{th} consumer group:

$$\eta(k) = c_k \times \frac{\overline{m}_k}{m} \times \frac{m}{Q} = c_k \times \frac{\overline{m}_k}{Q} \quad (10\text{-k})$$

With respect to the validity of the term (8), we can naturally determine the coefficients (10-1), (10-2) to (10-k) from the market demand model in the form (7) by a routine method. Furthermore, the introduced coefficients of the income elasticity of the market demand (10-1), (10-2) to (10-k) may also be achieved from the coefficients (4-1), (4-2) to (4-k), i.e. from the coefficients of the income elasticity of the demand functions of differentiated consumers groups. If we multiply the elasticity coefficients (4-1), (4-2) to (4-k) by the related demand shares (q_1/Q, q_2/Q, ..., q_k/Q), then we obtain the coefficients of income elasticity at the level (10-1), (10-2) to (10-k):

$$\eta_1 \times \frac{q_1}{Q} = c_1 \times \frac{\overline{m}_1}{q_1} \times \frac{q_1}{Q} = \eta(1) \quad (11\text{-}1)$$

$$\eta_2 \times \frac{q_2}{Q} = c_2 \times \frac{\overline{m}_2}{q_2} \times \frac{q_2}{Q} = \eta(2) \quad (11\text{-}2)$$

.....

$$\eta_k \times \frac{q_k}{Q} = c_k \times \frac{\overline{m}_k}{q_k} \times \frac{q_k}{Q} = \eta(k) \quad (11\text{-k})$$

Due to the introduced terms (11-1), (11-2) to (11-k), the derived equation for the income-elasticity decomposition, respectively, aggregation (9) may consequentially be rewritten as:

$$\eta = \eta_1 \times \frac{q_1}{Q} + \eta_2 \times \frac{q_2}{Q} + ... + \eta_k \times \frac{q_k}{Q} \quad (12)$$

RESULTS AND DISCUSSION

The relationship between the level of the income elasticity of the entire market demand and the values of the income elasticities of the demand functions of consumers' groups buying on the target market was studied. In accordance with the above-described way, the following formula was obtained:

$$\eta = \eta_1 \times \frac{q_1}{Q} + \eta_2 \times \frac{q_2}{Q} + ... + \eta_k \times \frac{q_k}{Q}$$

The achieved formula (12) defines that the income elasticity level of the entire market demand equals to the weighted sum of the income elasticities of demand functions of the differentiated consumer groups buying on the given market. The weights in the sum (12) are defined as the related demand shares, i.e. as the proportions of the groups' demands to the entire market demand. The aggregation term (12) was determined under the assumptions of the linearity of all related demand functions and no demand interactions among the differentiated consumer groups. Thus, the non-linearity of demands and/or

the demand interactions would lead to a difference of the obtained results by the equation (12) and the real values of income elasticity of the studied demand functions.

In accordance with the specified assumptions, the derived aggregation equation (12) may be used for the estimations of the income elasticity level of the studied market demand. For these estimations, we need to know all the levels of income elasticities of the differentiated groups' demands and their demand shares too. Under the analogical conditions, we can also use the term (12) to the determination of income elasticity in the selected group's demand. The suggested applications of the defined equation (12) were examined in the field of estimations of the income elasticity of the market demand of Czech households for meat and meat products. For this purpose, there the data and some results from the dissertation work, Syrovátka (1999), were used. In the dissertation work, the regression models of the Engel's demand were developed and applied for the behaviour simulation of four categories of households: employees (i = 1), farmers (i = 2), self-employed (i = 3) and pensioners (i = 4), i.e. for four consumer groups on the target market. The consumer behaviour of the studied households' categories was analysed using quarterly data from the Czech Household Budget Survey for the period from 1994 to 1998. The demand models of the Engel's type with the introduced explicit dynamics were based on the linear construction:

$$q_{it} = A_i + B_i \times {}_r m_{it} + C_i \times t \qquad (i = 1, 2, 3, 4); (t = 1, 2, ..., 20) \qquad (13)$$

where

q_{it} = quantity of the quarterly purchase of meat and meat products by the i^{th} households' categories (consumers' groups) at time t

${}_r m_{it}$ = average level of the real income with i^{th} the households' categories (consumers' groups) at time t

t = time variable.

The received values of the models' parameters (A_i), (B_i), (C_i), the determination coefficients (r_i^2) and the results of F-tests are illustrated in Table 1.

Using the Engel's demand models displayed in Table 1, we can calculate the levels of real income elasticities of the investigated groups' demands for meat and meat products between 1994 and 1998 (η_{it}). If we do not concentrate on the development of income elasticity of the studied market demand during the observed period, we can estimate the level of its income elasticity on basis of the equation (12) from the average values of η_{it}. These average levels of the income elasticity of differentiated groups' demands in the observed period (1994–1998) were determined in accordance with the formula for the arithmetic mean:

$$\overline{\eta}_i = \frac{1}{20} \times \sum_{t=1}^{20} \eta_{it} \qquad (i = 1, 2, 3, 4) \qquad (14)$$

With respect to the suggested way of the income-elasticity estimation of the studied market demand (without development of this elasticity coefficient), we also need to calculate the average levels of the related demand shares:

$$\overline{w}_i = \frac{1}{20} \times \sum_{t=1}^{20} w_{it} = \frac{1}{20} \times \sum_{t=1}^{20} \frac{q_{it}}{Q_t} \qquad (i = 1, 2, 3, 4) \qquad (15)$$

The achieved values of $\overline{\eta}_i$ and $\overline{w}_i$ are displayed in Table 2.

We input the calculated average values ($\overline{\eta}_i$), ($\overline{w}_i$) into the derived aggregation equation (12) and thus we determine the total average income elasticity of

Table 1. The Engel's demand models for the consumers' groups buying on the market for meat and meat products

Households' categories	Linear dynamic model $q_{it} = A_i + B_i \times {}_r m_{it} + C_i$; ($t$ = 1, 2, ..., 20)	Statistical verification	
		r_i^2	F-test
Employees (i = 1)	$q_{1t} = -0.5125 + 1.0257 \times 10^{-3} \times {}_r m_{1t} - 0.1161 \times t$	0.6384	15.0068
Farmers (i = 2)	$q_{2t} = -2.221 + 1.0447 \times 10^{-3} \times {}_r m_{2t} - 1.0537 \times 10^{-2} \times t$	0.7644	27.5710
Self-employed (i = 3)	$q_{3t} = -1.2468 + 1.0507 \times 10^{-3} \times {}_r m_{3t} - 0.1015 \times t$	0.6144	13.5417
Pensioners (i = 4)	$q_{4t} = +13.6682 + 2.9278 \times 10^{-4} \times {}_r m_{4t} + 0.1125 \times t$	0.5564	10.6598

Table 2. The average level of real income-demand elasticity and the average level of demand shares between 1994 and 1998 within the investigated consumers' groups

Households' categories	$\bar{\eta}_i$	$\bar{w}_i$
Employees (i = 1)	1.1409	0.2327
Farmers (i = 2)	1.2406	0.1888
Self-employed (i = 3)	1.1938	0.2278
Pensioners (i = 4)	0.1796	0.3507

the entire market demand of Czech households for meat and meat products ($\bar{\eta}$):

$$\bar{\eta} = \bar{\eta}_1 \times \bar{w}_1 + \bar{\eta}_2 \times \bar{w}_2 + \bar{\eta}_3 \times \bar{w}_3 + \bar{\eta}_4 \times \bar{w}_4 = \\ = 1.1409 \times 0.2327 + 1.2406 \times 0.1888 + 1.1938 \times 0.2278 + \\ + 0.1796 \times 0.3507 = 0.8346 \qquad (16)$$

Under the introduced assumptions of the linearity of all demand functions and no demand interactions among observed consumer groups, in accordance with (16), it is possible to say that the average level of real income elasticity of the market demand for meat and meat products in the observed time period equals 0.8346. Thus between 1994 and 1998, the 1% rise in the real incomes of Czech households brought the increase in their purchases of meat and meat products by approximately 0.83%.

In addition to the above-mentioned application, the derived formula for the aggregation of the income elasticity coefficients (12) is also useable for the estimations of income elasticities of the demand functions within the regions and their subregions. Thus, we can determine the income elasticity of the entire region demand in relation to the income elasticities of subregion-demand functions or, analogically, we can estimate income elasticity some of the subregion demands on the basis of the relevant values of the related coefficients of income elasticities. However, the accuracy of the elasticity estimations according to the equation (12) is also restricted by using the linear approximations of the real demand functions and not taking into account the interactions among individual demands, which are aggregated, see the initial assumptions of this derivation process.

Further, the obtained aggregation principle that is defined in the equation (12) can be applied within the object aggregation of demand functions of the individual consumer or the consumer's group. This application of the equation (12) is very useful for the two-stage or the multi-stage analysis of the consumer's demand system or the group's demand system, see Moschini (2000). However, there are some differences in the derivation process of the studied aggregation term. In this case, we do not need to differentiate the consumer incomes, because the system of n demand functions of the only one consumer or one consumer group is analysed. Thus, the income-elasticity coefficients are given as:

$$\eta_1 = c_1 \times \frac{m}{q_1} \qquad (17\text{-}1)$$

$$\eta_2 = c_2 \times \frac{m}{q_2} \qquad (17\text{-}2)$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$\eta_n = c_n \times \frac{m}{q_n} \qquad (17\text{-n})$$

With respect to the eventual heterogeneous units within the aggregation of demanded quantities, the individual-demand functions and the aggregate demand are investigated in the expenditure terms. The expenditure analysis of the demand systems requires the initial transformation of the nominal expenditures and the incomes into their real levels. In this context, it is possible to bring in the aggregation equation in another form:

$$1 = \eta_1 \times \frac{x_1}{m} + \eta_2 \times \frac{x_2}{m} + \ldots + \eta_k \times \frac{x_k}{m} \qquad (18)$$

This aggregation formula (18) is based on the assumption that the total expenditures for all consumed goods (x_1), (x_2), …, (x_n) are equal to the disposal income of the given consumer subject (m), thus:

$$m = x_1 + x_2 + \ldots + x_n = p_1 \times q_1 + p_2 \times q_2 + \ldots + p_n \times q_n \qquad (19)$$

In the theory of consumer's behaviour, the equation (18) is termed by the Engel's aggregation condition (adding up) and it is thoroughly examined within the development of the theoretical consistent models of the demand systems. Pursuant to the Engel aggregation condition, the average level of the income-demand elasticity within the income-expenditure well-balanced consumer bundle (19) equals 1, see Maurice et al. (1998).

CONCLUSION

Under the assumption of the linearity of all related demand functions, the coefficient of income elasticity of the entire market demand equals the weighted sum of the income-demand elasticities of the differentiated consumer groups buying on the given market. The weights in the aggregation equation are defined as the related demand shares, i.e. as the proportions of the groups' demands to the entire

market demand. The derived aggregation equation holds fully if no demand interactions (e.g. the effect of snob and fashion consumption) are recorded among the differentiated consumer groups. However, within the analysis of consumer's behaviour, the application potential of the achieved term for the income-elasticity aggregations is much broader. In addition to the subject aggregations of the demand functions, we can also apply the derived formula for the analysis and estimations of the income elasticities within the demand-object aggregations, i.e. the multistage analysis of the income elasticity of consumer demand. Another possibility of the use of the aggregation equation is within the evaluations and estimations of the income elasticity of the region demand functions in relation to the subregion demands or reversely, within the demand income-elasticity evaluations in the some subregion.

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Arrived on 13th June 2006

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