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Risk-adjusted approaches for planning sustainable agricultural development

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Abstract In this paper we show that explicit treatment of risks and uncertainties in agricultural production planning may considerably alter strategies for achieving robust outcomes with regard to sustainable agricultural developments. We discuss production planning models under uncertainties and risks that may assist in planning location-specific production expansion within environmental and health risk indicators and constraints. The proposed approaches are illustrated with the example of spatially explicit livestock production allocation in China to 2030.

Keywords Agricultural production · Risks · Safety constraints · CVaR risk measure · Downscaling · Robust decisions

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1 Introduction

Environmental impacts and health hazards associated with intensive agricultural production have increased awareness and established the need to identify pathways towards sustainable agriculture. Undesirable impacts of intensification include environmental pollution, input-intensive mono-cropping, and the marginalization and decline of smallholder farms, causing abandonment of land and migration of rural population to cities. These are further exacerbated by various risks such as climate change and variability, natural catastrophes, market distortions and instabilities.

In this paper we show the need to account for risks and uncertainties when planning sustainable agricultural development. Awareness of risks may considerably alter production decisions. This fact is illustrated in Sect. 2 with a simple model of two agricultural producers characterized by different levels of efficiency and exposure to risks. The example captures, in a nutshell, the features of a geographically detailed and dynamic model for agricultural production planning under risks and uncertainties, as adopted for the analysis of livestock production development in China to 2030 (Fischer et al. 2006b, c).

Concentration of intensive livestock production is an important cause of environmental pollution and health hazards. The analysis in Fischer et al. (2006c) has shown that the development of China's livestock production sector cannot just continue along past intensification trends. The goal of this paper is to discuss model-based approaches to guide decisions regarding the inevitable and significant future expansion of livestock production with respect to economic conditions at locations accounting for sustainability and risk indicators. Indicators of sustainability and risks are defined by various interdependent factors including the spatial distribution of people and incomes, the current levels of livestock production and intensification, and the conditions and current use of land resources. Combinations of these factors are used in proposed models to distinguish different locations by the degree of their risk exposure in order to achieve robust solutions.

In Sect. 3 we introduce a spatially explicit and dynamic simulation model used for planning livestock production expansion coherently with projected demand increases to 2030. It allows for spatio-temporal and risk-adjusted analysis of production developments under alternative scenarios. The meaningful specification of indicators and constraints to define alternative allocation scenarios is often constrained by the paucity of data at required resolutions. In this case, specific downscaling (disaggregating) and upscaling (aggregating) procedures (Fischer et al. 2006a) provide a tool for estimation of dependencies between the geographical factors, constraints, and economic-environmental policy responses. In Sect. 3 we analyze the main features of these procedures for spatial production allocation with respect to risks and suitability constraints in locations.

Section 4 introduces a new stochastic optimization approach for planning production allocation when some of the risks in the model of Sect. 3 are explicitly taken into account by stochastic scenarios. In fact, the Sects. 3 and 4 distinguish two types of uncertainties: endogenous uncertainties associated with behavioral principles regarding production expansion and exogenous uncertainties associated with parameters of models. Section 5 describes alternative allocation scenarios and presents selected numerical results. Section 6 concludes and indicates directions for future work.

2 Cooperation and co-existence for risk sharing

Ricardo (1822) stipulated that trading nations will gain by specialization in goods of comparative advantage. Accordingly, we may expect that production should be undertaken by the most efficient agent, with intensified production on large farms. This is true only under idealized conditions when risks are not accounted for. In reality, the outcomes of production intensification are causing great concerns regarding the pollution of natural resources and ecosystems, health problems, and lack of sustainability.

Agricultural production facilities may be exposed to various risks, but also may cause different negative impacts. Depending on the location and intensity, values of the facilities are interdependent subject to contingencies, and are determined endogenously. For this reason, of particular interest are production chains with large and small units to stabilize the aggregate production. Such diversification of producers by scale and location hedges against economic and environmental risks, improves welfare and ensures continuous supply of agricultural products to markets. Explicit accounting of risks may considerably alter the composition of production units and their intensification levels in a chain.

Let us illustrate this with a stylized model of only two producers, $i = 1, 2$, which in Sect. 4 will be extended to a multi-producer case. Let x_i denote the production level of ith producer and assume that only one good is produced, e.g., meat; c_i is the cost per unit of produce. The product can also be imported from an external source with price b per unit of produce. Assume $c_1 < c_2 < b$, i.e., the cheapest source is the first producer. The production has to satisfy the exogenous inelastic demand d of a given region.

In the *absence of risks*, the model is formulated as the minimization of the total cost function:

$$c_1 x_1 + c_2 x_2 \tag{1}$$

subject to

$$x_1 + x_2 = d, \quad x_1 \geq 0, \;\; x_2 \geq 0, \tag{2}$$

where x_1, x_2 are production capacities. The optimal solution to the problem is $x_1^* = d$, $x_2^* = 0$, i.e., the production is undertaken by the more efficient producer, which accords with Ricardo's views.

In case of *risk exposure*, the endogenous supply (2) is expressed, for example, as a linear function

$$a_1 x_1 + a_2 x_2 = d, \tag{3}$$

where a_1, a_2 are contingencies or "supply" shocks to x_1, x_2, e.g., due to outbreaks of diseases, weather risks, or other hazardous events. We assume that a_1, a_2 are random variables $0 \leq a_i \leq 1$, which may reduce the supply from $i = 1, 2$. If endogenous supply $a_1 x_1 + a_2 x_2$ falls short of demand d, the residual amount $d - a_1 x_1 - a_2 x_2$ must be obtained from external sources at unit import cost b. The planning of production capacities x_1, x_2 can be evaluated from the minimization of total production costs and potential import cost, i.e., the minimization of the function

$$F(x) = c_1 x_1 + c_2 x_2 + bE \max\{0, d - a_1 x_1 - a_2 x_2\},$$

where $x_1 \geq 0$, $x_2 \geq 0$, and the expected import cost when the demand d exceeds the supply $a_1 x_1 + a_2 x_2$ is $bE \max \{0, d - a_1 x_1 - a_2 x_2\}$. In this case, the role of a less efficient producer for stabilizing supply is clearly visible.

Assume that only the efficient producer is at risk, that is $a_2 = 1$. Let function $F(x)$ have continuous derivatives, e.g., the cumulative distribution function of a_1 has a continuous density function. It is easy to see that the optimal positive decisions $x_1^* > 0$, $x_2^* > 0$ exist in the case when partial derivatives meet $F_{x_1}(0,0)<0$, $F_{x_2}(0,0)<0$. We have $F_{x_1}(0,0)=c_1-bEa_1$, $F_{x_2}(0,0)=c_2-b$, and, perhaps counter intuitively, the less efficient producer 2 is active unconditionally (since $c_2-b<0$). The cost efficient producer 1 is inactive in the case $c_1-bEa_1\geq 0$, leaving production entirely to the higher-cost producer 2. Only in the case $c_1-bEa_1<0$ both producers are active. Hence, in this example the less cost-efficient producer is able to stabilize the aggregate production in the presence of contingencies affecting the more cost-effective producer 1.

To derive the market share of the producer 2, take the derivative

$$F_{x_2}(x,x_2)=c_2-bP[d>a_1x+x_2]$$

according to optimality conditions of stochastic minimax problems (Ermoliev et al. 1988). This means that the optimal production level $x_2^*>0$ of producer 2 is a quantile defined by the equation $P[d>a_1x_1^*+x_2^*]=c_2/b$, assuming $x_1^*>0$ (otherwise $x_2^*=d$). It also depends on x_1^* and all conditions ensuring a positive share x_1^* of producer 1. Although not at risk ($a_2=1$), the optimal production level of producer 2 is defined by (3) through interdependencies among producers participating in the same market with demand d. This example illustrates that the production level of the producer 1 at risk is implicitly constrained by interdependencies among contingencies, production costs and the import price.

In this example, the contingencies that determine each producer's market share are characterized by probability distributions. Often, the contingencies and other factors that guide production allocation (e.g., livestock diseases, environmental pollution, spatial distribution of population and incomes, current level of production) may have complex and often intractable geographical and temporal patterns. This requires specific stochastic simulation models as in Sect. 3 and downscaling procedures allowing for estimation of required values based on available auxiliary statistics and model-derived results.

3 Risk adjusted approaches for agricultural planning

3.1 A simulation model

The stochastic and dynamic livestock and crop production model developed by the land use change and agriculture (LUC) program at the International Institute for Applied Systems Analysis (IIASA) (Fischer et al. 2006b) integrates demographic, economic, agricultural and environmental modeling components. The IIASA model is essentially an accounting GIS-based model, which allows to incorporate inherent processes in an endogenized manner.

The model is developed with the aim to assist in planning sustainable agricultural developments combining various national, subnational and regional interacting agricultural activities, production, processing, consumers. Together with reasonable scales of biophysical modeling, this allows for production planning within limited resources and possibilities to improve or recover production potentials, against uncertainties of weather, climate change, market situation or other risks such as the contamination of land or pasture. Simplicity of model's structure enables to incorporate individual and collective risks combined with proper equity, fairness and safety constrains, which leads to welfare generating policies. Contrary to traditional linear programming (Arrow et al. 1958) and general equilibrium approaches (Kaldor 1996), the model allows to deal with economies of scales, time dynamics and increasing returns. This phenomenon is typical for practical problems of production and resource allocation, however, the discussion of these topics is beyond the scope of the paper. In contrast to general equilibrium and standard growth theory, the proposed risk-adjusted approach permits to deal with issues involving externalities, inherent uncertainties, non-monetary values such as environmental degradation, non-marketable risks of high consequences, social heterogeneities regarding various representative agents.

Allocation of production facilities have to reasonably confirm to the distribution of current and future consumers including evaluation of the option "make versus buy" typically addressed in spatial production planning models (Karlqvist et al. 1978; Fujita et al. 1999; Hotelling 1931). This implies the analysis of main production and demand driving forces such as population growth, urbanizations, energy provision, infrastructure, markets and market access. The discussed model can easily address regional "behavioral" aspects of production planning if these are determined by criteria other than pure cost–benefit or risks analysis. For example, rebalancing production allocation procedure in Sect. 3.3 allows to account for heterogeneous cultural traditions, complex interactions of behavioral, socio-economic, cultural and technological factors (Ermoliev and Leonardi 1982; Fischer et al. 1996; Wilson 1970), or specific fairness and equity considerations (Manne 1967).

Within a project on "Policy decision support for sustainable adaptation of China's agriculture to globalization" (CHINAGRO; Huang et al. 2003), the model included specifics of China agricultural developments and has been applied for the spatial analysis of future livestock sector expansions. Using alternative economic and demographic projections (Cao 2000; CCAP 2002; Huang et al. 2003; Huang and Liu 2003; Keyzer and van Veen 2005; Liu et al. 2003), the model estimates per capita demand increases and consumption of major agricultural products, e.g., cereals, meat, milk, etc. Demand patterns differ between urban and rural areas, between geographical regions, and vary with income. Thus, with increasing incomes higher quality low-fat meat, e.g., poultry is preferred. In fact, evolution of consumption is modeled as a function of group-specific per capita income increases by applying income elasticities and distinguishing urban and rural consumers.

Agricultural supply is represented at county level, i.e., for about 2,430 spatial units. Smallholders and specialized livestock farms adjust the livestock herd structure and production in response to the demand increase and the changes of consumption patterns. The model distinguishes the following livestock types: poultry, pigs, dairy, cattle, buffaloes, yaks, sheep and goats, and other large animals (combining horses, donkeys, and camels). To examine the current situation and the production intensification trends, modeling of livestock production considers three management systems: traditional, specialized/industrial, and grazing.

In the environmental module, the environmental loads caused by intensive crop and livestock production are evaluated against admissible environmental and health thresholds (which can be proposed by stakeholders and environmental experts). Indicators used for measuring environmental impacts and human health risk are: the density of livestock, nutrients from manure and chemical fertilizers in excess of a location's nutrient uptake by crop production, urbanization share, density of population, and others. Combinations of these and other factors (see Sect. 5) reflect different degrees of socio-economic and environmental risk exposures and can be used to guide sustainable production allocation.

The model simulates different paths of demand increase, which induces respective location-specific production adjustments. In some locations, the environmental and health risk indicators may already exceed admissible thresholds, which signals that further production growth in these locations should not take place. This raises the question of how to adjust the composition and allocation of livestock production facilities in response to increasing demand but without exacerbating environmental and health problems. The detailed description of the model and the allocation procedure (Fischer et al. 2006b) is rather lengthy. Therefore, in the following we provide only rather aggregate representation of their main constraints.

3.2 A simplified production model

When planning livestock development, the objective is to allocate the foreseeable increases of demand for livestock products among the locations and the main production systems in the best possible way while accounting for various risks. In the following model the risks are treated as constraints restricting production expansion. In Sect. 4 we introduce a stochastic model that allows to account for risks and uncertainties in a more explicit manner.

Denote the expected national demand increase (to be satisfied by supply increase) in livestock product i by d_i, $i = \overline{1:m}$. Let x_{ijl} be the unknown supply increase in product i at location j and by management system l. In its simplest form, the problem is to find x_{ijl} satisfying the following system of equations:

$$\sum_{l,j} x_{ijl} = d_i, \tag{4}$$

$$x_{ijl} \geq 0, \tag{5}$$

$$\sum_{i} x_{ijl} \leq b_{jl}, \quad l = \overline{1:L},\ j = \overline{1:n},\ i = \overline{1:m}, \tag{6}$$

where b_{jl} are thresholds aggregating environmental and health risks and imposing limitations to expand production in system l and location j. Apart from b_{jl}, there may be additional limits on x_{ijl}, $x_{ijl} \leq r_{ijl}$, which can be associated with legislation, for example, to restrict production i within a production "belt", or to exclude from urban or protected areas, etc. Thresholds b_{jl} and r_{ijl} may either indicate that livestock in excess of these values is strictly prohibited or it incurs penalties such as taxes or premiums, for mitigation of the risks, say, livestock disease outbreaks or environmental pollution. Equations (4)–(6) belong to the type of transportation problems. However, there may be more general constraints of type $\sum_{il} a_{ijl} x_{ijl} \leq d_i$ as in Sect. 2, $0 \leq a_{ijl} \leq 1$, which require extensions of the proposed approach.

In general, there exist infinitely many solutions of (4)–(6). The aim is to derive a solution that ensures appropriate balance between the efficiency and the risks. We can distinguish two sources of uncertainties generating potential risks: behavioral or endogenous uncertainties associated with allocation of new production capacities and exogenous uncertainties related to parameters of the model. In this section we consider only the first type of uncertainties. Section 4 addresses the second type of constraints.

The information on the current production facilities, threshold values b_{jl}, r_{ijl}, and costs are used to derive a prior probability q_{ijl} reflecting our belief that a unit of demand d_i should be allocated to management system l in location j. The use of priors is consistent with spatial economic theory [see discussion, e.g., in Ermoliev and Leonardi (1982), Karlqvist et al. (1978), Wilson (1970)]. The likelihood q_{ijl} can be inversely proportional to production costs and inherent risks r_{ijl} (Ermoliev and Fischer 1993; Ermoliev and Leonardi 1982). In Sect. 3.3 we show how it is used in a rebalancing procedure to determine the solution of (4)–(6) relying on behavioral, in a sense, risk-averse and cost-minimizing principles defined by this prior as in (10).

3.3 A rebalancing production–allocation algorithm

For simplicity of exposition, let us renumerate all pairs (l, j), $l = \overline{1, L}$, $j = \overline{1, n}$ by $k = 1, 2, \ldots, K$. In this new notation, the problem is formulated as finding y_{ik} satisfying constraints:

$$\sum_k y_{ik} = d_i, \tag{7}$$

$$y_{ik} \geq 0, \tag{8}$$

$$\sum_i y_{ik} = b_k, \quad i = \overline{1, m},\ k = \overline{1, K}, \tag{9}$$

consistent with a prior q_{ik} belief that a unit of demand for product i should be supplied by activity k. For instance, it is reasonable to allocate more livestock to locations with a larger demand increase, higher productivity, or better feed access. Assume that this preference structure is expressed in prior q_{ik}, $\sum_k q_{ik} = 1$ for all i. In this case, the initial amount of production i allocated to k can be derived as $q_{ik} d_i$. But this may lead to violation of constraints (9). Sequential rebalancing (Fischer et al. 2006a) proceeds as follows. Assume that relying on prior probability q_{ik}, the *expected* initial allocation of d_i to k is $y_{ik}{}^0 = q_{ik} d_i$, $i = \overline{1, m}$. However, this allocation may not satisfy constraint $\sum_i y_{ik}^0 \leq b_k$, $j = \overline{1, n}$. Derive the relative imbalances $\beta_k^0 = b_k / \sum_i y_{ik}^0$ and update $z_{ik}^0 = y_{ik}^0 \beta_k^0$, $i = \overline{1, m}$. Now the constraint $\sum_i y_{ik} \leq b_k$ is satisfied, $k = 1, 2, \ldots$, but the estimate z_{ik}^0 may cause imbalance for (7), i.e., $\sum_k z_{ik}^0 \neq d_i$.

Continue with calculating $\alpha_i^0 = d_i / \sum_k z_{ik}^0$, $i = \overline{1, m}$, and updating $y_{ik}^1 = z_{ik}^0 \alpha_i^0$, an so on. The estimate y_{ik}^s can be represented as

$$y_{ik}^s = q_{ik}^k d_i, \quad q_{ik}^s = \left(q_{ik} \beta_k^{s-1}\right) \Big/ \left(\sum\nolimits_j q_{ik} \beta_k^{s-1}\right),$$

$i = \overline{1, m}$, $k = 1, 2, \ldots$ Assume $y^s = \{y_{ik}^s\}$ has been calculated. Find $\beta_k^s = \overline{b}_k / \sum_i y_{ik}^s$ and

$$q_{ik}^{s+1} = \left(q_{ik} \beta_j^s \Big/ \sum\nolimits_i q_{ik} \beta_j^s\right), \quad i = \overline{1, m},$$

$k = 1, 2, \ldots$, and so on.

In this form the procedure can be viewed as a redistribution of required supply d_i among producers $k = 1, 2, \ldots$ by applying sequentially adjusted q_{ik}^{s+1}, e.g., by using a Bayesian type of rule for updating the prior distribution:

$$q_{ik}^{s+1} = q_{ik} \beta_k^s / \sum\nolimits_i q_{ik} \beta_k^s, \quad q_{ik}^0 = q_{ik}.$$

The update is done on an observation of imbalances of basic constraints rather than observations of random variables. A rebalancing procedure, similar to the one described above for Hitchcock–Koopmans transportation constraints (7)–(9), was proposed by G.V. Sheleikovskii [see a proof and references in Bregman (1967)] for estimation of passenger flows between regions. A proof of its convergence to the optimal solution maximizing the cross-entropy function

$$\sum_{i,k} y_{ik} \ln \frac{y_{ik}}{q_{ik}} \tag{10}$$

is given in Fischer et al. (2006a) for rather general types of constraints. It should be noted that in our model we use equality constraints (9). The general inequality constraints are reduced to this model by introduction of a fictitious demand constraint.

4 Stochastic production allocation model

The approach presented in Sect. 3 evaluates the increase of livestock production relying on individual behavioral principles set by priors. There, the risks are characterized in a simplified deterministic way by imposing certain standards as additional "safety" constraints. In general, these constraints may depend on some scenarios of potential future shocks. The behavioral uncertainty in Sect. 3 can also be treated in a stochastic manner as allocation of random demand $d_i(\omega)$ among points $k = \overline{1 : K}$ with respect to the prior q_{ik}, which is a topic of a separate paper.

Let us consider now a more general multi-producer model in a stochastic environment analogous to the example of Sect. 2. We may assume that there is a coordinating agency. The goal of this agency is to maximize the overall performance of the production chain with large and small units to stabilize the aggregate production and increase the facility values.

Suppose that the agency has to determine levels y_{ik} of livestock product i in locations k in order to meet stochastic demand $d_i(\omega)$, where $\omega = (\omega_1, \omega_2, \ldots)$ is a vector of all contingencies affecting demand and production. Naturally to assume that the decision on production expansion has to be made before the information on contingencies arrives. In this case, the total ex-ante production may not exactly correspond to the real demand, i.e., we may face both oversupplies and shortfalls. In other words, the amount of production y_{ik}, $k = 1, \ldots, K$, which is planned ex-ante to satisfy the demand $d_i(\omega)$, $y_i(\omega) = \sum a_{ik}(\omega) y_{ik}$ may underestimate $(y_i(\omega) < d_i(\omega))$ or overestimate $(y_i(\omega) > d_i(\omega))$ the real demand $d_i(\omega)$ under revealed contingencies ω and the safety constraints imposed by strict thresholds b_k in (9). The constraint (9) necessitates, in general, additional supply of ex-ante production $z_i \geq 0$ from external sources (say, through international trade). It may also require the ex-post redistribution of the production from internal producers, $k = \overline{1, K}$, to eliminate arising shortfalls and oversupplies in locations. For now, let us ignore these ex-post redistributional aspects assuming that the most significant impacts are associated with ex-ante decisions y_{ik}

and z_i. In fact, the presented further model can be easily extended to represent the ex-post adjustments of decisions y_{ik}, z_i, as well as temporal aspects of production planning.

Let c_{ik} be the unit production cost. In more general model formulation, c_{ik} may also include the unit transportation cost for satisfying location-specific demand. Then the model of production planning among the facilities can be formulated as the minimization of the total cost function:

$$f(y,z) = \sum_{i,k} c_{ik} y_{ik} + \sum_{i=1}^{m} e_i z_i,$$

subject to constraints (8), (9), and the following additional safety constraints

$$P\left[\sum_{k=1}^{K} a_{ik}(\omega) y_{ik} + z_i \geq d_i(\omega)\right] \geq p_i, \quad z_i \geq 0,\ i = \overline{1,m}, \tag{11}$$

where $e_i > 0$, $i = \overline{1,m}$, denotes the unit import cost. A safety level p_i, $0 < p_i < 1$, defines (ensures) the stability of the supply–demand relations for all possible scenarios (contingencies) ω. The introduction of constraints of type (11) is a standard approach for characterizing stability in case of the insurance business, operations of nuclear power plants and other risky activities especially when involving catastrophic risks (Ermolieva and Ermoliev 2005). Safety constraints of type (11) are usually used in cases where impacts of random interruptions cannot be easily evaluated. In this case, the value p_i is selected such that an expected shortfall occurs only, say, once in 100 months, i.e., $p_i = 1/100$.

The main methodological challenge is concerned with the lack of convexity of constraints (11). Yet, the remarkable fact is that the model defined by (8)–(11) can be effectively solved by linear programming methods due to the following equivalent convex form of this model.

Let us consider the minimization of the expectation function

$$F(y,z) = f(y,z) + \sum_{i=1}^{m} \alpha_i E \max\left\{0, d_i(\omega) - \sum_{k=1}^{K} a_{ik}(\omega) y_{ik} - z_i\right\}, \tag{12}$$

subject to constraints (8), (9), and $z_i \geq 0$, $i = \overline{1,.m}$. The minimization of function $F(y, z)$ is a rather specific case of stochastic minimax models analyzed (both optimality conditions and solution procedures) in Ermoliev et al. (1988). In particular, if $F(y, z)$ has continuous derivatives with respect to z_i, e.g., the probability distribution function of ω has continuous density function, then

$$\frac{\partial F}{\partial z_i} = e_i - \alpha_i EI\left(d_i(\omega) - \sum_{k=1}^{K} a_{ik}(\omega) y_{ik} - z_i \geq 0\right),$$

where $I(\xi \geq 0)$ is the indicator function: $I(\xi \geq 0) = 1$, if $\xi \geq 0$, and $I(\xi \geq 0) = 0$ otherwise. Therefore, we can rewrite $\frac{\partial F}{\partial z_i}$ as

$$\frac{\partial F}{\partial z_i} = e_i - \alpha_i P\left[d_i(\omega) - \sum_{k=1}^{K} a_{ik}(\omega) y_{ik} - z_i \geq 0\right], \tag{13}$$

which allows to establish connections between the original model defined by (8)–(11) and the minimization of convex function $F(y, z)$ defined by (12).

Assume (y^*, z^*) minimizes $F(y, z)$ subject to constraints (8), (9), and $z_i \geq 0$, $i = \overline{1,.m}$. Assume also that $e_i < \alpha_i$, $i = \overline{1,.m}$. Then from (13) it follows that for all i with positive components $z_i^* > 0$, i.e., when $\frac{\partial F}{\partial z_i} = 0$, the optimal solution (y^*, z^*) satisfies the following safety constraints

$$P\left[d_i(\omega) - \sum_{k=1}^{K} a_{ik}(\omega) y_{ik} - z_i \geq 0\right] = e_i/\alpha_i. \tag{14}$$

Moreover, for all i with $z_i^* = 0$, i.e., when $\frac{\partial F(y*,z*)}{\partial z_i} \geq 0$, the optimal (y^*, z^*) satisfies the following safety constraint

$$P\left[d_i(\omega) - \sum_{k=1}^{K} a_{ik}(\omega) y_{ik} \geq 0\right] \leq e_i/\alpha_i. \tag{15}$$

If we choose α_i as $e_i/\alpha_i = 1 - p_i$, i.e., $\alpha_i = e_i/(1 - p_i)$, then (14) and (15) become equivalent to the safety constraint (11) of the original model (8)–(11). In other words, the minimization of convex function $F(y, z)$ defined by (12) subject to (8), (9), and $z_i \geq 0$, $i = \overline{1,m}$, yields the optimal solution of the original model (8)–(11). Efficient computational procedures for solving stochastic minimax problems with objective functions defined as in (12) can be found in Ermoliev et al. (1988) and Rockafellar and Uryasev (2000). In particular, Rockafellar and Uryasev (2000) discussed the applicability of linear programming methods in cases where the original model defined by a general probability distributions of ω can be sufficiently approximated by models with discrete probability distributions. This paper establishes also important connections between the minimization of (12)-type functions and Conditional-Value-at-Risk risk measure.

The minimization of function (12) can also be solved by a stochastic quasi-gradient method (Ermoliev et al. 1988). In applying this method to minimization of (12), the differentiability of $F(y)$ and any assumption on probability distribution of ω is not required. Also, the probability distribution of ω may only be given implicitly. For instance, only observations of random $d_i(\omega)$ and $a_{ik}(\omega)$ may be available or only a Monte Carlo procedure (pseudo-sampling simulation model such as described in Sect. 3.1) is used to simulate supply and demand. In the following section we illustrate some applications by using only the rebalancing algorithm described in Sect. 3.3; elaboration of the outlined stochastic allocation algorithm is a topic for future implementation.

5 Numerical experiments

The model in Sect. 3 is used in the analysis of current and plausible future livestock production allocation and intensification in China. Namely, in each time period the simulation model generates levels and geographic distribution of demand for livestock products coherent with urbanization processes (Liu et al. 2003), demographic change (Cao 2000) and expected growth of incomes (Huang and Liu 2003; Huang et al. 2003). Production allocation and intensification levels are projected from the base year data for the main livestock types (pigs, poultry, sheep, goat, cattle) and management systems (grazing, industrial/specialized, traditional) at the level of counties (2,434 administrative units). For production allocation, we used the sequential rebalancing procedure described in Sect. 3.3.

Two scenarios of future production allocation corresponding to different priors q_{ik}, $i = \overline{1,m}, k = \overline{1,K}$, are compared: (a) an intensification scenario, when production is allocated proportionally to the geographical patterns of demand increases, and (b) a risk-adjusted scenario that combines the preference structure as defined by the geographical distribution of demand with indicators of environmental pressure.

Intensification scenario. Currently, common practice is to allocate intensive livestock production in areas with good access to consumers, close to high demand and high population density (Anderson and van Wincoop 2003). In many practical problems of large dimensionality, to describe the "profitability" of a location it has been standard practice to use an ad hoc but reasonable measure referred to as market access function. The typical market access function measures the potential of location k as a weighted sum of purchasing power of all other locations in some vicinity of the given k. The weights are defined either as a function of distance or as a function of other factors, say, costs or losses. In these studies, each county is characterized by its market access $\tilde{\Delta}_{ik}$, $k = \overline{1:K}$, calculated as a weighted sum of demand for product i in nearby counties within some vicinity M_k: $\tilde{\Delta}_{ik} = \sum_{m \in M_k} f(g_{km}) \Delta_{im(t)}$, where $\Delta_{im(t)} = d_{im(t)} - d_{im(t-1)}$ denotes the demand change in the location m and time t, $d_{im(t)}$ is the demand for product i, and $k = \overline{1:K}$ are counties. Weights $f(g_{km})$ are equal to inverse distance between locations k and m with a discounting factor α, $f(g_{km}) = \alpha/g_{km}$, g_{km} is distance between k and m, $f(g_{kk}) = 1$.

For each location, the definition of the vicinal area M_k has to ensure, in a sense, the best coverage of consumers. Optimal coverage can be derived either from rather complex spatial optimization models (Hotelling 1931), or from spatial estimation procedures, and sensitivity analysis. Values $\tilde{\Delta}_{ik}$, $k = \overline{1:K}$, determine a profit-based prior q_{ik} for allocation of demand increase among production units in locations as it is described in Sect. 3.3.

Risk-based sustainability scenario. The objective of this scenario is to care for the balance between profitability of the agricultural production, rural welfare, and the respect of nature and the environment. Challenges of spatially explicit planning for sustainability are related to the choice of adequate location-specific indicators to guide rural development within defined socio-economic and environmental objectives. While information on economic and livelihood conditions at location may be available from statistics and census data, estimation of agricultural pollution and health risks (for example, related to livestock diseases) is a more challenging task. The agricultural pollution falls into the category of non-point source pollution, which is geographically disperse, and the likelihood of disease occurrences is determined by a combination of factors. Measurements of the pollution level, health risks, and related impacts or losses are hardly possible as they depend on multiple highly uncertain socio-economic and environmental factors: weather patterns, population density, level of development, agricultural inputs and intensification levels, etc.

In many practical situations when the target variable is impossible or impractical to measure, it is possible to use context-specific proxies or even a set of proxies that can considerably well represent the state of the non-measurable variable. In livestock production planning, instead of pollution levels of intensive crop fertilization and livestock production, such variables as nutrients in excess of crop uptakes, density of livestock biomass, etc., are used to characterize environmental risks. Health norms and associated health risks are introduced by a combination of urbanization share (share of urban population in total population) and availability of non-residential area suitable for further production expansion in each location.

Feasible domains of these variables are subdivided into subdomains representing different levels of risk exposure. For example, it is intuitively clear that levels of pollution and associated losses may be higher in areas with higher livestock concentration. One of the primary goals is to impose critical thresholds that identify subdomains of different risk exposure classes. A riskier class automatically incurs higher losses and, consequently, expenditures and penalties. To distinguish different risk classes, we introduce a risk function. It characterizes the degree of risk exposure at each location. Such risk functions can be viewed as having a close relationship with membership functions of the fuzzy logic theory.

As an example, the risk function related to a location's urbanization share is defined as

$$R_k^1 = \begin{cases} 1, & \text{if} \quad s_k < s_{\min}, \\ (s_{\max} - s_k)/(s_{\max} - s_{\min}), & s_k \in [s_{\min}, s_{\max}], \\ 0, & s_k > s_{\max}, \end{cases}$$

where s_k is the location-specific urbanization share, k is a location. Here, critical thresholds are $s_{\min}$ and $s_{\max}$, e.g., equal to 10 and 80%, respectively. These values were derived applying statistical analysis combined with expert opinion. The implied risk function tells that a location with an urbanization share below $s_{\min}$ may allocate livestock facilities with no constraints. In other words, there are no (population) risks associated with production expansion. In locations with urban population share in the interval [10,80], losses associated with the allocation are increasing proportional to the share, and counties with urbanization share beyond 80 percent are not allowed to increase their livestock production at all. In this case, the risk function is of linear segment-wise continuous form. Alternatively, the function may have a non-linear and even discontinuous shape.

A health risk function can depend on more than one criterion. For example, in some locations where the urban population share is high, there may still be land in abundance to justify expansion of livestock production. To include this consideration, we introduce a risk function associated with the amount of non-residential area suitable for production allocation, defined as:

$$R_k^2 = \begin{cases} 1, & \text{if} \quad b_k < b_{\min} \\ (b_{\max} - b_k)/(b_{\max} - b_{\min}), & b_k \in [b_{\min}, b_{\max}] \\ 0, & b_k > b_{\max} \end{cases}$$

where b_k is the proportion of build-up area in total area of a location. The values of 25 and 50% were used for thresholds $b_{\min}$ and $b_{\max}$, respectively.

Accounting for health risks, the preference weights for allocating production increases by location are computed by adjusting profit-driven shares $\tilde{\Delta}_i = \{\tilde{\Delta}_{ik}\}$,, with risk function $R_k{}^{1,2} = \max\ \{R_k^1, R_k^2\}$, where $\tilde{\Delta}_{ik}$ corresponds to q_{ik}. The compound risk function $R_k{}^{1,2} = \max\ \{R_k^1, R_k^2\}$ combines two criteria. Depending on the purposes of the analysis, function $R_k^{1,2}$ may be further extended to include other factors affecting health risks. The function $R_k^{1,2}$ may be viewed as a union of two risk functions.

In these studies, we assume that environmental pollution and risks associated with livestock production can be approximated by variables measuring nutrients in excess of crop uptake, density of livestock biomass, etc. A risk function associated with nutrients in excess of crop uptake is defined as

$$R_k^3 = \begin{cases} 1, & \text{if} \quad u_k < u_{\min} \\ 1 + (0.5 - 1)(u_k - u_{\min})/(u_{\text{med}} - u_{\min}), & u_k \in [u_{\min}, u_{\text{med}}] \\ 0.5(u_{\max} - u_k)/(u_{\max} - u_{\text{med}}), & u_k \in [u_{\text{med}}, u_{\max}] \\ 0, & u_k > u_{\max} \end{cases}$$

where, $u_{\min}$, u_{med} (median), and $u_{\max}$ were chosen as, respectively, 90, 110 and 150% of nutrients supply in relation to crop uptake in the location j. The risk function accounting for animal density in locations is defined as

$$R_k^4 = \begin{cases} 1, & \text{if} \quad m_k \le m_{\min} \\ 1 + (0.5 - 1)(m_k - m_{\min})/(m_{\text{med}} - m_{\min}), & m_k \in [m_{\min}, m_{\text{med}}] \\ 0.5(m_{\max} - m_k)/(B_{\max} - B_{\text{med}}), & m_k \in [m_{\text{med}}, m_{\max}] \\ 0, & m_k > m_{\max} \end{cases}$$

where $m_{\min}$, m_{med}, $m_{\max}$ are, respectively, 300, 600, and 1,000 kg of livestock biomass per ha cultivated land in location k.

In the risk-adjusted sustainability scenario the prior for distributing production increases is computed by adjusting the profit-driven allocation shares with a compound risk function

$$R_k^{(1,2)3,4} = R_k^{(1,2)} \min(R_k^3, R_k^4) = \max(R_k^1, R_k^2) \min(R_k^3, R_k^4).$$

The intensification scenario (a) implicitly minimizes the transportation costs as production concentrates in the vicinity of urban areas with high demand. In the alternative scenario (b), the production is shifted to more distant locations characterized by availability of cultivated land, lower livestock and population density, but at the expense of additional transportation. Environmental sustainability aspects of the two scenarios were compared with respect to the share of people in China's regions exposed to different severity classes of environmental risks. Environmental risks are measured in terms of environmental pressure in relation to the coincidence of three factors: density of confined livestock, human population density, and availability of cultivated land. For year 2000, the estimates suggest that about 20% of China's population lives in counties characterized as having high or extreme severity of environmental pressure from intensive livestock production. In the "intensification" scenario, by 2030 this population share increases to 37%, while in the second, environmentally friendly scenario, it stays below 30%. To finally compare the two scenarios, it is necessary to "normalize" gains due to improved life conditions with expenses of additional transportation.

6 Conclusions

This paper addresses some important aspects of agricultural production planning under risks, uncertainties and incomplete information. When planning agricultural developments, the objective is to allocate the foreseeable increases of demand in the best possible way while accounting for various risks associated with production and suitability criteria for profitability, transport, health and environmental impacts. Models for production allocation under risks and uncertainties may have considerable implications. In particular, the allocation of livestock production away from urban peripheries where pressure is highest to regions where feed grains are in abundance could decrease the income gaps between the regions. Similarly, establishment of agricultural pollution regulations, e.g., taxation, at locations with high environmental loads may change the balance of agricultural market attracting imports from abroad.

In Sect. 3, the production allocation procedure is proposed for situations when the available information is given in the form of aggregate values without providing necessary local perspectives. Therefore, the main issue is to downscale these values to the local levels consistently with location specific behavioral principles based on some priors. Yet, many practical situations may require more rigorous probabilistic treatment of priors and safety constraints. Section 4 proposes an allocation mechanism with more general treatment of uncertainties and risks based on principles of stochastic optimization. This is a promising approach for a coordinating agency aiming to improve the overall performance of the production chain. By diversifying large and small units the agency may stabilize the aggregate production and increase individual facility's values. The application of this allocation procedure is a topic for future research.

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