Crisis and Commitment: Inflation Credibility and the Vulnerability to Sovereign Debt Crises

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Abstract

We propose a continuous time model of nominal debt and investigate the role of inflation credibility in the potential for self-fulfilling debt crises. Inflation is costly, but reduces the real value of outstanding debt without the full punishment of default. With high inflation credibility, which can be interpreted as joining a monetary union or issuing foreign currency debt, debt is effectively real. By contrast, with low inflation credibility, sovereign debt is nominal and in a debt crisis a government may opt to inflate away a fraction of the debt burden rather than explicitly default. This flexibility potentially reduces the country's exposure to self-fulfilling crises. On the other hand, the government lacks credibility not to inflate in the absence of crisis. This latter channel raises the cost of debt in tranquil periods and makes default more attractive in the event of a crisis, increasing the country's vulnerability. We characterize the interaction of these two forces. We show that there is an intermediate inflation credibility that minimizes the country's exposure to rollover risk. Low inflation credibility brings the worst of both worlds—high inflation in tranquil periods and increased vulnerability to a crisis.

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1 Introduction

Several countries' debt-to-GDP ratios are near or above record levels. These include the U.S, U.K, Japan, Greece, Spain, Portugal, Ireland, and Italy, among others. Some of these countries, such as those on the periphery of the euro area, have experienced dramatic spikes in yields on their debt, while others, such as the U.S., U.K, and Japan, have not. One factor that is often held responsible for this difference is that the latter countries directly control the supply of the currency in which they issue debt.[1] The euro-area economies, as well as emerging markets that issue debt in foreign currency, must repay debt solely through real fiscal surpluses. The US, UK, and Japan, on the other hand, have the option of lowering the real burden of nominal debt through inflation. A plausible conjecture is that the availability of this additional instrument makes domestic-currency debt less susceptible to outright default, and therefore less susceptible to self-fulfilling debt crisis.[2] In this paper we explore the validity of this conjecture.

We develop a tractable, continuous-time model of self-fulfilling debt crises with nominal bonds, building on the canonical models of Barro and Gordon (1983), Calvo (1988) and Cole and Kehoe (2000).[3] A benevolent government in a small open economy makes decisions over time without commitment. In every period, it chooses inflation, a level of borrowing, and whether to repay or default. Explicit default incurs real costs, modeled here as a drop in endowment and permanent exclusion from financial markets. Exploiting nominal bonds' vulnerability to ex post inflation is not costless, either, as in practice inflation involves real economic distortions as well. We embed these costs in the government's objective function, and refer to the relative weight on inflation disutility as the economy's "inflation credibility." Our environment nests foreign currency debt as the limiting case in which inflation costs

[1] See De Grauwe (2011) and Krugman (2011) for recent policy discussions.

[2] The fact that European Central Bank's Outright Monetary Transactions (OMT) announcement significantly reduced the spreads in Southern European countries in 2012 is consistent with the view that high spreads were to a large extent self fulfilling. Under this view, the ECB President Draghi's statement to "do whatever it takes to preserve the euro," including purchases of sovereign bonds in secondary markets, would be interpreted as a signal of the ECB's off-equilibrium willingness to intervene in the event of a run on euro sovereign debt. Such a policy in an environment of self-fulfilling crises can eliminate or reduce the possibility of a crisis, explaining the immediate reduction in spreads.

[3] The literature on self-fulfilling debt crises is large, some of which is surveyed and discussed in Aguiar and Amador (in progress). In addition to Calvo (1988) and Cole and Kehoe (2000), other classic references include Alesina et al. (1992) and Giavazzi and Pagano (1989). Our paper is also related to Da-Rocha et al. (forthcoming) which models the interplay of devaluation expectations and default in a model in which debt is denominated in foreign goods and the government chooses both a real exchange rate and a debt policy. Another closely related paper is Araujo et al. (2012), which considers the welfare gains or costs from issuing debt in local versus foreign currency. They model the costs of local currency debt as arising from an exogenous shock to inflation. Our model focuses on the joint dynamics of debt and inflation. Recent papers exploring themes involving currency denomination of debt or self-fulfilling crises include Corsetti and Dedola (2013), Jeanne (2011), Jeanne and Wang (2013), Kocherlakota (2011) and Roch and Uhlig (2011).

become arbitrarily large, rendering nominal bonds effectively real.

Our model highlights the fact that partial default via inflation versus explicit default may have asymmetric costs, and the key comparative static is in regard to the relative costs of inflation. A useful feature of separating the costs of full default from those of inflation is that a country can credibly commit not to partially default through inflation by issuing bonds in foreign currency; a similar commitment technology for explicit default is not as readily available. We therefore can address the positive and normative implications of issuing domestic versus foreign currency bonds in an environment of limited commitment, and how this tradeoff varies with the level of inflation commitment when the government issues domestic-currency bonds.

In equilibrium, risk-neutral foreign investors purchase sovereign bonds at prices that reflect anticipated government decisions to repay, default, or inflate. In turn, the government's optimal policy depends on the equilibrium interest rate, raising the possibility of self-fulfilling debt crises. Our environment allows us to explore how the degree of inflation credibility alters the country's vulnerability to self-fulfilling debt crises. A main finding of the analysis is that inflation credibility—and by implication the choice of domestic versus foreign currency bonds—has an ambiguous impact on the possibility of a self-fulfilling debt crises and on welfare.

To provide intuition for the ambiguous role of inflation credibility in preventing self-fulfilling debt crises, consider the case of real bonds and a zero-one default decision. If creditors fail to roll over bonds, the government is faced with a choice of default versus repaying the entire principal on all maturing debt. For large enough debt levels, outright default is preferable, and this may be the case even if the government were willing to service interest payments rather than default, raising the possibility of multiple equilibria. On the other hand, if the debt is denominated in domestic currency, the government has a third option; namely, inflate away part of the principal and repay the rest. What is perhaps the conventional wisdom regarding debt crises is that this third option lowers the burden of repayment and eliminates the desirability of full default, at least for a range of debt stocks. That is, adding another policy instrument (partial default through inflation) reduces the occurrence of outright default. However, this conclusion must be tempered by the fact that the lack of commitment to bond repayments also extends to inflation. If the commitment to low inflation is weak, then high inflation will be the government's policy even in the absence of a crisis. This drives up the nominal interest rate in the non-crisis equilibrium, making default relatively attractive in all equilibria. This latter effect can generate an environment in which nominal bonds are more vulnerable to self-fulfilling runs; that is, the option for partial default makes outright default more likely.

More precisely, we establish a threshold for inflation credibility below which an economy is more vulnerable to crises for a larger range of debt. A middle-range of inflation credibility generates the conventional wisdom of less vulnerability. It is this level of credibility at which the economy can best approximate the state-contingent policy of low inflation in tranquil periods and high inflation in response to a liquidity crisis. High inflation credibility renders nominal bonds into real bonds, recovering the Cole and Kehoe (2000) analysis.

In terms of welfare, when inflation credibility is low issuing foreign currency (real) bonds is preferable to domestic currency (nominal) bonds. This follows because with domestic currency debt, the vulnerability to a crisis is greater and inflation is high in all equilibria. This rationalizes the empirical fact that emerging markets typically issue bonds to foreign investors solely in foreign currency, the so-called "original sin." Borrowing in domestic currency also reduces the country's equilibrium borrowing limit. On the other hand, a moderate level of inflation credibility makes nominal bonds strictly preferable for intermediate levels of debt, where the reduction in crisis vulnerability is at work.

In some contexts, there may exist a richer set of options in designing institutions that govern monetary and fiscal policy. Delegation of certain economic decisions to agents with different objectives has long been understood to be a possible solution to lack of credibility. In the event such delegation is feasible, our analysis suggests that an attractive option is to delegate the conduct of policy to an institution that places a very high cost on inflation in normal times and a very low cost in crisis times. Such an institution delivers inflation only when it is needed, when confronted with a rollover crises. If it is successful at doing so then it can eliminate rollover crises altogether and guarantee no inflation in equilibrium. However, such solutions are confronted by the inherent difficulty of building institutions that follow objectives that conflict with those of the government.

Finally, we also make a technical contribution in this paper. We show that, in our continuous time formulation, the government's problem can be represented as an optimal control problem on a stratified domain: that is, a situation where both the payoffs to the government and the choice set for the control may change discontinuously as a function of the state variable. In our model this occurs because of two reasons: first, the probability of a self-fulfilling debt run discontinuously switches from zero to a strictly positive number as the country accumulates debt, generating both a discontinuity in the payoff function as well as in the equilibrium interest rate. But also, our model features an equilibrium inflation choice that is discontinuous, generating an additional discontinuity in the equilibrum interest rate. Recent advances on the existence and uniqueness of the value function for problems on stratified domains (see Bressan and Hong, 2007) allow us to provide a complete characterization of the equilibria in our environment.

The remainder of the paper is organized as follows. Section 2 introduces the environment; section 3 analyzes the equilibrium in the absence of self-fulfilling crises; section 4 introduces the possibility of self-fulfilling rollover crises and performs our main comparative statics and welfare comparisons; and section 5 concludes. All proofs are in the appendix.

2 Environment

2.1 Preferences and Endowment

We consider a continuous-time, small-open-economy environment. There is a single, freely-traded consumption good which has an international price normalized to 1. The economy is endowed with y units of the good each period. We consider an environment in which income is deterministic, and for simplicity assume that y is independent of time. The local currency price (relative to the world price) at time t is denoted $P_t = P(t) = P(0)e^{\int_0^t \pi(t)dt}$, where $\pi(t)$ denotes the rate of inflation at time t. To set a notational convention, we let $\boldsymbol{\pi} : [0, \infty) \to \mathbb{R}_+$ denote inflation as function of time and let $\pi(t)$ or π_t denote the evaluation of $\boldsymbol{\pi}$ at time t. When convenient, we use $\pi \in R_+$ to denote a particular inflation choice. A similar convention is used for other variables of interest, like consumption and debt.

The government has preferences over paths for aggregate consumption and domestic inflation, $x(t) = (c(t), \pi(t)) \in \mathbb{R}^2_+$, given by:

$$U = \int_0^\infty e^{-\rho t} v(x(t)) dt = \int_0^\infty e^{-\rho t} \left(u\left(c(t)\right) - \psi(\pi(t)) \right) dt. \tag{U}$$

Utility over consumption satisfies the usual conditions, $u' > 0, u'' < 0$, $\lim_{c \downarrow 0} u'(c) = \infty$, plus an upper bound restriction: $\lim_{c\to\infty} u(c) \le \bar{u} < \infty$ needed for technical reasons.[4] Power utility with a relative risk aversion coefficient greater than one satisfies these conditions.

The disutility of inflation is represented by the function $\psi : \mathbb{R}_+ \to \mathbb{R}_+$, with $\psi' > 0$ and $\psi'' \ge 0$. In the benchmark model discussed in the text, we let $\psi(\pi) = \psi_0 \pi$, $\psi_0 \ge 0$, and we restrict the choice of inflation to the interval $\pi \in [0, \bar{\pi}]$. We retain this functional form for tractability reasons and discuss later how our results extend to the case with strictly convex inflation costs.

While we do not micro-found preferences over inflation, a natural interpretation is that ψ is a reduced-form proxy for a reputational cost to the government of inflation. A large cost represents an environment in which the government has a relatively strong incentive for (or

[4] In particular this is needed to apply the results of Bressan and Hong (2007) to our set up. See Appendix.

commitment to) low inflation.[5] The cost ψ is not state contingent; in particular, the costs of inflation will be independent of the behavior of creditors, although we discuss implications of relaxing this assumption in section 4.5.

When performing comparative statics with respect to ψ_0, we have in mind institutional features of monetary and fiscal policy that vary across countries, such as the extent of inflation indexing in the private sector and the flexibility of prices; the political economy that governs the interaction of monetary and fiscal policy; the legislative mandate of the central bank and how readily this can be amended; and the ability to raise revenue through taxation in a non-distortionary manner. As discussed below, inflation serves as a device to partially default on certain bonds. Another interpretation of ψ_0 is the extent of creditor protection when bonds are issued under domestic law. That is, how easily can terms of the original bond contract be amended through legislation or litigation in courts.

The government chooses $x = (c, \pi)$ from a compact set $X \equiv [0, \bar{c}] \times [0, \bar{\pi}]$. The upper bound on consumption $\bar{c}$ is assumed to never bind.[6] The upper bound on π will bind in the benchmark case of linear cost, and as we shall see it yields a discrete choice between low (zero) inflation or high ($\bar{\pi}$) inflation. Let $\mathbf{X}$ denote admissible controls: the set of measurable functions of time $\mathbf{x} : [0, \infty) \to X$.

2.2 Bond Contracts and Budget Sets

The government can trade a nominal non-contingent bond. Let B_t denote the outstanding stock of nominal bonds, and let $b_t \equiv B_t/P_t$ denote the real value of outstanding debt. The initial $P(0)$ is assumed to be pre-determined. This, plus the fact that $P(t)$ is a continuous function of time, implies that b_t can be treated as a state variable.

The government contracts with competitive (atomistic) risk-neutral lenders who face an opportunity cost in real terms given by the world interest rate $r^\star = \rho$. We assume the wealth of the lenders in aggregate is sufficient to finance the stock of sovereign bonds in our equilibrium, and foreign lenders are willing to hold these bonds as long as the expected real return is $r^\star$. Bonds carry an instantaneous interest rate that is conditional on the outstanding stock of real debt. In particular, we consider stationary equilibria in which the government faces a time-invariant interest rate schedule $r : \overline{\Omega} \to R_+$, where $\overline{\Omega} = [0, b_{max}]$ denotes the domain of real debt permissible in equilibrium. The debt domain is characterized

[5] The reputational cost can be augmented by real distortions to a good that enters separably from tradable consumption. Allowing for inflation to reduce the (instantaneous) tradable endowment as well would pose no difficulties; for example, replacing $y(t) = y$ with $y(t) = (1 - \pi(t))y$.

[6] As we discuss in the next sub-section, we impose an upper bound on assets (or lower bound on debt), so an upper bound on consumption does not become an issue. The upper bound on assets is not restrictive for the analysis.

by a maximal debt level $b_{max} \in R_+$ above which the government cannot borrow. The value of b_{max} will be an equilibrium object. For expositional convenience, we put a lower bound on debt of zero; the analysis is not sensitive to allowing the economy to accumulate a finite amount of foreign assets. As the government is the unique supplier of its own bonds, it understands the effects of its borrowing decisions on the cost as given by the entire function r.

The evolution of nominal debt is governed by:

$$\dot{B}(t) = P(t)(c(t) - y) + r(b(t))B(t).$$

Dividing through by $P(t)$ and using the fact that $\dot{B}/B = \dot{b}/b + \pi$ gives the dynamics for real debt:

$$\dot{b}(t) = f(b(t), x(t)) \equiv c(t) - y + (r(b(t)) - \pi(t))b(t). \tag{1}$$

A key feature of (1) is that inflation reduces the real burden of debt repayment, conditional on $r(b)$. This reflects that ex post inflation and partial default are equivalent to the bond holder in terms of real returns. In practice, one could think of the central bank "printing money" to repay bond holders; in our environment, to the extent that printing money generates inflation and its associated costs, such effects are captured by π and $\psi(\pi)$, respectively. In a non-cashless economy there would an additional term in the budget constraint corresponding to seignorage revenues.[7] We abstract from these effects because seignorage revenues tend to be small. However, their inclusion will not fundamentally change our results.

We are interested in environments in which r may not be a continuous function. For technical reasons, we need to place some restrictions on the nature of these discontinuities.

Definition 1. *Given a domain $\overline{\Omega} = [0, b_{max}]$, the set $\mathcal{R}(\overline{\Omega})$ consists of functions $r : \overline{\Omega} \to \mathbb{R}_+$ such that*

(i) r is bounded and lower semi-continuous on $\overline{\Omega}$;

(ii) r is such that $y - (r(b) - \bar{\pi})b \geq M > 0$ for all $b \in \overline{\Omega}$; that is, it is always feasible to have $\dot{b} = 0$ with strictly positive consumption;

(iii) r contains a finite number of discontinuities denoted by $b_1, b_2, ..., b_N$ with $0 < b_n < b_{n+1} < b_{max}$ for all $n \in \{1, 2, .., , N - 1\}$;

[7]If real money demand is inelastic and equal to κ then the additional term on the right hand side of equation (1) will be $(-\kappa\pi)$.

(iv) r is Lipschitz continuous on sets Ω_n for all $n \in 0, ..., N$, where $\Omega_0 \equiv (0, b_1)$; $\Omega_n \equiv (b_n, b_{n+1})$ for $n = 1, ..., N-1$; and $\Omega_N = (b_N, b_{max})$.[8]

Denote the closure of Ω_n as $\overline{\Omega}_n$, and note that $\overline{\Omega} = \cup_{n=0}^{N} \overline{\Omega}_n$. The debt-dynamics equation (1) implies that $b(t)$ is always continuous in time; however, $f(b,x) = c + (r(b) - \pi)b - y$ may not be continuous in b. For $r \in \mathcal{R}(\overline{\Omega})$, continuous policies imply continuous dynamics except at finitely many points $\{b_1, ..., b_N\}$, at which the dynamics can change discretely.

2.3 Limited Commitment

The government cannot commit to repay loans or commit to a path of inflation. At any moment, it can default and pay zero, or partially inflate away the real value of debt. As noted above, we model the cost of inflation with the loss in utility $\psi(\pi)$. We model outright default as follows. If the government fails to repay outstanding debt and interest at a point in time, it has a grace period of length δ in which to repay the bonds plus accumulated interest. During this period, it cannot issue new debt, but is also not subject to the full sanctions of default. If it repays within the grace period, the government regains access to bond markets with no additional repercussions. If the government fails to make full repayment within the grace period, it is punished by permanent loss of access to international debt markets plus a potential loss to output.[9]

The grace period allows a tractable, continuous time representation such that it is feasible to repay and partly inflate away a positive stock of debt if creditors do not purchase new bonds. The length of the grace period δ can be thought of as proxying for debt maturity.[10] As with the costs of inflation and default, we treat δ as a primitive of the environment.

We let $\underline{V}$ represent the continuation value after a default, which we assume is independent of the amount of debt at the time of default.[11] As we shall see, in equilibrium the government will opt for full repayment only if the payoff to doing so weakly dominates $\underline{V}$. We assume that $\underline{V} > u(0)/\rho$, so the country prefers default to consuming zero forever.[12] We discuss the payoff to utilizing the grace period in section 4.1.

[8] That is, for all n, there exists $K_n < \infty$ such that $r(b') - r(b'') \leq K_n|b' - b''|$ for all $(b', b'') \in \Omega_n \times \Omega_n$.

[9] In practice countries can exit default status by repaying outstanding debt in full. We proxy this with a grace period, which allows the government to avoid the full punishment of default by repaying outstanding principal and interest.

[10] An alternative formulation is the one in He and Xiong (2012) in which each debt contract has a random maturity, which generates an explicit *iid* sequencing of creditors at any point in time. Long-maturity debt poses tractability issues in solving for an equilibrium given that the interest rate charged to new debt is a function of the inflation policy function over the bond's maturity horizon.

[11] For concreteness, we can define $\underline{V} = u((1-\tau)y)/\rho$ as the autarky utility, where $\tau \in [0,1)$ represents the reduction in endowment in autarky.

[12] We also assume $u(y)/\rho > \underline{V}$, so that strictly positive debt can be sustained in equilibrium.

Modeling limited commitment in this manner has a number of advantages. First, by separating the costs of inflation from the costs of outright default, we can consider environments in which the two are treated differently by market participants. It may be the case that the equilibrium costs or "punishment" of inflation may be greater or less than that of outright default, and the model encompasses both alternatives. For example, the high inflation of the 1970s in the US and Western Europe eroded the real value of outstanding bonds; however, the governments did not negotiate with creditors or lose access to bond markets, as typically occurs in cases of outright default. A short-coming of the analysis is we do not present a micro-founded theory of why these costs may differ in practice; we take them as primitives, and explore the consequences for debt and inflation dynamics. Second, our modeling allows us to compare the implications of issuing domestic currency debt ($\psi_0 < \infty$) which can be inflated away, versus issuing foreign currency debt ($\psi_0 = \infty$) which cannot be inflated away.

We can also interpret $\pi(t)$ as capturing a partial default technology. Some forms of debt contracts such as those issued under domestic law may be more pliable to partial restructuring as opposed to those issued under foreign law. The $\psi(\pi)$ function can then be interpreted as capturing this variation in the ability to partially default. Reinhart and Rogoff (2009) identify several historical episodes of overt default on domestic debt (as opposed to only inflating it away). Du and Schreger (2012) estimate the credit risk associated with local currency debt and find it to be consistently positive and in certain countries and time periods of the order of magnitude of a few hundred basis points.

As noted in the introduction, there is a readily available commitment technology for ruling out partial default through inflation; namely, issuing bonds in a foreign currency. Whether opting for such commitment is welfare improving will be taken up in section 4.6.

3 No-Crisis Equilibria

In this section we characterize equilibria in which creditors can commit to (or coordinate on) rolling over debt. In particular, we assume that the government can always trade bonds at an equilibrium schedule r with no risk of a rollover crisis. There remains limited commitment on the part of the government with regard to inflation and default. We solve the government's problem under the restriction that default (with or without subsequent repayment) is never optimal on the domain $\overline{\Omega}$. This is not restrictive in equilibrium.In particular, in the deterministic environment under consideration in this section, the equilibrium restricts debt to a domain on which it is never optimal to default.

Limited commitment with respect to inflation and linear inflation costs gives rise to a threshold level of debt b_π above which a country chooses high inflation $\bar{\pi}$ (and where interest rates are high) and below which inflation is zero (and interest rates are low). Since the government internalizes the impact of its choice of inflation on the interest rate it faces, for debt levels above b_π the government has an incentive to save so as to escape high interest rates and high inflation. There is no other incentive to save/borrow because $r^\star = \rho$ and y is fixed. We then describe the impact of the level of inflation commitment, ψ_0, on the inflation threshold, on debt dynamics and on welfare. Besides being of independent interest, this comparative static is an important ingredient of the analysis in Section 4 when we introduce rollover risk.

For a given $\overline{\Omega}$; $r \in \mathcal{R}(\overline{\Omega})$; and for all $b_0 \in \overline{\Omega}$; the government's value function can be written as

$$
\begin{aligned}
V(b_0) = \max_{\mathbf{x}\in\mathbf{X}} & \int_0^\infty e^{-\rho t} v(x(t))dt, \qquad \text{(P1)}\\
& \text{subject to:}\\
b(t) &= b_0 + \int_0^t f(b(t), x(t))dt\\
&= b_0 + \int_0^t \left(c(t) + \left(r(b(t)) - \pi(t)\right) b(t) - y\right) dt, \text{ and}\\
b(t) &\in \overline{\Omega} \text{ for all } t.
\end{aligned}
$$

Posing the government's problem in sequence form raises the question of whether the solution is time consistent, both in regard to default and inflation. Before discussing this and other aspects of the solution to the government's problem, we define our equilibrium concept:

Definition 2. *A* Recursive Competitive Equilibrium *is an interval $\overline{\Omega} = [0, b_{max}]$, an interest rate schedule $r : \overline{\Omega} \to \mathbb{R}_+$, a consumption policy function $C : \overline{\Omega} \to [0, \bar{c}]$, an inflation policy function $\Pi : \overline{\Omega} \to [0, \bar{\pi}]$, and a value function $V : \overline{\Omega} \to \mathbb{R}$ such that*

(i) $r \in \mathcal{R}(\overline{\Omega})$*;*

(ii) given $(\overline{\Omega}, r)$ and for any $b_0 \in \overline{\Omega}$, the policy functions combined with the law of motion (1) and initial debt b_0 generate sequences $x(t) = (C(b(t)), \Pi(b(t)))$ that solve the government's problem (P1) and deliver $V(b_0)$ as a value function;

(iii) given $C(b)$ and $\Pi(b)$, bond holders earn a real return $r^\star$, that is, $r(b) = r^\star + \Pi(b)$ for all $b \in \overline{\Omega}$; and

(iv) $V(b_0) \geq \underline{V}$ for all $b \in \overline{\Omega}$.

The final equilibrium condition imposes that default is never optimal in equilibrium. In the absence of rollover risk, there is no uncertainty and any default would be inconsistent with the lender's break-even requirement. As we shall see, condition (iv) imposes a restriction on the domain of equilibrium debt levels.[13] It also ensures that problem (P1), which imposes no default, is without loss of generality. That is, by construction the state constraint $b(t) \in \bar{\Omega}$ in (P1) ensures that the government would never exercise its option to default in any equilibrium.

The time consistency of optimal inflation policy in the above sequence formulation is more subtle. The potential for time inconsistency is embedded in the equilibrium interest rate function $r(b) = r^\star + \Pi(b)$. The government takes this function as given and does not internalize that its policies are ultimately determining the equilibrium interest rate schedule. We can view this expression as the limit of a discrete time environment in which the relevant inflation for bond pricing is that chosen by the "next period's" government, which is not under the current incumbent's control. Given an $r(b)$ function, the government is indifferent between choosing inflation ex-ante or period by period. Therefore, the sequence problem written above, for a given $r(b)$, satisfies the recursive problem when choices are made period-by-period under limited commitment. This contrasts with the full commitment Ramsey solution in which the government commits to a path of inflation at time 0, with the understanding that its choices will affect the equilibrium interest rate; that is, the Ramsey government does not take $r(b)$ as given. The Ramsey solution would be to set $\pi(t) = 0$ for all t and lock in $r(t) = r^\star$. This solution however may not be time consistent under limited commitment, as the government in the future may choose to deviate from its promises.

In recursive form, we solve the government's problem using the continuous time Bellman equation. Let $H(b, q) : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ be defined as

$$\begin{aligned} H(b,q) &= \max_{x \in X} \{v(x) + q f(b,x)\} \\ &= \max_{(c,\pi) \in X} \{u(c) - \psi(\pi) + q\,(c - y + (r(b) - \pi)b)\}. \end{aligned} \tag{2}$$

Note that H is defined conditional on an equilibrium interest rate schedule, which we suppress in the notation. The Hamilton-Jacobi-Bellman equation is:

$$\rho V(b) - H(b, V'(b)) = 0. \tag{HJB}$$

We proceed to show that the value function is the unique solution to (HJB). There

[13] It must also be the case that the government never prefers to default and then repay within the grace period. We postpone that discussion until section 4.1.

are two complications. The first is that r may not be continuous, so the HJB may be discontinuous in b. The second is that the value function may not be differentiable at all points, so its derivative, $V'(b)$, may not exist. Nevertheless, the value function is the unique solution to (HJB) in the viscosity sense. We use the definition of viscosity introduced by Bressan and Hong (2007) for discontinuous dynamics adapted to our environment:

Definition 3. *For a given $\overline{\Omega}$ and $r \in \mathcal{R}(\overline{\Omega})$, a **viscosity solution** to (HJB) is a continuous function $w \in C^0(\overline{\Omega})$ such that for any $\varphi \in C^1(\overline{\Omega})$ we have:*

(i) If $w - \varphi$ achieves a local maximum at b, then

$$\rho w(b) - H(b, \varphi'(b)) \leq 0;$$

(ii) If the restriction of $w - \varphi$ to Ω_n achieves a local minimum at $b \in \Omega_n$, then

$$\rho w(b) - H(b, \varphi'(b)) \geq 0,$$

where Ω_n is defined in Definition 1;

(iii) For $b \in \{0, b_1, b_2, ..., b_{max}\}$,

$$\rho w(b) - \max_{\pi \in \{0, \bar{\pi}\}} \{u(y - (r(b) - \pi)b) - \psi(\pi)\} \geq 0.$$

We make a few remarks on these conditions, and how we use them. First, suppose V is differentiable at b and r is continuous at b. In this case, a local max or min of $V - \varphi$ implies $V'(b) = \varphi'(b)$. The first two conditions then are equivalent to the classical Bellman equation $\rho V - H(b, V'(b)) = 0$.

Aside from points where V is differentiable, condition (i) concerns points where V may have a concave kink. As we show below, the equilibrium value function is smooth at points of continuity in $r(b)$. However, condition (i) allows for a concave kink in the value function V at points of discontinuity in $r(b)$. As we will see below, such kinks are indeed a feature of our solution, and we will use condition (i) when characterizing the value function at discontinuity points in $r(b)$.[14]

Condition (ii) applies only on the open sets Ω_n; that is, only at points at which $r(b)$ is continuous. We refer to this condition above, along with condition (i), to obtain the classical Bellman equation at points of differentiability in V. Where V is non-differentiable, then

[14]We also use condition (i) in the proof of Lemma 1 to rule out equilibra with interior inflation choices.

condition (ii) applies if there is a convex kink. The condition places a lower bound on the value function at such points.

The government always has the option of staying put at the point of discontinuity, and thus the value function is weakly greater than the steady state value function, which is condition (iii). Note that condition (ii) only refers to the open sets on which the interest rate is continuous, and thus condition (iii) provides the relevant floor on the value function at the points of discontinuity.

The following proposition states that we can confine attention to the viscosity solution of (HJB):

Proposition 1. *For a given $\overline{\Omega}$ and $r \in \mathcal{R}(\overline{\Omega})$, the government's value function is the unique bounded Lipschitz-continuous viscosity solution to* (HJB).

It is a natural restriction to consider equilibria where the interest rate is weakly increasing with the level of debt: that is, a government is more tempted to inflate at higher levels of debt. For the rest of the paper, we will restrict attention to *monotone equilibria*:[15]

Definition 4. *An equilibrium is* motone *if r is a non-decreasing function of b for all $b \in \bar{\Omega}$.*

We can now characterize monotone equilibria in the no-rollover-crisis environment. At points where the value function is differentiable the HJB is given by,

$$\rho V(b) = \max_{(c,\pi)\in X} \{u(c) - \psi_0 \pi + V'(b)\,(c - y + (r(b) - \pi)b)\}$$

Where V is differentiable, the first order conditions are:

$$u'(c) = -V'(b) \tag{3}$$

$$\pi = \begin{cases} 0 & \text{if } \psi_0 \geq -V'(b)b = u'(c)b \\ \bar{\pi} & \text{if } \psi_0 < u'(c)b \end{cases} \tag{4}$$

The first condition is the familiar envelope condition equating the marginal value of an additional unit of debt (more consumption today) to the the marginal cost of repaying that debt going forward ($V'(b)$). The second condition captures the trade off between inflating away the debt or repaying it through lower consumption. The marginal cost of inflation is ψ_0. The marginal benefit is that the entire stock of debt is reduced in real terms, which is

[15] In general, non-monotone equilibria may exist but the non-monotonicity is restricted to appear within the interval $\{\underline{b}_\pi, \overline{b}_\pi\}$, where these values are defined below.

why b is represented on the right hand side of (4). This reduction in debt is translated into utility terms via $V'(b)$ (or, equivalently, $u'(c)$). The first order condition has the intuitive implication that the government is tempted to inflate if the stock of outstanding debt is high. It also implies that inflation is preferable when the cost of raising real resources is high; this is captured in our framework by the marginal utility of consumption.

The assumption that the marginal cost of inflation is constant, together with the restriction to monotone equilibria, generates a "bang-bang" solution for the inflation rate in the government's problem: for low levels of debt, zero inflation is optimal, while high levels of debt involve high inflation. The following lemma, whose proof exploits the viscosity conditions of Definition 3, states this formally:

Lemma 1. *In any monotone no-crisis equilibrium, $r(b) \in \{r^*, r^* + \bar{\pi}\}$. In particular, in any such equilibrium there exists a b_π such that $r(b) = r^\star$ for $b \in [0, b_\pi]$ and $r(b) = r^\star + \bar{\pi}$ for $b \in (b_\pi, b_{max}]$.*

The threshold b_π characterizes the equilibrium $r(b)$ and is not uniquely determined. Instead, we can define an interval $[\underline{b}_\pi, \bar{b}_\pi]$ which contains all possible b_π. The upper threshold $\bar{b}_\pi$ is the highest value of debt below which the government chooses zero inflation when faced with the interest rate $r^\star$. The lower threshold $\underline{b}_\pi$ is similarly defined, but when the government faces the interest rate $r^\star + \bar{\pi}$.

Definition 5. *The values $\bar{b}_\pi$, $\underline{b}_\pi$ are given by the unique solutions to:*

$$\psi_0 = u'(y - r^\star \bar{b}_\pi)\bar{b}_\pi, \qquad \textit{and} \qquad \psi_0 = u'(C_\pi(\underline{b}_\pi))\underline{b}_\pi$$

where $b \mapsto C_\pi(b) \in (0, y - r^\star b)$ is defined uniquely by[16,17]

$$u(y - r^\star b) - u(C_\pi(b)) + \psi_0 \bar{\pi} + u'(C_\pi(b))(C_\pi(b) - y + r^\star b) = 0. \tag{5}$$

When faced with an interest rate of $r^\star$, since $r^\star = \rho$, there is no incentive to save or borrow if inflation is zero and $c = y - r^\star b$. From the first order conditions we have that low inflation is optimal as long as $u'(y - r^\star b)b - \psi_0 \geq 0$. For $b > \bar{b}_\pi$, this condition is violated.

The threshold $\underline{b}_\pi$ and the associated function C_π relate to the solution of (HJB) when the interest rate is $r^\star + \bar{\pi}$. In particular, as discussed below, $C_\pi(b_\pi)$ denotes optimal consumption

[16] To see that $C_\pi(b)$ exists, fix b and consider the function $G(c) = u(y - r^\star b) - u(c) + \psi(\bar{\pi}) + u'(c)(c - y + r^\star b)$, which is the left hand side of (5). Note that $G'(c) > 0$ for $c < y - r^\star b$, $G(y - r^\star b) = \psi(\bar{\pi}) > 0$, and $\lim_{c \downarrow 0} G(c) < 0$ by the condition that $\lim_{c \downarrow 0} u'(c) \to \infty$.

[17] Note that if we do not restrict attention to $c < y - r^\star b$, there is another solution to (5), at which $c > y - r^\star b$. As we shall see, the solution with $c < y - r^\star b$ is the appropriate consumption level that satisfies the Bellman equation.

assuming high inflation in the neighborhood above b_π, and the condition defining $\underline{b}_\pi$ ensures that optimal consumption is consistent with high inflation. Note that both $\bar{b}_\pi$ and $\underline{b}_\pi$ exist and are such that $y/r^\star > \bar{b}_\pi > \underline{b}_\pi > 0$.

The following proposition characterizes the set of recursive competitive equilibria and the associated equilibrium objects:

Proposition 2. *All monotone recursive competitive equilibria can be indexed by $b_\pi \in [\underline{b}_\pi, \bar{b}_\pi]$ and are characterized as follows. For a given b_π, define the following extended-domain value function $\hat{V} : (0, y/r^\star) \to \mathbb{R}$,*

$$\hat{V}(b) = \begin{cases} \frac{u(y-r^\star b)}{\rho} & \text{if } b \leq b_\pi \\ \hat{V}(b_\pi) - u'(C_\pi(b_\pi))(b - b_\pi) & \text{if } b \in (b_\pi, b^*] \\ \frac{u(y-r^\star b)-\psi_0\bar{\pi}}{\rho} & \text{if } b \in (b^*, y/\rho), \end{cases}$$

where $b^* = (y - C_\pi(b_\pi))/r^\star$. Define $\bar{b} = \max\{b \leq y/r^\star \,\big|\, \underline{V} \leq \hat{V}(b)\}$. Then for any $0 \leq b_{max} \leq \bar{b}$, define $\overline{\Omega} = [0, b_{max}]$, and the following constitutes a recursive equilibrium:

(i) The interest rate schedule $r : \overline{\Omega} \to \{r^\star, r^\star + \bar{\pi}\}$ defined by

$$r(b) = \begin{cases} r^\star & \text{if } b \leq b_\pi \\ r^\star + \bar{\pi} & \text{if } b \in (b_\pi, b_{max}]; \end{cases}$$

(ii) The value function $V : \overline{\Omega} \to \mathbb{R}$ defined by $V(b) = \hat{V}(b)$ for $b \in \overline{\Omega}$;

(iii) The consumption policy function $C : \overline{\Omega} \to \mathbb{R}_+$ defined by

$$C(b) = \begin{cases} y - r^\star b & \text{if } b \leq b_\pi \text{ or } b \geq b^* \\ C_\pi(b_\pi) & \text{if } b \in (b_\pi, b^*). \end{cases}$$

(iv) The inflation policy function $\Pi : \overline{\Omega} \to \{0, \bar{\pi}\}$ defined by:

$$\Pi(b) = \begin{cases} 0 & \text{if } b \leq b_\pi \\ \bar{\pi} & \text{if } b \in (b_\pi, \bar{b}]. \end{cases}$$

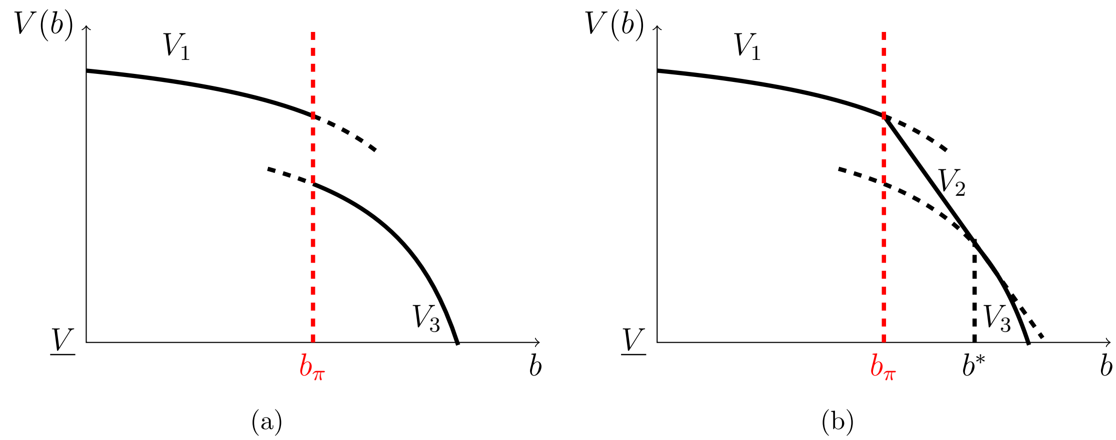
Proposition 2 characterizes the set of possible equilibria, in which each equilibrium is indexed by the value of b_π. That is, each equilibrium corresponds to an interest rate function which has a jump at b_π. If $\underline{b}_\pi \geq \bar{b}$, then inflation is zero for the entire domain $\overline{\Omega}$ as default is preferable to the consequences of inflation. More generally, each value $b_\pi \in [\underline{b}_\pi, \bar{b}_\pi] \cap \overline{\Omega}$ specifies a distinct equilibrium with an interest rate function that jumps up at b_π.

To provide some intuition for the construction of the equilibrium we use Figure 1. Figure 1(a) depicts two steady state value functions. For $b \leq b_\pi$, $V_1 = u(y - r^* b)/\rho$ is the steady-state value function with low inflation when the government faces a low interest rate. As noted above, low inflation is indeed chosen when $r(b) = r^\star$ for $b \leq \bar{b}_\pi$. Moreover, this value function and $c = y - r^\star b$ satisfy (HJB) for $b < b_\pi$. The second function V_3 is the steady state value function with high inflation when the government faces a high interest rate, $V_3 = (u(y - r^* b) - \psi_0 \bar{\pi})/\rho$. V_3 satisfies (HJB) for $b > b_\pi$.

While V_1 and V_3 satisfy (HJB) locally, they are not a viscosity solution over the entire domain $\overline{\Omega}$. This is due to the fact that they are not equal at b_π. The difference between V_1 and V_3 at b_π is equal to the discounted cost of inflation $\frac{\psi_0 \bar{\pi}}{\rho}$. As a result, stitching V_1 and V_3 together gives rise to a discontinuity, and is therefore not a solution. In the neighborhood above b_π, the government's optimal response to the jump in the interest rate is to reduce debt to b_π, and not to remain in the high-inflation zone indefinitely. By doing so it can attain discretely higher welfare. It therefore will consume less than the steady state consumption level $y - (r(b) - \bar{\pi})b = y - r^\star b$. Given the value function at b_π, we can solve for optimal consumption from (HJB). It is given by $C_\pi(b_\pi)$, introduced in Definition 5, which uses the value matching condition $\hat{V}(b_\pi^-) = \hat{V}(b_\pi^+)$ and the envelope condition $-\hat{V}'(b_\pi^+) = \lim_{b \downarrow b_\pi} u'(C(b)) = u'(C_\pi(b_\pi))$. Note that $\hat{V}'(b_\pi^-) \neq \hat{V}'(b_\pi^+)$, so the value function has a kink at b_π. This kink reflects that consumption equals $y - r^\star b$ to the left of b_π, but is strictly lower to the right given the incentive to save. To ensure that this consumption is indeed the solution to (HJB) at b_π, high inflation must be optimal. This is the case if $\psi_0 < u'(C_\pi(b_\pi))b$ for $b > b_\pi$, which motivates the definition of $\underline{b}_\pi$ in Definition 5.

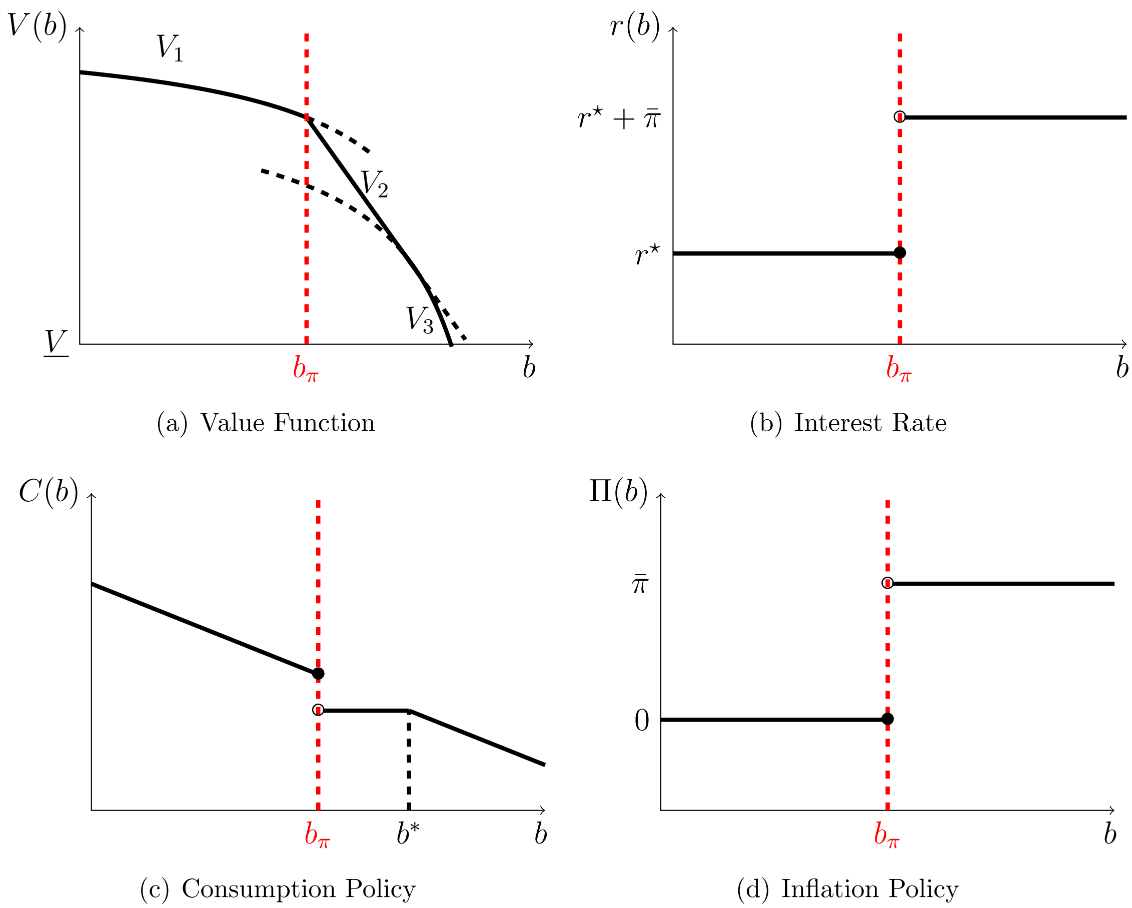
As $r^\star = \rho$, there is no incentive to vary consumption while the government saves. That is, the desire to save is in response to the discontinuity in the interest rate at b_π, not because the current (real) interest rate is high relative to impatience. Thus $C(b) = C_\pi(b_\pi)$ over the domain of active savings, and then jumps to $y - r^\star b_\pi$ at b_π. The domain of active savings extends to b^*, at which point $C_\pi = y - r^\star b^*$, and consumption is equal to the steady state consumption level. At this level of debt, the government is indifferent between saving towards b_π or remaining at that debt level forever. From the envelope condition, $-\hat{V}'(b) = u'(C_\pi(b_\pi))$ for $b \in (b_\pi, b^*)$; that is, the slope of the value function is constant over this region. This is represented by the linear portion $V_2(b)$ depicted in Figure 1(b). Note that V_2 is tangent to V_3 at b^*, as by definition $C_\pi(b_\pi)$ is the steady state consumption at b^*.

For a given $b_\pi \in [\underline{b}_\pi, \bar{b}_\pi]$ the solution for the value function, interest rates, consumption and inflation policy are depicted in Figure 2.



(a) (b)

Figure 1: Construction of Value Function



(a) Value Function (b) Interest Rate

(c) Consumption Policy (d) Inflation Policy

Figure 2: Equilibrium with No Crisis

3.1 Comparative Statics

In this section we evaluate how debt dynamics depend on the inflationary regime; that is, as we vary ψ_0. An increase in ψ_0 increases the gains from reaching the low-inflation region. This increases the incentive to save, while reducing the utility in the high inflation region. At the same time a higher ψ_0 gives rise to a larger low inflation region, shifting some debt levels from high to low inflation, and thus reducing the need to save. As a result, the implications for savings and welfare are ambiguous. Consequently the impact on the debt limit is also ambiguous. We now provide a detailed analysis.

Consider an increase in the cost of inflation ψ_0 to $\psi_0' > \psi_0$. To characterize what happens to the set of monotone equilibria, note that the expressions in Definition 5 imply that $\underline{b}_\pi$ and $\bar{b}_\pi$ increase.[18] Let $\underline{b}_\pi'$ and $\bar{b}_\pi'$ denote the new thresholds, respectively.

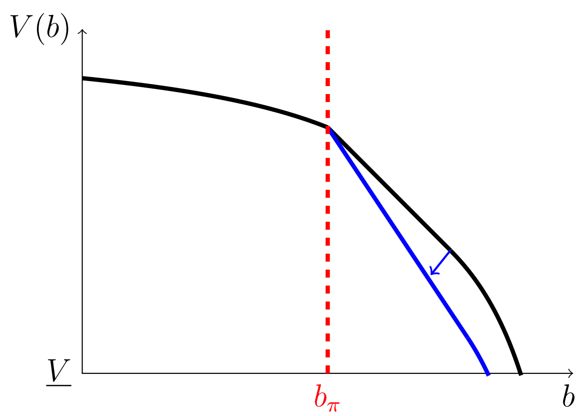
First, consider a b_π that is consistent with equilibrium under both ψ_0 and ψ_0', and that the shift in ψ_0 does not change the equilibrium b_π. This is possible for $b_\pi \in [\underline{b}_\pi', \bar{b}_\pi]$. The low-inflation steady state value function remains unaffected by the increase in ψ_0, while the high-inflation steady state value function shifts down in a parallel fashion by the amount $\frac{(\psi_0'-\psi_0)\bar{\pi}}{\rho}$. From the expression for C_π in Definition 5, $C_\pi(b_\pi)$ declines, which means a higher savings rate and steeper slope associated with the linear portion of the value function. The decline in C_π implies that $b^* = (y - C_\pi(b_\pi))/r^\star$ increases as well, so the domain for the linear portion increases. The steeper slope and larger domain for the linear segment is consistent with the shift down and strict concavity of the high-inflation steady state value function. The new value function is strictly below the original for all $b > b_\pi$. For a given value of $\underline{V}$, this implies that the amount of debt that can be sustained has decreased (as long as $\bar{b}$ is higher than b_π). This is shown in panel (a) of Figure 3.

Consider now what happens when b_π shifts in response to the change in ψ_0. For example, suppose the initial equilibrium feataured $b_\pi = \underline{b}_\pi < \underline{b}_\pi'$, which cannot survive the increase in ψ_0. In panel (b) of figure 3 we contrast the value function for an initial equilibrium b_π with a new equilibrium $b_\pi' > b_\pi$. The region $(b_\pi, b_\pi']$ shifts from being a high-interest rate zone to a low-interest rate zone. The new optimal policy of low inflation in this zone implies higher welfare, as the government avoids the costs of inflation. That is, the value function is now higher in that region, and by continuity will be higher even at debt levels in which the interest rate jumps up. This reflects the increased proximity to the low-inflation zone. However, given that the linear portion of the value function has a steeper slope under ψ_0',

[18] The increase in $\bar{b}_\pi$ follows directly from the definition $u'(y - r^\star \bar{b}_\pi)\bar{b}_\pi = \psi_0$. The definition of $u'(C_\pi(\underline{b}_\pi))\,\underline{b}_\pi = \psi_0$ implies that the lower threshold depends on ψ_0 directly from this first-order condition, and indirectly through the definition of the function C_π given in equation (5). Nevertheless, manipulating these expressions yields an unambiguous implication that $\underline{b}_\pi$ is increasing in ψ_0.

eventually the new value function intersects the original one from above (see panel (b) of Figure 3). Note that depending on the level of $\underline{V}$, the borrowing limit $\bar{b}$ can shift up or down.

The implication for savings of an increase in ψ is therefore mixed. In panel (a), the savings rate is always weakly greater when ψ_0 is higher, and strictly so for the range $(b_\pi, b^{*\prime})$. In panel (b), when b_π shifts up as a result of the increase in ψ_0, there is a region $(b_\pi, b'_\pi]$ in which the low-ψ economy is saving while the high-ψ economy is not. This reflects that the inflation rate is higher in this region for the low-ψ economy, and savings is the method to regain commitment to a low inflation rate. As we let ψ_0 go to infinity, the low inflation zone covers the entire space, and savings is zero everywhere. In this limiting case, a strong commitment to low inflation is consistent with weakly higher steady state debt levels and a higher maximal debt limit.



(a) No Change in b_π

(b) Change in b_π

Figure 3: The Role of Inflation Commitment Absent a Crisis: An Increase in Inflation Costs ψ_0

As discussed, b_π is not uniquely determined and is contained in the interval $[\underline{b}_\pi, \bar{b}_\pi]$. Going forward we consider equilibria in which the low inflation zone is as large as possible. As creditors are indifferent and the government prefers a low interest rate, the maximal domain is weakly Pareto superior. We focus on these upper-bound thresholds, tracing out the Pareto-dominant equilibrium interest rate function, conditional on parameters. In the no-crisis case this implies $b_\pi = \bar{b}_\pi$. The important feature of this equilibrium selection is that comparative statics of b_π with respect to ψ can be pinned down unambiguously. While ruling out switching among possible non-crisis equilibria, the fact that the entire range of possible equilibria shifts up with ψ suggests our focus on $\bar{b}_\pi$ is representative when it comes to comparative statics. [19]

[19]With the piecewise-linear $\psi(\pi)$ function, we can characterize the non-crisis interest rate function by the

4 Equilibria with Rollover Crises

The preceding analysis constructed equilibria in which bonds were risk free. We now consider equilibria in which investors might refuse to purchase new bonds and the government defaults in equilibrium. This links the preceding analysis of nominal bonds with Cole and Kehoe (2000)'s real-bond analysis of self-fulfilling crises. Importantly, it allows us to explore the role of inflation credibility in the vulnerability to debt crises.

In the no-crisis case we demonstrated a threshold b_π such that when debt exceeded this threshold inflation was high and interest rates were high. Now with rollover risk we construct a second threshold b_λ. When debt exceeds this threshold the government defaults whenever the investors refuse to purchase new bonds. In keeping with the terminology of Cole and Kehoe (2000) we refer to the region $b \in (b_\lambda, b_{\max}]$ as the "crisis zone" and its complement $b \in [0, b_\lambda]$ as the "safe zone". Unlike the safe zone, in the crisis zone the government is exposed to self-fulfilling debt crises that occur with exogenous Poisson probability λ.

Interest rates in the crisis zone are higher than in the safe zone because of the probability of default. In the safe zone, which mimics the analysis of the no-crisis equilibria, the government may choose to save to escape high inflation and high nominal interest rates. In the crisis zone there is an additional incentive to save so as to escape self-fulfilling debt crisis and the associated higher interest rates. By saving out of the crisis zone they trade off temporarily lower consumption for higher steady state consumption when they enter the safe zone.

In the following sections we characterize the impact of inflation credibility on the vulnerability to debt crises, that is we determine how the threshold b_λ is impacted by changes in ψ_0. The answer depends in important ways on how b_π is impacted by changes in ψ_0. A main result is that the impact of ψ_0 on b_λ is non-monotonic.

We proceed by first characterizing the grace-period problem of the government. Recall that bonds mature at every instant. If investors refuse to roll over outstanding bonds, the government will be unable to repay the debt immediately. However, the government has the option to repay the *nominal* balance within the grace period δ to avoid the full punishment of default (in real terms the government can use a combination of inflation and real repayments). After characterizing this sub-problem of a government that enters the default state but repays the debt within the grace period, we characterize the government's

single threshold b_π, the potential domain of which is increasing in ψ_0. For the family of quadratic inflation costs that are parameterized by ψ_0: $\psi(\pi) = \psi_0 \pi^2$, a similar comparative static applies, although the entire interest rate schedule over the domain $b > 0$ shifts up with ψ_0. The convenience of the piecewise-linear cost function is the complete characterization of the schedule, and associated comparative statics, based on a single threshold.

full problem and characterize equilibria with rollover crises.

4.1 The Grace-Period Problem

To set notation, let $W(b_0, r_0)$ denote the government's value at the start of the grace period with outstanding real bonds b_0 carrying a nominal interest rate r_0. We re-normalize time to zero at the start of the grace period for convenience. To avoid the costs of outright default, the government is obligated to repay the nominal balance on or before date δ, with interest accruing over the grace period at the original contracted rate r_0. This r_0 embeds equilibrium inflation expectations. The government can reduce its real debt burden by resorting to inflation.

We impose the *pari passu* condition that all bond holders have equal standing; that is, the government cannot default on a subset of bonds, while repaying the remaining bondholders. Therefore, the relevant state variable is the entire stock of outstanding debt at the time the government enters the grace period.

The function $W(b_0, r_0)$ is the solution to the following problem:

$$W(b_0, r_0) = \max_{\mathbf{x}\in\mathbf{X}} \int_0^\delta e^{-\rho t} v(x(t))dt + e^{-\rho\delta} V(0), \tag{6}$$
$$\text{subject to :}$$
$$\dot{b}(t) = c(t) + (r_0 - \pi(t))b(t) - y$$
$$b(0) = b_0, b(\delta) = 0, \dot{b}(t) \leq -\pi(t)b(t),$$

where for the grace-period problem the controls $\mathbf{x}$ and admissible set $\mathbf{X}$ refer to measurable functions $[0, \delta] \to X$. The $V(0)$ in the objective function represents the equilibrium value of returning to the markets with zero debt (which is to be determined below in equilibrium) at the end of the grace period. Note that if the government repays before the end of the grace period, it could exit default sooner. However, as it has no incentive to borrow again once $b = 0$, it is not restrictive to impose no new debt for the entire grace period. The final constraint, $\dot{b}(t) \leq -\pi(t)b(t)$ is equivalent to the constraint of no new nominal bonds, $\dot{B}(t) \leq 0$.

The grace period problem is a simple finite-horizon optimization with a terminal condition for the state variable. We do not discuss the solution in depth, but highlight a few key implications. An important feature of (6) is that $W(b_0, r_0)$ is strictly decreasing in both arguments. Moreover, W is decreasing in ψ_0, and strictly decreasing if positive inflation is chosen for a non-negligible fraction of the grace period. In order to repay its debt quickly, the government has an incentive to inflate away a portion of the outstanding debt. The cost of doing so is governed by ψ_0.

Regarding a piece of unfinished business left over from the no-crisis analysis, with W in hand we can state explicitly why the government would never choose to enter default in the non-crisis equilibria discussed in the previous section. In particular, the government could always mimic the grace period policy in equilibrium. The one caveat is that r_0 is held constant in the grace period, while the equilibrium interest rate varies with b outside of default. However, as debt is strictly decreasing and $r(b)$ must be monotone in any no-crisis equilibrium, this caveat works against choosing to default.

4.2 Rollover Crises

If investors do not roll over outstanding bonds, the government will be forced to default, but may decide to repay within the grace period to avoid the full punishment inherent in $\underline{V}$. If such an event occurs at time t, then the government will repay within the grace period if and only if $W(b, r(b)) \geq \underline{V}$. The weak inequality assumes that the government repays if indifferent.

We assume that a rollover crisis is an equilibrium possibility only if $W(b_t, r_t) < \underline{V}$. This assumption is motivated as follows. Suppose that lenders refuse to issue new bonds and the government repays within the grace period (that is, $W(b_t, r_t) \geq \underline{V}$). The outstanding bonds would carry a positive price in a secondary market and individual lenders would be willing to purchase new bonds at the margin from the government at a positive price (which would incorporate the grace-period inflation dynamics).[20]

On the other hand, a rollover crisis when $W(b_t, r_t) < \underline{V}$ has a natural interpretation. If this inequality holds and all other investors refuse to roll over their bonds, an individual lender would have no incentive to extend new credit to the government. Assuming each lender is infinitesimal, such new loans would not change the government's default decision. Moreover, as the government would not repay this new debt, such lending would not be challenged by outstanding bondholders.

Similar to Cole and Kehoe (2000) we assume that, as long as $W(b_t, r(b_t)) < \underline{V}$, a rollover crisis occurs with a Poisson arrival probability equal to λ. The value of λ will be taken as a primitive in the definition of an equilibrium below, as is δ, the grace period. We can define an indicator function for the region in which outright default is preferable to repayment within the grace period:

[20]We choose to focus on self-fulfilling *default* crises and not self-fulfilling *inflation* crises. The latter is also interesting but is not the main focus of this paper and something we leave for future research.

Definition 6. *Let $I : \mathbb{R}^2 \to \{0, 1\}$ be defined as follows:*

$$I(b_0, r_0) = \begin{cases} 1 & \text{if } W(b_0, r_0) < \underline{V} \\ 0 & \text{otherwise} \end{cases}$$

The Poisson probability of a crisis at time t can then be expressed as $\lambda I(b_t, r_t)$. Given an equilibrium $r(b)$, we shall refer to the set $\{b \in \overline{\Omega} | I(b, r(b)) = 1\}$ as the "crisis zone," and its complement in $\overline{\Omega}$ as the "safe zone."

4.3 The Government's Problem

We now state the problem of the government when not in default. As in the no-crisis equilibrium of section 3, we assume the government faces a bond-market equilibrium characterized by domain $\overline{\Omega}$ and a $r \in \mathcal{R}(\overline{\Omega})$, as well as the parameters δ and λ defining the duration of the grace period and the Poisson probability of a rollover crisis conditional on $I(b_t, r(b_t)) = 1$. Let $T \in (0, \infty]$ denote the first time a rollover crisis occurs. From the government's and an individual creditor's perspective, T is a random variable with a distribution that depends on the path of the state variable. In particular, $\Pr(T \leq \tau) = 1 - e^{-\lambda \int_0^\tau I(b(t), r(b(t)))dt}$. The realization of T is public information and it is the only uncertainty in the model. The government's problem is:

$$\begin{aligned} V(b_0) = \max_{\mathbf{x} \in \mathbf{X}} & \left\{ \int_0^\infty e^{-\lambda \int_0^t I(b(s), r(b(s))ds - \rho t} v(x(t))dt \right. \\ & \left. + \lambda \underline{V} \int_0^\infty e^{-\lambda \int_0^t I(b(s), r(b(s)))ds - \rho t} I(b(t), r(b(t)))dt \right\} \\ & \text{subject to:} \\ & b(t) = b_0 + \int_0^t f(b(t), x(t))dt \text{ and} \\ & b(t) \in \overline{\Omega} \text{ for all } t. \end{aligned} \tag{P2}$$

As in the non-crisis case, we impose the equilibrium restriction on $\overline{\Omega}$ that default is never optimal in the absence of a rollover crisis.[21]

[21] This implies that $V(b) \geq \max\langle \underline{V}, W(b, r(b)) \rangle$ for all $b \in \overline{\Omega}$ in any equilibrium.

The associated Bellman equation is:

$$
\begin{aligned}
(\rho+\lambda I_b)V(b)-\lambda I_b\underline{V} &= \max_{x\in X}\left\{v(x)+V'(b)f(b,x)\right\} \\
&= \max_{(c,\pi)\in X}\left\{u(c)-\psi(\pi)+V'(b)\left(c+(r(b)-\pi)b-y\right)\right\},
\end{aligned} \tag{HJB'}
$$

where I_b is shorthand for the crisis indicator $I(b, r(b))$. We state the viscosity definition for this case in Appendix B, as it is similar to the one for the no-crisis case (Definition 3). As in the no-crisis case, the government's value function is the unique viscosity solution to the (HJB'):

Proposition 3. *For a given $\overline{\Omega}$ and $r \in \mathcal{R}(\overline{\Omega})$, the government's value function defined in (P2) is the unique bounded Lipschitz-continuous viscosity solution to (HJB').*

4.4 Crisis Equilibrium

We can now state the definition of equilibrium with crisis:

Definition 7. *A* Recursive Competitive Equilibrium with Crisis *is an interval $\overline{\Omega} = [0, b_{max}]$, an interest rate schedule r, a consumption policy function $C : \overline{\Omega} \to [0, \bar{c}]$, an inflation policy function $\Pi : \overline{\Omega} \to [0, \bar{\pi}]$, and a value function $V : \overline{\Omega} \to \mathbb{R}$ such that*

(i) $r \in \mathcal{R}(\overline{\Omega})$;

(ii) given $(\overline{\Omega}, r)$ and for any $b_0 \in \overline{\Omega}$, the policy functions combined with the law of motion (1) and initial debt b_0 generate sequences $x(t) = (C(b(t)), \Pi(b(t)))$ that solve the government's problem (P2) and deliver $V(b_0)$ as a value function;

(iii) given $C(b)$ and $\Pi(b)$, bond holders earn a real return $r^\star$, that is, $r(b) = r^\star + \Pi(b) + \lambda I(b, r(b))$ for all $b \in \overline{\Omega}$; and

(iv) $V(b_0) \geq \underline{V}$ for all $b \in \overline{\Omega}$.

Note that when $\lambda = 0$ this equilibrium corresponds to the equilibrium in Definition 2.

As in section 3, we restrict attention to monotone equilibria, that is, where the interest rate is a non-decreasing function of b. Just as in that section, monotone equilibria feature a bang-bang inflation policy,

Lemma 2. *In any monotone equilibrium with crisis, $r(b) \in \{r^\star, r^\star + \bar{\pi}, r^\star + \lambda, r^\star + \bar{\pi} + \lambda\}$ for all $b \in \overline{\Omega}$.*

There are four discrete values for the equilibrium interest rate (as opposed to two in the no-crisis case) because of the equilibrium probability of default λ in the crisis zone.

Thresholds for the safe zone b_λ: As W is strictly decreasing in both arguments, monotonicity in $r(b)$ ensures that $I(b, r(b))$ is non-decreasing as well, and the safe zone can be defined as an interval $[0, b_\lambda]$ for some $b_\lambda \in \mathbb{R}_{++}$. This threshold for the safe zone can be characterized as follows. Define $\underline{b}_\lambda$ and $\bar{b}_\lambda$ by:

Definition 8. *Let*

$$\underline{b}_\lambda \equiv \max\left\{ b \le \frac{(1-e^{-r^\star\delta})y}{\rho} \middle| W(b, r^\star + \bar{\pi}) \ge \underline{V} \right\}; \textit{and}$$
$$\bar{b}_\lambda \equiv \max\left\{ b \le \frac{(1-e^{-r^\star\delta})y}{\rho e^{-\bar{\pi}\delta}} \middle| W(b, r^\star) \ge \underline{V} \right\}.$$

These two thresholds correspond to the maximal debt the government is willing to repay within the grace period if the interest rate is $r^\star + \bar{\pi}$ and $r^\star$, respectively. This is depicted in Figure 4. Note that we have only to consider these two interest rates because we are defining the upper threshold for the safe zone where there is no rollover crisis in equilibrium. From the government's problem described in section 4.1, we have $\underline{b}_\lambda < \bar{b}_\lambda$. This follows from the fact that W is strictly decreasing in both arguments. The equilibrium threshold for a rollover crisis b_λ lies in $\in [\underline{b}_\lambda, \bar{b}_\lambda]$, the exact value within this interval being determined by equilibrium inflation.

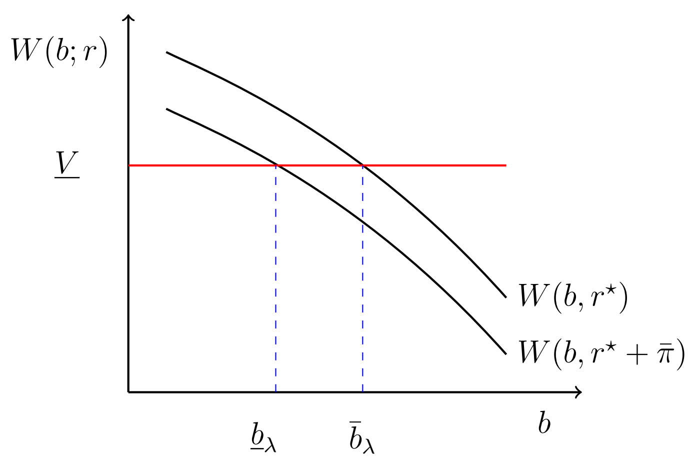


Figure 4: Threshold for Safe Zone

Thresholds for Low Inflation b_π: We now turn to two thresholds that determine the optimal inflation policy. As stated in section 3, we consider equilibria in which the low inflation zone is as large as possible. In the no-crisis equilibria, the maximum threshold is $\bar{b}_\pi$ from Definition 5, which is the maximal debt consistent with zero inflation when the government is offered an interest rate of $r^\star$. With the possibility of a rollover crisis, we introduce a second threshold, $\tilde{b}_\pi$. This threshold concerns the best response when the interest rate is $r^\star + \lambda$. That is, it is the maximum debt when there is the possibility of a crisis and yet the government opts for low inflation.

The cut-off $\tilde{b}_\pi$ is once again determined by the condition $\psi_0 = u'(C(\tilde{b}_\pi))\tilde{b}_\pi$. The particular value for consumption $C(\tilde{b}_\pi)$ however depends on whether the government is saving to exit the crisis zone or if it chooses to stay. Specifically,

Definition 9. *Let $\tilde{b}_\pi$ be defined as:*

$$\tilde{b}_\pi = \begin{cases} \frac{\psi_0}{u'(c_\lambda)} & \text{if } c_\lambda \leq y - \frac{(r^\star+\lambda)\psi_0}{u'(c_\lambda)} \\ \frac{\psi_0}{u'(y-(r^\star+\lambda)\tilde{b}_\pi)} & \text{otherwise,} \end{cases}$$

where $c_\lambda \in (0, y - (r^\star + \lambda)\bar{b}_\lambda)$ is defined uniquely by

$$\frac{(\rho+\lambda)u(y - r^\star \bar{b}_\lambda)}{\rho} = u(c_\lambda) - u'(c_\lambda)(c_\lambda - y + (r^\star + \lambda)\bar{b}_\lambda) + \lambda \underline{V}.$$

To motivate this definition, we consider the choice of inflation in the crisis zone. In particular, suppose that $\pi = 0$ were optimal for some b above $\bar{b}_\lambda$. The associated consumption level is given by $c = \min\{c_\lambda, y - (r^\star + \lambda)b\}$, where the c_λ is the consumption level if the government is saving, and $y - (r^\star + \lambda)b$ if the government is not. To verify that $\pi = 0$ is optimal, we require that $u'(c)b \leq \psi_0$. The threshold $\tilde{b}_\pi$ is the maximum debt where this inequality is satisfied. Note that $\tilde{b}_\pi < \bar{b}_\pi$ in the range of interest, as $c_\lambda < y - (r^\star + \lambda)\bar{b}_\lambda < y - r^\star \bar{b}_\lambda$, where the last term is the steady-state consumption when $r(b) = r^\star$.

4.4.1 Inflation Credibility and the Crisis Zone: An Intuition

Let us now provide some intuition for the impact of inflation credibility, ψ_0, on the determination of the crisis zone. In particular, the crisis zone is composed of levels of debt where, for the given equilibrium interest rate schedule r, we have

$$W(b, r(b)) < \underline{V},$$

so that the country is vulnerable to a roll-over crisis.

A change in the inflation cost, ψ_0, has two effects on the above inequality. First, an increase in ψ_0 lowers W for a given b and $r(b)$. Because the value of defaulting, $\underline{V}$, is independent of ψ_0, it follows that an increase in the inflation cost will tend to enlarge the levels of debt that lie within the crisis region. This is because, for a given interest rate, it is more difficult to inflate at the higher ψ_0 in a case of a crisis, making default relatively attractive and creating room for a self-fulfilling rollover crisis. This is the conventional wisdom: higher costs of inflation make the country more vulnerable to crisis.[22]

However, there is another effect of ψ_0 on the inequality, which works through the equilibrium interest rate function r. In particular, consider an increase in ψ_0. One might expect that such an increase will lead to a reduction of the equilibrium inflation rate, implying a fall in the interest rate schedule, r. This resulting equilibrium reduction in r increases $W(b, r(b))$ for some levels of debt (as the cost of repaying the debt in case a rollover crisis occurs has been reduced). Thus, an increase in ψ_0 tends to shrink the crisis region through its effect on lowering the equilibrium interest rate.

Which of the two effects described above dominates depends on the parameters of the model (and we will provide a full characterization below). However, there are two extreme cases where we can be more precise before moving on. Suppose for example that the inflation costs are really low, so that the equilibrium inflation rate is at its maximum level for most levels of debt. Then, in this case, a marginal increase in the ψ_0 has no significant impact on the equilibrium interest rate schedule (as inflation will remain high), and the first effect dominates; that is, an increase in inflation costs will tend to enlarge the crisis region. A similar situation occurs when ψ_0 is quite large, as in that case, the equilibrium inflation rate will be zero for most debt levels (although it may still be used in case of a rollover crisis). A marginal increase in the inflation cost will also tend to enlarge the crisis region for this parameter region. For intermediate levels of the inflation cost ψ_0, the equilibrium inflation rate will however be sensitive to changes in ψ_0, bringing back the second effect into play.

To summarize, the decision to default depends both on the level of debt and the equilibrium interest rate. The ability to inflate is useful in a crisis to avoid the need to default, perhaps eliminating the bad equilibrium at a particular debt level. On the other hand, the temptation to inflate drives up the nominal interest rate, creating a vulnerability where perhaps none exists with foreign currency bonds. We shall see below that in the latter case, the particularly weak commitment to inflation makes issuing foreign bonds a dominant strategy, lowering inflation in equilibrium and at the same time shrinking the crisis zone relative to domestic currency debt.

[22] It is important to note that this result uses the assumption that the value of defaulting is independent of ψ_0. This is a reasonable assumption within the context of our model, as the only reason why inflation arises in equilibrium is because of the fiscal need to repay the nominal debt; a need that disappears after a default. More generally, we could allow the function $\underline{V}$ to be affected by ψ_0, and the result would hold as long as ψ_0 affects $\underline{V}$ less than it does W.

In what follows, we flesh out more completely how the various thresholds are impacted by inflation credibility. Further, we fully characterize the dynamics for consumption and savings and the value functions under various scenarios for inflation credibility. With this we can address welfare issues.

4.5 Inflation Commitment and Crisis Vulnerability

Any monotone equilibrium $r(b)$ is characterized by $\{b_\pi, b_\lambda\}$ that determine the edge of the low-inflation and safe zones, respectively. The values $\{b_\pi, b_\lambda\}$ depends on the relative magnitudes of the four thresholds $\{\underline{b}_\lambda, \overline{b}_\lambda\}$ and $\{\tilde{b}_\pi, \overline{b}_\pi\}$. From the above discussion, $b_\pi \in [\tilde{b}_\pi, \overline{b}_\pi]$ and $b_\lambda \in [\underline{b}_\lambda, \overline{b}_\lambda]$. While we know $\underline{b}_\lambda < \overline{b}_\lambda$ and $\tilde{b}_\pi < \overline{b}_\pi$, the position of the inflation thresholds relative to the crisis thresholds depends on parameters, specifically on ψ_0.

The four thresholds as functions of the parameter ψ_0 are depicted in Figure 5. Recall that the inflation cutoffs are strictly increasing in ψ_0. The crisis thresholds are strictly decreasing in ψ_0 as long as inflation is optimal in the grace-period problem, which is the case for low ψ_0. Eventually the crisis thresholds flattens out for high enough ψ_0 when the government chooses not to inflate in the grace period.[23] The portions in bold refer to the equilibrium threshold for crisis b_λ (panel (a)) and inflation b_π (panel (b)). There are three values of ψ_0 that are of interest:

Definition 10. *Define $\boldsymbol{\psi_1}$ as the cost of inflation such that $\overline{b}_\pi = \underline{b}_\lambda$; define $\boldsymbol{\psi_2}$ as the cost of inflation such that $\overline{b}_\pi = \overline{b}_\lambda$; and define $\boldsymbol{\psi_3}$ as the cost of inflation such that $\tilde{b}_\pi = \overline{b}_\lambda$.*

Note that $\boldsymbol{\psi_1} < \boldsymbol{\psi_2} < \boldsymbol{\psi_3}$. These three values divide the parameter space into four regions.

We now discuss the general properties regarding inflation and vulnerability to rollover crises of increases in ψ_0. Start with the first region where $\psi_0 < \psi_1$. In this region the commitment to inflation is so weak that the government inflates even in the safe zone. The relevant crisis threshold is therefore $b_\lambda = \underline{b}_\lambda$. That is, the crisis threshold is determined by $W(b, r^\star + \bar{\pi})$, as inflation is high at the relevant debt level.

As ψ_0 increases in this region the crisis threshold decreases as it traces out the downward sloping curve b_λ, and the inflation threshold increases. The intuition for the decline in b_λ is as follows. Given the high temptation to inflate even in tranquil times, inflation gets priced into equilibrium interest rates. In a crisis then the government cannot generate surprise inflation. So in the grace period while it pays the cost of higher inflation it gets none of the

[23] If the grace period is long enough and ψ_0 high enough, the government may not inflate during the grace period. In this parameter space, $\underline{b}_\lambda$ and $\overline{b}_\lambda$ have slope zero. Figure 5 depicts the case in which the thresholds are decreasing at the points of intersection with the inflation cutoffs. The crisis thresholds are strictly decreasing at $\psi_0 = 0$, as inflation will always be optimal for low enough costs. The eventual flattening out of the crisis thresholds as $\psi_0 \to \infty$ is implied as $\overline{b}_\lambda$ converges to the horizontal dashed line in panel (a).

benefit. Since W is decreasing in ψ_0 when the government inflates in the grace period the crisis cut-off decreases.

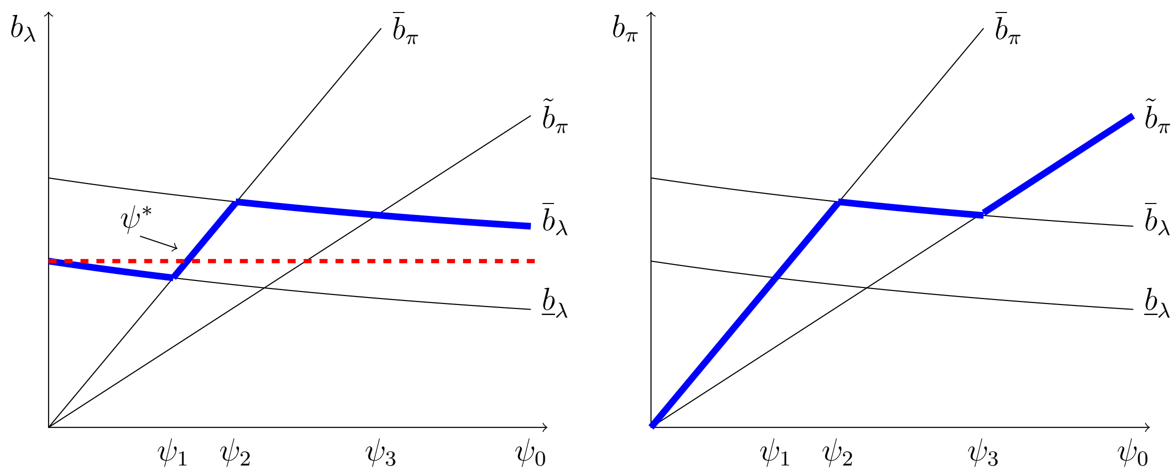
When $\psi_0 > \psi_1$, the threshold $\underline{b}_\lambda$ is no longer relevant because $r(b) = r^\star$ at $\underline{b}_\lambda$. In the region $\psi_0 \in (\psi_1, \psi_2]$ we have $\underline{b}_\lambda < \bar{b}_\pi \leq \bar{b}_\lambda$. Therefore, the jump in inflation and the associated increase in interest rate is sufficient to generate a crisis. The negative relationship between b_λ and ψ_0 is therefore reversed and the safe zone starts to expand with inflation commitment. This reflects the fact that the temptation to inflate absent a crisis creates the vulnerability to a crisis. The stronger the commitment to inflation in tranquil periods, the less vulnerable the economy is to a rollover crisis. The size of the safe zone peaks when $\psi_0 = \psi_{\mathbf{2}}$, at which point the safe zone begins to shrink again.

When $\psi_0 > \psi_2$ $\bar{b}_\pi > \bar{b}_\lambda$. In this case a crisis becomes possible even if $\pi = 0$. The equilibrium crisis threshold traces $\bar{b}_\lambda$. In the region $\psi_0 \in (\psi_2, \psi_3]$ we have $\bar{b}_\pi > \bar{b}_\lambda \geq \tilde{b}_\pi$. This implies that the optimal response to being in the crisis zone involves inflation. Therefore $\bar{b}_\lambda = b_\pi$ also defines the inflation zone. The reason the safe zone begins to shrink again as ψ_0 increases is because in this region the costs of inflation not only reduces inflation in tranquil periods, but also makes responding to a rollover crisis with inflation very costly.

As ψ_0 becomes very large, the cost of inflation is so great that the government does not inflate even in a crisis. This is the fourth region where $\psi_0 > \psi_3$, $\tilde{b}_\pi > \bar{b}_\lambda$, and the inflation threshold b_π tracks $\tilde{b}_\pi$. In the limit, the size of the safe zone converges to that of $\psi_0 = 0$, as in both cases the real value of bonds is independent of the arrival of a crisis.

As Figure 5 makes clear there is a non-monotonic relation between the size of the safe zone and inflation credibility. It is useful to focus on the two extremes, when $\psi_0 = 0$ and when $\psi_0 = \infty$. The latter extreme is analogous to the case when debt is in foreign currency and cannot be inflated away. At this extreme the cost of inflation is so high that the government does not inflate in tranquil or in crisis periods. The crisis threshold corresponds to the case of real debt. At the other extreme when $\psi_0 = 0$ it is costless to inflate so inflation is always high, both in tranquil and crisis times. The high inflation gets priced into equilibrium interest rates and there is no benefit from inflating in a crisis. Since the cost of inflation is zero the crisis threshold is exactly the same as the case when debt is in foreign currency. For intermediate ranges of ψ_0 we have b_λ first decreasing, then increasing before decreasing again. For values of ψ_0 near the left of ψ_1 there are no benefits from inflating in the crisis period as it is already priced into interest rates. The costs of inflation are however incurred and W declines. Consequently the safe zone is now smaller than what it would be if the debt was in foreign currency. That is, issuing nominal bonds enlarges the range in which a rollover crises is possible relative to foreign currency bonds, contrary to the conventional wisdom.

For values of ψ_0 between ψ_1 and ψ_2 the safe zone increases with inflation credibility. At some threshold ψ^*, nominal bonds generate a larger safe zone. This is the happy medium in which inflation is not high in normal times, but the option to increase inflation in response to a crisis provides insurance. For ψ_0 above ψ^*, therefore, the economy can approximate state-contingent inflation relatively well and is reminiscent of the conventional wisdom.[24]



(a) b_λ as a function of ψ_0

(b) b_π as a function of ψ_0

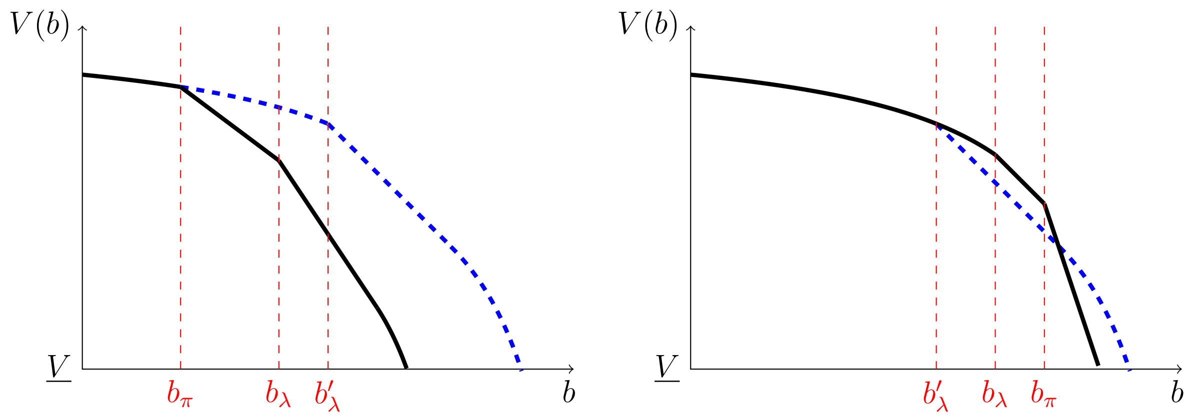
Figure 5: Thresholds as a Function of Inflation Commitment

4.6 Welfare Implications of Foreign Currency Debt

The cutoffs depicted in Figure 5 allow us to answer the question of whether an economy is better off issuing nominal (domestic currency) or real (foreign currency) debt. We depict two cases in Figure 6. In each panel, the dashed line is the value function for $\psi_0 = \infty$, which corresponds to issuing foreign currency debt. The solid line is the value from issuing domestic currency debt, where the two panels differ by the costs of inflation. All lines coincide for low b as inflation is zero and there is no risk of a crisis in this region.

Panel (a) is such that $\psi_0 \leq \psi^*$, so the safe zone is smaller with domestic currency debt.

[24]The fact that ψ_0 has a non-monotonic impact on b_λ does not require the piecewise-linear costs of inflation or the upper bound $\bar{\pi}$. As noted in footnote 19, in the safe zone $r(b)$ is decreasing in ψ_0 for quadratic inflation costs of the type $\psi(\pi) = \psi_0\pi^2$. All else equal, a reduction in r reduces the incentive to default in a rollover crisis, generating a mechanism for extending the safe zone via an increase in ψ_0. On the other hand, it is clear that for a fixed b and r, $W(b, r)$ is decreasing in ψ_0 in the quadratic case, increasing the vulnerability to a crisis. Note that these are the same two opposing forces highlighted in the benchmark case. Numerical simulations in the quadratic cost case show that the impact of ψ_0 is non-monotonic, as first one and then the other effect dominates, generating an intermediate ψ_0 that minimizes the range of debt vulnerable to rollover crises.



(a) $\psi_0 < \psi^*$ vs. $\psi_0 = \infty$

(b) $\psi_0 > \psi^*$ vs. $\psi_0 = \infty$

Figure 6: Government Welfare as a Function of Inflation Commitment

In particular, b_π, the point at which the economy begins inflating, is within the safe zone. At this point, the domestic currency debt economy becomes worse off due to the inability to deliver low inflation. At b_λ, the economy becomes vulnerable to a rollover crisis, while the crisis threshold is b'_λ for the foreign currency debt scenario. The safe zone is smaller with domestic currency debt as debt carries with it the burden of inflation, making default relatively attractive. In this case, the economy is always strictly better off with foreign currency debt. The incentive to inflate is high in equilibrium, lowering welfare without reducing the exposure to a rollover crisis. Most emerging markets rely solely on foreign currency debt for international bond issues. The analysis rationalizes this so-called "original sin" as the optimal response to a weak inflationary regime, with or without self-fulfilling debt crises.

Panel (b) depicts a case in which $\psi_0 > \psi^*$. That is, domestic currency debt reduces the exposure to a rollover crisis, but at the expense of higher equilibrium inflation for very large debt levels. This makes domestic currency debt optimal for intermediate stocks of debt, but sub-optimal for high levels of debt. The closer ψ_0 is to the peak-safe-zone level $\psi_{\mathbf{3}}$, the greater the range for which domestic currency debt strictly dominates. Thus governments that have a moderate degree of inflation commitment strictly prefer domestic currency debt over a non-negligible interval of debt. For extremely high levels of debt, the economy will inflate (and face a crisis), and so the commitment to zero inflation in this region is preferable.

4.7 Delegation

Figure 6 considered the option of issuing bonds in foreign currency, a policy readily available in practice. In some contexts, there may exist a richer set of options in designing institutions that govern monetary and fiscal policy, which in our environment will be reflected in ψ_0. Delegation of certain economic decisions to agents with different objectives has long been understood to be a possible solution to lack of credibility. However, such solutions are sometimes met with skepticism because of the inherent difficulty in building institutions that follow objectives that conflict with those of the government.

In the event such delegation is feasible, our analysis suggest that an attractive option is to delegate the conduct of policy to an institution with a per-period objective function given by $u(c) - \tilde{\psi}_0\pi$ where the perceived cost of inflation $\tilde{\psi}_0$ is (1) potentially different from the true cost of inflation ψ_0, and (2) can be state contingent. Indeed, by choosing $\tilde{\psi}_0 = \infty$ in normal times and $\tilde{\psi}_0 = 0$ in case of a rollover crisis, we eliminate rollover crisis altogether as long as $\bar{\pi}$ is high enough, and we also guarantee no inflation in equilibrium. Such an institution delivers inflation only when it is needed, when confronted with a rollover crises. It is so successful at doing so that it staves off rollover crises altogether.

Of course the inherent fragility of this solution is that the delegation of policy might be challenged ex-post by the government in case of a rollover crisis, as the economy actually has to bear the cost of high inflation. Moreover, while this solution eliminates rollover crises, it might increase the vulnerability of the economy to self-fulfilling shifts in expectations in inflation, which we have not explored in detail in this paper, but would be important to explore if such delegation solutions were to be viable options.[25]

4.8 Full Characterization of Crisis Equilibria

The next four propositions fully characterize the equilibria in the four regions of the ψ_0 parameter space. As the propositions share many similarities, we redefine notation when convenient. After each proposition, we discuss the characteristics of the equilibrium before moving to the next case.

[25] This allows us to connect with the analyses of Jeanne (2011) and Corsetti and Dedola (2013). Jeanne (2011) considers a central bank with an exogenous objective function parametrized by the degree of monetary dominance (probability of not backstopping the government in case of a fiscal crisis). They emphasize that if the degree of monetary dominance is equal to 0, then self-fulfilling debt crises can be eliminated. Corsetti and Dedola (2013) consider the possibility that the central bank buys up the debt of the government and issues its own debt, which they assume is default-free. One rationale for this assumption is that the central bank has more commitment than the government. Another interpretation, closer to the analysis in this section, is that the central bank would always choose to print money to repay these liabilities rather than default.

Case 1: $\psi_0 \in [0, \psi_{\mathbf{1}}]$

We now characterize equilibria for $\psi_0 < \psi_{\mathbf{1}}$:

Proposition 4. *Suppose $\bar{b}_\pi \leq \underline{b}_\lambda$ (that is, $\psi_0 \in [0, \psi_{\mathbf{1}}]$). Define $c_\pi = C_\pi(\bar{b}_\pi)$, where $C_\pi(b)$ is as in Definition 5. Define $b^*_\pi = (y - c_\pi)/r^\star$. For $b \leq \underline{b}_\lambda$, define $\hat{V}(b)$ by*[26]

$$\hat{V}(b) = \begin{cases} \frac{u(y - r^\star b)}{\rho} & \text{if } b \leq \bar{b}_\pi \\ \frac{u(y - r^\star \bar{b}_\pi)}{\rho}) - u'(c_\pi)(b - \bar{b}_\pi) & \text{if } b \in (\bar{b}_\pi, \min\langle b^*, \underline{b}_\lambda\rangle) \\ \frac{u(y - r^\star b) - \psi_0 \bar{\pi}}{\rho} & \text{if } b \in [b^*, \underline{b}_\lambda], \end{cases}$$

Define $c_\lambda \in (0, y - (r^\star + \lambda)\underline{b}_\lambda)$ as the solution to

$$(\rho + \lambda)\hat{V}(\underline{b}_\lambda) = u(c_\lambda) - \psi_0 \bar{\pi} - u'(c_\lambda)(c_\lambda + (r^\star + \lambda)\underline{b}_\lambda - y) + \lambda \underline{V}.$$

Let $b^*_\lambda = (y - c_\lambda)/(r^\star + \lambda)$. For $b > \underline{b}_\lambda$, define $\hat{V}(b)$ by

$$\hat{V}(b) = \begin{cases} V(\underline{b}_\lambda) - u'(c_\lambda)(b - \underline{b}_\lambda) & \text{if } b \in (\underline{b}_\lambda, b^*_\lambda) \\ \frac{u(y - (r^\star + \lambda)b) - \psi_0 \bar{\pi}}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda}\underline{V} & \text{if } b \geq b^*_\lambda. \end{cases}$$

Define $b_{max} = \max\{b \leq y/(r^\star + \lambda) | \underline{V} \leq \hat{V}(b)\}$. Then define $\overline{\Omega} = [0, b_{max}]$ for $0 \in \mathbb{R}_-$, and the following constitutes a recursive equilibrium with crisis parameter λ:

(i) The interest rate schedule $r : \overline{\Omega} \to \{r^\star, r^\star + \bar{\pi}, r^\star + \bar{\pi} + \lambda\}$ defined by

$$r(b) = \begin{cases} r^\star & \text{if } b \in [0, \bar{b}_\pi] \cap \overline{\Omega} \\ r^\star + \bar{\pi} & \text{if } b \in (\bar{b}_\pi, \underline{b}_\lambda] \cap \overline{\Omega} \\ r^\star + \bar{\pi} + \lambda & \text{if } b \in (\underline{b}_\lambda, b_{max}] \cap \overline{\Omega}; \end{cases}$$

(ii) The value function $V : \overline{\Omega} \to \mathbb{R}$ defined by $V(b) = \hat{V}(b)$ for $b \in \overline{\Omega}$;

[26] In defining $\hat{V}$ in each proposition, for notational ease we do not include the restrictions on debt that ensure consumption is non-negative. As we later truncate $\hat{V}$ to a domain on which consumption is positive, this extended domain is not relevant to the equilibrium characterization.

(iii) The consumption policy function $C : \overline{\Omega} \to \mathbb{R}_+$ defined by

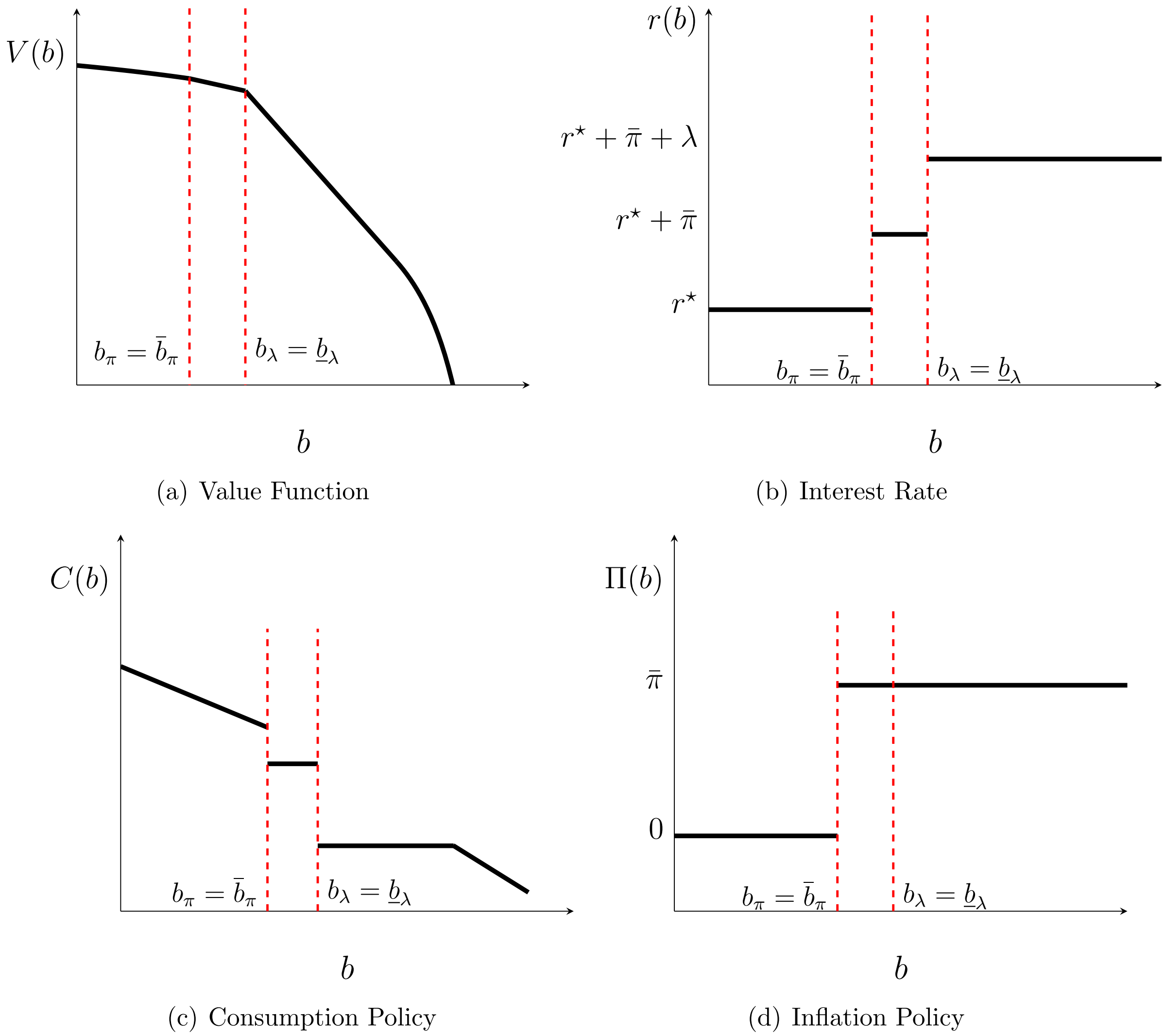
$$C(b) = \begin{cases} y - r^\star b & \text{if } b \in [0, \bar{b}_\pi] \cap \overline{\Omega} \\ c_\pi & \text{if } b \in (b_\pi, \min\langle b^*, \underline{b}_\lambda \rangle] \cap \overline{\Omega} \\ y - r^\star b & \text{if } b \in (b^*, \underline{b}_\lambda] \cap \overline{\Omega} \\ c_\lambda & \text{if } b \in (\underline{b}_\lambda, b^*_\lambda] \cap \overline{\Omega} \\ y - (r^\star + \lambda) b & \text{if } b \in (b^*_\lambda, b_{max}] \cap \overline{\Omega}; \end{cases}$$

(iv) The inflation policy function $\Pi : \overline{\Omega} \to \{0, \bar{\pi}\}$ defined by:

$$\Pi(b) = \begin{cases} 0 & \text{if } \in b \in [0, \bar{b}_\pi] \cap \overline{\Omega} \\ \bar{\pi} & \text{if } b \in (\bar{b}_\pi, b_{max}] \cap \overline{\Omega}. \end{cases}$$

The equilibrium is depicted in Figure 7. In the case of $\bar{b}_\pi < \underline{b}_\lambda$, the government has an incentive to inflate in a region in which there is no probability of a crisis, reflecting the low level of inflationary commitment. This implies that in the region $b \leq \underline{b}_\lambda$, the analysis is the same as in section 3. For low debt, the government does not inflate and enjoys steady-state utility. This is the first segment of the value function depicted in figure 7. Low inflation is no longer optimal for $b > \bar{b}_\pi$, and inflation and the interest rate respond accordingly. As in the no-crisis case of section 3, this jump in inflation and the corresponding increase in the interest rate provides an incentive to save. In the neighborhood above $\bar{b}_\pi$, consumption is constant at c_π as the economy saves towards this threshold, with consumption satisfying the corresponding Bellman equation. If the distance between $\bar{b}_\pi$ and $\underline{b}_\lambda$ is large enough (which is not the case depicted in Figure 7), there may be a high-inflation/no-crisis region where the government sets $\dot{b} = 0$ (i.e., $(b^*, \underline{b}_\lambda]$). Given the high debt levels and the low consumption, the government's optimal policy is to inflate, rationalizing the jump in the interest rate as an equilibrium.

At debt greater than $\underline{b}_\lambda$, the economy is vulnerable to a rollover crisis. The interest rate jumps again to $r^\star + \bar{\pi} + \lambda$. This provides the government with a greater incentive to save, and reflects the kink at b_λ, after which the value function declines more rapidly. The corresponding consumption level is $c_\lambda < c_\pi$, which satisfies the Bellman equation at $\underline{b}_\lambda$. Note that consumption is discretely lower at $\underline{b}_\lambda$, so inflation is weakly greater, verifying that $\bar{\pi}$ is optimal in the crisis zone as well. The equilibrium behavior of the government therefore is to save in a neighborhood above $\underline{b}_\lambda$ to eliminate the possibility of a crisis as well as reduce inflation; at $\underline{b}_\lambda$, it may continue to save at a slower rate in order to reduce inflation, eventually



(a) Value Function

(b) Interest Rate

(c) Consumption Policy

(d) Inflation Policy

Figure 7: Case 1: Crisis Equilibrium if $\psi_0 \in [0, \psi_1]$

reaching $\bar{b}_\pi$.

Case 2: $\psi_0 \in (\psi_{\mathbf{1}}, \psi_{\mathbf{2}}]$

Proposition 5. *Suppose $\bar{b}_\pi \in (\underline{b}_\lambda, \bar{b}_\lambda]$ (that is, $\psi_0 \in (\psi_{\mathbf{1}}, \psi_{\mathbf{2}}]$). Define $c_\pi \in (0, y - r^\star \bar{b}_\pi)$ as the solution to*

$$\frac{(\rho+\lambda)u(y - r^\star \bar{b}_\pi)}{\rho} = u(c_\pi) - \psi_0 \bar{\pi} - u'(c_\pi)(c_\pi + (r^\star + \lambda)\bar{b}_\pi - y) + \lambda \underline{V}.$$

Let $b^* = (y - c_\pi)/(r^\star + \lambda)$. Define $\hat{V}(b)$ by

$$\hat{V}(b) = \begin{cases} \frac{u(y - r^\star b)}{\rho} & \text{if } b \leq \bar{b}_\pi \\ \frac{u(y - r^\star \bar{b}_\pi)}{\rho} - u'(c_\pi)(b - \bar{b}_\pi) & \text{if } b \in (\bar{b}_\pi, b^*) \\ \frac{u(y-(r^\star+\lambda)b) - \psi_0\bar{\pi}}{\rho+\lambda} + \frac{\lambda}{\rho+\lambda}\underline{V} & \text{if } b \geq b^*. \end{cases}$$

Define $b_{max} = \max\{b \leq y/(r^\star + \lambda) | \underline{V} \leq \hat{V}(b)\}$. Then define $\overline{\Omega} = [0, b_{max}]$ for $0 \in \mathbb{R}_-$, and the following constitutes a Recursive Equilibrium with Crisis:

(i) The interest rate schedule $r : \overline{\Omega} \to \{r^\star, r^\star + \bar{\pi} + \lambda\}$ defined by

$$r(b) = \begin{cases} r^\star & \text{if } b \in [0, \bar{b}_\pi] \cap \overline{\Omega} \\ r^\star + \bar{\pi} + \lambda & \text{if } b \in (\bar{b}_\pi, b_{max}] \cap \overline{\Omega}; \end{cases}$$

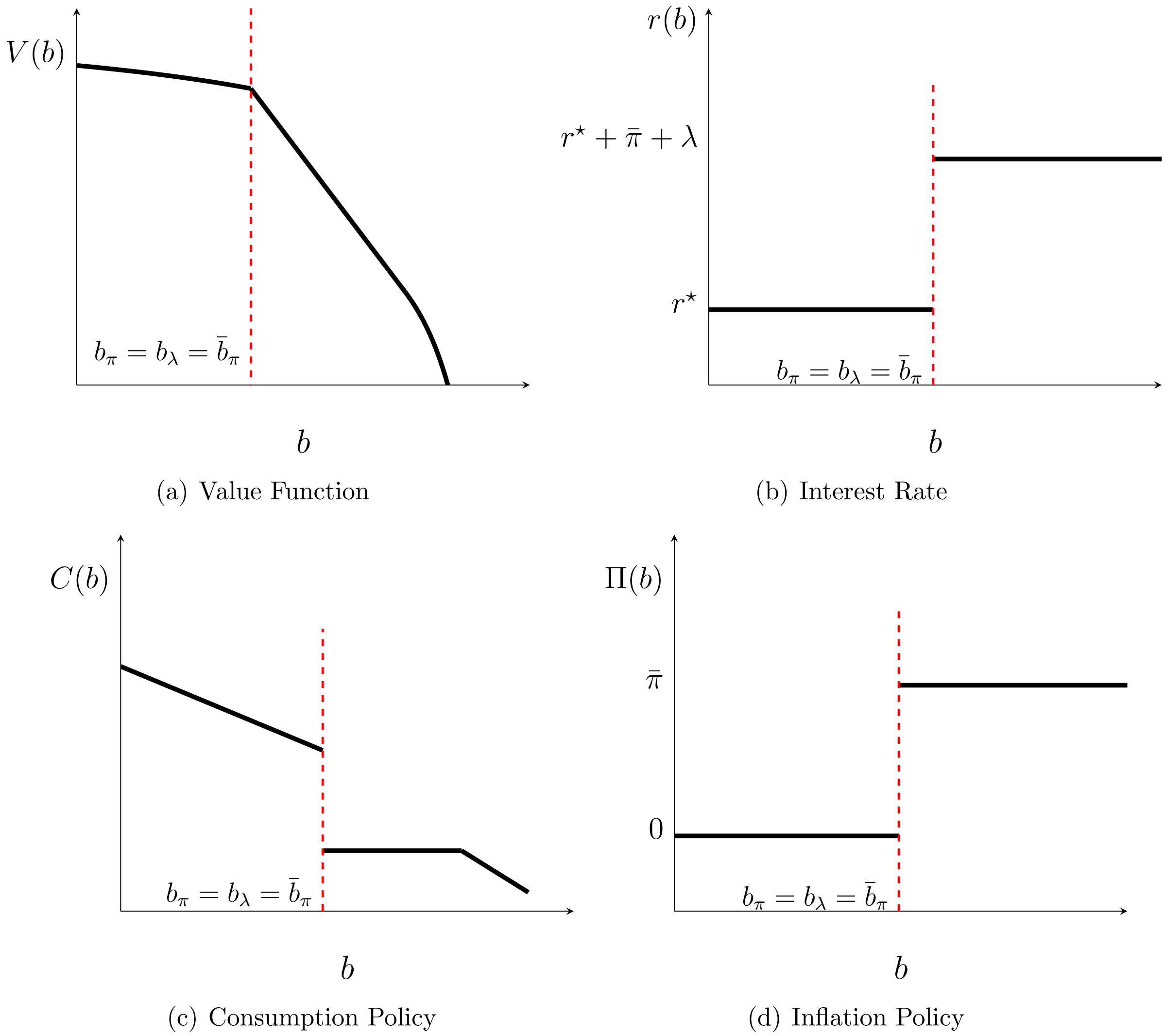
(ii) The value function $V : \overline{\Omega} \to \mathbb{R}$ defined by $V(b) = \hat{V}(b)$ for $b \in \overline{\Omega}$;

(iii) The consumption policy function $C : \overline{\Omega} \to \mathbb{R}_+$ defined by

$$C(b) = \begin{cases} y - r^\star b & \text{if } b \in [0, \bar{b}_\pi] \cap \overline{\Omega} \\ c_\pi & \text{if } b \in (\bar{b}_\pi, b^*] \cap \overline{\Omega} \\ y - (r^\star + \lambda)b & \text{if } b \in (b^*, b_{max}] \cap \overline{\Omega}; \end{cases}$$

(iv) The inflation policy function $\Pi : \overline{\Omega} \to \{0, \bar{\pi}\}$ defined by:

$$\Pi(b) = \begin{cases} 0 & \text{if } b \in [0, \bar{b}_\pi] \cap \overline{\Omega} \\ \bar{\pi} & \text{if } b \in (b_\pi, b_{max}] \cap \overline{\Omega}. \end{cases}$$



(a) Value Function

(b) Interest Rate

(c) Consumption Policy

(d) Inflation Policy

Figure 8: Case 2: Crisis Equilibrium if $\psi_0 \in (\psi_1, \psi_2]$

In this case, the economy has low inflation at $\underline{b}_\lambda$, so this is not the relevant threshold for the safe zone. However, inflation may be high in equilibrium at $\bar{b}_\lambda$, making this an irrelevant threshold as well. We have instead that the equilibrium threshold for a crisis is $b_\lambda = \bar{b}_\pi$, so the jump in the interest rate due to high inflation creates room for a crisis. The government's value function is depicted in figure 8. The government is at a low inflation steady state for $b \leq \bar{b}_\pi = b_\lambda$. At $b \in (b_\lambda, b_\lambda + \varepsilon)$ for some $\varepsilon > 0$ the economy saves towards the low inflation/safe zone, setting $\pi = \bar{\pi}$. Consumption is c_π with $\pi = \bar{\pi}$ and $V(b_\lambda) = \frac{u(y - r^\star b_\lambda)}{\rho}$.

Case 3: $\psi_0 \in (\psi_\mathbf{2}, \psi_\mathbf{3}]$

Proposition 6. *Suppose $\bar{b}_\pi > \bar{b}_\lambda \geq \tilde{b}_\pi$ (that is, $\psi_0 \in (\psi_\mathbf{2}, \psi_\mathbf{3}]$). Define $c_\lambda \in (0, y - r^\star \bar{b}_\lambda)$ as the solution to*

$$\frac{(\rho + \lambda) u(y - r^\star \bar{b}_\lambda)}{\rho} = u(c_\lambda) - \psi_0 \bar{\pi} - u'(c_\lambda)(c_\lambda + (r^\star + \lambda)\bar{b}_\lambda - y) + \lambda \underline{V}.$$

Let $b^* = (y - c_\lambda)/(r^\star + \lambda)$. Define $\hat{V}(b)$ by

$$\hat{V}(b) = \begin{cases} \frac{u(y - r^\star b)}{\rho} & \text{if } b \leq \bar{b}_\lambda \\ \frac{u(y - r^\star \bar{b}_\lambda)}{\rho} - u'(c_\lambda)(b - \bar{b}_\lambda) & \text{if } b \in (\bar{b}_\lambda, b^*) \\ \frac{u(y - (r^\star + \lambda)b) - \psi_0 \bar{\pi}}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda} \underline{V} & \text{if } b \geq b^*. \end{cases}$$

Define $b_{max} = \max\{b \leq y/(r^\star + \lambda) | \underline{V} \leq \hat{V}(b)\}$. Then define $\overline{\Omega} = [0, b_{max}]$ for $0 \in \mathbb{R}_-$, and the following constitutes a Recursive Equilibrium with Crisis:

(i) The interest rate schedule $r : \overline{\Omega} \to \{r^\star, r^\star + \bar{\pi} + \lambda\}$ defined by

$$r(b) = \begin{cases} r^\star & \text{if } b \in [0, \bar{b}_\lambda] \cap \overline{\Omega} \\ r^\star + \bar{\pi} + \lambda & \text{if } b \in (\bar{b}_\lambda, \bar{b}] \cap \overline{\Omega}; \end{cases}$$

(ii) The value function $V : \overline{\Omega} \to \mathbb{R}$ defined by $V(b) = \hat{V}(b)$ for $b \in \overline{\Omega}$;

(iii) The consumption policy function $C : \overline{\Omega} \to \mathbb{R}_+$ defined by

$$C(b) = \begin{cases} y - r^\star b & \text{if } b \in [0, \bar{b}_\lambda] \cap \overline{\Omega} \\ c_\lambda & \text{if } b \in (\bar{b}_\lambda, b^*] \cap \overline{\Omega} \\ y - (r^\star + \lambda)b & \text{if } b \in (b^*, b_{max}] \cap \overline{\Omega}; \end{cases}$$

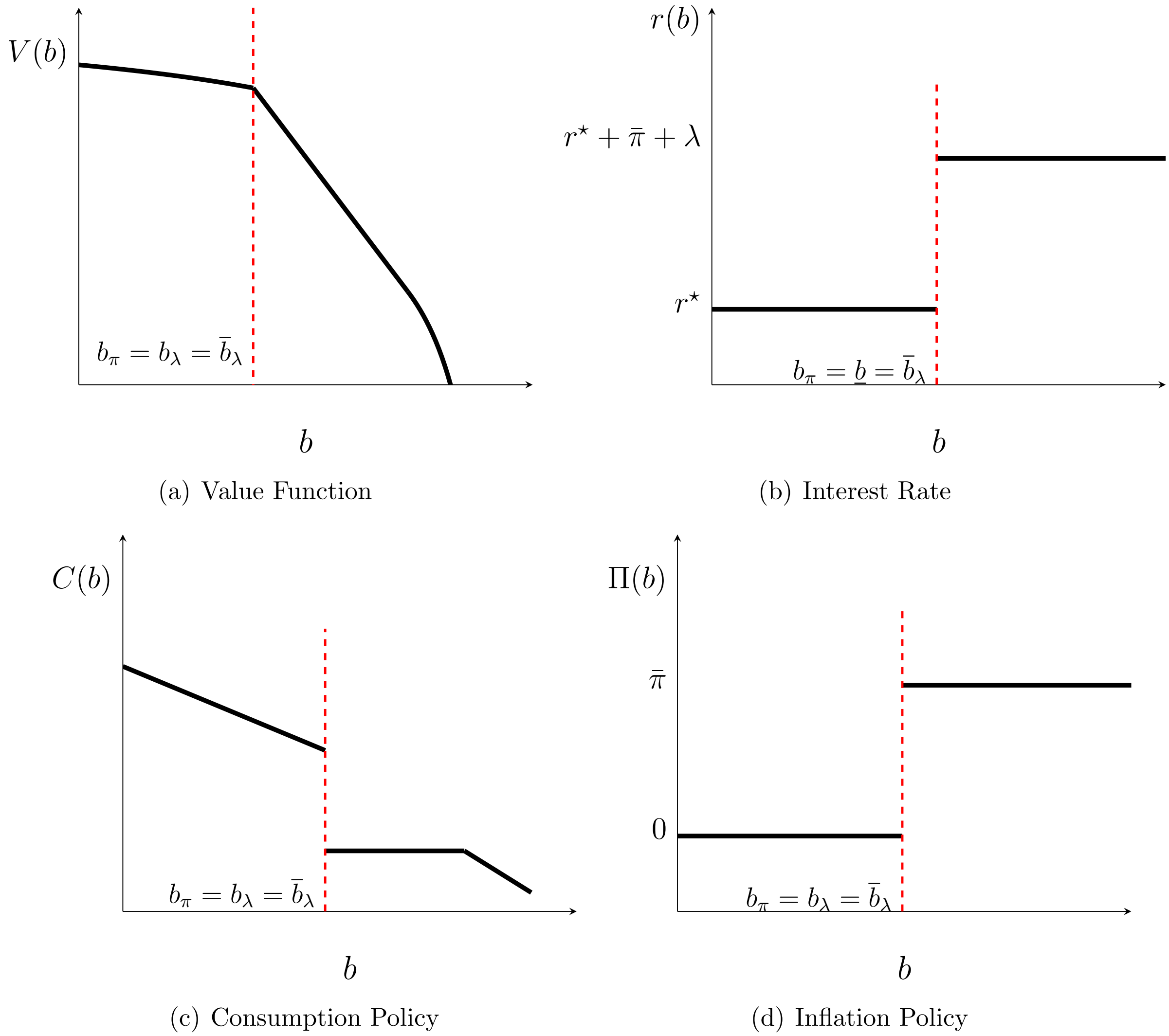


Figure 9: Case 3: Crisis Equilibrium if $\psi \in (\psi_2, \psi_3]$

(iv) The inflation policy function $\Pi : \overline{\Omega} \to \{0, \bar{\pi}\}$ *defined by:*

$$\Pi(b) = \begin{cases} 0 & \textit{if } b \in [0, \bar{b}_\lambda] \cap \overline{\Omega} \\ \bar{\pi} & \textit{if } b \in (\bar{b}_\lambda, b_{max}] \cap \overline{\Omega}. \end{cases}$$

This case is the mirror-image of case 2. In particular, the equilibrium crisis threshold and the inflation threshold are equivalent, but the reason is reversed. That is, the government increases inflation at $\bar{b}_\lambda$ because it faces a rollover crisis and wishes to reduce debt quickly. Therefore, the jump in interest rate due to a crisis leads the government to high inflation, rather than vice versa, as was the situation in case 2. Given this symmetry, the value function and policy functions in case 3 (Figure 9) take the same form as those in case 2.

Case 4: $\psi_0 > \psi_3$

Proposition 7. *Suppose $\tilde{b}_\pi > \bar{b}_\lambda$ (that is, $\psi > \psi_3$). Define $c_\lambda \in (0, y - (r^\star + \lambda)\bar{b}_\lambda)$ as the unique solution to:*

$$\frac{(\rho+\lambda)u(y - r^\star \bar{b}_\lambda)}{\rho} = u(c_\lambda) - u'(c_\lambda)(c_\lambda - y + r^\star \bar{b}_\lambda) + \lambda \underline{V}.$$

Define $b^*_\lambda = (y - c_\lambda)/(r^\star + \lambda)$. For $b \leq \tilde{b}_\pi$, define $\hat{V}(b)$ by

$$\hat{V}(b) = \begin{cases} \frac{u(y - r^\star b)}{\rho} & \text{if } b \leq \bar{b}_\lambda \\ \frac{u(y - r^\star \bar{b}_\lambda)}{\rho} - u'(c_\lambda)(b - \bar{b}_\lambda) & \text{if } b \in (\bar{b}_\lambda, \min\langle b^*_\lambda, \tilde{b}_\pi \rangle) \\ \frac{u(y-(r^\star+\lambda)b)}{\rho+\lambda} + \frac{\lambda}{\rho+\lambda}\underline{V} & \text{if } b \in [b^*_\lambda, \tilde{b}_\pi]. \end{cases}$$

Define $c_\pi \in (0, y - (r^\star + \lambda)\tilde{b}_\pi)$ as the solution to

$$(\rho+\lambda)\hat{V}(\tilde{b}_\pi) = u(c_\pi) - \psi_0 \bar{\pi} - u'(c_\pi)(c_\pi + (r^\star + \lambda)\tilde{b}_\pi - y) + \lambda \underline{V}.$$

Let $b^*_\pi = (y - c_\pi)/(r^\star + \lambda)$. For $b > \tilde{b}_\pi$, define $\hat{V}(b)$ by

$$\hat{V}(b) = \begin{cases} V(\tilde{b}_\pi) - u'(c_\pi)(b - \tilde{b}_\pi) & \text{if } b \in (\tilde{b}_\pi, b^*_\pi) \\ \frac{u(y-(r^\star+\lambda)b) - \psi_0\bar{\pi}}{\rho+\lambda} + \frac{\lambda}{\rho+\lambda}\underline{V} & \text{if } b \geq b^*_\pi. \end{cases}$$

Define $b_{max} = \max\{b \leq y/(r^\star+\lambda) | \underline{V} \leq \hat{V}(b)\}$. Then define $\overline{\Omega} = [0, b_{max}]$ for $0 \in \mathbb{R}_-$, and the following constitutes a Recursive Equilibrium with Crisis:

(i) The interest rate schedule $r : \overline{\Omega} \to \{r^\star, r^\star + \lambda, r^\star + \bar{\pi} + \lambda\}$ defined by

$$r(b) = \begin{cases} r^\star & \text{if } b \in [0, \bar{b}_\lambda] \cap \overline{\Omega} \\ r^\star + \lambda & \text{if } b \in (\bar{b}_\lambda, \tilde{b}_\pi] \cap \overline{\Omega} \\ r^\star + \bar{\pi} + \lambda & \text{if } b \in (\tilde{b}_\pi, b_{max}] \cap \overline{\Omega}; \end{cases}$$

(ii) The value function $V : \overline{\Omega} \to \mathbb{R}$ defined by $V(b) = \hat{V}(b)$ for $b \in \overline{\Omega}$;

(iii) The consumption policy function $C : \overline{\Omega} \to \mathbb{R}_+$ *defined by*

$$C(b) = \begin{cases} y - r^\star b & \textit{if } b \in [0, \bar{b}_\lambda] \cap \overline{\Omega} \\ c_\lambda & \textit{if } b \in (\bar{b}_\lambda, \min\langle b^*_\lambda, \tilde{b}_\pi \rangle] \cap \overline{\Omega} \\ y - (r^\star + \lambda) b & \textit{if } b \in (b^*_\lambda, \tilde{b}_\pi] \cap \overline{\Omega} \\ c_\pi & \textit{if } b \in (\tilde{b}_\pi, b^*_\pi] \cap \overline{\Omega} \\ y - (r^\star + \lambda) b & \textit{if } b \in (b^*_\pi, b_{max}] \cap \overline{\Omega}; \end{cases}$$

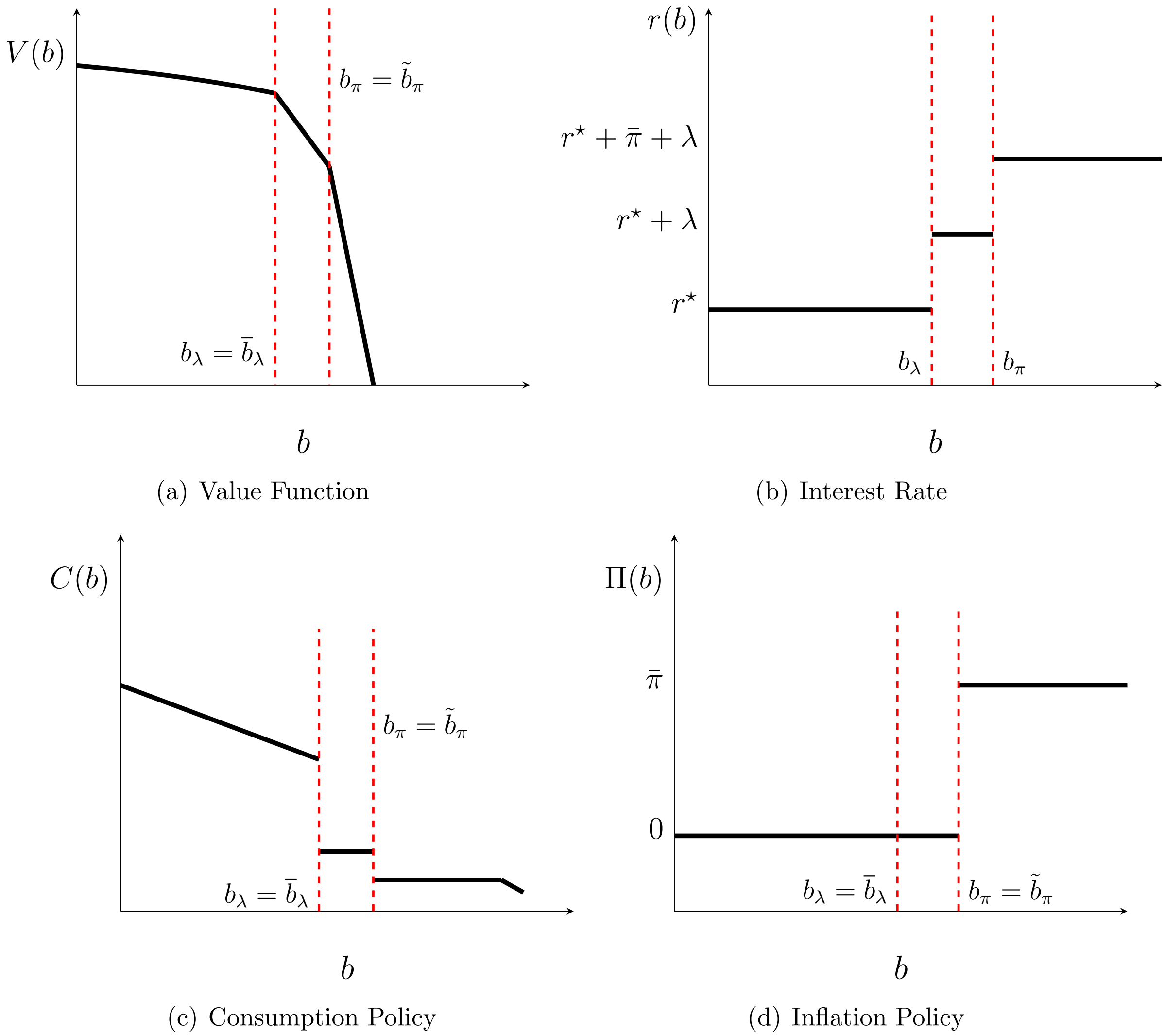
(iv) The inflation policy function $\Pi : \overline{\Omega} \to \{0, \bar{\pi}\}$ *defined by:*

$$\Pi(b) = \begin{cases} 0 & \textit{if } \in b \in [0, \tilde{b}_\pi] \cap \overline{\Omega} \\ \bar{\pi} & \textit{if } b \in (\tilde{b}_\pi, b_{max}] \cap \overline{\Omega}. \end{cases}$$

Case 4 is an environment with a strong commitment to low inflation. It is optimal to set inflation to zero even in part of the crisis zone ($b \in (b_\lambda, \tilde{b}_\pi]$), despite the strong incentive to reduce debt in the neighborhood of b_λ. As $\psi_0 \to \infty$, $\tilde{b}_\pi \to \infty$, and there is zero inflation over the entire domain $\overline{\Omega}$ and in response to a rollover crisis. This corresponds to the environment of Cole and Kehoe (2000) in which debt is real, both on and off the equilibrium path. The value and policy functions depicted in Figure 10 indicate the typical incentives to save at each increase in the interest rate, with the value function being linear in these regions.

5 Conclusion

In this paper we explored the role inflation commitment plays in vulnerability to a rollover crisis. We confirmed that for an intermediate level of inflationary commitment, an economy is less vulnerable to a crisis with domestic currency debt. The intermediate commitment provides the missing state contingency, delivering low inflation in tranquil periods but high inflation in response to a crisis. Extreme commitment to low inflation eliminates the option to inflate in a crisis. In the model, strong commitment can be seen as equivalent to issuing foreign currency debt; such commitment may also arise by being a small member of a monetary union subject to idiosyncratic rollover risk. On the other hand, weak commitment to inflation renders an economy more vulnerable to a rollover crisis if it issues domestic currency bonds. This rationalizes the exclusive issuance of foreign currency bonds to international investors by governments with limited inflation credibility.



(a) Value Function

(b) Interest Rate

(c) Consumption Policy

(d) Inflation Policy

Figure 10: Case 4: Crisis Equilibrium if $\psi_0 > \psi_3$

Appendices

A Proofs

A.1 Proof of Proposition 1

Proof. Our model is a particular case of the general environment studied by Bressan and Hong (2007) (henceforth, BH) in their analysis of solutions to Hamilton-Jacobi-Bellman equations on stratified domains, and our proof relies on their results.

The outline of the proof is as follows. First we show that the conditions for Theorem 1 in BH are satisfied in our environment, guaranteeing the existence of an optimal solution to the government's problem. Then, we further argue that our environment gives the government enough controllability of the state so that the resulting value function is Lipschitz continuous. Lipshchitz continuity is taken as a premise in BH, as apposed to an outcome. Finally, we argue that the conditions for Corollary 1 in BH hold, which guarantee that the value function is the unique Lipschitz continuous solution to (HJB) as previously defined in Definition 3.

We alter some of the BH notation to be consistent with our set up, and translate their minimization cost problem into a maximization of utility problem. In their paper, BH consider the state space to be the entire real line, while in our case we restrict attention to the compact set $\bar{\Omega}$. However, we can extend the space by following BH's Example 2 and letting the payoff in the complement of $\bar{\Omega}$ be sufficiently low, in effect forcing the solution of the problem to lie in $\bar{\Omega}$ at all times when the initial state lies in $\bar{\Omega}$. BH also restrict attention to non-negative costs (non-positive utility), which we incorporate by re-defining, while abusing notation, $v(x) \equiv v(x) - \bar{u}$ for all $x \in X$, where $\bar{u}$ is the upper bound on utility from consumption.

BH decompose the state space ($\bar{\Omega}$ in our case) into $M < \infty$ disjoint manifolds (intervals in our case): $\bar{\Omega} = \mathcal{M}_1 \cup \mathcal{M}_2 \cup ... \cup \mathcal{M}_M$. In our environment, this corresponds to the points of discontinuity $\{0, b_1, ..., b_N, b_{max}\}$ as well as the intervening open sets, $(0, b_1), ..., (b_N, b_{max})$.[27] This decomposition of the state space satisfies the requirements of BH: if $j \neq k$, then $\mathcal{M}_j \cap \mathcal{M}_k = \emptyset$; and if $\mathcal{M}_j \cap \overline{\mathcal{M}}_k \neq \emptyset$, then $\mathcal{M}_j \in \overline{\mathcal{M}}_k$. Let $i(b)$ denote the index of the interval that contains b.

Following BH, define a subset of controls $X_i \subset X$ for each interval $\mathcal{M}_i$ that produce tangent trajectories. That is,

$$X_i \equiv \left\{ x \in X \,\middle|\, \lim_{h \to 0} \frac{\inf_{b' \in \mathcal{M}_i} |b + f(b,x)h - b'|}{h} = 0 \;, \forall b \in \mathcal{M}_i \right\}.$$

[27] Given our previous discussion, we will ignore the "extended regions", $(-\infty, 0)$ and (b_{max}, ∞).

Let $T_{M_i}(b)$ denote the set of feasible tangent trajectories for $b \in M_i$. For the open sets between points of discontinuity, all admissible controls produce tangent trajectories, and so $X_i = X$ and $T_{M_i}(b) = [\min_{x \in X} f(b,x), \max_{x \in X} f(b,x)]$. For the boundaries, $\{0, b_1, ..., b_{max}\}$, we have the steady state controls: $X_i = \{x | f(b_n, x) = 0\}$ if $\mathcal{M}_i = \{b_n\}$ and $T_{M_i}(b_n) = \{0\}$. BH consider the following sets. Define

$$\hat{F}(b) \equiv \left\{ (h,w) \,\middle|\, h = f(b,x), w \leq v(b,x), x \in X_{i(b)} \right\} \subset \mathbb{R}^2.$$

for all $b \in \bar{\Omega}$. This is the set of feasible tangent trajectories $f(b,x), x \in X_i$ paired with the payoff interval $(-\infty, v(b,x)]$. For a point b, we consider the convex combinations of tangent trajectories and associated utility in the neighborhood of b. In particular, let $\overline{co}S$ denote the convex hull of a set S. Define

$$G(b) \equiv \bigcap_{\epsilon > 0} \overline{co} \left\{ (h,l) \in \hat{F}(b') \, || b' - b| < \epsilon \right\} \subset \mathbb{R}^2.$$

BH define the Hamilton-Jacobian-Bellman equation as:

$$\begin{gathered} \rho V(b) - \tilde{H}(b, V'(b)) = 0 \\ \text{where } \tilde{H}(b,q) \equiv \sup_{(h,w) \in G(b)} \{w + qh\}. \end{gathered} \tag{BH:HJB}$$

We now map the (BH:HJB) equation into our equation (HJB). Let

$$f_*(b,x) \equiv c - \pi b - y + \lim \inf_{b' \to b} r(b')b',$$

and

$$f^*(b,x) \equiv c - \pi b - y + \lim \sup_{b' \to b} r(b')b',$$

where $x = (c,b)$. Because $r(b)$ is lower semi-continuous $f_*(b,x) = f(b,x)$. Also note that $f_*(b,x) \leq f^*(b,x)$.

Let $\mathcal{H}(b) \equiv [\min_{x \in X} f(b,x), \max_{x \in X} f^*(b,x)]$, that is, $\mathcal{H}(b)$ is the relevant interval of debt dynamics for a given b. Given b, and for $h \in \mathcal{H}(b)$, define

$$\hat{W}(h,b) \equiv \max_{x \in X} v(x), \text{ subject to } f(b,x) \leq h.$$

$\hat{W}(h,b)$ represents the maximum utility of generating a change in debt less than or equal to h. Note that $\hat{W}(h,b)$ is non-decreasing and concave in its first argument. If follows then that we can rewrite G as follows,

$$G(b) = \left\{ (h,w) \,\middle|\, h \in \mathcal{H}(b), w \leq \hat{W}(h,b) \right\}.$$

Moreover, for $q \leq 0$, we have that the Hamiltonian in equation (16) of BH can be written as:

$$\tilde{H}(b,q) = \sup_{(h,w)\in G(b)} \{w + qh\} = \max_{x\in X}\{v(x) + qf(b,x)\} = H(b,q).$$

which corresponds to the Hamiltonian defined in our paper in equation (2). With this equivalence, the definition of a viscosity solution given in the text corresponds to that used in BH.[28] To see this note that the definition of an upper-solution in BH (Definition 1 in BH) is equivalent to requirement (i) of our Definition 3; and that the definition of a lower-solution in BH (Definition 2 in BH) corresponds to requirements (ii) and (iii) of our Definition 3. This last point follows because of the following:

- When $b \in \Omega_n$, the set of feasible tangent trajectories is composed of all feasible trajectories in Ω_n, and the restriction to tangent trajectories has no bite in equation (26) of BH, and in this case, Definition 2 in BH is equivalent to our condition (ii).[29]

- When $b \in \{0, b_1, ..., b_N, b_{max}\}$, condition (ii) of the definition of the set $\mathcal{R}$ (in our Definition 1) guarantees that the set of feasible tangent trajectories is nonempty and equal to $\{0\}$, and hence, equation (26) of BH collapses to a bound on the stationary value, which is our condition (iii).

Given this mapping from our environment into that of BH, we now verify the BH assumptions. The definition of the set $\mathcal{R}$ ensures that condition **H1** in BH hold on $\overline{\Omega}$. BH condition **H2** holds in our environment as the tangent trajectories are either all trajectories (on the open sets of continuity) or the steady-state dynamics on the points of discontinuity. Note that the stationary trajectory generates a finite utility, and hence we can use Theorem 1 in BH to show that the optimization problem has an optimal solution.

Finally, to apply Corollary 1 in BH, note that given that our value function is Lipschitz continuous (as stated and proved in Lemma 3 below), it follows that it satisfies condition **H3** in BH. Condition **H4** of BH requires that $V(b)$ is globally bounded, which is satisfied in our environment as $\frac{\bar{u}}{\rho} \geq V(b) \geq \underline{V}$ for all b. Finally, equation (46) in BH requires that

[28] BH define the concept of a viscosity solution in the context of a cost minimization problem. We redefine their definition to conform to a utility maximization problem.

[29] There is an important misprint in the published version of equation (26) of BH. See the correction in Bressan (2013).

the flow utility function be Lipschitz continuous with respect to b. As $v(x)$ is independent of b in our environment, this is satisfied trivially. It follows then, from Corollary 1 in BH and Lemma 3, that the value function is the unique Lipschitz continuous viscosity solution to (HJB). $\square$

Proof of the Lipschitz continuity of the value function

We now state and proof the following lemma, which exploits the controllability of the system in our model.

Lemma 3. *The value function, $V(b)$, which solves problem (P1) is Lipschitz continuous on $\overline{\Omega}$; that is, there exists a $K \in (0, \infty)$ such that $|V(b') - V(b)| \leq K|b' - b|$ for all $(b, b') \in \overline{\Omega}^2$.*

Proof. Choose $\kappa \in [\underline{\kappa}, \bar{\kappa}]$, where $\underline{\kappa} > 0$ and $\bar{\kappa} < y - r(b_{max})b_{max}$. This is a non-empty interval as otherwise $y = r(b_{max})b_{max}$ and consumption must be zero at b_{max}, which is not consistent with equilibrium given that $\underline{V} > u(0)/\rho$. Define $\bar{c} = y - r(b_{max})b_{max} - \kappa$, where r is the equilibrium interest rate schedule faced by the government. Note that $\bar{c} > 0$, and so $u(\bar{c})$ is finite. Given our upper-bound assumptions on u, we can assume that $u \leq 0$ (and so $V \leq 0$). Now consider $(b, b') \in \overline{\Omega}^2$, and without loss we can take $b' > b$. Let τ be the time it takes to transition from b' to b pursing a policy of $c(t) = \bar{c}$ and $\pi(t) = 0$ along the transition. Note that $\dot{b}(t) = \bar{c} + r(b(t))b(t) - y \leq \bar{c} + r(b_{max})b_{max} - y = -\kappa$, where the $\leq$-step uses the monotonicity of r. As debt falls at a rate weakly less than κ along the transition, we have $\tau \leq \frac{|b'-b|}{\kappa}$, where the fact that $\kappa > 0$ implies this latter term is well defined. As $c(t) = \bar{c}$ and $\pi(t) = 0$ is feasible in equilibrium, we have:

$$
\begin{aligned}
V(b') &\geq \int_0^\tau e^{-\rho\tau} u(\bar{c}) dt + e^{-\rho\tau} V(b) \\
&\geq \int_0^\tau u(\bar{c}) dt + V(b) \\
&= \tau u(\bar{c}) + V(b) \\
&\geq \frac{u(\bar{c})|b' - b|}{\kappa} + V(b),
\end{aligned}
$$

where the second and fourth lines use the fact that $u \leq 0$ and $V \leq 0$. Rearranging, we have

$$
\frac{-u(\bar{c})}{\kappa}|b' - b| \geq V(b) - V(b') = |V(b') - V(b)|,
$$

where the last equality follows from monotonicity of V. Let $K \equiv \frac{-u(\bar{c})}{\kappa} \in (0, \infty)$, and we have that V is Lipschitz continuous. $\square$

A.2 Proof of Lemma 1

The proof the Lemma proceeds as follows. First we show that in a monotone equilibrium, inflation can be interior only for levels of $b > \bar{b}_\pi$, where $\bar{b}_\pi$ is defined in 5. We show however that if inflation is interior for $b > \bar{b}_\pi$, then the government's best response is to borrow at b; that is, $C(b) > y - \rho b$. When $r(b)$ is at its maximum, $r^\star + \bar{\pi}$, the government's best response is to not borrow (either save or keep debt constant); that is, $C(b) \leq y - \rho b$ when $r(b) = r^\star + \bar{\pi}$. Hence we can find a b such that the government borrows from below and saves from above this debt level. We then show that such a point generates a violation of the viscosity conditions.

Recall the Hamiltonian:

$$H(b, q) = \max_{c,\pi}\{u(c) - \psi_0\pi + q(c - y + (r(b) - \pi)b)\}$$

Let $c^\star(b, q)$ and $\pi^\star(b, q)$ be in the argmax of the above maximization. There are two cases to consider.

CASE 1: Suppose that there is a b_b such that the equilibrium interest rate is $r(b) = r^\star + \bar{\pi}$ for $b > b_b$ and $r^\star < r(b) < r^\star + \bar{\pi}$ for $b \in (b_a, b_b)$. Note that the value function is differentiable almost every where, and thus for almost all $b \in (b_a, b_b)$, $V'(b) = -\psi_0/b$ given interiority of π. Given that any Lipschitz continuous function is absolutely continuous, it follows that the value function can be written as

$$V(b) = V(b_a) - \int_{b_a}^{b} \frac{\psi_0}{t} dt \tag{7}$$

for $b \in (b_a, b_b)$, implying that $V(b)$ is differentiable in (b_a, b_b). Recall as well that conditions (i) and (ii) of the definition of a viscosity solution imply that

$$\rho V(b) = H(b, -\psi_0/b)$$

because of differentiability of the value function. Using the equilibrum condition, $r(b) = r^\star + \Pi(b)$, the above equation implies that

$$r(b) = r^\star + \pi^\star(b, -\psi_0/b)x = r^\star + \frac{u(c^\star(b, -\psi_0/b)) - \psi_0/b(c^\star(b, -\psi_0/b) - y + r^\star b) - \rho V(b)}{\psi_0} \tag{8}$$

for $b \in (b_a, b_b)$.

From the first order condition, it follows that $c^\star(b, -\psi_0/b) = \hat{C}(\psi_0/b)$ where $\hat{C}$ is the inverse of u', and hence $c^\star(b, -\psi_0/b)$ is continuous and differentiable in b. Equation (8) implies that the interest rate is differentiable in (b_a, b_b) with a first derivative equal to:

$$r'(b) = \frac{c^\star(b, -\psi_0/b) - y + r^\star b}{b^2}$$

Note that $c^\star(b, -\psi_0/b)$ is strictly increasing in b.

Now, suppose that $b_a < \bar{b}_\pi$. Note that $c^\star(\bar{b}_\pi, -\psi_0/\bar{b}_\pi) = \hat{C}(\psi_0/\bar{b}_\pi) = y - r^\star\bar{b}_\pi$ (from the definition of $\bar{b}_\pi$), so for $b < \bar{b}_\pi$ we have that $c^\star(b, -\psi_0/b) < c^\star(\bar{b}_\pi, \psi_0/\bar{b}_\pi) < y - r^\star\bar{b}_\pi < y - r^\star b$ which implies that $r'(b) < 0$, a violation of monotonicity. Hence, it must be the case that $b_a \geq \bar{b}_\pi$.

It follows then $b_b > b_a \geq \bar{b}_\pi$. So consider the points $b > b_b$. For such points, the interest rate is at his highest possible value and will be so for any higher debt level. Thus, the optimal solution in this range will either keep debt constant or decrease it. This implies that $V'(b) \leq -u'(y - r^\star b)$, whenever V is differentiable.

Let φ_1 be defined as the following differentiable function

$$\varphi_1(b) = p(b - b_b) + V(b_b)$$

for some $p \in [-u'(y - r^\star b_b), -\psi_0/b_b]$. First, note that $\varphi_1(b_b) - V(b_b) = 0$. We can then write, $V(b) - \varphi(b) = \int_{b_b}^{b}(V'(t) - \varphi'(t))dt$, exploiting the Lipschitz continuity of V.

For $b > b_b$, we have that $V'(b) \leq -u'(y - r^\star b) \leq p = \varphi_1'(b)$ whenever V is differentiable, and thus $V(b) - \varphi_1(b) \leq 0$ for $b > b_b$.

For $b < b_b$, we have that $V'(b) = -\psi_0/b \geq p = \varphi_1'(b)$ (where the strict inequality follows from the definition of $\bar{b}_\pi$ and $b_b > \bar{b}_\pi$) which implies that $V(b) - \varphi_1(b) \leq 0$ for $b < b_b$.

Hence $V(b) - \varphi_1(b)$ achieves a local maximum at b_b. The definition of a viscosity solution, condition (i), implies that

$$\rho V(b_b) \leq H(b_b, p)$$

Now consider a point to left of b_b, $b_\epsilon = b_b - \epsilon$, for $\epsilon > 0$. Given that the value function is differentiable in this region, we have that the HJB holds in the classical sense:

$$\rho V(b_\epsilon) = H(b_\epsilon, -\psi_0/b_\epsilon)$$

Taking the limits as $\epsilon \to 0$ (exploiting the lower semi-continuity of $r(b)$), we get that

$$\rho V(b_b) = H(b_b, -\psi_0/b_b)$$

Now recall that $\hat{C}(\psi_0/b_\epsilon)$ and $\pi(b) = r(b) - r^\star$ are in the argmax of $H(b_\epsilon, -\psi_0/b_\epsilon)$. Given that $H(b_\epsilon, -\psi_0/b_\epsilon)$ is continuous in b_ϵ for $b_\epsilon \leq b_b$ then it follows that that the policies

$\hat{C}(\psi_0/b_b)$ and $\pi(b_b) = r(b_b) - r^\star$ are in the argmax of $H(b_b, -\psi_0/b_b)$. The envelope condition implies that:

$$\left.\frac{\partial H(b_b, x)}{\partial x}\right|_{x=-\psi_0/b_b} = \hat{C}(\psi_0/b_b) - y + r^\star b_b > 0$$

It follows then that there exists $p \in [-u'(y-r^\star b_b), -\psi_0/b_b)$ such that $H(b_b, p) < H(b_b, -\psi_0/b_b)$, generating a contradiction.

CASE 2: Suppose that there is a b_a such that the equilibrium interest $r^\star < r(b) < r^\star + \bar{\pi}$ for $b \in (b_a, b_{max})$. Same argument as before, implies that $b_a \geq \bar{b}_\pi$. But recall that we can extend the domain (as discussed in the Proof of Proposition 1) to guarantee that the dynamics to the rigth of b_{max} point towards b_{max}, and hence a similar argument to the one in case 1 for the existence of the differentiable function φ_1 applies, which generates the same contradiction of the viscosity conditions.

Taken together, the two cases above imply that any monotone equilibrium cannot have an interior equilibrium inflation rate.

A.3 The proof of Proposition 2

Proof. The proposition characterizes by construction all equilibria with $b_\pi \in [\underline{b}_\pi, \bar{b}_\pi]$. Equilibria for b_π outside this interval can be ruled out using the definition of the intervals. In particular, equilibrium requires that $\Pi(b) = r(b) - r^\star$. Impose this condition on the government's problem and solve for optimal consumption. At b_π, implied inflation is zero and $r(b) = r^\star = \rho$. The government's optimal policy response is to set $C(b_\pi) = y - r^\star b_\pi$, so that $\dot{b} = 0$ and $V(b_\pi) = u(y - r^\star b)/\rho$. We now check whether consumption is consistent with implied inflation using the HJB equation at b_π. Optimal consumption in the neighborhood above b_π is given by $C_\pi(b_\pi)$ from equation (5). If $b_\pi < \underline{b}_\pi$, this consumption is inconsistent with high inflation, violating the equilibrium requirement to the right of b_π. Conversely, if $b_\pi > \bar{b}_\pi$, then zero inflation is inconsistent with the steady state consumption at b_π, violating the equilibrium requirement that $\Pi(b_\pi) = 0$. □

A.4 Proof of Proposition 3

Proof. The proof follows the arguments of the proof of Proposition 1, which relies on Bressan and Hong (2007). □

A.5 Proof of Lemma 2

Proof. The proof of this lemma is the same as the proof of Lemma 1. □

A.6 Proofs of Propositions 4, 5, 6 and 7

Proof. These propositions follow by construction, verifying that the conditions for a Recursive Competitive Equilibrium (i.e., that $r \in \mathcal{R}$, that v satisfies the viscosity conditions; that bond holders earn a real return $r^{\star}$; and that the government does not default without a roll-over crisis, $V(b_0) \geq \underline{V}$). □

B Viscosity Definition for the Equilibrium With Rollover Crises

As discussed in Section 3, here we state the viscosity solution definition for equation (HJB'). Recall that I_b is shorthand notation for the crisis indicator $I(b, r(b))$. The definition below uses the equilibrium knowledge that I_b is discontinuous only at points where $r(b)$ is also discontinuous, so that (HJB') is continuous on sets Ω_n (as defined by $\mathcal{R}$ in Definition 1).

Definition 11. *For a given $\overline{\Omega}$ and $r \in \mathcal{R}(\overline{\Omega})$, a **viscosity solution** to (HJB') is a continuous function $w \in C^0(\overline{\Omega})$ such that for any $\varphi \in C^1(\overline{\Omega})$ we have:*

(i) If $w - \varphi$ achieves a local maximum at b, then

$$(\rho + \lambda I_b)w(b) - \lambda I_b \underline{V} - H(b, \varphi'(b)) \leq 0;$$

(ii) If the restriction of $w - \varphi$ to Ω_n achieves a local minimum at $b \in \Omega_n$, then

$$(\rho + \lambda I_b)w(b) - \lambda I_b \underline{V} - H(b, \varphi'(b)) \geq 0,$$

where Ω_n is defined in Definition 1;

(iii) For $b \in \{0, b_1, b_2, ..., b_{max}\}$,

$$(\rho + \lambda I_b)w(b) - \lambda I_b \underline{V} - \max_{\pi \in \{0, \bar{\pi}\}} \{u(y - (r(b) - \pi)b) - \psi(\pi)\} \geq 0.$$

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