Analysis of tangential curve equation describing mechanical behaviour of rapeseeds (*Brassica napus* L.) mixture under compression loading

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Abstract

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The analysis of tangential curve equation for describing mechanical behaviour of rapeseeds (*Brassica napus* L.) mixture under compression loading were discussed. Parameters considered in this study include both measured and theoretical amounts of deformation, volume compression and deformation energy. The measured amounts denoted as true deformation, true volume compression and true deformation energy were determined directly from the compression test of the rapeseeds pressed mixture whiles the theoretical amounts expressed as max. deformation, max. volume compression and max. deformation energy were determined by mathematical adjustment of the general tangent curve equation which were verified from the measured data. Also the theoretical amounts further explain the limit deformation, limit volume compression and relationship between strains and limit volume compressions which were statistically analysed to describe true situation of pressing. The results indicate that the tangent curve equation by mathematical modification can be used to determine accurately theoretical amounts of maximal deformation, maximal volume compression and maximal deformation energy.

Keywords: force; deformation; oil bearing crops; mathematical model; compression; volume; energy; limit

Not much literature information is available on the pressing theories for describing mechanical behaviour of oil bearing crops such as rapeseeds (*Brassica napus* L.) under compression loading. Most of these theories are based on the Darcy's Law (Fomin 1978; Fasino, Ajibola 1990), for fluid flow through porous media (Singh, Kulshreshtha 1969; Mrema, Mc Nulty 1985) and these studies combine with the rheological properties of the deformable solid matrix of compressed seeds which are fundamental for further research on mathematical model for the description of mechanical behaviour of oil bearing crops (Omobuwajo et al. 1998; Raji, Favier 2004; Očenášek, Voldřich 2009). Also deformation characteristics of pressed mixture can be described by methods based on the Terzagi's model (Shirato et al. 1986; Willems et al. 2008) or energetic balance model (Zheng et al. 2005). To describe the mechanical behaviour of pressed mixture under compression loading using circular shape vessel, the tangent curve method was derived (Herák et al. 2010, 2011a,b; Kabutey et al. 2011; Petrů et. al. 2012) and it was also validated for different types oil bearing crops and other materials such as waste paper and wood chips. Considering the few models available there is a need to generate other models such as the tangential curve equation model to completely describe the mechanical behaviour of oil bearing crops under compression loading. The advantage of using this model is that it is not necessary to resolve individual particles and their properties and relationships between particles because this method uses the mixtures of the seed as a unit that is affected by constrains between the pressing vessel and seed's mixture and the process of the pressing. From the previous article published (Herák et al. 2011a), the general tangent curve equation Eq. (1) was determined and verified; it describes the dependency between compressive force $F(x)$ (N) and deformation x (mm) of rapeseeds mixture under compression loading:

$$F(x) = A \times \tan\left(\frac{G}{H} \times x\right) \quad (1)$$

where:

A – force coefficient of mechanical behaviour (N)

G – compressive coefficient (–)

H – initial height of the pressing (mm)

The relationship between compressive coefficient and deformation coefficient of mechanical behaviour B per mm was also derived, which is given by Eq. (2) (Herák et al. 2011a).

$$G = B \times H \quad (2)$$

From the previous article published by Herák et al. (2011a), the amounts of compressive coefficient $G = 3.347 \pm 0.196$ and force coefficient of mechanical behaviour $A = (2.039 \pm 0.139) \times 10^4$ N were also determined. The objective of this article is to determine true max. deformation, true deformation energy and true volume compression of the pressed mixture and also to compare these measured parameters with the theoretical amounts determined from mathematical modifications of tangent curve equation.

MATERIAL AND METHODS

Compression test. In this experiment, cleaned rapeseeds (*Brassica napus* L.) originated from the Czech Republic were used. Samples for each compressed mixtures were repeated three times and their moisture content determination Mc (%) was done using the moisture equipment Farm Pro (Supertech Agroline, Bogense, Denmark). Compression devices ZDM 50 (VEB, Dresden, Germany), pressing vessel with inner diameter D = 76 mm (Fig. 1) and pressing plunger were used to determine the relationship between the magnitude of the pressing force and deformation function of the seven initial pressing heights H = 80, 70, 60, 50, 40, 30, and 20 mm, respectively, of rapeseeds pressed mixture. The mixtures were pressed under the temperature of 20°C and the pressing rate was 1 mm/s. The experiment was repeated for each pressed mixture three times and averaged values were used in subsequent calculations. The measuring range of force was between 0 and 100 kN, in which the test was stopped. Individual points of measurements were digitally recorded and analysed with each new addition of deformation of 0.5 mm. The initial volume of pressing vessel was determined by Eq. (3):

$$V = \frac{\pi \times D^2}{4} \times H \quad (3)$$

where:

H – initial pressing height (mm)

D – inner diameter of pressing vessel (mm)

The weight of the pressed mixtures m (g) was determined using the equipment Kern 440–35N (Kern & Sohn GmbH, Balingen, Germany) and it is presented in Table 1. The porosity P_f (%) was calculated from bulk and true density using the relationship given by porosity formula Eq. (4) (Blahovec 2008).

$$P_f = \left(1 - \frac{\rho_b}{\rho_t}\right) \times 100 \quad (4)$$

where:

P_f – porosity (%)

ρ_b – bulk density (kg/m^3)

ρ_t – true density (1,080 kg/m^3)

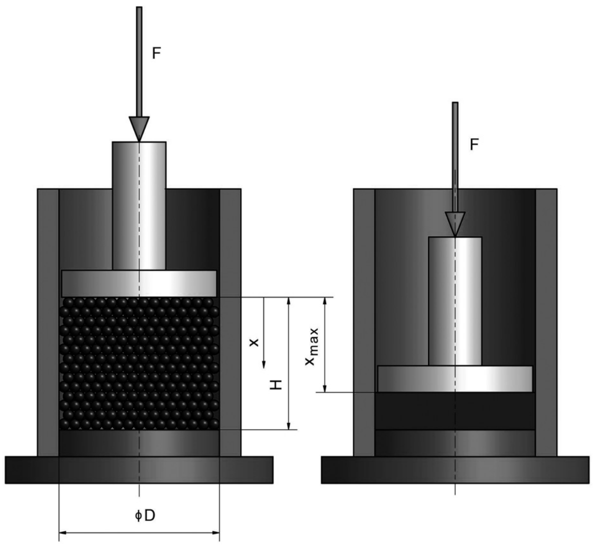

Fig. 1. Scheme of pressing equipment

Table 1. Determined mechanical properties of rapeseeds pressed mixture

H (mm)	m (g)	x_{max} (mm)	x^*_{max} (mm)	δ (mm)	C (–)	C^* (–)	K (–)	E (J)	W (J)
20	48.1 ± 0.9	8.56 ± 0.84	8.18 ± 0.48	9.39 ± 0.55	0.572 ± 0.042	0.591 ± 0.024	0.531 ± 0.028	223.3 ± 25.2	241.5 ± 41.6
30	82.3 ± 0.7	12.09 ± 1.12	12.28 ± 0.72	14.08 ± 0.83	0.564 ± 0.037	0.591 ± 0.024	0.531 ± 0.028	245.2 ± 34.6	276.6 ± 26.8
40	108.8 ± 1.8	16.54 ± 1.75	16.37 ± 0.96	18.77 ± 1.10	0.587± 0.048	0.591 ± 0.024	0.531 ± 0.028	390.1 ± 38.1	410.2 ± 19.2
50	144.2 ± 1.7	20.72 ± 2.13	20.46 ± 1.21	23.47 ± 1.38	0.586 ± 0.043	0.591 ± 0.024	0.531 ± 0.028	502.3 ± 35.3	517.4 ± 20.4
60	166.5 ± 1.2	24.59 ± 2.46	24.55 ± 1.45	28.16 ± 1.66	0.590 ± 0.041	0.591 ± 0.024	0.531 ± 0.028	563.4 ± 42.0	592.6 ± 28.4
70	202.4 ± 1.9	28.90 ± 2.96	28.65 ± 1.69	32.85 ± 1.93	0.587 ± 0.042	0.591 ± 0.024	0.531 ± 0.028	658.6 ± 33.7	713.1 ± 43.6
80	224.6 ± 2.1	33.17 ± 3.02	32.74 ± 1.93	37.55 ± 2.21	0.585 ± 0.038	0.591 ± 0.024	0.531 ± 0.028	780.3 ±61.6	830.3 ± 48.9

data in the table are means ± SD; H – inital pressing height, m – weight of the sample, x_{max} – true max. deformation, x^*_{max} – theoretical max. deformation, δ – limit deformation, C – true volume compression, C^* – theoretical volume compression, K – limit volume compression, E – deformation energy, W – theoretical deformation energy

Bulk density was determined as the weight of the sample m (g) divided by initial volume of pressing vessel Eq. (3) and true density was determined by hydrostatics method (Blahovec 2008).

True determined amounts. The true max. deformation x_{max} (mm) (Table 1) was determined as the deformation appropriate to the max. compressive force being the force in which the experiment was stopped. The true volume compression C (–) was determined by Eq. (5) as the ratio of volume of pressed mixture to initial volume of pressing Eq. (3).

$$C = \frac{H - x_{max}}{H} \tag{5}$$

The deformation energy is the area under the compressive force-deformation curve from the zero deformation to maximum deformation. The deformation energy E (J) was calculated using the software Engauge Digitizer 4.1 (Mark Mitchell, New York, USA) which gives measurement of all points under the curve with respect to compressive force and maximum deformation.

Theoretical determined amounts. The dependency between compressive force and deformation (Eq. 6) can be expressed from equation Eq. (1). Putting max. compressive force in the Eq. (6) the theoretical max. deformation x^*_{max} (mm) was also determined (Table 1).

$$x = \frac{1}{B}\arctan\left(\frac{F(x)}{A}\right) \tag{6}$$

Solving the previous equation Eq. (6) for compressive force approaching to the infinity it is possible to obtain equation Eq. (7) for limit deformation δ (mm) expressed by deformation coefficient of mechanical behaviour.

$$\delta = \lim_{F(x)\to\infty} x = \frac{\pi}{2\times \mathrm{B}} \tag{7}$$

Using Eq. (2) in Eq. (7) equation (8) can be also obtained describing limit deformation expressed by compressive coefficient and initial pressing height.

$$\delta = \frac{\pi \times H}{2\times G} \tag{8}$$

The pressed volume is determined by Eq. (9) and it belongs to the limit deformation.

$$\Delta V = \frac{\pi \times D^2}{4} \times \frac{\pi}{2\times B} \tag{9}$$

Limit volume compression K (–) is given by Eq. (10) and it is ratio between pressed volume Eq. (9) and initial volume of pressing Eq. (3).

$$K = \frac{\Delta V}{V} = \frac{H-\delta}{H} \tag{10}$$

Simplifying previous equation Eq. (10) and Eq. (8) the equation Eq. (11) describing limit volume compression expressed by the compressive coefficient can be derived.

$$K = 1 - \frac{\pi}{2\times G} \tag{11}$$

Putting Eq. (2) in Eq. (11) equation (12) can be derived for the limit volume compression expressed by deformation coefficient of mechanical behaviour and initial pressing height.

$$K = 1 - \frac{\pi}{2\times B\times H} \tag{12}$$

Eq. (12) can be also formulated by Eq. (14) with aid of limit strain ε (–) Eq. (13) which is derived as ratio between limit deformation Eq. (8) and initial height of pressing:

$$\varepsilon = \frac{\delta}{H} = \frac{\pi}{2 \times G} \tag{13}$$

Based on Eq. (11) and Eq. (13), dependency between limit volume compression and limit strain is given by Eq. (14):

$$K = 1 - \varepsilon \tag{14}$$

Also modifying equation Eq. (5) and the amount of the theoretical deformation, the theoretical volume compression C^* (–) can be obtained by equation Eq. (15):

$$C^* = \frac{H - x_{max}}{H} \tag{15}$$

The area under compressive force-deformation curve, which is bound by appropriate deformation of pressed mixture and axis of deformation, presents the theoretical deformation energy W (J). This deformation energy Eq. (16) can be derived by integration of compressive force function Eq. (1), where D (J) which is integration constant depends on the boundary conditions of the pressing:

$$W(x) = \int F(x)dx = \frac{A}{2 \times B} \times \ln\left[1 + \tan(B \times x)^2\right] + D \tag{16}$$

Putting equation (2) in Eq. (16) deformation energy Eq. (17) can be also expressed by compression coefficient and initial pressing height, where integration constant must be equal to zero which implies that from boundary conditions of the pressing, zero deformation equally must be zero deformation energy:

$$W(x) = \frac{A \times H}{2 \times G} \times \ln\left[1 + \tan\left(\frac{G}{H} \times x\right)^2\right] + D \tag{17}$$

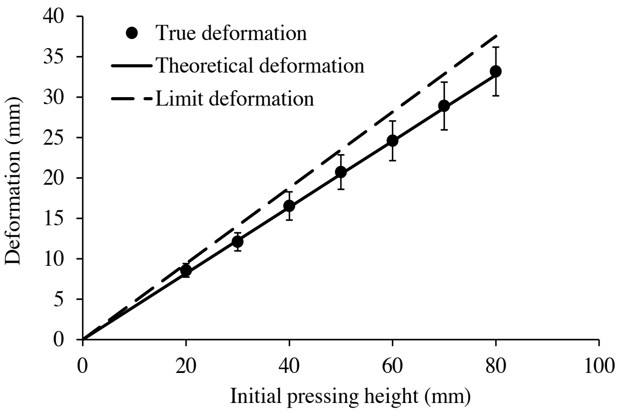

Fig. 2. Dependency between deformation and initial pressing height, the marks belong to the points of the true deformation indicating the standard deviation

RESULTS AND DISCUSSION

From the physical properties, the mean values of moisture content in dry basis M_c = (7.0 ± 0.1) % (d.b.) and porosity P_f = (42.97 ± 0.18) % were statistically significant at the 0.05 significance level (Herák et al. 2011a) which was also used at all statistical analyses conducted in this experiment. In all calculations these physical properties of the rapeseeds pressed mixture remained constant. The amounts determined from the relationship between compressive force and deformation for each initial pressing height of the rapeseeds pressed mixture were fitted by the tangent curve equation Eq. (1) and mathematical calculations involving the derivation and verification of the tangent curve equation Eq. (1) were described in detail in the preceding articles (Herák et al. 2011a). Also, the Marquardt Levenberg process (Marquardt 1963) was used for their approximation. The amounts of the true max. deformation for each initial pressing height obtained from the compressive test are presented in Table 1 and graphically displayed in Fig. 2. Using Eq. (6) for max. compressive force 100 kN theoretical max. deformation was determined and by Eq. (8) the limit deformation was also determined (Table 1). The results show that the dependency between theoretical max. deformation and initial height of pressing can be described by Eq. (18) which is quantified by Eq. (6) and also the dependency between limit deformation and initial pressing height is described by Eq. (18):

$$x^*_{max} = 0.428 \times H \tag{18}$$

Eq. (6) and Eq. (18) were also used for fitting measured amounts of true max. deformation (Fig. 2) and it is clear that the amounts of deformation calculated from these equations are statistically significant to the measured amount of deformation obtained from the compressive test. The statistical analyses using ANOVA (Table 2) show that equations Eq. (6) and Eq. (18) can be used for fitting measured amounts. Also the coefficients of determination R^2 were highly significant, which shows that fitted equations accurately describe deformation for all initial pressing heights of the rapeseeds pressed mixture.

Table 2. Data of the ANOVA statistical analyses for the level of significance 0.05

Compared data	F_{crit} (–)	F_{ratio} (–)	P_{value} (–)	R^2 (–)
C to C^*	4.747	0.197	0.665	–
C to K	4.747	47.228	1.725×10^{-05}	–
E to W	4.747	0.081	0.781	–
Eq. (19) to E	4.747	0.000493	0.982654	0.99
Eq. (18) to x_{max}	4.474	0.023692	0.88023	0.99
x_{max} to x^*_{max}	4.747	0.004	0.948	0.99
x_{max} to δ	4.747	0.426	0.526	0.99

F_{crit} – critical value that compares a pair of models, F_{ratio} – value of the F test, P_{value} – the significance level at which it can be rejected the hypothesis of equality of models, R^2 – coefficient of determination

It is also evident that limit deformation (Fig. 2) presents the border of deformation which is not possible to achieve in the real conditions of the pressing because in this limit deformations the amounts of compressive force and also deformation energy are in infinity for instance in equations Eq. (1) and Eq. (17). Comparison of the statistical analyses (Table 2) for theoretical max. deformation and limit deformation shows the margin of deformation between limit deformation and true max. deformation which confirms the previous claim about unreachable limit deformation. In this study the true max. deformation approaches to the limit deformation which determines the max. compressive force. The true volume compression was calculated by Eq. (5) and it is presented in Table 1 and Fig. 3. The amounts of the theoretical volume compression Eq. (15) and limit volume compression Eq. (11) are quantified and also shown in Table 1 and Fig. 3. From Table 1 it is clear that theoretical volume compression and limit compression are constant, which depends on the initial pressing height. True max. volume compression can be also considered as constant equal to the theoretical max. volume compression $C^* = 0.591 \pm 0.021$ (Fig. 3) which is confirmed by statistical analyses (Table 2). By comparing true max. volume compression and limit volume compression (Fig. 3) it was shown that the values of these two volume compressions are not statistically significant and that the pressed rapeseeds mixture can be more compressed with bigger compressive force greater than energy from equations Eq. (1) and Eq. (17). Because the rigidity of the pressing vessel is higher than the rigidity of pressed mixture of rapeseeds it can be considered that the inner diameter of pressing vessel as well as its other dimensions are non deformable in the range of the compressive forces applied in this study. Thereafter the presumption about relationship between limit strain Eq. (13) and limit volume compression Eq. (11) which is given by Eq. (14) must be also valid. The amounts of true deformation energy and theoretical deformation energy Eq. (17) for each initial pressing height were determined (Table 2) and (Fig. 4). The ANOVA implies that the determined amounts were statistically significant (Table 2). Using the Gauss quadratic law of errors accumulation (Freedman et al. 1998) the standard deviations of theoretical deformation energy were identified and they are shown in Table 1. Substituting theoretical max. deformation Eq. (6) in deformation energy equation Eq. (17) and its quantification, linear function Eq. (19) can be derived which theoretically describes the dependency between deformation energy and initial pressing height.

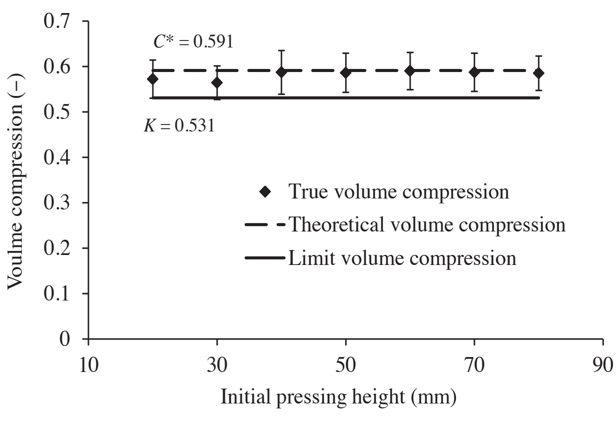

Fig. 3. Dependency between volume compression and initial pressing height, the marks belong to the points of the true volume compression indicating the standard deviation

$$E = U \times H \quad (19)$$

where:

U – numerical coefficient (9.560 J/mm)

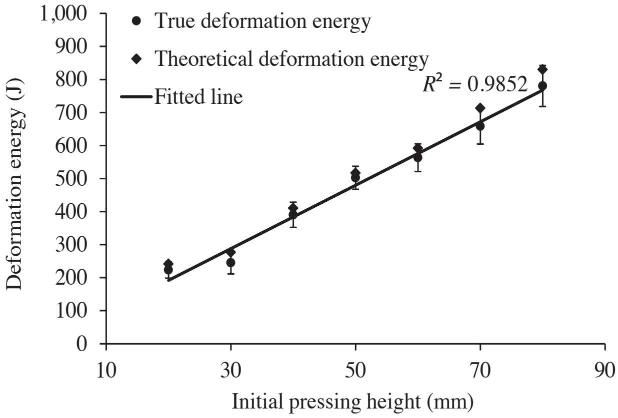

Fig. 4. Dependency between deformation energy and initial pressing height, the marks belong to the points of the true deformation energy indicating the standard deviation

Numerical coefficient was determined by the method of least squares (Freedman et al. 1998) for fitting the measured amounts of true deformation energy (Table. 1, Fig. 4). This function Eq. (19) was also statistically analyzed using ANOVA (Table 2) and it is evident that true deformation energy (Table 1) and calculated deformation energy Eq. (19) are statistically significant. This formulated dependency of equation Eq. (19) confirmed the previous assumption about linear dependency between max. deformation and initial pressing height. Again if the true max. deformation linearly depends on the initial pressing height then the true max. deformation energy equally has the same relationship. The results from this study show that tangent curve equation Eq. (1) can be accurately used for the description of mechanical behaviour of rapeseeds mixture and its course corresponds to the physical situation of the rapeseeds pressing which is in accordance with previous works (Herák et al. 2010, 2011a,b; Kabutey et al. 2011; Petrů et al. 2012). To explain further, it was confirmed that the force coefficient of mechanical behaviour mainly influences the magnitude of the compressive force from Eq. (1) and rigidity of the pressed mixture. From this fact the max. deformation of the pressing is indirectly affected by this coefficient and strongly influenced by deformation coefficient of mechanical behaviour. Also the limit deformation Eq. (7) as well as limit volume compression Eq. (11) and Eq. (12) depend only on deformation coefficient of mechanical behaviour or on the compressive coefficient Eq. (8) from previous calculated deformation coefficient of mechanical behaviour with respect to the initial pressing height Eq. (2). This coefficient strongly affects rigidity of the pressed mixture and also indirectly affects the magnitude of the compressive force. In practical terms it can be supposed that the force coefficient of mechanical behaviour presents mechanical properties of pressed mixture and the deformation coefficient of mechanical behaviour as well as compression coefficient present physical properties of the pressed mixture. Yet, in the real situation the coefficients influence each other, which means that they are given by physical and mechanical properties together. From the analysis described in this study it results that this theory can be used not only for determining mechanical behaviour of rape seeds mixture under compression loading but also for describing mechanical behaviour of different mixtures of plant seeds (Blahovec, Řezníček 1980), biological waste (Wilaipon 2009), wood chips (Vieira, Rocha 2007) and paper chips (Lehtikangas 2001). It can be assumed that the utilization of this method is not limited only for oil bearing crops where the rigidity of the pressing vessel is greater than the rigidity of the pressed mixture.

CONCLUSION

The true deformation, true volume compression and true deformation energy were determined from the compression test of rapeseeds mixture of seven initial pressing heights. These measured amounts were compared with theoretical amounts derived from mathematical adjustment of the general tangent curve equation Eq. (1). Both results obtained were statistically significant at the significance level 0.05 and theoretical amounts can be accurately used for determining max. deformation, max. volume compression and max. deformation energy. Limit deformation, limit volume compression and relationship between strain and limit volume compression were also discussed in this study and from statistical analysis their values accurately describe true situation during the pressing.

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