

1996

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John Donoghue

*University of Massachusetts Amherst*, [donoghue@physics.umass.edu](mailto:donoghue@physics.umass.edu)

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Donoghue, John, "Final state interactions in the decay of heavy quarks" (1996). *Physics Department Faculty Publication Series*. Paper 244.

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# FINAL STATE INTERACTIONS IN THE DECAY OF HEAVY QUARKS<sup>1</sup>

JOHN F. DONOGHUE

*Department of Physics and Astronomy,  
University of Massachusetts, Amherst MA 01003 USA*

## Abstract

I discuss some recent results on the systematic behavior of final state rescattering which makes use of the limit of a heavy B meson. The results suggest that soft final state interactions do not disappear in the large  $m_B$  limit. Soft and hard final state phases can both contribute to CP violating asymmetries in B decay. The way that the soft phases occur is interesting theoretically and suggests the violation of local quark-hadron duality.

Some of the CP violating asymmetries that can occur in the Standard Model require the presence of strong final state interaction phases. Very little is known about final state interactions at the high energies that are relevant for B decay. What I will describe in this talk does not “solve” the problem of these interactions. However, my collaborators (Eugene Golowich, Alexey Petrov and João Soares) and I have obtained some insights that at least taught me something about this frustrating topic<sup>1</sup>. I would like to share these with you. This has taken us on a path into unfashionable but surprisingly interesting physics and may shed new light on weak decays. As is now common, we use the mass of the B meson as an organizing parameter and consider the limit where the mass is very large. We can show that soft final state interactions survive in this limit and can isolate their leading cause, although in the end we cannot provide a specific number describing the magnitude of the phase.

Our main conclusions rest on a few simple facts. These are:

1) The Unitarity Relation. Final state interactions in B decay involve the rescattering of physical final state particles. Unitarity of the  $\mathcal{S}$ -matrix,  $\mathcal{S}^\dagger \mathcal{S} = 1$ , implies that the  $\mathcal{T}$ -matrix,  $\mathcal{S} = 1 + i\mathcal{T}$ , obeys

$$\text{Disc } \mathcal{T}_{B \rightarrow f} \equiv \frac{1}{2i} [\langle f | \mathcal{T} | B \rangle - \langle f | \mathcal{T}^\dagger | B \rangle] = \frac{1}{2} \sum_I \langle f | \mathcal{T}^\dagger | I \rangle \langle I | \mathcal{T} | B \rangle . \quad (1)$$

Of interest are all physical intermediate states which can scatter into the final state  $f$ .

2) The optical theorem. The optical theorem relates the forward invariant amplitude  $\mathcal{M}$  to the total cross section,

$$\text{Im } \mathcal{M}_{f \rightarrow f}(s, t = 0) = 2k\sqrt{s}\sigma_{f \rightarrow \text{all}} \sim s\sigma_{f \rightarrow \text{all}} , \quad (2)$$

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<sup>1</sup> Talk presented at the 3rd German-Russian Workshop on Heavy Quark Physics, Dubna, May 1996

where  $s$  is the squared center-of-mass energy and  $t$  is the squared momentum transfer.

3) The exponential fall-off in momentum transfer. Soft hadronic interactions are categorized by a limited momentum transfer, and all of the high energy hadronic reactions have an exponential damping of the form

$$\mathcal{M}(s, t) \simeq f(s)e^{bt} . \quad (3)$$

Recall that  $t$  is negative. This damping limits the momentum transfer to be of order  $0.5 \text{ GeV}$ .

4) The measured cross sections. The asymptotic total cross sections are known experimentally to rise slowly with energy. All known cross sections can be parameterized by fits of the form<sup>2</sup>

$$\sigma(s) = X \left( \frac{s}{s_0} \right)^{0.08} + Y \left( \frac{s}{s_0} \right)^{-0.56} , \quad (4)$$

where  $s_0 = \mathcal{O}(1) \text{ GeV}$  is a typical hadronic scale. Combined with the optical theorem, this implies that the imaginary part of the forward elastic scattering amplitude rises asymptotically as  $s^{1.08}$ .

This growth with  $s$  is an important ingredient in our results. (Note that the emphasis is on the factor of  $s^1$ ; the extra factor of  $s^{0.08}$  that occurs repeatedly below is not particularly important.) It is a surprising feature in that it cannot be generated by a perturbative mechanism at any finite order. In particular, standard calculations based on the quark model or perturbative  $QCD$  would completely miss this feature.

These indisputable facts can be combined to show that final state rescattering does not disappear in the limit of large  $m_B$ . In order to arrive most simply at this result, let us consider first only the imaginary part of the amplitude. Building in the features described above one has

$$i\mathcal{I}m \mathcal{M}_{f \rightarrow f}(s, t) \simeq i\beta_0 \left( \frac{s}{s_0} \right)^{1.08} e^{bt} . \quad (5)$$

It is then an easy task to calculate the contribution of the imaginary part of the elastic amplitude to the unitarity relation for a final state  $f = a + b$  with kinematics  $p'_a + p'_b = p_a + p_b$  and  $s = (p_a + p_b)^2$ . We find

$$\begin{aligned} \text{Disc } \mathcal{M}_{B \rightarrow f} &= \frac{1}{2} \int \frac{d^3 p'_a}{(2\pi)^3 2E'_a} \frac{d^3 p'_b}{(2\pi)^3 2E'_b} (2\pi)^4 \delta^{(4)}(p_B - p'_a - p'_b) \\ &\cdot -i\beta_0 \left( \frac{s}{s_0} \right)^{1.08} e^{b(p_a - p'_a)^2} \mathcal{M}_{B \rightarrow f} \\ &= -\frac{i}{32\pi} \int d(\cos\theta) e^{-\frac{bs}{2}(1-\cos\theta)} \beta_0 \left( \frac{s}{s_0} \right)^{1.08} \mathcal{M}_{B \rightarrow f} \\ &= -\frac{1}{16\pi} \frac{i\beta_0}{s_0 b} \left( \frac{m_B^2}{s_0} \right)^{0.08} \mathcal{M}_{B \rightarrow f} , \end{aligned} \quad (6)$$

where  $t = (p_a - p'_a)^2 \simeq -s(1 - \cos\theta)/2$  and we have taken  $s = m_B^2$ . There are two competing effects that are important in this result. The first is a kinematic suppression of soft final state interactions because of the limited angular region corresponding to the soft region. The integration over the angle involving the direction of the intermediate state is seen to introduce a suppression factor to the final state interaction of  $s^{-1} = m_B^{-2}$ . This is because the soft final state rescattering can take place only if the intermediate

state has a transverse momentum  $p_{\perp} \leq 1$  GeV with respect to the final particle direction. This would naively suggest a result consistent with conventional expectations, *i.e.* an FSI which falls as  $m_B^{-2}$ . However, the second feature is the fact, mentioned above, that the forward scattering amplitude *grows* with a power of  $s$  which overcomes this suppression and leads to elastic rescattering which does not disappear at large  $m_B$ .

In fact, we can make a more detailed estimate of elastic rescattering because the phenomenology of high energy scattering is well accounted for by Regge theory<sup>3</sup>. Scattering amplitudes are described by the exchanges of Regge trajectories (families of particles of differing spin) which lead to elastic amplitudes of the form

$$\mathcal{M}_{f \rightarrow f} = \xi \beta(t) \left( \frac{s}{s_0} \right)^{\alpha(t)} e^{i\pi\alpha(t)/2} \quad (8)$$

with  $\xi = 1$  for charge conjugation  $C = +1$  and  $\xi = i$  for  $C = -1$ . Each such trajectory is described by a straight line,

$$\alpha(t) = \alpha_0 + \alpha' t \quad . \quad (9)$$

The leading trajectory for high energy scattering is the Pomeron, having  $C = +1$ ,  $\alpha_0 \simeq 1.08$  and  $\alpha' \simeq 0.25$  GeV<sup>-2</sup>. Using known features of Pomeron physics and taking  $s = m_B^2 \simeq 25$  GeV<sup>2</sup>, we obtain for the Pomeron contribution

$$\mathcal{D}isc \mathcal{M}_{B \rightarrow \pi\pi}|_{\text{Pomeron}} = -i\epsilon \mathcal{M}_{B \rightarrow \pi\pi} \quad , \quad (10)$$

where we find from our computation,

$$\epsilon \simeq 0.21 \quad . \quad (11)$$

The simplest conclusion from this calculation is that final state interactions survive in the large  $m_B$  limit and are reasonably large.

This calculation also tells us more: it requires that the inelastic channels be at least equally important, and that they are the key to the origin of the final state phases. This is due to the fact that the elastic effect calculated above is purely imaginary. In the limit of T-invariance, the discontinuity  $\mathcal{D}isc\mathcal{M}$  is a real number up to an irrelevant rephasing invariance of the B-state. The factor of  $i$  in the elastic amplitude must be removed by the effects of the inelastic rescattering channels. This implies that inelastic rescattering cannot be vanishingly small and must share the same power behavior in  $m_B$  as the elastic amplitude. At a physical level this is not at all surprising since a two body initial state scatters primarily inelastically at high energy. In fact, the elastic calculation implies even more, in that the inelastic channels can be considered systematically *larger* than the elastic channel. We have a  $\mathcal{T}$ -matrix element  $\mathcal{T}_{ab \rightarrow ab} = 2i\epsilon$ , which directly gives  $\mathcal{S}_{ab \rightarrow ab} = 1 - 2\epsilon$ . However, the constraint of the  $\mathcal{S}$ -matrix be unitary can be shown to imply that the off-diagonal elements must be  $\mathcal{O}(\sqrt{\epsilon})$ . Since  $\epsilon$  is approximately  $\mathcal{O}(m_B^0)$  in powers of  $m_B$  and numerically  $\epsilon < 1$ , the inelastic amplitude must also be  $\mathcal{O}(m_B^0)$  and of magnitude  $\sqrt{\epsilon} > \epsilon$ . Therefore, the presence of inelastic effects is seen to be necessary.

It is possible to illustrate the systematics of inelastic scattering by means of a simple two-channel model. This pedagogic example involves a two-body final state  $f_1$  undergoing elastic scattering and a final state  $f_2$  which is meant to represent ‘everything else’. We assume that the elastic amplitude is purely imaginary. Thus, the scattering can be

described in the one-parameter form

$$S = \begin{pmatrix} 1 - 2\epsilon & 2i\sqrt{\epsilon} \\ 2i\sqrt{\epsilon} & 1 - 2\epsilon \end{pmatrix}, \quad T = \begin{pmatrix} 2i\epsilon & 2\sqrt{\epsilon} \\ 2\sqrt{\epsilon} & 2i\epsilon \end{pmatrix}, \quad (12)$$

(These are approximate forms valid to order  $\epsilon$ . It is not hard to use exactly unitary forms, but I find it instructive to explicitly display the powers of  $\epsilon$ .) The unitarity relations become

$$\begin{aligned} \text{Disc } \mathcal{M}_{B \rightarrow f_1} &= -i\epsilon \mathcal{M}_{B \rightarrow f_1} + \sqrt{\epsilon} \mathcal{M}_{B \rightarrow f_2}, \\ \text{Disc } \mathcal{M}_{B \rightarrow f_2} &= \sqrt{\epsilon} \mathcal{M}_{B \rightarrow f_1} - i\epsilon \mathcal{M}_{B \rightarrow f_2} \end{aligned} \quad (13)$$

If, in the limit  $\epsilon \rightarrow 0$ , the decay amplitudes become the real numbers  $\mathcal{M}_1^0$  and  $\mathcal{M}_2^0$ , these equations are solved by

$$\mathcal{M}_{B \rightarrow f_1} = \mathcal{M}_1^0 + i\sqrt{\epsilon} \mathcal{M}_2^0, \quad \mathcal{M}_{B \rightarrow f_2} = \mathcal{M}_2^0 + i\sqrt{\epsilon} \mathcal{M}_1^0. \quad (14)$$

As a check, we can insert these solutions back into Eq. (13). Upon doing so and bracketing contributions from  $\mathcal{M}_{B \rightarrow f_1}$  and  $\mathcal{M}_{B \rightarrow f_2}$  separately, we find

$$\text{Disc } \mathcal{M}_{B \rightarrow f_1} = \frac{1}{2} \left[ \left( -2i\epsilon \mathcal{M}_{B \rightarrow f_1}^0 + \mathcal{O}(\epsilon^{3/2}) \right) + \left( 2\sqrt{\epsilon} \mathcal{M}_{B \rightarrow f_2}^0 + 2i\epsilon \mathcal{M}_{B \rightarrow f_1}^0 \right) \right]. \quad (15)$$

The first of the four terms comes from the elastic channel  $f_1$  and is seen to be canceled by the final term, which arises from the inelastic channel  $f_2$ . The third term is dominant, being  $\mathcal{O}(\sqrt{\epsilon})$ , and comes from the inelastic channel.

In this example, we have seen that the phase is given by the inelastic scattering with a result of order

$$\frac{\text{Im } \mathcal{M}_{B \rightarrow f}}{\text{Re } \mathcal{M}_{B \rightarrow f}} \sim \sqrt{\epsilon} \frac{\mathcal{M}_2^0}{\mathcal{M}_1^0}. \quad (16)$$

Clearly, for physical  $B$  decay, we no longer have a simple one-parameter  $\mathcal{S}$  matrix. However, the main feature of the above result is expected to remain — that inelastic channels *cannot* vanish because they are required to make the discontinuity real and that the phase is systematically of order  $\sqrt{\epsilon}$  from these channels. Of course, with many channels, cancellations or enhancements are possible for the sum of many contributions. However the generic expectation remains — that inelastic soft final-state-rescattering arising from Pomeron exchange will generate a phase which does not vanish in the large  $m_B$  limit.

What about nonleading effects? It is not hard to see that these may be significant at the physical values of  $m_B$ . For example, the fit to the  $\bar{p}p$  total cross section is

$$\sigma(p\bar{p}) = \left[ 22.7 \left( \frac{s}{s_0} \right)^{0.08} + 140 \left( \frac{s}{s_0} \right)^{-0.56} \right] \text{ (mb)} \quad (17)$$

with  $s_0 = 1 \text{ GeV}^2$ . At  $s = (5.2 \text{ GeV})^2$ , the nonleading coefficient is a factor of six larger than the leading effect, effectively compensating for the  $s^{-0.56} = m_B^{-1.12}$  suppression. The subleading terms are then comparable in the elastic forward  $\bar{p}p$  scattering amplitude. There are several next-to-leading trajectories, both those with  $C = -1$  ( $\rho(770)$  &  $\omega(782)$  trajectories) and those with  $C = +1$  ( $a_2(1320)$  &  $f_2(1270)$  trajectories). Roughly, these have  $\alpha_0 \simeq 0.44$ ,  $\alpha' \simeq 0.94 \text{ GeV}^{-2}$  and lead collectively to the  $s^{-0.56}$  dependence in the asymptotic cross section of Eq. (4). If we estimate the  $\beta$  coefficient of the  $\rho$  trajectory in

$\pi\pi$  by relating it to  $\bar{p}p$  via a factor of  $\beta_{\pi\pi} \simeq 4\beta_{\bar{p}p}$  and then perform the integration over the intermediate state momentum we find

$$Disc \mathcal{M}_{B \rightarrow \pi\pi} \Big|_{\rho\text{-traj}} = i\epsilon_\rho \mathcal{M}_{B \rightarrow \pi\pi} \quad , \quad (18)$$

with  $\epsilon_\rho \simeq 0.11 - 0.05 i$ . It is likely that the  $f_2(1270)$  trajectory could be somewhat larger, as it is in  $\bar{p}p$  and  $\pi p$  scattering.

In addition to the soft physics described above, one may expect that hard physics also may generate final state interaction phases. Hard physics is characterized by larger momentum transfer and is best described by the exchanges of quarks and gluons. The final state interactions then correspond to rescattering of intermediate states which are modeled by on-shell quarks and gluons. These arise as imaginary parts in the Feynman diagrams relevant for the decays. It might be thought that such phases are always of order  $\alpha_s$ , but this need not be always true. For example, the best known counterexample occurs in the penguin diagram. Here the physical intermediate state is the  $q\bar{q}$  (with  $q = u, c$ ) in the loop of the penguin diagram which can go onshell, yielding an imaginary part to the diagram. The hard rescattering is the transition of this intermediate state into the final  $q\bar{q}$  pair (eq.  $c\bar{c} \rightarrow s\bar{s}$ ) through a gluon. Both the real part of the penguin diagram and the imaginary part are of order  $\alpha_s$  and therefore the phase occurs at the zeroth order in  $\alpha_s$ . A similar situation can occur even in the W-exchange class of operators, for the operator that is often called the color-octet operator. For this case, the quarks that are to emerge in a particular final hadron (eg. a  $c\bar{d}$  for a  $D^+$ ) occur in the operator in a color octet combination. Therefore in a factorization scheme, the matrix element would vanish since the quarks in the hadron are in a color singlet. However, with the exchange of a gluon the matrix element can be non-zero. The same gluon intermediate state can generate an imaginary part to the amplitude, so that again both the real and imaginary parts of the diagram can end up being of the same order in  $\alpha_s$ . (This case is not as clear as that of the penguin diagram because it assumes the the real part of these operator matrix elements is generated in a perturbative fashion, which has not been carefully explored.) The reverse situation might occur for what is called the color singlet operator, where the quarks are in a singlet state allowed by the factorization hypothesis. Here the intermediate states with an imaginary part from single gluon exchange in the final state are forbidden by the color structure, and the lowest gluonic rescattering occurs with two gluon exchange, leading to a hard phase naively of order  $\alpha_s^2$ .

Both the soft and hard phases can contribute to some of the CP violating asymmetries. As an example consider the decays  $B \rightarrow K\pi$ . Both W-exchange diagrams and penguin diagrams can contribute to the amplitude, and these diagrams have different weak phases<sup>4,5</sup>. What is then required for an observable CP violating asymmetry is for these two sets of diagrams to also have different final state interaction phases. The asymmetry is generated by an interference of the two types of phases, with

$$\begin{aligned} \mathcal{M}(B \rightarrow K\pi) &= A_w e^{i\phi_w} e^{i\delta_w} + A_p e^{i\phi_p} e^{i\delta_p} \\ \mathcal{M}(\bar{B} \rightarrow \bar{K}\pi) &= A_w e^{-i\phi_w} e^{i\delta_w} + A_p e^{-i\phi_p} e^{i\delta_p} \end{aligned} \quad (19)$$

leading to

$$\Delta\Gamma \sim A_w A_p \sin(\phi_w - \phi_p) \sin(\delta_w - \delta_p) \quad . \quad (20)$$

The hard interactions in the penguin diagram can generate the required FSI phase, as described above. What is perhaps not as obvious is that soft interactions can also generate this phase difference. This occurs because the W-exchange and penguin diagrams will in general populate the elastic and inelastic channels in different ratios. Even if the soft rescattering is the same, this would lead to different FSI phases for the two classes of diagrams (see Eq. (16)). Both the hard and soft contributions are of the same order in the parameters ( $m_B$  and  $\alpha_s$ ) which we are using to characterize the transitions. This means that one cannot simply calculate the final state phase difference by a perturbative calculation of the penguin diagram.

The above situation is also of interest theoretically, as it is an example of a violation of the loose notion of local quark hadron duality, which would have implied that a calculation of the process at the quark-gluon level would have given the hadron level answer even for exclusive quantities when suitably averaged. The soft final state phases are quantities that would not arise in conventional quark level calculations, and hence are always outside of the realm of local quark hadron duality. This has occurred because of the growth of the forward amplitude with  $s$ . The soft interactions will limit the accuracy of models which are based on quark level ideas ignoring final state interactions. It is also an intriguing possibility that the assumption of local quark hadron duality can be questioned in other aspects of weak decays also. This is a topic which deserves more study. The final state interactions in general are a subject about which little is understood. The scaling properties described by our study give at least a little insight into the physics of these interactions.

## References

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