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NONPERTURBATIVE METHODS IN KAON PHYSICS (ASIDE FROM THE LATTICE)

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I discuss the progress in the use of analytic techniques for low energy QCD, in particular as applied to kaon physics. These methods are becoming increasingly powerful and we have gained a good deal of control over the difficult hadronic interactions. There are continuing developments, and I speculate on the ways that these techniques may become yet more sophisticated in the future.

The title of this talk (suggested by the organizers) and the scheduling of it immediately after the fine lattice review by Greg Kilcup[1] suggest a competition between analytic and computer methodologies, man vs. machine. This dichotomy has always reminded me of the American folk tale of John Henry. The story is from a time before the LEP tunneling machines, when tunnels were dug by men swinging 20 pound hammers. John Henry was the best of these until one day someone arrived with a machine to do the job, and challenged John Henry to a contest. John Henry replied that a man is only a man, but that he would accept the challenge. The event unfolds with the machine pulling ahead, whereupon John Henry grabs a second hammer and, swinging one in each hand, begins to catch up. The machine eventually breaks down, John Henry keeps going and he wins at the end of the day. Unfortunately, the effort was too much even for the big heart of John Henry, and he collapses and dies on the spot. At this stage, all versions have the refrain that "he died with a hammer in his hand". As a child, I used to think that the dying spoiled the story. However as adults, knowing what we do about men and machines, it is clear that dying is the only possible ending. The nobility was not the victory but the effort - that he died with a hammer in his hand. For perhaps obvious reasons, this story resonates with those of us who work with analytic techniques. In any case, it is my task to take the side of "man", and I am pleased to report that it is still a contest.

Our goal is to tame the low energy strong interactions in order to be able to make predictions for the weak decays of kaons, with the hope of extracting information about CP violation or rare weak interaction processes. However, at low energies one cannot use QCD perturbation theory so that these processes have always posed challenges. We do have at our disposal a set of rigorous techniques, and these have allowed us to make some real progress in the treatment of low energy physics. The analytic methods that are presently employed have an extended history going back to the sixties. Although in their modern incarnations they are more powerful than previously, I will review briefly their roots, using a few papers from 1967 as examples[2]. Then I turn to our modern usage, and to our hope for the future. I would like to convince you that, when looked at with the right tools, this physics is not too complicated, and that we have hopes to produce more solid calculations in the future.

1   Roots

a) Chiral Symmetry:

The first of our "root" examples is a seminal paper by C. Bouchiat and Ph. Meyer[3] relating the reactions $K \to 3\pi$ to $K \to 2\pi$. They predict that the $\Delta I = 1/2$ amplitude for $K \to \pi^+\pi^-\pi^0$ is

$$A(K_L^0 \to \pi^+\pi^-\pi^0) = \frac{A_0}{6F_\pi} \left[1 + 3\frac{(s_3 - s_0)}{m_K^2}\right] , \tag{1}$$

where $s_3 = (k - p_0)^2$, $s_0 = m_K^2 + m_\pi^2/3$ and A_0 is the amplitude for $K_S \to 2\pi$. The input to this result is chiral symmetry, more specifically chiral SU(2) symmetry which is an exact symmetry in the limit of vanishing up and down quark masses[4]. This may seem to be an unusual type of prediction for a *symmetry*, in that it gives dynamical information about the kinematic structure of the amplitude. In understanding why this occurs, one gets to the heart of chiral techniques. Symmetries relate different states, with the most familiar situation being the relation between single particle states. For example, isospin symmetry relates the properties of neutrons and protons, which live in the same isospin multiplet. However for the axial vector rotations inherent in chiral symmetries, there are no multiplets of particles that are related by a symmetry transformation. Instead, if you transform a proton state you obtain a state consisting of a proton plus a zero-energy pion. Therefore the typical predictions of axial rotations are those

relating a process to one with an extra pion. If one provides a Taylor expansion of the amplitude in terms of the various particle momenta, the symmetry fixes the zero-momentum limit. The specific technique used in this context are the so-called soft-pion theorems, where "soft" refers to the zero-momentum limit. In the case of $K \to 3\pi$ there are three separate zero momentum limits. The Bouchiat Meyer calculation allows an expansion to two powers of the external momenta, in which case the three limits are sufficient to completely determine the amplitude.

This allows us also to understand the nature of the corrections to the calculation. A simple class of corrections would occur because the chiral SU(2) symmetry is broken by the u,d quark masses, which would lead to corrections to the prediction by higher powers of m_π^2. However this is not the dominant effect. There are also corrections which come from the fact that, in the Taylor expansion of the amplitude, we stopped at a quadratic momentum dependence. There are quartic terms, such as $k \cdot p_0 p_+ \cdot p_-$, which vanish in all zero-momentum limits and hence have a coefficient which is not fixed by the symmetry alone. The kinematic factor cited in the previous sentence is of order m_K^4 compared to terms of order m_K^2 from the quadratic terms so that the correction is of relative order m_K^2/Λ^2, where Λ is some scale. In practice, Λ is of order 1 GeV.

This pattern is typical of chiral relations. One gets real dynamical information about processes with differing numbers of pions. The lowest order momentum dependence is often fixed by the symmetry in terms of another reaction. Then there occur unknown coefficients corresponding to higher order momentum dependence. Sometimes one can in turn relate these coefficients to ones measured in other processes. In the context of this talk it is useful to point out that these results are fully nonperturbative, and constitute low energy theorems of QCD.

2) Effective Lagrangians

A 1967 paper by Jeremiah Cronin[5] provides a good example of an effective way to organize the predictions of chiral symmetry. By writing a Lagrangian involving the pion fields which has the chiral symmetry, one can easily read off the symmetry predictions in a way much simpler than using the soft pion theorem. This can work because if the predictions are to be consequences of the symmetry alone, then all Lagrangians with the same symmetries will share the same predictions. Since the chiral predictions involve differing numbers of pions, and this process can be continued until any number of pions are present, the effective Lagrangian for chiral chiral symmetry must be nonlinear, involving all numbers of pion fields. Cronin's paper contains the Lagrangian for kaon decays which yields the result quoted above

$$L_W = g_8 Tr \left(\lambda_6 D_\mu U D^\mu U^\dagger \right) \tag{2}$$

where

$$U = exp \left(i \frac{\lambda^A \phi^A}{F_\pi} \right). \tag{3}$$

[Technically, the soft pion theorems use chiral SU(2) while this Lagrangian is defined in chiral SU(3). In this case, the difference is not particularly important.] This Lagrangian is determined by the symmetry of the weak interactions, in which only left-handed fields participate in the charged current processes, plus the fact that we know that the octet nonleptonic interaction is much larger than the 27-plet. [A similar Lagrangian can be written for the 27-plet.]

One interesting extension of the use of effective lagrangians is to include the consequences of the corrections to the lowest order chiral relations, such as I described above as coming from quartic terms in the Taylor expansion of the decay amplitude. There exist effective Lagrangians which have the same symmetry as the one in Eq.3, but which have more derivatives. An example is

$$L_{h.o.} = g'Tr\left(\lambda_6 D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger\right) \tag{4}$$

Because it has four derivatives on the meson fields, it will lead to corrections which have four powers of the momenta, such as were described above. The coefficient of such a Lagrangian would not be known ahead of time, but similar four derivative Lagrangians could be useful in categorizing the deviations form the lowest order predictions. Chiral symmetry constrains not only the zero-momentum limit, but also the corrections to that limit. This latter role is much easier to study systematically using effective Lagrangians.

3) Dispersion relations

Another topic which was active both in 1967 and at present is the use of dispersion relations as a calculational tool. Examples of this are the Weinberg sum rules[6] and the calculation of the pion electromagnetic mass difference by Das et al[7]. In 1967 these involved rather bold assumptions about the short distance/high energy behavior of the theory, but we now know that these assumptions are satisfied in QCD with massless quarks. These calculations involve the vector and axial vector spectral functions ρ_V and ρ_A, which are measurable in e^+e^- annihilation and in τ decays, and whose high energy behavior is known in QCD. Specifically, we have the Weinberg sum rules

$$F_\pi^2 = \int_0^\infty ds(\rho_V(s) - \rho_A(s)) \tag{5}$$

$$0 = \int_0^\infty dss(\rho_V(s) - \rho_A(s)) \tag{6}$$

as well as a sum rule for one of the chiral parameters

$$-4\bar{L}_{10} = \int_{4m_\pi^2}^\infty \frac{ds}{s}(\rho_V(s) - \rho_A(s)) \tag{7}$$

with

$$\begin{aligned}\bar{L}_{10} &= L_{10}^r(\mu) + \frac{1}{192\pi^2}\left[ln\frac{m_\pi^2}{\mu^2} + 1\right] \\ &= (-0.7 \pm 0.03) \times 10^{-2} \; (Expt : \pi \to e\nu\gamma)\end{aligned} \tag{8}$$

and a sum rule for the pion electromagnetic mass difference

$$m_{\pi^+}^2 - m_{\pi^0}^2 = -\frac{3e^2}{16\pi^2 F_\pi^2}\int_0^\infty dsslns\left[\rho_V(s) - \rho_A(s)\right]. \tag{9}$$

These are theorems of QCD in the chiral limit. The first and third of these remain true even in the presence of quark masses, but the other two are no longer convergent. The spectral functions are pretty well known[8] and are consistent with these sum rules.

Dispersion relations derive their validity from the general analytic properties of amplitudes in field theory. They relate the real and imaginary parts of various amplitudes, with the general structure

$$f(s) = \frac{1}{\pi}\int_0^\infty ds'\frac{Imf(s')}{s' - s - i\epsilon} \tag{10}$$

or possibly with subtractions to make them convergent. The imaginary parts correspond to real on-shell intermediate states, and can often be measured directly by experiment. These can then be used to predict the full amplitude at all values of the kinematic variables.

The importance of this technique lies in the fact that dispersion relations are the only rigorous analytic methods that is able to handle the intermediate energy regions in QCD. However, of course, the output of a dispersion relation is only as good as the input, and only sometimes we do know the relevant imaginary part of the amplitude.

2   Effective field theory

The techniques described above form the basis for much of the work on low energy QCD which is required for the study of kaon interactions. However, there is one very important extra ingredient which was not present in 1967 but which is a key to modern applications. This technique is effective field theory[9].

Most of us learned field theory in the context of renormalizable field theories, and these are certainly attractive for the fundamental interactions. However at low energies one is often faced with an effective theory which need not be renormalizable. Do we have to solve the full high energy theory in order to know about quantum predictions at low energy? Intuitively we know that the most important physics at low energies is that involving whatever light degrees of freedom are present in the low energy theory. This is why condensed matter physics or atomic physics can proceed without knowing about quarks and gluons. Effective field theory is a formalism which has been developed to handle such a situation. It allows one to calculate the quantum effects of the light degrees of freedom and encodes one's ignorance of the ultimate high energy theory into a set of parameters in an effective Lagrangian. It then brings the full field theoretic apparatus to bear on non-renormalizable theories when treated at low enough energies.

The importance of this for low energy QCD is that it converts the constraints of chiral symmetry into a fully dynamical field theory, called chiral perturbation theory[4,10], which one can justifiably claim is rigorously equivalent to QCD when applied at low energy. Chiral symmetry dictates the low energy couplings of the light pions and kaons, and the propagation and rescattering of these are calculated in effective field theory. The usual infinities of perturbation theory are absorbed into the unknown coefficients of the effective Lagrangian, and the renormalized values of these must be determined from experiment. This gives a systematic method for calculating not only the lowest order amplitude but higher order corrections in the energy expansion as well as quantum corrections. Chiral perturbation theory is now a well developed technology, and serves as a model for a complete effective field theory in much the way that QED serves as a model for a complete renormalizable field theory.

3   Where are we now?

The development of chiral perturbation theory has radically transformed our treatment of low energy QCD. It has given us a calculational tool which has rigor and which is useful for phenomenology. Although there are some important limitations to the utility of the method, it has justifiably become the standard basis for calculations of low energy reactions.

The present frontier in mesonic chiral perturbation theory involves two-loop calculations. These now exist in the literature for several quantities[11,12,13]. A two-loop calculation is equivalent to an order E^6 in the energy expansion, and brings in new chiral parameters from the Lagrangian at that order. The special physics that is revealed in these calculations appears to me to be the iteration of final state rescattering. In several reactions, especially involving the $I = 0$, $J = 0$ two pion state, the rescattering is quite strong at one-loop and thus it is relevant to calculate the two-loop effect. My favorite example is the reaction $\gamma\gamma \to \pi^0\pi^0$ studied by

Bellucci, Gasser and Sanio[11]. There is no tree level contribution at orders E^2 and E^4, although there is a finite one loop effect which reflects a difference in the $\pi^+\pi^- \to \pi^0\pi^0$ rescattering in the $I = 0$ and $I = 2$ channels. The one loop result is a bit low even near threshold, but the two-loop result nicely corrects this flaw. The important ingredient near threshold does appear to be the more correct treatment of the large $I = 0$ rescattering, as the threshold region is quite insensitive to any new chiral parameters. In general, however, most two-loop calculations will always have some model dependence, at least at higher energies, in that the new chiral parameters will not be able to determined experimentally for use in making predictions (there are too many of them), so that they will need to be estimated using less rigorous models.

Another development in recent years is the increased use of dispersive techniques in connection with chiral calculations. I have reviewed this in greater depth elsewhere[14], so that I will mainly state the main points here. Any chiral loop calculation can be reformulated as a dispersion relation, since the Feynman diagrams have the same analyticity structure assumed for the dispersion relation. Use of the lowest order chiral prediction for the imaginary parts of diagrams reproduces the usual one-loop result. Then any improvement in the imaginary part, especially through the use of data, will lead to improved predictions. The matching of the low and moderate energy regions is known, with chiral symmetry providing information on the subtraction constants and dispersion relations providing the extrapolation to higher energy. Again $\gamma\gamma \to \pi^0\pi^0$ can provide a nice example[15]. By matching to the chiral calculation we remove all free parameters in the dispersive treatment, and the dispersion corrections provide the iteration of the rescattering diagrams to all orders. Again some modest model dependence enters at high energies. However, the dispersive and two-loop calculations agree extremely well, and I feel the the physics of this process is well under control. The dispersive techniques involving a matching with chiral perturbation theory are quite promising as calculational tools.

This technique also opens up completely new forms of calculations. Here the Das et al[7] calculation of the pion electromagnetic mass difference in the chiral limit from 1967 is a model. The electromagnetic effect involves the integration over all loop momenta, but the dispersive method converts this into a sum rule involving $\rho_V(s) - \rho_A(s)$. Since we know these spectral functions well enough, we can convert this into a calculation of the mass difference. This type of calculation goes beyond what can be done in pure chiral perturbation theory, where the electromagnetic mass difference is described by a unknown parameter. This has also been used to calculate a weak nonleptonic matrix element in the chiral limit[17].

4   Man vs machine

Computer methods are also used to address some of the same problems we have described above. However, there are some constraints on these methods, both temporary and long term. In the short term, the quenched truncation is an issue of unknown severity, as it is not even a valid approximation scheme in QCD (in the sense that there is no small parameter such as $1/N_c$ that controls the size of corrections). We know from chiral calculations that loop effects are important in some matrix elements such as the B parameter-these are misrepresented by the quenched truncation. The inability to reach very low energies/masses is another short term problem. There is also a more difficult issue that will always remain in that lattice results are in the Euclidian region. For processes where physical intermediate states play an important role, these effects will be missed by a Euclidian simulation. It is not clear that this part of the continuation to the Minkowski region can ever be built in in a rigorous way.

For these areas, the rigorous analytic methods described above are still superior to computer simulations. In fact, even in some intermediate energy applications man may still do better

than machine. For example, in the dispersive calculation of the weak matrix element mentioned above, the Monte Carlo methods can be thought of as producing simulations to the relevant intermediate states, while the dispersive method uses real data for the same. A comparison here may be more a test of the Monte Carlo method.

5   Models

Phenomenologically, the physics on the low energy region is not very complicated. The structure of almost any amplitude as a function of energy involves a few visible resonances merging into a high energy continuum. Particularly clear examples of this are the vector and axial-vector spectral functions, and the structure functions of deep inelastic scattering. The primary resonances that are involved are few in number, with the rho playing the prominent role and the resonances up to 1.4 GeV occasionally being visible. The simplicity of this physics has led to the development of models which attempt to provide a useful description of this dynamics. These models invariably drop some aspects of the full dynamics, and so they are not rigorous techniques. However, to the extent that they capture some of the correct physics, they may be convenient and reasonably accurate ways to handle the intermediate energy region.

a) Vector Dominance:

Vector dominance or resonance saturation may be considered as a poor man's dispersion relations. If one takes a dispersion relation and replaces the integrand by a zero-width resonance, one obtains the resonance saturation approximation. As noted above, this most often involves vector mesons such as the rho. The result can be turned onto a field theory calculation, with the various transitions being described by measured coupling constants in a Lagrangian. The specific application of vector dominance turns out to be surprisingly subtle, keeping referees and authors busy correcting the multiple mistakes which are possible. However, done properly it does capture a good deal of the right low energy physics. For example, the major chiral parameters in the chiral effective Lagrangian can be predicted by resonance saturation[18].

b) Quark Loops:

Resonances are reasonably well understood as $q\bar{q}$ bound states, and so intermediate states with a resonance could be thought of as having $q\bar{q}$ propagation. An extreme limit of this propagation is a free quark loop[19], and this model has been used to compute various low energy processes. When thought of in the resonance plus continuum language this limit is the reverse of vector dominance, being all continuum and no resonance. In practice, this model is less successful than vector dominance, but still it does surprisingly well considering the naivity of the approximation.

c) NJL models:

There is a whole subfield devoted to Nambu Jona-Lasinio models[20,21]. The idea is to include a four-quark contact interaction to model the QCD interaction between quarks. Using this with a cutoff produces a model with a light pion with nontrivial chiral interactions. Of all the variants of this idea, my favorite is that of Ref 21, because they use it in a way that connects easily to both chiral and dispersion techniques. By summing up classes of diagrams, they generate intermediate states that contain both resonance-like bumps and continuum contributions (see for example the vector spectral function of Ref 21.). This is approaching the right physics. The cutoff may sometimes make it difficult to perform a valid matching to high energy, but this is certainly an improvement over the use of free quark loops.

None of these models is yet the full story. We need a way to capture both resonance and continuum physics as accurately as possible. My feeling is that this is best done in a dispersion theory context.

6 Where are we going?

There are clearly many applications of analytic methods to specific reactions, and more will be studied in the future. However, my focus here is not on these but on the way that techniques as a whole are developing. To my biased eye, there actually is new direction in techniques which has the potential for great importance if we can develop it sufficiently. This involves the calculation of "nonleptonic" processes which involve current matrix elements integrated over all scales. Such calculations are still in the exploratory stages, but the issues that are being studied, such as the matching of short and long distance physics, are the final frontier of analytic methods.

Many of processes that we have presently mastered are matrix elements of a single current. However, typical of the new more difficult class is the electromagnetic mass differences of the mesons, which involves two electromagnetic currents in which the intermediate states are integrated over together with the photon propagator. Since these intermediate states involve all scales, it is not sufficient to know the short distance physics from QCD perturbation theory, nor the long distance physics from chiral perturbation theory. In addition to these, we must be able to bring them together in the intermediate energy region in an accurate way. This is the challenge.

The weak nonleptonic matrix elements are the most difficult of these. The $\Delta I = 1/2$ rule still has not been definitively explained in a way that is convincing to the full community. In some ways, this is the "John Henry challenge". Lattice methods, despite a promising start, have been unable to resolve the problem satisfactorily. In the analytic arena, the work of Buras, Bardeen and Gerard[22] has taught us the right questions to ask and has stimulated a modern way of approaching the problem, even if may feel that theirs is not the final answer.

The work on electromagnetic mass differences illustrates how this field is developing. The early calculation of pion mass difference in the chiral limit via dispersive sum rules is a benchmark, and when combined with QCD indicates that high energy effects vanish in this limit. In attempting to deal with on-shell pions and kaons, the first approachs have been models of the intermediate energy region[23]. Subsequently more attention is being paid to a more systematic treatment with more realistic matching with higher energies. My student Antonio Perez[24] has done what I feel is the best job in this area by using a dispersive treatment related to the Cottingham method. One can identify all of the ingredients of the chiral limit approach, and these come with identifiable on-shell corrections. The constraints of QCD determine the high energy matching, those of chiral symmetry fix the low energy structure, and data fix much of the intermediate region. This provides a reasonably solid description of these matrix elements.

Another recent calculation in this pioneering area is the B-parameter work of Bijnens and Prades using the NJL model[25]. While this is still a model calculation, the approach is instructive. They use the model as a guide to the matching in the intermediate energy region, which is the way that one can remove the scale ambiguity of the calculations of chiral loop corrections. The result has finite calculable corrections when compared to the lowest order chiral prediction. While the model is not a complete characterization of the intermediate energy region, it does indicate what the physics is that we need to do better if we are to calculate this amplitude reliably.

I expect that this type of calculation will continue to develop through an interplay of model and dispersive techniques. My expectation is that we will rely increasingly on dispersive methods for the final answers, perhaps with a bit of modeling thrown in to account for the intermediate states that we cannot measure. These type of calculations are also at the frontier of lattice work.

7   Summary

We certainly cannot claim to have completely tamed the strong interactions yet. However we have made progress, in that both the high energy and low energy regions have reliable methods that are now well developed. In the intermediate energy region, we know the basic physics and have ways to incorporate some of it into calculations. The next step is to gain more control over these effects. My own vision of how this can take place is to learn how to do a good job of modeling the ingredients to dispersive calculations. Overall, I am optimistic that we are still progressing in our calculational ability, and I personally find this progress to be one of the most interesting aspects of the field at the moment.

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2) It is just a curiosity that the papers that I chose for the examples were all from 1967. Another pattern of selection would have been to use all papers of Steven Weinberg who, in addition to another well known 1967 paper and his effective field theory influence (see Ref 9 below), produced seminal works in all the areas that I am citing.

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