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Chiral Symmetry and Electroweak π, K, η Processes

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Abstract

Recently, the development of chiral perturbation theory has allowed the generation of rigorous low-energy theorems for various hadronic processes based only on the chiral invariance of the underlying QCD Lagrangian. Herein we examine the experimental implications of chiral symmetry in the regime of electroweak Goldstone boson interactions.

1 Introduction

It has long been the holy grail for particle and nuclear knights to generate rigorous predictions from the Lagrangian of QCD

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}G_{\mu\nu}G^{\mu\nu} + \bar{q}(i\gamma_{\mu}D^{\mu} - m)q. \quad (1)$$

Despite the ease with which one can write this equation, because of its inherent nonlinearity progress in this regard has been slow. In recent years, however, procedure has been developed—chiral perturbation theory (χ PT)—which exploits the (broken) chiral symmetry of QCD and allows rigorous predictive power in the case of low energy reactions. This technique, based on a suggestion due to Weinberg[1] was developed (at one loop level) during the last decade in an important series of papers by Gasser and Leutwyler and others,[2] and is based on the feature that the QCD Lagrangian (Eq. 1) has a global $SU(3)_L \times SU(3)_R$ (chiral) invariance in the limit of vanishing quark mass. Such invariance is manifested in the real world not in the conventional fashion but rather is spontaneously broken, resulting in eight light Goldstone bosons— π, K, η —which would be massless if the corresponding quark masses also vanished. While the identification of this symmetry is apparent in terms of quark/gluon degrees of freedom, it is not so simple to understand the implications of chiral invariance in the arena of experimental meson/baryon interactions.

Early attempts in this direction were based on current algebra/PCAC methods, yielding relationships between processes differing in the number of pions, e.g.

$$\lim_{q \rightarrow 0} \langle B\pi_q^a | \mathcal{O} | A \rangle = \frac{-i}{F_{\pi}} \langle B | [F_5^a, \mathcal{O}] | A \rangle \quad (2)$$

where $F_{\pi} = 92.4 \text{ MeV}$ is the pion decay constant. However, recently we have learned how to study chiral strictures using so-called effective Lagrangian techniques, and the development of χ PT has opened up an important window on the low energy interactions of these Goldstone particles which has heretofore been unavailable and which succinctly expresses all information about such reactions in the energy regime $E \lesssim 0.6 \text{ GeV}$ in terms of just ten phenomenological constants L_1, \dots, L_{10} . Because of space limitations we shall not be able here to outline this chiral technology but rather refer the interested reader to the relevant literature. We shall have to be satisfied in the next sections to outline the results of such calculations within the electroweak interactions of π, K, η mesons respectively, indicating where possible problems and challenges lie for future experimental and/or theoretical work.

2 π, K Reactions

The simplest pionic process for which chiral perturbation theory makes a prediction is that of radiative pion decay— $\pi^+ \rightarrow e^+\nu_e\gamma$ —for which the decay amplitude assumes the form

$$M_{\mu\nu}(p, q) = \int d^4x e^{iq \cdot x} \langle 0 | T(V_\mu^{\text{em}}(x) J_\nu^{\text{wk}}(0)) | \pi^+(\mathbf{p}) \rangle = \text{Pole} \\ + h_A [(p - q)_\mu q_\nu - g_{\mu\nu} (p - q) \cdot q] + r_A (q_\mu q_\nu - g_{\mu\nu} q^2) + i h_V \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \quad (3)$$

In general then there exist two axial (h_A, r_A) and one vector (h_V) structure functions. However, there is a catch. Chiral symmetry does *not* predict the size of h_A , for which a rather precise number is available. Rather this parameter is used as input for determination of one of the GL parameters— L_{10} . In order to determine h_V and r_A the rather rare ($\sim 10^{-9}$) Dalitz mode— $\pi^+ \rightarrow e^+\nu_e e^+e^-$ —must be employed and the limits obtained thereby are somewhat imprecise[3]

$$h_V|_{\text{theo}} = \frac{1}{4\sqrt{2}\pi^2 F_\pi} = 0.027 m_\pi^{-1} \quad \text{vs.} \quad h_V|_{\text{exp}} = (0.029 \pm 0.017) m_\pi^{-1} \\ \frac{r_A}{h_V}|_{\text{theo}} = \frac{8\pi^2 F_\pi^2}{3} \langle r_\pi^2 \rangle \simeq 2.6 \quad \text{vs.} \quad \frac{r_A}{h_V}|_{\text{exp}} = 2.3 \pm 0.6 \quad (4)$$

Thus while both numbers are in agreement with the chiral restrictions there is also plenty of room for improvement in experimental precision.

An important probe of pion structure is provided by of its electric (magnetic) polarizability $\alpha_E(\beta_M)$, which measures the constant of proportionality between induced dipole moments and applied electric (magnetic) fields.[4] The polarizability may be probed via the Compton scattering process, which to lowest order must assume the form

$$T_{\text{Compton}} = \hat{\epsilon} \cdot \hat{\epsilon}' \left(\frac{-Q^2}{m} + \omega\omega' 4\pi\alpha_E \right) + \hat{\epsilon} \times \mathbf{q} \cdot \hat{\epsilon}' \times \mathbf{q}' 4\pi\beta_M + \dots \quad (5)$$

Chiral symmetry predicts the size of both electric and magnetic polarizabilities in terms of the axial structure function h_A via[5]

$$\alpha_E = -\beta_M = \frac{\alpha}{8\pi^2 m_\pi F_\pi^2} \frac{h_A}{h_V} = (2.8 \pm 0.3) \times 10^{-4} \text{fm}^3 \quad (6)$$

The sum of electric and magnetic polarizabilities is predicted to vanish, in agreement with experiment—

$$\alpha_E + \beta_M = (1.4 \pm 3.1) \times 10^{-4} \text{fm}^3 [6] \quad (7)$$

but there is not yet agreement on the experimental size of the electric polarizability, which has been measured in three different fashions

$$\alpha_E = (6.8 \pm 1.4) \times 10^{-4} \text{fm}^3 \quad (\text{radiative pion scattering}) [7] \\ = (20 \pm 12) \times 10^{-4} \text{fm}^3 \quad (\text{radiative pion production}) [8] \\ = (2.2 \pm 1.6) \times 10^{-4} \text{fm}^3 \quad (\gamma\gamma \rightarrow \pi\pi) [9] \quad (8)$$

Clearly there exists a lack of agreement here and clarifying experimental work is called for on this very fundamental aspect of the pion.

Moving to the kaon sector, there is a complete correspondence between pion quantities and their kaonic analogs, and the latter are completely predicted by chiral symmetry—

$$h_A^K = h_A^\pi, \quad r_A^K = r_A^\pi, \quad h_V^K = h_V^\pi, \quad \alpha_E^K = -\beta_M^K = \frac{m_\pi F_\pi^2}{m_K F_K^2} \alpha_E^\pi. \quad (9)$$

However, the experimental information is very limited and is restricted to the sum of vector and axial structure functions in radiative kaon decay

$$h_A^K + h_V^K|_{\text{exp}} = (0.043 \pm 0.003)m_\pi^{-1}[10] \quad \text{vs.} \quad h_A^K + h_V^K|_{\text{theo}} = 0.038m_\pi^{-1}. \quad (10)$$

The existence of a high intensity kaon factory could clearly have a significant impact in this regard.

Although there is much more which we could discuss such as $K_{\ell 3}, K_{\ell 4}$ decays and their radiative partners, for space reasons we move on to consider the nonleptonic kaon sector, where chiral symmetry also is a powerful tool. Besides the dominant modes $K \rightarrow 2\pi, 3\pi$ which are closely related via soft pion theorems, there is special interest in the nonleptonic-radiative processes $K \rightarrow \gamma\gamma, \pi^0\gamma\gamma$, which occur at one loop order in the chiral expansion. In the former case one finds a prediction[14]

$$\begin{aligned} \Gamma(K_S \rightarrow \gamma\gamma) &= \frac{\alpha^2 m_K^3 g_8^2 F_\pi^2}{16\pi^3} \left(1 - \frac{m_\pi^2}{m_K^2}\right) \left|F\left(\frac{m_K^2}{m_\pi^2}\right)\right|^2 \\ \text{where } F(z) &= 1 - \frac{1}{z} \ln^2\left(\frac{\beta(z) + 1}{\beta(z) - 1}\right) \end{aligned} \quad (11)$$

with $\beta = \sqrt{1 - \frac{4}{z}}$ being the pion velocity in $K_S \rightarrow \pi\pi$ and $g_8 \approx 7.8 \times 10^{-8} F_\pi^2$ is a parameter determined via the $K_S \rightarrow \pi\pi$ decay rate. The branching ratio predicted in this way is in good agreement with the value recently measured at CERN[12]

$$B(K_S \rightarrow \gamma\gamma)|_{\text{theo}} = 2.0 \times 10^{-6} \quad \text{vs.} \quad B(K_S \rightarrow \gamma\gamma)|_{\text{exp}} = (2.4 \pm 1.2) \times 10^{-6}. \quad (12)$$

Not so simple, on the other hand, is the process $K_L \rightarrow \pi^0\gamma\gamma$ for which we predict

$$\begin{aligned} \frac{d\Gamma}{dz}(K_L \rightarrow \pi^0\gamma\gamma) &= \frac{\alpha^2 m_K^5 g_8^2}{(4\pi)^5} \lambda^{\frac{1}{2}}\left(1, z, \frac{m_\pi^2}{m_K^2}\right) \left| \left(z - \frac{m_\pi^2}{m_K^2}\right) F\left(z \frac{m_K^2}{m_\pi^2}\right) + \left(1 - z + \frac{m_\pi^2}{m_K^2}\right) F(z) \right|^2 \\ \text{with } z &= \frac{m_{\gamma\gamma}^2}{m_K^2}, \quad \lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc) \end{aligned} \quad (13)$$

The expected spectrum is very distinctive, with most of the events predicted to occur in the high z range and this is in agreement with present experimental indications. However, the predicted branching ratio is *not*

$$B(K_L \rightarrow \pi^0\gamma\gamma)|_{\text{theo}} = 6.8 \times 10^{-7} \quad \text{vs.} \quad B(K_L \rightarrow \pi^0\gamma\gamma)|_{\text{exp}} = (2.0 \pm 0.5) \times 10^{-6}[13] \quad (14)$$

indicating the need for additional experimental and theoretical effort. While we could continue this discussion of the nonleptonic kaon sector considerably, space limitations require that we move now to the realm of eta decay.

3 Eta Decay Processes

The challenge of dealing with decay of $\eta(547)$ involves inclusion of the mixing with $\eta'(958)$, which lies outside the simple chiral $SU(3)_L \times SU(3)_R$ framework. To lowest order things are simple—in the chiral limit the pseudoscalar mass spectrum would consist of a massless octet of Goldstone bosons plus a massive $SU(3)$ singlet (η_0). With the breaking of chiral invariance the octet pseudoscalar masses become nonzero, and are related at first order in symmetry breaking by the Gell-Mann-Okubo formula

$$m_{\eta_8}^2 = \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 \quad (15)$$

where η_8 is the eighth member of the octet. At this same order in symmetry breaking the singlet η_0 will in general mix with η_8 producing the physical eigenstates η, η' given by

$$\eta = \cos \theta \eta_8 - \sin \theta \eta_0, \quad \eta' = \sin \theta \eta_8 + \cos \theta \eta_0. \quad (16)$$

The mixing angle θ can be determined via diagonalization of the mass matrix

$$m^2 = \begin{pmatrix} m_{\eta_8}^2 & \kappa \\ \kappa & m_{\eta_0}^2 \end{pmatrix} \quad (17)$$

Taking m_{η_8} from Eq. 15 and fitting m_{η_0}, κ with the two known masses yields the prediction $\theta = -9.4^\circ$. However, there is good reason not to trust this traditional analysis, since higher order chiral symmetry breaking terms can make important modifications. For example, inclusion of the leading chiral log correction from meson-meson scattering, we find[14]

$$m_{\eta_8}^2 = \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 - \frac{2}{3} \frac{m_K^2}{(4\pi F_\pi)^2} \ln \frac{m_K^2}{\mu^2} \quad (18)$$

for which diagonalization of the mass matrix yields $\theta \approx -19.5^\circ$. Of course, this is just an approximate result. However, a full one loop calculation using χ PT yields essentially the same value.[2] At this same (one-loop) level of symmetry breaking there is generated a shift in the lowest order value of the pseudoscalar decay constant F_P —

$$\begin{aligned} F_\pi &= \bar{F} \left[1 - \frac{1}{2} \frac{m_K^2}{(4\pi F_\pi)^2} \ln \frac{m_K^2}{\mu^2} \right] \approx 1.12\bar{F} \\ F_{\eta_8} &= \bar{F} \left[1 - \frac{3}{2} \frac{m_K^2}{(4\pi F_\pi)^2} \ln \frac{m_K^2}{\mu^2} \right] \approx 1.25F_\pi \quad \text{for } \mu \approx 1\text{GeV}. \end{aligned} \quad (19)$$

With this introductory material in hand we can now confront the remaining subject of our report—that of eta decay. First consider the dominant two-photon decay mode, which to leading order arises due to the anomaly. In the analogous $\pi^0 \rightarrow \gamma\gamma$ case we find

$$\text{Amp} \equiv F_{\pi\gamma\gamma}(0) \epsilon^{\mu\nu\alpha\beta} \epsilon_\mu k_\nu \epsilon'_\alpha k'_\beta \quad \text{with} \quad F_{\pi\gamma\gamma}(0) = \frac{N_c \alpha}{3\pi F_\pi} = 0.025 \text{GeV}^{-1}. \quad (20)$$

General theorems guarantee that this result is not altered in higher orders of chiral symmetry breaking and the experimental value[14]

$$F_{\pi\gamma\gamma} = (0.0250 \pm 0.0005) \text{GeV}^{-1} \quad (21)$$

is in excellent agreement with its theoretical analog, eloquently confirming the value $N_c = 3$ as the number of colors. The $\eta, \eta' \rightarrow \gamma\gamma$ couplings also arise from the anomalous component of the effective chiral Lagrangian, and in an extended χ PT approximation have the values

$$\begin{aligned} F_{\eta\gamma\gamma}(0) &= \frac{F_{\pi\gamma\gamma}(0)}{\sqrt{3}} \left(\frac{F_\pi}{F_8} \cos \theta - 2\sqrt{2} \frac{F_\pi}{F_0} \sin \theta \right) \\ F_{\eta'\gamma\gamma}(0) &= \frac{2\sqrt{2}F_{\pi\gamma\gamma}(0)}{\sqrt{3}} \left(\frac{1}{2\sqrt{2}} \frac{F_\pi}{F_8} \sin \theta + \frac{F_\pi}{F_0} \cos \theta \right). \end{aligned} \quad (22)$$

The experimental numbers

$$F_{\eta\gamma\gamma}(0) = 0.0249 \pm 0.0010 \text{GeV}^{-1} \quad F_{\eta'\gamma\gamma}(0) = 0.0328 \pm 0.0024 \text{GeV}^{-1} \quad (23)$$

are fit well by $F_8/F_\pi \approx 1.24$ and $F_0/F_\pi \approx 1.04$. Note also that the value of F_8/F_π is in good agreement with that expected from chiral arguments given above.

Processes involving a photon coupled to three pseudoscalar mesons also involve the anomaly and at zero four-momentum are completely determined. First consider the case of $\gamma \rightarrow \pi^+\pi^-\pi^0$. At zero four-momentum the anomaly requires[15]

$$\begin{aligned} \text{Amp}(3\pi - \gamma) &= A(s_{+-}, s_{+0}, s_{-0})\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu p+\nu p-\alpha p_0\beta} \\ \text{where } A(0, 0, 0) &= \frac{eN_c}{12\pi^2 F_\pi^3} = 9.7\text{GeV}^{-3} \quad \text{and} \quad s_{ij} = (p_i + p_j)^2 \end{aligned} \quad (24)$$

Inclusion of additional diagrams yields

$$A(s, t, u) = \frac{eN_c}{12\pi^2 F_\pi^3} \left[1 + \frac{1}{2} \left(\frac{s}{m_\rho^2 - s} + \frac{t}{m_\rho^2 - t} + \frac{u}{m_\rho^2 - u} \right) \right] \quad (25)$$

which has the structure required by vector dominance, and agrees with the value required by the chiral anomaly at zero four-momentum. The $\gamma - 3\pi$ reaction has been studied experimentally via pion pair production by the pion in the nuclear Coulomb field and yields a number[16]

$$A(0, 0, 0)_{\text{exp}} = 12.9 \pm 0.9 \pm 0.5\text{GeV}^{-3} \quad (26)$$

in apparent disagreement with Eq. 24 and suggesting the value $N_c \approx 4!$ The most likely conclusion is that this is an experimental problem associated with this difficult-to-measure process, but in any case a new high-precision experiment would be of great interest.

Having warmed up on the $\gamma - 3\pi$ process, it is now straightforward to construct the analogous $\eta \rightarrow \pi^+\pi^-\gamma$ amplitude, for which we find

$$\text{Amp}(\eta \rightarrow \pi^+\pi^-\gamma) = B(s_{+-}, s_{+\gamma}, s_{-\gamma})\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu p+\nu p-\alpha k\gamma\beta} \quad (27)$$

with

$$\begin{aligned} B(s, t, u) &= B(0, 0, 0) \times \left[1 + \frac{3}{2} \frac{s}{m_\rho^2 - s} \right] \quad \text{and} \\ B(0, 0, 0) &= \frac{eN_c}{12\sqrt{3}\pi^2 F_\pi^3} \left(\frac{F_\pi}{F_8} \cos\theta - \sqrt{2} \frac{F_\pi}{F_0} \sin\theta \right) = 6.81\text{GeV}^{-3} \end{aligned} \quad (28)$$

The $\eta \rightarrow \pi^+\pi^-\gamma$ reaction was studied in the experiment of Layter et al.[17] and yielded

$$|B(0, 0, 0)|_{\text{exp}} = (6.47 \pm 0.25)\text{GeV}^{-3} \quad (29)$$

which is in reasonable agreement with Eq. 28 and reinforces the validity of the numbers obtained in the two photon analysis.

Having above confirmed the basic correctness of the predictions of the anomaly (and thereby of this important cornerstone of QCD) we move now to the important three pion decay of the eta, which rather probes the *conventional* two- and four-derivative piece of the chiral Lagrangian. The decay of the isoscalar eta to the predominantly I=1 final state of the three pion system occurs primarily due to the u-d quark mass difference, and the result arising from lowest order chiral perturbation theory is well-known

$$\text{Amp}(\eta_8 \rightarrow \pi^+\pi^-\pi^0) = \frac{-B_0(m_d - m_u)}{3\sqrt{3}F_\pi^2} \left[1 + \frac{3(s - s_0)}{m_\eta^2 - m_\pi^2} \right]. \quad (30)$$

The d-u quark mass difference has traditionally been extracted from the experimental $K^+ - K^0$ mass splitting with the electromagnetic component eliminated via use of Dashen's theorem—[18]

$$(m_{\pi^+}^2 - m_{\pi^0}^2) = (m_{K^+}^2 - m_{K^0}^2)_{\text{EM}}. \quad (31)$$

This assumption results in a prediction in serious contradiction to the experimental result

$$\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)_{\text{theo}} = 66eV \quad \text{vs.} \quad \Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)_{\text{exp}} = 310 \pm 50eV. \quad (32)$$

At first sight this would appear to be a rather strong and irreparable violation of a lowest order chiral prediction and therefore not salvagable by the expected $\mathcal{O}(m_\eta^2/(4\pi F_\pi^2)^2) \sim 30\%$ corrections from higher order effects. However, this is not at all the case. The one-loop and counterterm contributions were calculated by Gasser and Leutwyler and were found to enhance the lowest order prediction by a factor 2.6, and recent work has suggested a significant violation of Dashen's theorem—

$$(m_d - m_u)_{\chi\text{-broken}} \approx 1.2(m_d - m_u)_{\text{Dashen}}[19] \quad (33)$$

which corresponds to an additional 40% enhancement of the chiral estimate, *i.e.* $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \sim 240eV$, and puts the result now in the right ballpark.

In order to decide the origin of any remaining discrepancy, it is necessary to make careful spectral shape measurements. Phenomenologically, we expand the decay amplitude about the center of the Dalitz plot as

$$\text{Amp} \equiv \alpha \left[1 + \beta Y + \gamma(Y^2 + \frac{1}{3}X^2) + \delta(Y^2 - \frac{1}{3}X^2) \right] \quad (34)$$

where X, Y are the usual Dalitz variables. These parameters have been determined phenomenologically to be[20]

$$\begin{aligned} \beta &= 0.216 \pm 0.003 & \gamma &= -0.0067 \pm 0.003 & \delta &= -0.0139 \pm 0.003 \\ \beta &= 0.234 \pm 0.004 & \gamma &= -0.0006 \pm 0.003 & \delta &= -0.0099 \pm 0.003 \end{aligned} \quad (35)$$

to be compared to the one-loop chiral prediction

$$\beta = 0.266 \quad \gamma = 0.0054 \quad \delta = -0.0072 \quad (36)$$

We see that there is general though certainly not excellent agreement.

An additional test of the validity of the chiral approach lies in our ability to predict the $\eta \rightarrow 3\pi^0$ reaction, for which one finds

$$\frac{\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)}{\Gamma(\eta \rightarrow \pi^0\pi^0\pi^0)} = \begin{cases} 1.5 & \mathcal{O}(p^2) \\ 1.43 & \mathcal{O}(p^4) \\ 1.3 & \text{experiment} \end{cases} \quad (37)$$

Clearly there is plenty of challenge—both theoretical and experimental—in the eta system.

4 Conclusions

We have seen above that chiral symmetry provides an important link between experimental low energy physics within the Goldstone boson sector and the QCD Lagrangian which presumably underlies it. Despite the evident success of such methods, however, a number of challenges remain. These include

i) pions: clearing up apparent discrepancies in the anomalous process $\gamma \rightarrow 3\pi$ and in the charged pion polarizability; ii) kaons: providing experimental numbers which are presently unmeasured in radiative semileptonic decay and clarifying the nonleptonic-radiative $K_L \rightarrow \pi^0\gamma\gamma$ process; iii) etas: a precise measurement of the eta lifetime would be of interest in that present values obtained via different methods disagree, and a new and more precise experiment on the $\eta \rightarrow 3\pi$ spectrum would be very useful as a test of chiral methods. In summary, there is plenty of interesting physics here for experimentalists and theorists alike.

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