1997

Systematic 1/M expansion for spin 3/2 particles in baryon chiral perturbation theory

TR Hemmert

BR Holstein
holstein@physics.umass.edu

J Kambor

IPNO/TH 95-60

Systematic 1/M Expansion for Spin 3/2 Particles in Baryon Chiral Perturbation Theory

Thomas R. Hemmert and Barry R. Holstein[1]

Department of Physics and Astronomy
University of Massachusetts
Amherst, MA 01003, U.S.A.

Joachim Kambor[2]

Division de Physique Théorique
Institut de Physique Nucléaire
F-91406 Orsay Cedex, France

Abstract

Starting from a relativistic formulation of the pion-nucleon-delta system, the most general structure of 1/M corrections for a heavy baryon chiral lagrangian including spin 3/2 resonances is given. The heavy components of relativistic nucleon and delta fields are integrated out and their contributions to the next-to-leading order lagrangians are constructed explicitly. The effective theory obtained admits a systematic expansion in terms of soft momenta, the pion mass m_π and the delta-nucleon mass difference Δ. As an application, we consider neutral pion photoproduction at threshold to third order in this small scale expansion.

[1]Research supported in part by the National Science Foundation
[2]Laboratoire de Recherche des Universités Paris XI et Paris VI, associé au CNRS.

1 Introduction

Chiral symmetry provides important restrictions on the interactions of pions, nucleons and photons [1]. The consequences are most conveniently summarized by the use of an effective field theory, valid in the low energy regime. This simultaneous expansion in small momenta and light quark masses is known as Chiral Perturbation Theory (ChPT) [2, 3, 4, 5, 6]. Unlike in the sector of Goldstone bosons, the mass of the nucleon is large and nonvanishing in the chiral limit. Nevertheless, a consistent chiral power counting, known as Heavy Baryon Chiral Perturbation Theory, HBChPT, can be maintained by performing also a systematic $1/M$-expansion, M being the mass of a baryon [7, 8].[3] In principle, any observable of the pion-nucleon system can be calculated to a given order in the chiral expansion—the price to be paid is the introduction of new low energy coupling constants which are not fixed by the symmetry requirement alone. Any parameter-free prediction of HBChPT is, however, also a prediction of low energy QCD. Many of these resulting "low energy theorems" (LET) have been discussed in the recent literature [11].

The spin 3/2 delta resonances play a special role in the pion-nucleon system, since the mass difference $\Delta = M_\Delta - M_N$ is not large compared to the typical low energy scale m_π and because the $\pi N\Delta$-coupling constant is anomalously large. The more conventional version of HBChPT takes into account the effect of the delta (and of other resonances) only through contributions to the coupling constants of higher order operators in the chiral expansion. This approach is in particular well suited to derive low energy theorems. A concern, however, is that, in the physical world of nonvanishing quark masses, the perturbation series might converge slowly due to the presence of large coupling constants driven by small denominators—*i.e.* by terms proportional to $1/\Delta$. An alternative approach to HBChPT includes the delta degrees of freedom explicitly [8, 12]. In addition to solving the problems mentioned above, this technique has the advantage that the range of applicability can in principle be extended into the delta-region.

In this letter we sketch explicitly the steps necessary for a systematic low energy expansion in the presence of the spin 3/2 delta resonance. A full presentation will appear shortly.[13] We begin with a covariant formulation

[3]Such techniques have been developed and used extensively in the parallel case of heavy quark physics—*cf.* [9, 10].

of an effective theory of the $\pi N\Delta$-system. The heavy degrees of freedom are identified and integrated out via a systematic $1/M$-expansion. We arrive at an effective field theory of nonrelativistic nucleons and deltas coupled to pions and external sources. The theory is manifestly Lorentz invariant and admits a low energy expansion in terms of small momenta q, the pion mass m_π and the delta-nucleon mass difference Δ, which we collectively denote by the symbol ϵ.[4] Of course, the procedure described in the next section is not unique—the general methods of such heavy mass expansions have been given previously [9, 10, 14]. However, it represents a useful starting point for the evaluation of higher order effects . Indeed, the 1/M corrections derived in this manner have a simple physical interpretation—exchange of the heavy degrees of freedom—and the formalism can straightforwardly be extended to deal with higher order terms in the ϵ expansion, as will be discussed in [13]. Furthermore, it is straightforward to treat other resonances than spin 3/2 along the same lines.

As a simple application of our formalism we shall consider neutral pion photoproduction at threshold . A one-loop calculation within the framework of HBChPT has produced a LET for the electric dipole amplitude E_{0^+} as an expansion in powers of $\mu = m_\pi/M_N$ [15, 16]

$$E_{0+}^{\pi^0 p}(s_{\rm thr}) = -\frac{e g_{\pi NN}}{8\pi M_N}\left[\mu - \frac{1}{2}(3+\kappa_p)\,\mu^2 - \left(\frac{M_N}{4F_\pi}\right)^2\,\mu^2 + \mathcal{O}(\mu^3)\right] \qquad (1)$$

Here κ_p is the anomalous magnetic moment of the proton while $g_{\pi NN}$ is the strong pion-nucleon coupling constant. Recently an $\mathcal{O}(p^4)$ calculation has been given[17] which reconciles the theoretical prediction with experiment [18]. However, each term in this expansion is large with alternating sign, thus making the convergence particularly slow. Below we calculate the correction of order ϵ^3 to Eq. (1), which arises due to a 1/M corrected vertex of the $\Delta(1232)$ resonance. However, first we set our formalism.

[4] We shall continue to refer to the conventional chiral treatment, wherein only q and m_π are taken as small quantities, as an expansion in the generic small quantity p.

2 1/M-expansion for spin 3/2 resonances

Consider the lagrangian for a relativistic spin 3/2 field Ψ_μ coupled in a chirally invariant manner to the Goldstone bosons

$$\mathcal{L}_{3/2} = \bar{\Psi}^\alpha O^A_{\alpha\mu} \Lambda^{\mu\nu} O^A_{\nu\beta} \Psi^\beta \tag{2}$$

with

$$O^A_{\alpha\mu} = g_{\alpha\mu} + \frac{1}{2} A \gamma_\alpha \gamma_\mu. \tag{3}$$

Following Pascalutsa [19] we have factored out the dependence on the unphysical free parameter A by use of the projection operator $O^A_{\alpha\mu}$. Then defining the physical spin 3/2 field as

$$\psi_\mu(x) = O^A_{\mu\nu} \Psi^\nu(x) \tag{4}$$

we see that Eq. (2) is manifestly invariant under point transformations

$$\begin{aligned} \Psi_\mu(x) &\rightarrow \Psi_\mu(x) + \lambda \gamma_\mu \gamma_\nu \Psi^\nu(x) \\ A &\rightarrow \frac{A - 2\lambda}{1 + 4\lambda} \end{aligned} \tag{5}$$

as required by general considerations [20].

To leading order in the derivative expansion, the relativistic spin 3/2 lagrangian with field-redefinition Eq.(4) then takes the form [5]

$$\mathcal{L}_\Delta = \bar{\psi}^\mu_i \; \Lambda^{ij}_{\mu\nu} \; \psi^\nu_j \tag{6}$$

with

$$\begin{aligned} \Lambda^{ij}_{\mu\nu} = & -\Big[(i \not{D}^{ij} - M_\Delta \, \delta^{ij}) g_{\mu\nu} - \frac{1}{4} \gamma_\mu \gamma^\lambda (i \not{D}^{ij} - M_\Delta \, \delta^{ij}) \gamma_\lambda \gamma_\nu \\ & + \frac{g_1}{2} g_{\mu\nu} \not{u}^{ij} \gamma_5 + \frac{g_2}{2} (\gamma_\mu u^{ij}_\nu + u^{ij}_\mu \gamma_\nu) \gamma_5 + \frac{g_3}{2} \gamma_\mu \not{u}^{ij} \gamma_5 \gamma_\nu \Big] \end{aligned} \tag{7}$$

[5] To take into account the isospin 3/2 property of $\Delta(1232)$ we supply the Rarita-Schwinger spinor with an additional isospin index i, subject to the subsidiary condition $\tau^i \; \psi^i_\mu(x) = 0$.

Following the conventions of SU(2) HBChPT in the spin 1/2 sector [6, 11], we have defined the following structures:

$$\begin{aligned}
D_\mu^{ij}\,\psi_j^\nu &= \left(\partial_\mu\,\delta^{ij}+\Gamma_\mu^{ij}\right)\psi_j^\nu \\
\Gamma_\mu^{ij} &= \Gamma_\mu\,\delta^{ij}-\frac{i}{2}\,\epsilon^{ijk}\,Tr[\tau^k\,\Gamma_\mu] \\
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger,\partial_\mu u\right]-\frac{i}{2}u^\dagger(\mathbf{v}_\mu+\mathbf{a}_\mu)u-\frac{i}{2}u(\mathbf{v}_\mu-\mathbf{a}_\mu)u^\dagger \\
u_\mu^{ij} &= u_\mu\delta^{ij}-i\epsilon^{ijk}w_\mu^k \\
w_\mu^i &= \frac{1}{2}Tr\left[\tau^i u_\mu\right] \\
u_\mu &= iu^\dagger\nabla_\mu U u^\dagger \\
\nabla_\mu U &= \partial_\mu U-i(\mathbf{v}_\mu+\mathbf{a}_\mu)U+iU(\mathbf{v}_\mu-\mathbf{a}_\mu) \\
U &= u^2=exp\left(\frac{i}{F_\pi}\vec{\tau}\cdot\vec{\pi}\right)
\end{aligned}\tag{8}$$

$\mathbf{v}_\mu$, $\mathbf{a}_\mu$ denote external vector, axial-vector fields and are the only external sources possible at this order. The first two pieces in Eq. (7) are the kinetic and mass terms of a free spin 3/2 lagrangian [19]. The remaining terms constitute the most general chiral invariant couplings to pions. Note that aside from the conventional $\pi\Delta\Delta$ coupling constant g_1 we have included two additional pion-couplings characterized by g_2, g_3 which contribute only if at least one of the spin 3/2 fields is off mass shell.

The next step consists of identifying the "light" and "heavy" degrees of freedom of the spin 3/2 fields, respectively. The procedure is analogous to the case of spin 1/2 fields, as pioneered in the case of heavy quark effective theory [9, 10] and later applied to spin 1/2 HBChPT [16]. For the case of spin 3/2 particles the problem is technically somewhat more challenging due to the off-shell spin 1/2 degrees of freedom associated with the Rarita-Schwinger field[6]. In order to separate the spin 3/2 from the spin 1/2 components it is convenient to introduce a complete set of orthonormal spin projection operators for fields with *fixed velocity* v_μ

$$P^{3/2}_{(33)\mu\nu} = g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu-\frac{1}{3}\left(\not{v}\gamma_\mu v_\nu+v_\mu\gamma_\nu\not{v}\right)$$

[6]The formalism for the case of heavy systems of arbitrary spin was given by Falk in [9].

$$
\begin{aligned}
P^{1/2}_{(11)\mu\nu} &= \frac{1}{3}\gamma_\mu\gamma_\nu - v_\mu v_\nu + \frac{1}{3}\left(\not{v}\gamma_\mu v_\nu + v_\mu\gamma_\nu \not{v}\right) \\
P^{1/2}_{(22)\mu\nu} &= v_\mu v_\nu \\
P^{1/2}_{(12)\mu\nu} &= \frac{1}{\sqrt{3}}(v_\mu v_\nu - \not{v} v_\nu \gamma_\mu) \\
P^{1/2}_{(21)\mu\nu} &= \frac{1}{\sqrt{3}}(\not{v} v_\mu \gamma_\nu - v_\mu v_\nu)
\end{aligned}
\tag{9}
$$

which satisfy

$$
\begin{aligned}
P^{3/2}_{(33)\mu\nu} + P^{1/2}_{(11)\mu\nu} + P^{1/2}_{(22)\mu\nu} &= g_{\mu\nu} \\
P^{I}_{(ij)\mu\nu}\, P^{J,\nu\delta}_{(kl)} &= \delta^{IJ}\,\delta_{jk}\, P^{J,\delta}_{(il)\mu}.
\end{aligned}
\tag{10}
$$

The four-velocity v_μ is related to the four-momentum p_μ of the spin 3/2 particle by

$$p_\mu = M v_\mu + k_\mu, \tag{11}$$

where M is a baryon mass scale and k_μ is taken to be a residual soft momentum. We now employ the familiar projection operators of the heavy mass formalism

$$P_v^\pm = \frac{1}{2}\left(1 \pm \not{v}\right). \tag{12}$$

and introduce heavy baryon fields for our spin 3/2 particles in order to eliminate the dependence on the large mass M_Δ in Eq.(7). In analogy to the spin 1/2 case we identify the "light" spin 3/2 degree of freedom via

$$T^i_\mu(x) \equiv P_v^+\, P^{3/2}_{(33)\mu\nu}\, \psi^\nu_i(x)\, \exp(iMv\cdot x) \tag{13}$$

whereas the remaining components

$$G^i_\mu(x) = \left(g_{\mu\nu} - P_v^+\, P^{3/2}_{(33)\mu\nu}\right)\psi^\nu_i(x)\, \exp(iMv\cdot x) \tag{14}$$

can be shown to be "heavy" [13] and are integrated out. We note in particular that G_μ includes both spin 1/2 *and* spin 3/2 components. Of course, the virtual effects of the heavy degrees of freedom G_μ are nevertheless accounted for in the heavy baryon formalism, they show up as higher order 1/M corrected vertices involving the remaining (on-shell) spin 3/2 fields T_μ, as we will show below. We also note that the T_μ degrees of freedom satisfy the constraints

$$v_\mu T_i^\mu = \gamma_\mu T_i^\mu = 0 \tag{15}$$

and correspond to the SU(2) version of the decuplet field introduced in ref.[8].

We now perform a systematic $1/M$-expansion, following an approach developed by Mannel et al. in HQET [10], which was later applied to spin 1/2 HBChPT by Bernard et al. [16]. Since we are interested in the interactions of nucleons with the spin 3/2 resonance, we must treat both fields simultaneously. We therefore write the most general lagrangian involving relativistic spin 1/2 (ψ_N) and spin 3/2 (ψ_μ) fields as

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\Delta + (\mathcal{L}_{\Delta N} + h.c.) \tag{16}$$

with $\mathcal{L}_\Delta$ given in Eq.(7) and

$$\begin{aligned} \mathcal{L}_N &= \bar{\psi}_N \left(i \not{D} - M_N + \frac{g_A}{2} \not{u} \gamma_5 \right) \psi_N + ... \\ \mathcal{L}_{\Delta N} &= g_{\pi N \Delta} \bar{\psi}_i^\mu \left(g_{\mu\nu} + z \gamma_\mu \gamma_\nu \right) w_i^\nu \psi_N + ... \end{aligned} \tag{17}$$

where the dots denote higher order counterterm contributions and z corresponds to the leading-order pion-nucleon-delta off-shell coupling constant.

Rewriting the lagrangians of Eq.(16) in terms of the spin 3/2 heavy baryon components T_μ and G_μ, and the corresponding "light" and "heavy" spin 1/2 components N, h, defined as

$$\begin{aligned} N(x) &= P_v^+ \psi_N \exp(iMv \cdot x) \\ h(x) &= P_v^- \psi_N \exp(iMv \cdot x), \end{aligned} \tag{18}$$

we find the general heavy baryon lagrangians

$$\begin{aligned} L_N &= \bar{N} \mathcal{A}_N N + \left(\bar{h} \mathcal{B}_N N + h.c. \right) - \bar{h} \mathcal{C}_N h \\ L_{\Delta N} &= \bar{T} \mathcal{A}_{\Delta N} N + \bar{G} \mathcal{B}_{\Delta N} N + \bar{h} \mathcal{D}_{N\Delta} T + \bar{h} \mathcal{C}_{N\Delta} G + h.c. \\ L_\Delta &= \bar{T} \mathcal{A}_\Delta T + \left(\bar{G} \mathcal{B}_\Delta T + h.c. \right) - \bar{G} \mathcal{C}_\Delta G. \end{aligned} \tag{19}$$

Note that we have used the *same* mass M in the definition of heavy delta and nucleon fields respectively. This is necessary in order that all exponential factors drop out in Eq. (19). The matrices $\mathcal{A}_N$, $\mathcal{B}_N$, ..., $\mathcal{C}_\Delta$ admit a small energy scale expansion of the form

$$\mathcal{A}_\Delta = \mathcal{A}_\Delta^{(1)} + \mathcal{A}_\Delta^{(2)} + ..., \tag{20}$$

where $\mathcal{A}_\Delta^{(n)}$ is of order ϵ^n. As emphasized in the introduction, we denote by ϵ small quantities of order p, like m_π or soft momenta, *as well as* the mass difference $\Delta = M_\Delta - M_N$. This mass difference is distinct from the pion mass in the sense that it stays finite in the chiral limit. However, in the physical world, Δ and m_π are of the same magnitude. We therefore adhere to a simultaneous expansion in both quantities. It is only through this small scale expansion that we obtain a systematic low energy expansion of the $\pi N\Delta$-system.

To make this more explicit, consider the leading order matrices $\mathcal{A}_N^{(1)}$, $\mathcal{A}_{\Delta N}^{(1)}$ and $\mathcal{A}_\Delta^{(1)}$. Choosing the heavy baryon mass parameter $M = M_N$, we obtain

$$\begin{aligned} \mathcal{A}_N^{(1)} &= i(v\cdot D) + g_A(S\cdot u) \\ \mathcal{A}_{\Delta N}^{(1)} &= g_{\pi N\Delta} w_\mu^i \\ \mathcal{A}_\Delta^{(1)} &= -\left[i\ v\cdot D^{ij} - \Delta\ \delta^{ij} + g_1\ S\cdot u^{ij}\right]\ g_{\mu\nu} \end{aligned} \tag{21}$$

where S_μ denotes the Pauli-Lubanski spin vector. One can easily see from Eq.(21) that our formalism produces the exact SU(2) analogues of the spin 1/2 [7] and spin 3/2 [8] lagrangians of Jenkins and Manohar. Furthermore, as expected, the O(ϵ) heavy baryon lagrangians Eq.(21) are free of the off-shell couplings z, g_2, g_3 [7] In our formalism off-shell couplings will only start contributing at $\mathcal{O}(\epsilon^2)$ via $\mathcal{B}$ and $\mathcal{D}$ matrices. Explicit expressions for the expansions of $\mathcal{B}_\Delta$, $\mathcal{C}_\Delta$ *etc.* will not be displayed here but can be found in ref. [13].

From Eq.(21) we determine the SU(2) HBChPT propagator for the delta field:

$$i\ S_{\mu\nu}^{3/2}(v\cdot k) = \frac{-i}{v\cdot k - \Delta + i0^+}\ P_{\mu\nu}^{3/2}\ \xi_{I=3/2}^{ij} \tag{22}$$

where $P_{\mu\nu}^{3/2}$ is a spin 3/2 projector [8]

$$P_{\mu\nu}^{3/2} = P_v^+\ P_{(33)\mu\nu}^{3/2}\ P_v^+ = g_{\mu\nu} - v_\mu v_\nu + \frac{4}{3}\ S_\mu S_\nu \tag{23}$$

[7] This point has also been emphasized in recent work by Napsuciale and Lucio [21].

and

$$\xi^{ij}_{I=3/2} = \delta^{ij} - \frac{1}{3}\,\tau^i\tau^j \tag{24}$$

denotes an isospin 3/2 projector. From Eq.(22) one can see that the delta propagator counts as ϵ^{-1} in our expansion scheme.

The final step is again in analogy to the heavy mass formalism for spin 1/2 systems. Shifting variables and completing the square, we obtain the effective action

$$S_{\rm eff} = \int d^4x \left\{ \bar{T}\tilde{\mathcal{A}}_\Delta T + \bar{N}\tilde{\mathcal{A}}_N N + \left[\bar{T}\tilde{\mathcal{A}}_{\Delta N} N + h.c.\right] \right\} \tag{25}$$

with

$$\begin{aligned} \tilde{\mathcal{A}}_\Delta &= \mathcal{A}_\Delta + \gamma_0 \tilde{\mathcal{D}}^\dagger_{N\Delta}\gamma_0 \tilde{\mathcal{C}}_N^{-1}\tilde{\mathcal{D}}_{N\Delta} + \gamma_0\mathcal{B}^\dagger_\Delta\gamma_0\mathcal{C}^{-1}_\Delta\mathcal{B}_\Delta \\ \tilde{\mathcal{A}}_N &= \mathcal{A}_N + \gamma_0\tilde{\mathcal{B}}^\dagger_N\gamma_0\tilde{\mathcal{C}}_N^{-1}\tilde{\mathcal{B}}_N + \gamma_0\mathcal{B}^\dagger_{\Delta N}\gamma_0\mathcal{C}^{-1}_\Delta\mathcal{B}_{\Delta N} \\ \tilde{\mathcal{A}}_{\Delta N} &= \mathcal{A}_{\Delta N} + \gamma_0\tilde{\mathcal{D}}^\dagger_{N\Delta}\gamma_0\tilde{\mathcal{C}}_N^{-1}\tilde{\mathcal{B}}_N + \gamma_0\mathcal{B}^\dagger_\Delta\gamma_0\mathcal{C}^{-1}_\Delta\mathcal{B}_{\Delta N} \end{aligned} \tag{26}$$

and

$$\begin{aligned} \tilde{\mathcal{C}}_N &= \mathcal{C}_N - \mathcal{C}_{N\Delta}\mathcal{C}^{-1}_\Delta\gamma_0\mathcal{C}^\dagger_{N\Delta}\gamma_0 \\ \tilde{\mathcal{B}}_N &= \mathcal{B}_N + \mathcal{C}_{N\Delta}\mathcal{C}^{-1}_\Delta\mathcal{B}_{\Delta N} \\ \tilde{\mathcal{D}}_{N\Delta} &= \mathcal{D}_{N\Delta} + \mathcal{C}_{N\Delta}\mathcal{C}^{-1}_\Delta\mathcal{B}_\Delta. \end{aligned} \tag{27}$$

Eq.(26) represents the master formula of our treatment of a coupled spin 1/2 - spin 3/2 system in HBChPT. All 1/M corrected vertices can be directly obtained by calculating the appropriate matrices $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ to any order desired. The new terms proportional to $\mathcal{C}_\Delta^{-1}$ and $\mathcal{C}_N^{-1}$ are given entirely in terms of coupling constants of the lagrangian for relativistic fields. This guarantees reparameterization[8] and Lorentz invariance [14, 23]. Furthermore, all such terms are $1/M$ suppressed. The effects of the heavy degrees of freedom (both spin 3/2 and 1/2) thus show up only at order ϵ^2. Note also that the effective NN-, $N\Delta$- and $\Delta\Delta$-interactions all contain contributions from both heavy N- and Δ-exchange respectively.

In the above formalism, it is understood that at each order one must also include the most general counterterm lagrangian consistent with chiral

[8]Imposing the conditions of reparameterization invariance on the leading order lagrangians Eq.(21) as developed by Luke and Manohar for the case of HQET [14] provides an alternative way to derive the 1/M corrected vertices.

symmetry, Lorentz invariance, and the discrete symmetries P and C. As should be clear, it is crucial to write this counterterm lagrangian in terms of relativistic fields. The choice of variables Eqs.(13,14) and Eq.(18) yields then automatically the contributions to matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$. Only these objects have a well defined small scale expansion—since $S_{\rm eff}$ is written entirely in terms of heavy baryon fields, derivatives count as order ϵ, quark masses as order ϵ^2 etc. In order to calculate a given process to order ϵ^n, it thus suffices to construct matrices $\mathcal{A}$ to the same order, ϵ^n, $\mathcal{B}$ and $\mathcal{D}$ to order ϵ^{n-1}, and $\mathcal{C}$, $\tilde{\mathcal{C}}$ to order ϵ^{n-2}. Note that since the propagator of the T_μ field counts as order ϵ^{-1}, one-particle reducible diagrams as in Fig. 1 have also to be considered. [9] Finally one has to add all loop-graphs contributing at the order one is working. The relevant diagrams can be found by straightforward power counting in ϵ.

3 1/M corrections to threshold π^0 photoproduction to O(ϵ^3)

As an elementary example of the use of this formalism, consider neutral pion photoproduction[10]. A phenomenological analysis of the influence of $\Delta(1232)$ in this process was given in [24]. At threshold in the small scale expansion as described above, to order ϵ^3, this amounts to calculating all 1-loop graphs with vertex insertions from $\tilde{\mathcal{A}}^{(1)}$ as well as all tree graphs with vertices derived from up to and including $\tilde{\mathcal{A}}^{(3)}$ to the process $\gamma p \to p\pi^0$. The electric dipole amplitude is then related to the cross section in the center of mass frame through [25]

$$(E_{0+})^2 = \frac{|\mathbf{k}|}{|\mathbf{q}|}\frac{d\sigma}{d\Omega}|_{|\mathbf{q}|\to 0}, \tag{28}$$

where $\mathbf{k}$ and $\mathbf{q}$ are the photon and pion three-momenta, respectively.

It is most convenient to break up the calculation into one-particle irreducible (1PI) diagrams. Possible loop graphs involving the leading order vertices of Eq. (21) start at order ϵ^3 in our counting. However, one can check

[9] This could be avoided by imposing systematically the equations of motion for asymptotic states, as done in ref. [23] for the nucleon sector.

[10] A complete HBChPT analysis of the effects of $\Delta(1232)$ on S and P-wave multipoles in pion photoproduction is in preparation.

explicitly that, aside from the well-known triangle graph contribution [15, 16], *at threshold* there exist no other loop effects to $E_{0+}^{\pi^0 p}$ involving $\Delta(1232)$ to this order in the ϵ-expansion.

The photoproduction amplitude is then given by the diagrams of Fig. 1 a)-c). Due to the structure of $\mathcal{A}_N^{(1)}$ and $\mathcal{A}_{\Delta N}^{(1)}$, several simplifications appear: at threshold the nonvanishing 1PI subgraphs of Fig. 1 are at least $O(\epsilon^2)$. For the nucleon-nucleon transition this is well known. For the nucleon-delta vertex, the situation is similar. To leading order, the $\gamma N\Delta$ vertex does not exist, the $\gamma\pi N\Delta$ coupling vanishes for the neutral pion, and the $\pi N\Delta$ vertex is proportional to q^μ, which, when contracted with projection operator $P_{\mu\nu}^{3/2}$ associated with the delta-propagator, vanishes at threshold. Moreover, 1PI-vertices without pions or photons attached, also only begin at $O(\epsilon^2)$; this is the reason why no tree diagrams with more than a single propagator need be considered. Thus the 1PI one-photon and one-pion vertices are needed to $O(\epsilon^2)$, while the 1PI $\pi\gamma$ vertices are to be calculated to $O(\epsilon^3)$.

Analyzing Eq.(26) we evaluate the following structures for our calculation of $E_{0+}^{\pi^0 p}$ at $\mathrm{O}(\epsilon^3)$:

vertex	lagrangian
$\mathrm{O}(\epsilon^2)$ γNN	$\mathcal{A}_N^{(2)}$ and $\gamma_0\mathcal{B}_N^{(1)\,\dagger}\gamma_0\mathcal{C}_N^{(0)\,-1}\mathcal{B}_N^{(1)}$
$\mathrm{O}(\epsilon^2)$ $\pi^0 NN$	$\gamma_0\mathcal{B}_N^{(1)\,\dagger}\gamma_0\mathcal{C}_N^{(0)\,-1}\mathcal{B}_N^{(1)}$
$\mathrm{O}(\epsilon^2)$ $\gamma\Delta N$	$\mathcal{A}_{\Delta N}^{(2)}$
$\mathrm{O}(\epsilon^2)$ $\pi^0\Delta N$	$\gamma_0\mathcal{B}_\Delta^{(1)\,\dagger}\gamma_0\mathcal{C}_\Delta^{(0)\,-1}\mathcal{B}_{\Delta N}^{(1)}$
$\mathrm{O}(\epsilon^2)$ $\pi^0\gamma NN$	$\gamma_0\mathcal{B}_N^{(2)\,\dagger}\gamma_0\mathcal{C}_N^{(0)\,-1}\mathcal{B}_N^{(1)} + h.c.$
$\mathrm{O}(\epsilon^3)$ $\pi^0\gamma NN$	vanishes at threshold

Vertices which do not involve spin 3/2 particles can be taken from [11, 23]. Summing up all the nucleon-only contributions, including the triangle graphs, we recover the LET Eq.(1), as expected. We now proceed to analyse the effects of 1/M corrected vertices involving $\Delta(1232)$.

Due to the fact that the photoexcitations of $\Delta(1232)$ only start with the M1 transition, there is no $\gamma N\Delta$ interaction at $\mathrm{O}(\epsilon)$. Consequently, there is also no 1/M corrected vertex at $\mathrm{O}(\epsilon^2)$. However, the well-known relativistic counterterm lagrangian [13, 22]

$$\mathcal{L}_{c.t.}^{\gamma N\Delta} = \frac{ib_1}{2M_N}\bar{\psi}_i^\mu\left(g_{\mu\nu} + y\gamma_\mu\gamma_\nu\right)\gamma_\rho\gamma_5\,\frac{1}{2}\,Tr\left[f_+^{\rho\nu}\,\tau^i\right]\psi_N, \tag{29}$$

provides a large part of the M1 transition strength and leads to the heavy baryon structure

$$\mathcal{A}_{\Delta N}^{(2)} = \frac{ib_1}{M_N} S_\nu \, \frac{1}{2} \, Tr\left[f_+^{\nu\lambda} \, \tau^i\right], \tag{30}$$

which we use in the diagrams of Fig. 1b.

The leading order $\pi N\Delta$ vertex does not contribute at threshold, but its $1/M$ *corrected structure* can provide a contribution to the s-waves ! Multiplying out the relevant matrices, we find:

$$\left(\gamma_0 \mathcal{B}_\Delta^{(1)\,\dagger} \gamma_0 \mathcal{C}_\Delta^{(0)\,-1} \mathcal{B}_{\Delta N}^{(1)}\right)_{\Delta N} = \frac{-i}{M_N} \, g_{\pi N\Delta} \, D_\mu^{ij} (v \cdot w^j). \tag{31}$$

These two vertices then lead to an O(ϵ^3) contribution of $\Delta(1232)$ to the process $\gamma p \to \pi^0 p$ at threshold, given by the diagrams in Fig. 1b:

$$E_{0+}^{\Delta} = \frac{e}{8\pi} \frac{4 b_1 g_{\pi N\Delta}}{9 F_\pi M_N^2} \frac{m_\pi^3}{m_\pi + \Delta}. \tag{32}$$

This new contribution of Eq. (32) is distinct in the following sense—in the chiral limit, it scales like m_π^3, *i.e.* the corresponding photoproduction amplitude is of order p^4. The LET Eq.(1) is therefore not violated by this term. There are *many* other terms which are of O(p^4) [17], but Eq.(32) is the *only* term which is of order ϵ^3 due to the 1/M corrections. In the physical world of finite pion mass, E_{0+}^{Δ} is in principle of the same order of magnitude as the p^3 effects. Moreover, it has the opposite sign to the large p^3 terms in the LET for E_{0+}.

In order to give a numerical estimate for E_{0+} to O(ϵ^3), we add Eq. (32) and Eq. (1). Utilizing $b_1 = -2.30 \pm 0.35, g_{\pi N\Delta} = 1.5 \pm 0.2$ [22], we find

$$E_{0+}^{\pi^0 p} = -\,\frac{e g_{\pi N}}{8\pi M_N} \, \mu \times \begin{cases} +1 & \text{O}(\epsilon^2) \text{ Kroll-Ruderman term} \\ -0.97 & \text{O}(\epsilon^3) \; N\pi \text{ loop graphs} \\ -0.35 & \text{O}(\epsilon^3) \text{ Nucleon Born terms} \\ +0.07 & \text{O}(\epsilon^3) \text{ 1/M corrected Delta terms} \end{cases}$$

$$\to E_{0+}^{\pi^0 p} \approx 0.8 \times 10^{-3}/m_\pi. \tag{33}$$

Comparing with the number extracted from the most recent experiment [18], $E_{0+} = (-1.31 \pm 0.08) \times 10^{-3}/m_{\pi^+}$, which is in agreement with a chiral O(p^4) calculation [17], we conclude that it is mandatory to calculate the E_{0+}

multipole to O(ϵ^4). As the O(p^4) calculation shows, $N\pi$ loop graphs at the next order cancel to a large extent the big contribution from the triangle graphs. It will be interesting to see how big the $\Delta(1232)$ effects are at that order. Work in this direction is under progress.

To conclude, we have presented a systematic low energy expansion of HBChPT including spin 3/2 resonances. As an application, we have considered π^0 photoproduction at threshold and have found the leading contribution of $\Delta(1232)$ to $E_{0+}^{\pi^0 p}$ to be of order $m_\pi^3/(m_\pi + \Delta)$. Many other processes associated with the $\pi N\Delta$-system can be treated with the formalism described here.

References

[1] See, *e.g.* J.F. Donoghue, E. Golowich and B.R. Holstein, *Dynamics of the Standard Model*, Cambridge University Press, New York (1992).

[2] H. Pagels, Phys. Rept. **C16** (1975) 219.

[3] S. Weinberg, Physica **96A** (1979) 327.

[4] H. Bijnens, H. Sonoda and M.B. Wise, Nucl. Phys. **B261** (1985) 185.

[5] J. Gasser and H. Leutwyler, Ann. Phys. (NY) **158** (1984) 142 and Nucl. Phys. **B250** (1985) 465.

[6] J. Gasser, M.E. Sainio, and A. Svarc, Nucl. Phys. **B307** (1988) 779.

[7] E. Jenkins and A.V. Manohar, Phys. Lett. **B255** (1991) 558.

[8] E. Jenkins and A.V. Manohar, Phys. Lett. **B259** (1991) 353 and "Baryon Chiral Perturbation Theory" in "Effective Field Theories of the Standard Model," ed. U.-G. Meißner, World Scientific, Singapore (1992).

[9] H. Georgi, Phys. Lett. **B240** (1990) 447; A. Falk, Nucl. Phys. **B378** (1992) 79.

[10] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. **B368** (1992) 204; M. Neubert, Phys. Rept. **245** (1994) 259.

[11] V. Bernard, N. Kaiser, and U.-G. Meißner, Int.J.Mod.Phys. **E4** (1995) 193.

[12] M.N. Butler, M.J. Savage and R.P. Springer, Nucl. Phys. **B399** (1993) 69.

[13] T.R. Hemmert, B.R. Holstein, and J. Kambor, "Chiral Lagrangians and $\Delta(1232)$ Interactions", IPN Orsay preprint IPNO/TH 95-61, in preparation.

[14] M. Luke and A.V. Manohar, Phys. Lett. **B286** (1992) 348.

[15] V. Bernard, J. Gasser, N. Kaiser, and U.-G. Meißner, Phys. Lett. **B268** (1991) 291.

[16] V. Bernard, N. Kaiser, J. Kambor, and U.-G. Meißner, Nucl. Phys. **B388** (1992) 315.

[17] V. Bernard, N. Kaiser, and U.-G. Meißner, Z.Phys. **C70** (1996) 483.

[18] M. Fuchs et al., Phys. Lett. **B368**, 20 (1996); J.C. Bergstrom et al., Phys. Rev. **C53**, R1052 (1996).

[19] V. Pascalutsa, "On the Interactions of Spin 3/2 Particles", University of Utrecht preprint THU-94/21.

[20] L.M. Nath, B. Etemadi, and J.D. Kimel, Phys. Rev. **D3** (1971) 2153; S. Kamefuchi, L. O'Raifeartaigh, and A. Salam, Nucl. Phys. **28** (1961) 529.

[21] M. Napsuciale and J.L. Lucio M., "Spin 3/2 Interacting Fields and Heavy Baryon Chiral Perturbation Theory", hep-ph/9605262.

[22] R.M. Davidson, N.C. Mukhopadhyay, and R.S. Wittman, Phys. Rev. **D43** (1991) 71.

[23] G. Ecker and M. Mojžiš, Phys. Lett. **B365** (1996) 312.

[24] R.M. Davidson and N.C. Mukhopadhyay, Phys. Rev. Lett. **60** (1988) 748.

[25] For reviews see *e.g.* D. Drechsel and L. Tiator, J. Phys. **G18** (1992) 449; U.-G. Meißner, Strange Twists in Neutral Pion Photo/Electro - Production, University of Bonn preprint TK 95 27.

Figure 1: Nucleon pole (a), delta pole (b), and contact (c) diagrams contributing to pion photoproduction in the heavy baryon approach.