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SU(3) Baryon Chiral Perturbation Theory and Long Distance Regularization*

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Abstract

The use of SU(3) chiral perturbation theory in the analysis of low energy meson-baryon interactions is discussed. It is emphasized that short distance effects, arising from propagation of Goldstone bosons over distances smaller than a typical hadronic size, are model-dependent and can lead to a lack of convergence in the SU(3) chiral expansion if they are included in loop diagrams. In this paper we demonstrate how to remove such effects in a chirally consistent fashion by use of a cutoff and demonstrate that such removal ameliorates problems which have arisen in previous calculations due to large loop effects.

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1 The Problem

The low energy phenomenology of baryons is relatively simple. In the 1960's, this simplicity was evidenced by the successes of SU(3) symmetry. Indeed, masses and couplings can be well described by SU(3) invariant interactions with SU(3) breaking at the 5 - 25% level. In the present era we have come to understand this invariance in terms of QCD and the underlying quark substructure of baryons—SU(3) relations work because the effects of the s-u-d mass splittings are relatively small. Moreover, the quark model even allows us to understand many details of the pattern of SU(3) symmetry breaking. Overall, most features of the static properties of baryons are reasonably well understood.

It has also been realized that the old SU(3) results represent merely the lowest order terms of an expansion in energy and quark masses in a rigorous effective field theory framework which exploits the (broken) $SU(3)_L \times SU(3)_R$ chiral symmetry of the QCD Lagrangian. The higher order terms in this expansion can be calculated via the technique called "chiral perturbation theory", which has already been highly developed and successfully applied within the sector of Goldstone boson interactions.[1] In the related case of baryon-Goldstone interactions, there has also been a great deal of activity using methods generalized from the purely mesonic situation.[2]

However, the problem is that traditional SU(3) baryon chiral perturbation theory does not appear to work well. As generally applied, it does not manifest the approximate SU(3) symmetry that one sees in the real world, in that SU(3) breaking corrections in loop diagrams often appear at the 100% level. It is particularly distressing that these effects come from the most apparently model-independent parts of the theory—the nonanalytic chiral loops. With some parameter fitting, it appears in practice that such effects can be compensated by positing equally large effects from the effective Lagrangian at higher order in the chiral expansion. However, this leads to worries about convergence. In any event, the simplicity evident in baryon physics has become lost. In its conventional manifestation then, SU(3) baryon chiral perturbation theory does not represent a good first approximation to baryon physics.

In this paper we will suggest a resolution to this problem in terms of a reformulation of baryon chiral perturbation theory within a framework which is better suited to phenomenological applications. Before we turn to a diagnosis, let us, however, demonstrate the nature of the problem by observing several pertinent results. In each case we defer the specifics of the chiral analysis until later in the paper and simply quote results in order to convince the reader that a problem exists.

i) Baryon masses can be understood by noting that the quark mass non-degeneracy arises from a component of $\mathcal{L}_{\rm QCD}$ which can be represented in terms of a Lorentz scalar SU(3) octet $\bar{q}\lambda_8 q$ operator. To first order in symmetry breaking one can then write the baryon octet masses in terms of an SU(3) invariant term $\hat{M}_0$ plus octet f_m, d_m couplings—

$$\begin{aligned} M_N &= \hat{M}_0 - 4m_K^2 d_m + 4(m_K^2 - m_\pi^2) f_m \\ M_\Lambda &= \hat{M}_0 - \frac{4}{3}(4m_K^2 - m_\pi^2) d_m \\ M_\Sigma &= \hat{M}_0 - 4m_\pi^2 d_m \\ M_\Xi &= \hat{M}_0 - 4m_K^2 d_m - 4(m_K^2 - m_\pi^2) f_m \end{aligned} \tag{1}$$

Since the four octet baryon masses are represented in terms of effectively three parameters there is a corresponding sum rule—that of Gell-Mann and Okubo[3]—

$$\begin{aligned} M_\Sigma - M_N &= \frac{1}{2}(M_\Xi - M_N) + \frac{3}{4}(M_\Sigma - M_\Lambda) \\ \text{Expt.}: \quad 254\text{MeV} &= 248\text{MeV} \end{aligned} \tag{2}$$

which is satisfied experimentally at the 3% level.

When analyzed in the usual fashion in chiral perturbation theory, however, this simplicity is lost. At one loop—$\mathcal{O}(q^3)$—order the chiral loop corrections are found to be extremely large[4]

$$\begin{aligned} &\delta M_N = -0.31\text{ GeV}; \quad \delta M_\Sigma = -0.67\text{ GeV}; \\ &\delta M_\Lambda = -0.66\text{ GeV}; \quad \delta M_\Xi = -1.02\text{ GeV} \end{aligned} \tag{3}$$

such that, *e.g.*, the Ξ mass receives a 100% correction. This calculation has also been carried out to—$\mathcal{O}(q^4)$—by Borasoy and Meissner[5], who quote their results as

$$M_N = \bar{M}(1 + 0.34 - 0.35 + 0.24)$$

$$\begin{aligned} M_\Sigma &= \bar{M}(1+0.81-0.70+0.44) \\ M_\Lambda &= \bar{M}(1+0.69-0.77+0.54) \\ M_\Xi &= \bar{M}(1+1.10-1.16+0.78) \end{aligned} \tag{4}$$

where the non-leading terms above refer to the contribution from $\mathcal{O}(q^2)$ counterterms, nonanalytic pieces of $\mathcal{O}(q^3)$, and $\mathcal{O}(q^4)$ counterterms respectively. Obviously, the contribution from higher order terms is far larger than one expects and the series does not display obvious convergence. Also, the Gell-Mann-Okubo deviation is found to be five times larger than experiment.

ii) Baryon axial couplings can be related by noting that the weak axial current arises from an SU(3) octet $\bar{q}'\gamma_\mu\gamma_5 q$ structure. Thus to leading order in SU(3) the various weak matrix elements can be represented in terms of simple f_A, d_A couplings. A fit to the ten experimentally measured semileptonic hyperon decay rates is found to yield reasonable results, with $\chi^2/$ d.o.f. ~ 1. SU(3) breaking in the decay rates is noticeable, but the amount of SU(3) breaking is never above 5%.[6] One can explore quark models and find that they generate breaking that is of about this magnitude, and the challenge then is to fit the pattern of breaking.

When chiral loops are calculated,[7] one finds logarithmic dependence on the meson masses that leads to significant SU(3) breaking. Typically these effects are too large. Numerically, choosing a renormalization scale $\mu \sim 1$ GeV, typical leading log corrections are found to be at the 30-50% level and a fit to the experimental hyperon decay rates finds a much increased chi-squared—the chiral corrections go in the wrong direction!

iii) S-wave nonleptonic hyperon decay amplitudes can be related by using the feature that the octet component of the weak Hamiltonian is dominant over its 27-dimensional counterpart by a factor of twenty or so, plus using chiral symmetry to relate the experimental pion decay amplitudes to simpler baryon to baryon matrix elements. This allows a fit in terms of octet f_w, d_w parameters: Such a representation yields a very good fit to the experimental amplitudes in that the two independent predictions:[1]

$$
\begin{aligned}
A(\Sigma^+_+) &= 0 \quad \text{vs. } 0.13\times 10^{-7} \text{ (expt.)} \\
\sqrt{3}A(\Sigma^+_0) &- 2A(\Xi^-_-) - A(\Lambda^0_-) = 0 \quad \text{vs. } 0.11\times 10^{-7} \text{ (expt.)}
\end{aligned} \tag{5}
$$

are, since the typical size of an s-wave amplitude is $\sim 4\times 10^{-7}$, both reasonably well satisfied by the data. [2]

In baryon chiral perturbation theory, the chiral loop corrections to individual terms are found to be at the 30-50% level,[7] and a large correction to the Lee-Sugawara relation is found

$$
\sqrt{3}A(\Sigma^+_0) - 2A(\Xi^-_-) - A(\Lambda^0_-) \approx -6.4\times 10^{-7} \tag{6}
$$

which is in considerable disagreement with the experimental number. The other lowest order prediction—$A(\Sigma^+_+) = 0$—is not affected by chiral logarithms.

One can also see the problem with chiral convergence of individual terms. Indeed, a comprehensive analysis of the problem up to second order counterterms has given[9]

$$
\begin{aligned}
A(\Lambda^0_0) &= 2.35(1 + 0.62 - 0.65)\times 10^{-7} \\
A(\Sigma^+_0) &= 3.09(1 + 0.30 - 0.32)\times 10^{-7} \\
A(\Sigma^+_+) &= 0\times 10^{-7} \\
A(\Xi^0_0) &= 3.06(1 + 0.40 - 0.36)\times 10^{-7}
\end{aligned} \tag{7}
$$

where the various contributions are from lowest order, nonanalytic components, and next order counterterms respectively.

iv) Hyperon magnetic moments can be related to one another since they arise from an SU(3) octet $\bar{q}'\gamma_\mu q$ structure. Then to leading order the

[1]Note that the second of these results is the Lee-Sugawara sum rule.[8]

[2]It is, of course, possible to apply a similar analysis to the corresponding P-wave amplitudes. However, in this case the leading piece of each amplitude involves a significant cancellation from from pairs of baryon pole diagrams, so that there is large and very model dependent sensitivity to higher order chiral contributions. Thus we do not analyze this case.

moments can be written in terms of simple f_μ, d_μ couplings—

$$\begin{aligned}
\mu_p &= \mu_{\Sigma^+} = \frac{1}{3}d_\mu + f_\mu \\
\mu_n &= 2\mu_\Lambda = \mu_{\Xi^0} = -\frac{2}{3}d_\mu \\
\mu_{\Sigma^-} &= \mu_{\Xi^-} = \frac{1}{3}d_\mu - f_\mu \\
\mu_{\Lambda\Sigma} &= \sqrt{3}\mu_{\Sigma^0} = \frac{1}{\sqrt{3}}d_\mu
\end{aligned} \tag{8}$$

The experimental moments are in approximate (but not outstanding) agreement with these predictions. (Although it is not relevant for our considerations here we note that the heavier mass, and hence smaller magnetic moment, of the strange quark explains most of the observed SU(3) breaking.)

Again the chiral corrections are large and harmful. Numerically, picking a renormalization scale $\mu = 1$ GeV, the nonanalytic corrections are at the 50-90% level, and make enormous modifications of the lowest order results. As shown by Caldi and Pagels, there remain three relations, which are independent of these corrections and are in fact reasonably well satisfied by the experimental numbers:[10]

$$\mu_{\Sigma^+} = -2\mu_\Lambda - \mu_{\Sigma^-}, \ \mu_{\Xi^0} + \mu_{\Xi^-} + \mu_n = 2\mu_\Lambda - \mu_p, \ \mu_\Lambda - \sqrt{3}\mu_{\Lambda\Sigma} = \mu_{\Xi^0} + \mu_n \tag{9}$$

However, other relations pose significant problems for experimental agreement. Meissner and Steininger have performed a $\mathcal{O}(q^4)$ analysis of the problem and have shown that it is possible to get good agreement via a careful choice of counterterms.[11] The convergence of the chiral expansion is again a possible problem, as the contributions of terms of successive orders is found to be

$$\begin{aligned}
\mu_p &= 4.69(1 - 0.57 + 0.16) = 2.79 \\
\mu_n &= -2.85(1 - 0.36 + 0.03) = -1.91 \\
\mu_{\Sigma^+} &= 4.69(1 - 0.72 + 0.24) = 2.46 \\
\mu_{\Sigma^0} &= 1.43(1 - 0.93 + 0.38) = 0.65 \\
\mu_{\Sigma^-} &= -1.83(1 - 0.41 + 0.04) = -1.16
\end{aligned}$$

$$\begin{aligned} \mu_{\Lambda\Sigma} &= 2.47(1-0.57+0.18) = 1.51 \\ \mu_{\Xi^0} &= -2.85(1-0.95+0.39) = -1.25 \\ \mu_{\Xi^-} &= -1.83(1-0.57+0.18) = -0.65 \end{aligned} \tag{10}$$

We see in each case then that the chiral corrections are large and in each situation the leading nonanalytic components destroy the good experimental agreement which exists at lowest order. There is something clearly ineffective about this procedure. For a technique that has aspirations of rigor, this is a dismaying situation We will show below that the problem resides in a spurious short-distance contribution that appears in loop diagrams when they are regularized dimensionally. We propose that we should keep only the long distance parts of the loops, and propose a cutoff regularization that accomplishes this.

2 Effective Field Theory: Separating Long and Short Distances

Effective field theory is a technique for describing the low energy limit of a theory. It is an "effective" description because it uses the degrees of freedom and the interactions which are correct at low energy. All the features of the high energy portion of the theory are captured in the parameters of a general local effective Lagrangian which describes the low energy vertices. Using these interactions one treats the low energy dynamics in a complete field theoretic description.

Within such a treatment, one encounters loop diagrams, in which the integration over the momenta includes both low energy and high energy components. While the low energy portion is fully correct within the effective theory, the high energy portion is not. One might worry then about the inclusion of such incorrect high-energy/short-distance physics present in loops. However, this is not a problem in general since this high energy effect has the same structure as the terms in the general local Lagrangian, meaning that any incorrect loop contribution can be compensated by a shift of the parameters of the Lagrangian. As an example, the ultraviolet divergences in the effective theory are all absorbed by defining renormalized parameters.

In practice, there is a situation where such loop effects *can* cause problems. This occurs if the residual short distance contributions are large even *after* renormalization. A large and incorrect short distance effect can still be removed by the adjustment of parameters, but those parameters must consequently also be large. We then obtain an expansion which is of the form

$$M \sim M_0(1 - 1 + 1 - 1 + ...) \tag{11}$$

where each term in the expansion is sizeable and there is no clear convergence. If one were able to carry out the process to all orders, one would, of course, still get the right answer. However, at any finite order, the incorrect short-distance phyiscs in loops has obscured the answer and the expansion is useless. While not formally "wrong", this procedure is ineffective, which is certainly a poor trait for an effective field theory.

In SU(3) baryon chiral perturbation theory, exactly this situation occurs when the theory is regularized dimensionally. We will show that the poor convergence described in the introduction follows largely from the short-distance component of loop diagrams. In order to provide a more effective description, we will then reformulate the theory using a cutoff which retains only the reliable—long-distance—portion of loop diagrams. This will result in improved phenomenology. Baryon effective field theory becomes even more effective with a long-distance regularization scheme!

In baryon chiral perturbation theory, the transition between short and long distance occurs around a distance scale of ~1 fermi, or a momentum scale of ~200 MeV. This corresponds to the measured size of a baryon and we will refer to it as the separation scale. The effective field theory treats the baryons and pions as point particles. This is appropriate for the very long distance physics - the "pion tail" is independent of whether the baryon is treated as a point particle or an extended object. However, for propagation at distances less then the separation scale, the point particle theory does not provide an accurate representation of the physics - the composite substructure becomes manifest below this point.

In the next section we focus on the specific Feynman integrals that arise in baryonic calculations. Our goal is to understand the structure of loops in this effective field theory by separating the short-distance and long-distance physics within the loop integral. The use of a cutoff representing the separation scale will allow us to show that the long distance physics is well behaved, and that dimensional regularization in practice contains large short distance contributions for these particular integrals.

3 Anatomy of Feynman integrals

We begin by performing an autopsy on a particular Feynman integral that appears in the baryon mass analysis. Consider the integral

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)(k^2 - m^2 + i\epsilon)} = -i\delta_{ij} \frac{I(m)}{24\pi} \tag{12}$$

where the right hand side simply defines the function $I(m)$. When regularized dimensionally this has the value

$$I_{dim-reg}(m) = m^3 \tag{13}$$

This integral is uniquely the source of nonanalytic corrections to baryon masses.

Some comments about the dimensionally regularized form are instructive.

i) The Feynman integral is cubicly divergent at high energy. However a peculiarity of the dimensionally regularized form is that the result is finite. This is not a problem and occurs at other times in dimensional regularization. However, it is one indication that this regularization scheme implies a particular short distance subtraction, which will in general leave behind finite effects from short distance.

ii) The only scale in the integral is the meson mass m. Therefore the relevant momenta in the integral all scale with m also. In the limit that m is very large, all of the relevant momenta correspond to high-energy/short-distance. This is an indication that as m grows the dimensionally regularized integral becomes totally dominated by short distance physics—below the separation scale.

iii) If we are interested in *only* the long distance component of the integral, this portion would fall off with increasing mass. At large m the meson progagator could be approximated by a constant (e.g., as we do for the W- boson mass in low energy weak interactions) and the low energy portion of the integral would fall as $1/m^2$.

iv) We would expect that the long distance portion of the integral would be largest for the smallest meson masses, and greatest for masssless Goldstone bosons. However the form Eq. 12 vanishes for massless particles and is very small for small meson masses.

These are all indications that an overall subtraction has taken place which confuses short and long distance physics. We cannot count on the dimensionally regularized form to yield only long distance physics—an implicit short-distance contribution is carried along also.

Now let us isolate the long distance component of the integral. Indeed it is possible to remove the short distance portion by use of a cutoff regularization, as we demonstrated in ref. [12]. Although an exponential cutoff in three-momentum was employed therein, for our purposes it is most convenient to employ a simple dipole regulator

$$(\frac{\Lambda^2}{\Lambda^2 - k^2})^2 \tag{14}$$

since it enables loop integration to be carried out in terms of simply analytic forms. However, the specific shape of the cutoff is irrelevant—a consistent chiral expansion can always be carried out to the order we are working.

The introduction of the dipole cutoff Eq. 14 yields

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)(k^2 - m^2 + i\epsilon)} (\frac{\Lambda^2}{\Lambda^2 - k^2})^2 = -i\delta_{ij} \frac{I_\Lambda(m)}{24\pi} \tag{15}$$

where

$$I_\Lambda(m) = \frac{1}{2}\Lambda^4 \frac{2m + \Lambda}{(m + \Lambda)^2} \tag{16}$$

Various comments on this form are appropriate

i) This integral is plotted in Figure 1 for $\Lambda = 400$MeV, along with its dimensionally regularized analog. We see that the cutoff result is much smaller than that of dimensional regularization for kaon and eta masses. Moreover, what matters for SU(3) breaking are *differences* in the intgral between pions kaons and etas, since a constant effect can be absorbed into chiral parameters. This difference is quite small for the cutoff version. We conclude that most of the dimensonally regularized Feynman integral for kaons and etas corresponds to short distance physics.

ii) The greatest contribution at long distance is seen in the cut-off scheme to come from massless mesons, as expected. As the meson mass increases, there is a decreasing effect from the long distance portion of the integral.

iii) We observe then that in the small mass limit

$$I(m) \stackrel{m<<\Lambda}{\longrightarrow} \frac{1}{2}\Lambda^3 - \frac{1}{2}\Lambda m^2 + m^3 + \ldots \tag{17}$$

i.e., $I(m)$ reduces to the dimensional regularization result—m^3—plus polynomial terms in Λ which are absent in the dimensional approach. In the next section, we will see explicitly how these polynomial terms can be absorbed in the renormalization of chiral parameters.

iv) In the opposite limit of a large mass compared to the cutoff

$$I(m) \stackrel{\Lambda<<m}{\longrightarrow} \frac{\Lambda^4}{m} - \frac{3}{2}\frac{\Lambda^5}{m^2} + \ldots \tag{18}$$

the function $I(m)$ is found to depend upon the pseudoscalar mass to inverse powers, meaning that the pion will contribute much more than its heavier eta or kaon counterparts, as we expect intuitively.

Our conclusion from studying the integral Eq. 12 is that the cut-off scheme picks out the long distance part of the integral, which behaves as expected on physical grounds. In contrast, the dimensional form carries with it implicit and large contributions from short distance physics. It is not suprising then that the large short distance effects dominate the analysis when dimensional regularization is employed and we will demonstrate this explicitly in the next sections.

Before returning to the physics, we analyze the other Feynman integrals which arise in the analysis of baryon physics. In the case of baryon axial couplings and s-wave hyperon decay the relevant heavy baryon integral which generates the nonanalytic terms in $m^2 \ln m^2$ is

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)^2 (k^2 - m^2 + i\epsilon)} = -i\delta_{ij} \frac{J(m^2)}{16\pi^2} \tag{19}$$

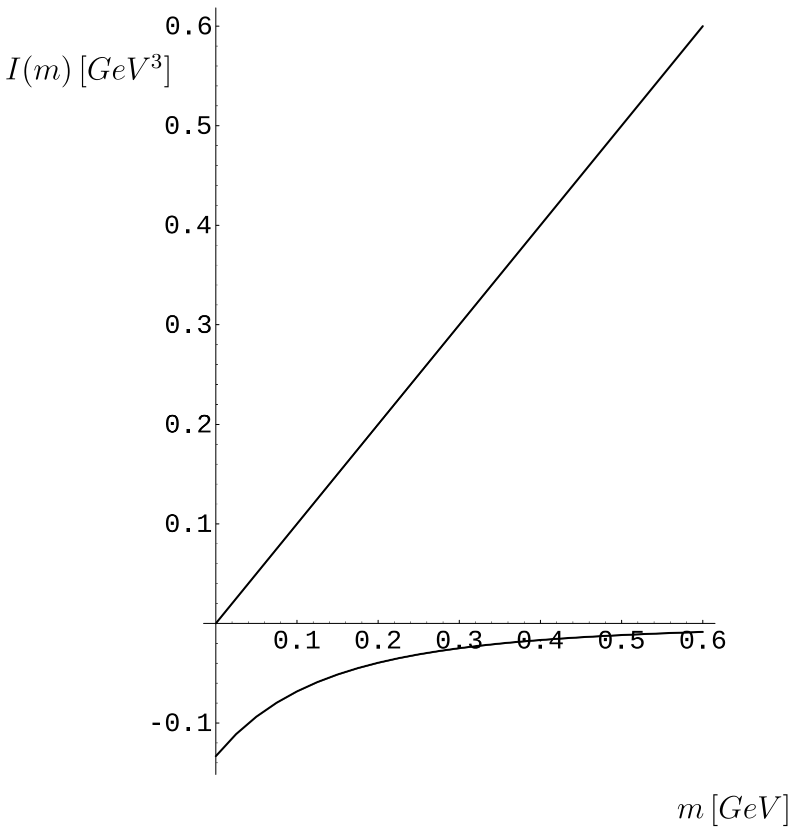

Figure 1: The integral $I(m)$ for the case of dimensional regularization ($I = m^3$) and in the cutoff scheme with $\Lambda = 400$ MeV.

In dimensional regularization the integral has the value

$$J_{dim-reg}(m^2) = m^2 \ln \frac{m^2}{\mu^2} \tag{20}$$

while the cutoff version is given by

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)^2 (k^2 - m^2 + i\epsilon)} \left(\frac{\Lambda^2}{\Lambda^2 - k^2}\right)^2 = -i\delta_{ij} \frac{J_\Lambda(m^2)}{16\pi^2} \tag{21}$$

with

$$J_\Lambda(m^2) = \frac{\Lambda^4 m^2}{(\Lambda^2 - m^2)^2} \ln \frac{m^2}{\Lambda^2} + \frac{\Lambda^4}{\Lambda^2 - m^2} \tag{22}$$

We plot these forms in Fig. 2. The behavior is qualitatively similar to that which occured with the previous integral—the dimensional form overstates the amount of SU(3) breaking. In addition the growth in the magnitude of the integral at large masses indicates that short distance physics dominates the dimensionally regulated form.

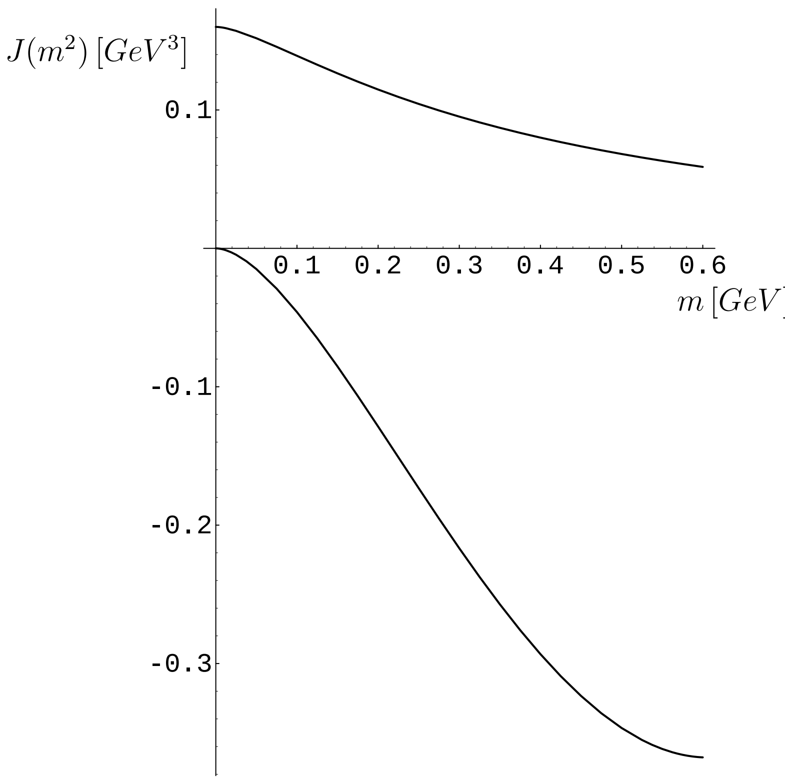

Figure 2: The integral $J(m^2)$. The lower curve is the result in dimensional regularization, whereas the upper curve shows the case of the cutoff scheme with $\Lambda = 400$ MeV.

The small and large mass limits of the cut-off form are given by

$$J(m^2) \overset{m^2 << \Lambda^2}{\longrightarrow} \Lambda^2 + m^2 \ln \frac{m^2}{\Lambda^2} + \ldots \tag{23}$$

and

$$J(m^2) \overset{m^2>>\Lambda^2}{\longrightarrow} \frac{\Lambda^4}{m^2} \ln \frac{m^2}{\Lambda^2} + \ldots \tag{24}$$

so that again our intutitive expectations are met.

Finally, we consider the integral which is relevant in the analysis of the magnetic moments

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)(k^2 - m^2 + i\epsilon)^2} = -i\delta_{ij} \frac{K(m)}{16\pi} \tag{25}$$

The dimensionally regularized form is given by

$$K_{dim-reg}(m) = m \quad . \tag{26}$$

Once again, the integral shows no sign of its true linear divergence, and grows at large values of m, indicating short distance dominance at large m. The use of the dipole cutoff yields

$$K(m) = -\frac{1}{3}\Lambda^4 \frac{1}{(\Lambda + m)^3} \quad , \tag{27}$$

which is plotted in Fig. 3 and is there compared to the dimensionally regularized form. Again we see that the long distance portion of the integral is well behaved and that dimensional regularization overstates the SU(3) breaking in the integral. The function $K(m)$ has the small and large mass limits

$$K(m) \overset{m<<\Lambda}{\longrightarrow} -\frac{1}{3}\Lambda + m + \ldots \tag{28}$$

and

$$K(m) \overset{m>>\Lambda}{\longrightarrow} -\frac{\Lambda^4}{3m^3} + \ldots \tag{29}$$

which have the expected qualitative forms.

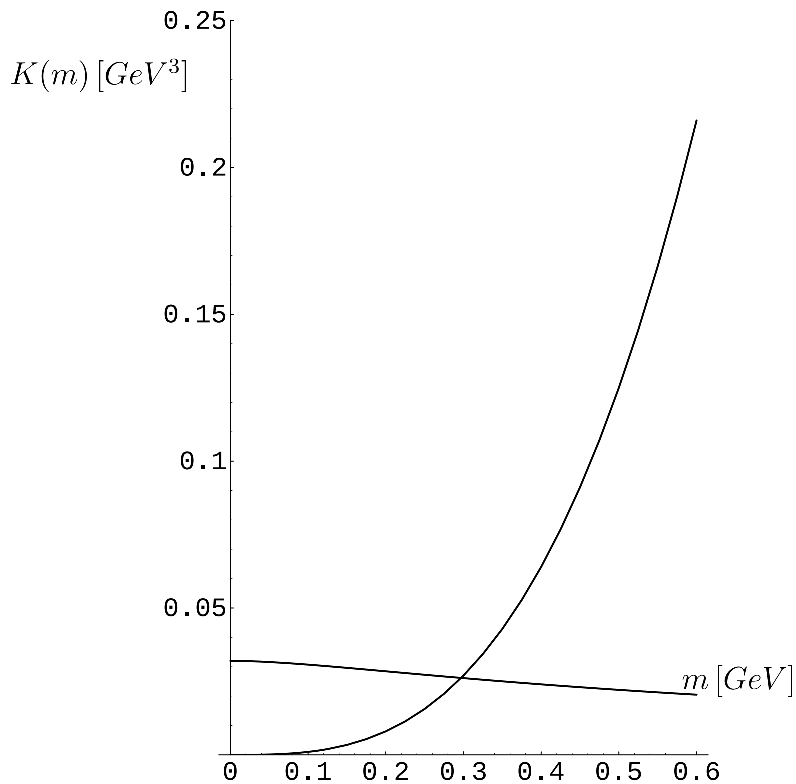

Figure 3: The integral $K(m)$. The upper curve is the result in dimensional regularization, whereas the lower one shows the case of the cutoff scheme with $\Lambda = 400$ MeV.

4 Theory and Phenomenology with a Cutoff

In the previous section, we used a cutoff as a tool to explore the long-distance portion of loop integrals. Since we find the long-distance portion to be well behaved, we suspect that the problems described in the introduction are in fact caused by the spurious inclusion of short-distance effects. We then turn to a different use for the cutoff - as a regularization technique for handling loop integrals.

Field theories can be applied with a variety of regularization methods. In the end, the resulting physics should be independent of the choice of regularization scheme. At first sight this suggests that it is unlikely that simply employing a change in regularization can have any impact on the problems mentioned in the introduction. However, we will see that the choice of a cutoff with a value around the separation scale will amount to a partial resummation of the chiral expansion and that this can be done without losing the generality of the effective field theory treatment. If we are right in our assesment that the problem is caused by spurious loop effects below the separation scale, this resummation can then lead to an improved procedure for phenomological applications.

We will first show explicitly how the standard chiral expansion is exactly reproduced for small values of the meson masses. A key ingredient of this demonstration is the renormalization of the chiral parameters. The loop integrals will often depend strongly on the value of the cutoff, and we will encounter integrals with Λ^3, Λ^2, Λ and $ln\Lambda$ dependences, where Λ represents the momentum space cutoff. However, this does not mean that the resulting physics will depend on the cutoff this strongly. Indeed, the final physics is independent of Λ. This occurs because the terms in Λ have a chiral SU(3) dependence which is the same as the various terms in the effective Lagrangian. Therefore, in physical processes one can absorb this Λ dependence into a renormalized value of these parameters, e.g.

$$c_i^{ren} = c_i + \frac{\gamma_i \Lambda^2}{16\pi^2} \tag{30}$$

for some specific coefficient c_i. (Here γ_i is a number to be calculated in the renormalization process.) All phenomenology can be expressed in terms of the renormalized parameters and the strong Λ^2 depencence of this example would have vanished. When the meson masses are small, we will Taylor expand the loop integrals, renormalize the chiral parameters and recover exactly the usual results.

For realistic phenomenology, however, we need to use the physical values of the meson masses. The kaon and eta masses are in reality not small compared to the separation scale. They are also not so large that all of their effects can reliably treated as short-distance and hence be built into the parameters of the effective Lagrangian. We do need to treat them as dynamical degrees of freedom and include at least their long distance effects. When we use a cutoff regularization, with a cutoff close to the separation scale, the loop integrals will be the nonlinear functions of the mass, as described in the previous section. When these are evaluated at the physical meson masses, this will generate effects that are equivalent to higher orders in the chiral expansion. Thus this form of regularization can be viewed as a partial resummation of the chiral series. If we continue to treat the problem in full generality, we will still need to include chiral parameters in the effective Lagrangian which will allow us to continue to be fully model-independent. In each of the sections that follow, we will explore the phenomenology at physical values of the meson masses.

The cutoff Λ should not be taken so low in energy that it removes any truly long distance physics. Also, while it can in principle be taken much larger than the separation scale, this will lead to the inclusion of spurious short distance physics which can upset the convergence of the expansion. It is ideal to take the cutoff slightly above the separation scale so that all of the long distance physics, but little of the short distance physics, is included.

This procedure is not a model. Indeed its purpose is to remove the model-dependent short distance portions of loops. However, it appears to do so at the cost of introducing a new parameter, the cutoff Λ, plus the dependence of the choice of cutoff function. If the form of this function or the value of Λ played a major role in the phenomenology, this would be a serious drawback for this approach. However, renormalization theory leads us to expect that the dependence on the cutoff should be quite mild in phenomenological applications. This is because the cutoff (and the functional dependence) can be absorbed in the renormalization of the chiral parameters. If one worked to all orders, all dependence would disappear. If one is working to a given finite order, the residual dependence is expected to occur only at the next order beyond that at which one is working at. Since it appears from the above analysis that the cutoff integrals are rather slowly varying functions of the mass, we expect that working to an order where one includes the first SU(3) breaking parameters should be sufficient to minimize the cutoff dependence to an acceptable value.

Another issue that we should address here is the nature of the energy expansion in such a procedure. When using a regularization scheme which does not contain any dimensionful parameters, there is a particularly simple power counting procedure which determines the order of contributions of loop diagrams. If the regularization scheme does involve a mass parameter, this counting will not directly apply. We will see this explicitly below as the loop process will renormalize chiral parameters at different orders in the energy expansion. As one goes to the next order in loops, one will have to perform this renormalization again order by order. This, however, is not a fundamental problem. As we show, for small values of the meson masses we obtain exactly the same results as in other regularization schemes. Therefore, we can use the small mass limit to set up the chiral expansion and determine the order of the loops that one should include. Subsequently taking the masses to their physical values will accomplish the partial resummation of effects described above. However, the procedure in terms of which loops to include need not be changed.

There is one special feature involved in doing chiral perturbation theory with a cutoff instead of dimensional regularization. This involves an occasional change in the Feynman rules due to the presence of derivative couplings. The analysis of this aspect goes back to a classic paper on the subject, [15]. Recall that in the canonical construction of a field theory, one forms the canonical momenta conjugate to the field variables via

$$\pi(x) = \frac{\delta \mathcal{L}}{\delta \partial_0 \phi(x)} \quad . \tag{31}$$

When the interaction piece of the Lagrangian involves time derivatives, the canonical momenta will also carry portions of the interaction so that in forming the Hamiltonian, the interaction Hamiltonian will no longer be simply the negative of the interaction Lagrangian. Since perturbation theory and the Feynman rules are formulated from the interaction Hamiltonian, the canonical formalism will involve some modified (and non-covariant) vertices. At the same time, the presence of time derivatives in interactions will act on the time ordering in propagators to produce further non-covariant contributions to loop processes[15]. These modifications do not always cancel but can leave a residual interaction. While one can simply calculate this using the straightforward but clumsy canonical formalism, the authors of Ref. [15] show that one can use the naive rules if one adds a specific contact interaction proportional to $\delta^4(0)$ to the Feynman rules of the mesonic part of the theory. When using dimensional regularization, one of the peculiarities is that the regularized value of $\delta^4(0)$ is equal to zero. Therefore the contact interaction vanishes and we may proceed using the naive Feynman rules when calculating dimensionally. However, with a momentum-space cutoff, one has $\delta^4(0) \sim \Lambda^4$ and one gets a nontrivial modification quartic in the cutoff. This influences the purely mesonic sector of the theory. We have verified, however, that the baryonic processes that we consider are not modified by this feature at the order that we are working.

We now explore several specific cases of the physics of loop processes in baryon chiral perturbation theory. Our procedure in each case is the same. We take the known results of a standard analysis of the one-loop amplitudes and re-express it in terms of the Feynman integrals that we have analysed. We then show how the renormalization procedure is accomplished with a cutoff, absorbing the leading cutoff dependence into renormalized parameters. In each case this reproduces the standard analysis for small values of the mesonic masses. Then we turn to the realistic case of the physical meson masses and a finite cutoff. In this situation, the presence of the cutoff only permits the long distance loop effects, and this leads to the much more moderate effect of loops compared to the results quoted in the introduction.

5 Baryon Masses

In this section we return to the physics of chiral loops, as illustrated in the analysis of baryon masses, and deal with specific numerical results. This has already been discussed in Ref [12], but it is pedagogically useful to revisit the analysis in the present context. This will clearly illustrate the renormalization program and the isolation of long-distance loop effects.

To lowest and next leading order in the derivative expansion the effective Lagrangian which descibes the interactions of baryons can be written, in the heavy baryon formalism, as

$$\begin{aligned} \mathcal{L}_M B &= \mathrm{Tr}\bar{B}iv\cdot DB + d_A\mathrm{Tr}\bar{B}S^\mu\{u_\mu, B\} + f_A\mathrm{Tr}\bar{B}S^\mu[u_\mu, B] \\ &+ d_m\mathrm{Tr}\bar{B}\{\chi_+, B\} + f_m\mathrm{Tr}\bar{B}[\chi_+, B] + b_0\mathrm{Tr}\bar{B}B\mathrm{Tr}\chi_+ \ldots \end{aligned} \quad (32)$$

where χ_+ is given in terms of the quark mass matrix m via $\chi_+ = 2B_0 m$,

$$D_\mu = \partial_\mu + \frac{1}{2}[u^\dagger, \partial_\mu u] \quad (33)$$

is the covariant derivative, and

$$S_\mu = \frac{i}{2}\gamma_5\sigma_{\mu\nu}v^\nu \quad (34)$$

is the Pauli-Lubanski spin vector. The nonlinear mesonic chiral constructs u, u_μ are given by

$$U = u^2 = \exp(\frac{i}{F_\pi}\sum_j \lambda_j \phi_j), \qquad u_\mu = iu^\dagger \partial_\mu U u^\dagger \tag{35}$$

Here M_0, f_m, d_m, b_0 are free parameters in terms of which the tree level contribution to the baryon masses can be written as given above in Eq. 1 with

$$\hat{M}_0 = M_0 - 2(2m_K^2 + m_\pi^2)b_0 \tag{36}$$

If we continue the analysis to higher order we include the effects of quark loops and of the higher order terms in a general Lagrangian. In an expansion in quark mass we have the schematic form

$$M_B = M_0 + \sum_q a_q m_q + \sum_q b_q m_q^{\frac{3}{2}} + \sum_q c_q m_q^2 + \dots \tag{37}$$

Here, the terms linear in the quark mass are those parametrized in Eq. 1, where we recall that $m_P^2 \sim m_q$. The next term in the expansion is nonanalytic in the quark mass and comes uniquely from loop diagrams. Finally the terms at order m_q^2 come from yet higher order effects which we will not explicitly consider here.

The one loop chiral corrections are well known and involve the integral given in Eq. 12 of the previous section. In dimensional regularization this yields terms in m_P^3 and can be represented as

$$\delta M_i = -\frac{1}{24\pi F_\pi^2}\sum_j \kappa_i^j m_j^3 \tag{38}$$

with

$$\begin{aligned}
\kappa_N^\pi &= \frac{9}{4}(d_A + f_A)^2, \quad \kappa_N^K = \frac{1}{2}(5d_A^2 - 6f_A d_A + 9f_A^2), \quad \kappa_N^\eta = \frac{1}{4}(d_A - 3f_A)^2 \\
\kappa_\Sigma^\pi &= (d_A^2 + 6f_A^2), \quad \kappa_\Sigma^K = 3(d_A^2 + f_A^2), \quad \kappa_\Sigma^\eta = d_A^2 \\
\kappa_\Lambda^\pi &= 3d_A^2, \quad \kappa_\Lambda^K = d_A^2 + 9f_A^2, \quad \kappa_\Lambda^\eta = d_A^2 \\
\kappa_\Xi^\pi &= \frac{9}{4}(d_A - f_A)^2, \quad \kappa_\Xi^K = \frac{1}{2}(5d_A^2 + 6d_A f_A + 9f_A^2), \quad \kappa_\Xi^\eta = \frac{1}{4}(d_A + 3f_A)^2
\end{aligned} \tag{39}$$

This produces the large mass shifts quoted in Eq. 3. The violation of the Gell-Mann-Okubo relation is given then by

$$\frac{1}{4}[3M_\Lambda + M_\Sigma - 2M_N - 2M_\Xi] = \frac{d_A^2 - 3f_A^2}{96\pi F_\pi^2}[4m_K^3 - 3m_\eta^3 - m_\pi^3] \qquad (40)$$

The deviation from the Gell-Mann-Okubo relation due to loops is found to be quite small, primarily due to the (accidental) feature that $d_A^2 - 3f_A^2 \approx 0.02 << 1$.

We now turn to an exploration of the analysis using a cutoff regulariation. The first task is to see how the renormalization program works, in order that we obtain exactly the same result in the limit of small masses. The diagrams involved are the same as in the previous analysis, but we utilize the cutoff form for the Feynman integral. This is simply done by replacing m_P^3 in Eq. 38 by the function $I_\Lambda(m_P^2)$, expanded as in Eq. 17. The one loop contribution to the mass then has the schematic form

$$\delta M_i = -\frac{1}{24\pi F_\pi^2}\sum_j(\frac{1}{2}\Lambda^3 - \frac{1}{2}\Lambda m_j^2 + m_j^3 + \ldots) \qquad (41)$$

Obviously the term in m_j^3 is identical to that arising in conventional dimensional regularization, but more interesting are the contributions proportional to Λ^3 and to Λm_P^2. The piece cubic in Λ has the form

$$\delta M_i^{\Lambda^3} = -\frac{\Lambda^2}{48\pi F_\pi^2}\sum_j \kappa_i^j \qquad (42)$$

and is independent of baryon type—it may be absorbed into a renormalization of M_0—

$$M_0^r = M_0 - (5d_A^2 + 9f_A^2)\frac{\Lambda^3}{48\pi F_\pi^2} \qquad (43)$$

On the other hand the terms linear in Λ

$$\delta M_i^\Lambda = \frac{\Lambda}{48\pi F_\pi^2}\sum_j \kappa_i^j m_j^2 \qquad (44)$$

must be able to be absorbed into renormalizations of the coefficients involving m_q, and indeed this is found to be the case—one verifies that

$$d_m^r \;=\; = d_m - \frac{3f_A^2 - d_A^2}{128\pi F_\pi^2}\Lambda$$

$$\begin{aligned} f_m^r &= f_m - \frac{5 d_A f_A}{192 \pi F_\pi^2} \Lambda \\ b_0^r &= b_0 - \frac{13 d_A^2 + 9 f_A^2}{576 \pi F_\pi^2} \Lambda \end{aligned} \tag{45}$$

That such renormalization can occur involves a highly constrained set of conditions and the fact that they are satisfied is a significant verification of the chiral invariance of the cutoff procedure. Of course, once one has defined renormalized coefficients, since they are merely phenomenological parameters which must be determined empirically, the procedure is identical to the results of the usual dimensionally regularized technique when the masses are smaller than the cutoff.

Having convinced ourselves of the chiral invariance of the cutoff procedure to the order we are working, we can now apply it to the case where masses are their physical values and the cutoff is taken to be phenomenologically relevant—-*i.e.*, $\Lambda \geq 1/ < r_B > \sim 300-600$ MeV. However, we first remove the asymptotic mass-independent component of the function $I(m)$ by defining

$$\tilde{I}(m) = I(m) - \frac{1}{2}\Lambda^3 \tag{46}$$

since these effects can be absorbed into M_0 and give misleading indications about the size of the nonanalytic effects in the large cutoff limit. The size of the long distance nonanalytic contributions to the baryon masses is then given by

$$\delta M_i = -\frac{1}{24\pi F_\pi^2} \sum_j \kappa_i^j \tilde{I}(m_j) \tag{47}$$

and the corresponding numerical results are given in Table 1. A careful look at these findings reveals that the quantitative results are in agreement with our qualitative expectations—for a reasonable value of the cutoff parameter Λ, the overall size of the nonanalytic corrections is much smaller that that found in the dimensionally regularized case since the short distance contribution from kaon, eta loops is much reduced. There is no longer any in principle problem with the convergence of the chiral expansion and the "mystery" of why the lowest order fit linear in m_q works so well is resolved. Of course, one still must include the model-dependent contribution from short distance effects, but there no longer exists a problem from the calculable and model-independent long distance component.

	dim.	$\Lambda = 300$	$\Lambda = 400$	$\Lambda = 500$	$\Lambda = 600$
N	-0.31	0.02	0.03	0.05	0.07
Σ	-0.62	0.03	0.05	0.08	0.12
Λ	-0.68	0.03	0.06	0.09	0.13
Ξ	-1.03	0.04	0.08	0.12	0.17

Table 1: Given (in GeV) are the nonanalytic contributions to baryon masses in dimensional regularization and for various values of the cutoff parameter Λ in MeV.

A good fit to the baryon masses can be accomplished for any value of the cutoff in the range that we consider. For example, with $\Lambda = 400$MeV, we have the masses described by

$$\begin{aligned} M_N &= 1.143 - 0.237 + 0.034 = 0.940 \\ M_\Sigma &= 1.143 - 0.005 + 0.053 = 1.191 \\ M_\Lambda &= 1.143 - 0.086 + 0.057 = 1.114 \\ M_\Xi &= 1.143 + 0.106 + 0.077 = 1.326 \end{aligned} \quad (48)$$

where all numbers are given in GeV. In Eq. 48, $\hat{M}_0$ is the first term, the second term comes from the leading tree level SU(3) breaking due to quark masses parameterized as in Eq. 1 and the last term from the residual loop effects. The tree level terms contribute 343 MeV to the Ξ-N mass splitting, while the loop effects contribute only 43 MeV. The chiral expansion is well-behaved—loops do not upset the basic pattern at lowest order and the approximate SU(3) invariance is manifest. In order to disentangle M_0 and b_0, one has also to take, *e.g*, the πN σ–term into account [13].

If we had used a different value of the cutoff in the regularization, the specific contributions would have been different, yet the final answers change by less that 1 MeV for Λ from 300 MeV to 600 MeV. This is a demonstration of the cutoff independence of this procedure. (Our previous discussion suggested that we should have found a cutoff dependence equivalent to neglected higher order terms, which in this case would have been of order 5 MeV. In practice we found less dependence than that.) We have also verified that we obtain identical results for another form of the cutoff function[12].

Having seen how the cutoff procedure can be successfully applied in the case of the baryon masses, we can now move on the the remaining applications – axial coupling, nonleptonic hyperon decay, and magnetic moments – to show how a chirally consistent picture emerges therein.

6 Axial Currents

The baryon axial couplings are parameterized in terms of the same f_A, d_A coofficients which appear in the Hamiltonian of Eq 5. Defining the lowest order contribution using the notation $g_A(\bar{i}j) = \alpha_{ij}$, we have

$$\begin{aligned}
\alpha_{pn} &= f_A + d_A \\
\alpha_{\Lambda\Sigma^-} &= \frac{2}{\sqrt{6}} d_A \\
\alpha_{p\Lambda} &= -\frac{1}{\sqrt{6}}(d_A + 3f_A) \\
\alpha_{\Lambda\Xi^-} &= -\frac{1}{\sqrt{6}}(d_A - 3f_A) \\
\alpha_{n\Sigma^-} &= d_A - f_A \\
\alpha_{\Sigma^0\Xi^-} &= \frac{1}{\sqrt{2}}\alpha_{\Sigma^+\Xi^0} = \frac{1}{\sqrt{2}}(d_A + f_A)
\end{aligned} \tag{49}$$

It is these forms which are used in SU(3) fits to hyperon beta decay.

The leading nonanalytic corrrections from loops are $\mathcal{O}(m_P^2 \ln m_P^2)$ and were first calculated by Bijnens, Sonoda, and Wise[7]. They have the form

$$g_A(\bar{i}j) = \sqrt{Z_i Z_j}[\alpha_{ij} + \frac{1}{16\pi^2 F_\pi^2}\sum_k \beta_{ij}^k m_k^2 \ln\frac{m_k^2}{\mu^2}] \tag{50}$$

with

$$\begin{aligned}
\beta_{pn}^\pi &= \frac{1}{4}(d_A^3 + f_A^3 + 3d_A^2 f_A + 3f_A^2 d_A) - (d_A + f_A), \\
\beta_{pn}^K &= \frac{1}{3}d_A^3 - \frac{1}{3}f_A d_A^2 + d_A f_A^2 - f_A^3 - \frac{1}{2}(d_A + f_A), \\
\beta_{pn}^\eta &= -\frac{1}{12}d_A^3 + \frac{5}{12}f_A d_A^2 - \frac{1}{4}d_A f_A^2 - \frac{3}{4}f_A^3
\end{aligned}$$

$$
\begin{aligned}
\beta^{\pi}_{p\Lambda} &= \frac{1}{\sqrt{6}}(-\frac{3}{2}d_A^3+\frac{3}{2}d_Af_A^2+\frac{3}{8}(d_A+3f_A)),\\
\beta^{K}_{p\Lambda} &= \frac{1}{\sqrt{6}}(\frac{5}{6}d_A^3-\frac{5}{2}d_A^2f_A-\frac{3}{2}f_A^2d_A+\frac{9}{2}f_A^3)+\frac{3}{4}(d_A+3f_A)),\\
\beta^{\eta}_{p\Lambda} &= \frac{1}{\sqrt{6}}(\frac{1}{6}d_A^3-\frac{3}{2}d_Af_A^2+\frac{3}{8}(d_A+3f_A))\\
\beta^{\pi}_{\Lambda\Sigma^-} &= \frac{1}{\sqrt{6}}(-\frac{2}{3}d_A^3+2d_Af_A^2-2d_A),\\
\beta^{K}_{\Lambda\Sigma^-} &= \frac{1}{\sqrt{6}}(d_A^3-d_Af_A^2-d_A),\\
\beta^{\eta}_{\Lambda\Sigma^-} &= \frac{1}{\sqrt{6}}(\frac{2}{3}d_A^3)\\
\beta^{\pi}_{n\Sigma^-} &= \frac{1}{6}d_A^3-\frac{1}{3}d_A^2f_A+\frac{2}{3}d_Af_A^2+f_A^3-\frac{3}{8}(d_A-f_A)),\\
\beta^{K}_{n\Sigma^-} &= \frac{1}{2}f_A^3+\frac{1}{2}d_Af_A^2+\frac{1}{6}d_A^2f_A+\frac{1}{6}d_A^3-\frac{3}{4}(d_A-f_A)),\\
\beta^{\eta}_{n\Sigma^-} &= \frac{1}{2}d_Af_A^2-\frac{2}{3}d_A^2f_A+\frac{1}{6}d_A^3-\frac{3}{8}(d_A-f_A))\\
\beta^{\pi}_{\Lambda\Xi^-} &= \frac{1}{\sqrt{6}}(-\frac{3}{2}d_A^3+\frac{3}{2}f_A^2d_A+\frac{3}{8}(d_A-3f_A)),\\
\beta^{K}_{\Lambda\Xi^-} &= \frac{1}{\sqrt{6}}(\frac{5}{6}d_A^3+\frac{5}{2}d_A^2f_A-\frac{3}{2}d_Af_A^2-\frac{9}{2}f_A^3+\frac{3}{4}(d_A-3f_A)),\\
\beta^{\eta}_{\Lambda\Xi^-} &= \frac{1}{\sqrt{6}}(\frac{1}{6}d_A^3-\frac{3}{2}d_Af_A^2+\frac{3}{8}(d_A-3f_A))\\
\beta^{\pi}_{\Sigma^0\Xi^-} &= \frac{1}{\sqrt{2}}(-f_A^3+\frac{1}{3}f_Ad_A^2+\frac{1}{2}f_A^2d_A+\frac{1}{6}d_A^3-\frac{3}{8}(d_A+f_A)),\\
\beta^{K}_{\Sigma^0\Xi^-} &= \frac{1}{\sqrt{2}}(\frac{1}{6}d_A^3-\frac{1}{6}f_Ad_A^2+\frac{1}{2}f_A^2d_A-\frac{1}{2}f_A^3-\frac{3}{4}(d_A+f_A)),\\
\beta^{\eta}_{\Sigma^0\Xi^-} &= \frac{1}{\sqrt{2}}(\frac{1}{6}d_A^3+\frac{2}{3}d_A^2f_A+\frac{1}{2}d_Af_A^2-\frac{3}{8}(d_A+f_A)) \qquad (51)
\end{aligned}
$$

Here Z_i are the wavefunction renormalization factors, whose leading nonanalytic form is

$$
Z_i = 1-\frac{1}{16\pi^2F_\pi^2}\sum_j \kappa_i^j m_j^2 \ln\frac{m_j^2}{\mu^2} \qquad (52)
$$

with κ_i^j given in Eq. 39. These forms generate the corrections discussed in the introduction.

When we apply the cutoff formalism we first note that all of the nonanalytic behavior of the form $m^2 \ln m^2$ comes uniquely from the integral that we labeled $J(m)$ in Section 3. This means that all that we need to do in order to convert the analysis above to our formalism is to replace $m_P^2 \ln m_P^2$ by $J(m_P)$ everywhere throughout these formulas. We may again check the chiral consistency of the renormalization program by verifying that the contribution quadratic in Λ—

$$\delta g_A^{\Lambda^2}(\bar{i}j) = \frac{\Lambda^2}{16\pi^2 F_\pi^2} \sum_k [\beta_{ij}^k - \frac{1}{2}\alpha_{ij}(\lambda_i^k + \lambda_j^k)] \tag{53}$$

can be absorbed into renormalizations of the lowest order axial couplings d_A, f_A via

$$\begin{aligned} d_A^r &= d_A - \frac{3}{2} d_A (3d_A^2 + 5f_A^2 + 1)\frac{\Lambda^2}{16\pi^2 F_\pi^2} \\ f_A^r &= f_A - \frac{1}{6} f_A (25 d_A^2 + 63 f_A^2 + 9)\frac{\Lambda^2}{16\pi^2 F_\pi^2} \end{aligned} \tag{54}$$

Since such coefficients are determined empirically the analysis with small meson masses becomes identical to that of the dimensionally regularized case.

In the case of a physically realistic cutoff—$\Lambda \sim 300-600$ MeV— and the physical meson masses, we have

$$\delta g_A(\bar{i}j) = \frac{1}{16\pi^2 F_\pi^2} \sum_k [\beta_{ij}^k - \frac{1}{2}\alpha_{ij}(\lambda_i^k + \lambda_j^k)]\tilde{J}(m_k^2) \tag{55}$$

where we have again removed the asymptotic mass-independent component of the function $J(M^2)$ via

$$\tilde{J}(m^2) = J(m^2) - \Lambda^2 \tag{56}$$

The numerical results using typical values of the cutoff are compared with those from dimensional regularization in Table 2 and again reflect the feature that the SU(3) chiral expansion is now under control at least as far as long distance effects are concerned—the "mystery" of the correctness of the simple SU(3) fit *without* chiral corrections is resolved. A complete discussion of axial-vector current matrix elements can be found in [14].

	dim.	Λ=300	Λ=400	Λ=500	Λ=600
$g_A(\bar{p}n)$	1.72	0.37	0.53	0.69	0.84
$g_A(\bar{p}\Lambda)$	-1.78	-0.34	-0.51	-0.67	-0.84
$g_A(\bar{\Lambda}\Sigma^-)$	1.17	0.23	0.34	0.45	0.56
$g_A(\bar{n}\Sigma^-)$	0.36	0.07	0.10	0.14	0.17
$g_A(\bar{\Lambda}\Xi^-)$	0.83	0.15	0.23	0.31	0.39
$g_A(\bar{\Sigma}^0\Xi^-)$	2.46	0.45	0.68	0.91	1.15

Table 2: Given are the nonanalytic contribtions to g_A for various transitions in dimensional regularization and for various values of the cutoff parameter Λ in MeV.

7 S-wave hyperon decay

Chiral invariance relates the S-wave nonleptonic decay amplitudes to the baryon-to-baryon matrix elements of the weak Hamiltonian. For the dominant octet Hamiltonian this can be paramterized in terms of two SU(3) coefficientsf_w, d_w:

$$A(\Upsilon_i^j) = \zeta(\Upsilon_j^i) \tag{57}$$

where

$$\begin{aligned} \zeta(\Lambda_0^0) &= -\frac{1}{\sqrt{2}}\zeta(\Lambda_-^0) = -\frac{1}{2\sqrt{3}}(d_w + 3f_w) \\ \zeta(\Sigma_0^+) &= -\frac{1}{\sqrt{2}}\zeta(\Sigma_-^-) = \frac{1}{\sqrt{2}}(d_w - f_w) \\ \zeta(\Sigma_+^+) &= 0 \\ \zeta(\Xi_0^0) &= -\frac{1}{\sqrt{2}}\zeta(\Xi_-^-) = -\frac{1}{2\sqrt{3}}(d_w - 3f_w) \end{aligned} \tag{58}$$

This yields a good fit to the data, including the chiral SU(3) results given in Eq. 5.

Proceeding to one-loop order, the leading nonanalytic corrections are dependent upon $m_P^2 \ln m_P^2$ and have the form

$$A(\Upsilon_j^i) = \sqrt{Z_i Z_j}[\zeta(\Upsilon_j^i) + \frac{1}{16\pi^2 F_\pi^2}\sum_k \rho(\Upsilon_j^i)^k m_k^2 \ln\frac{m_k^2}{\mu^2}] \tag{59}$$

with

$$\begin{aligned}
\rho(\Lambda^0_0)^\pi &= -\frac{1}{2\sqrt{3}}d_w(\frac{7}{24}-\frac{9}{2}d_A^2-\frac{9}{2}d_Af_A)-\frac{1}{2\sqrt{3}}f_w(\frac{7}{8}+\frac{9}{2}d_A^2+\frac{9}{2}d_Af_A)\\
\rho(\Lambda^0_0)^K &= -\frac{1}{2\sqrt{3}}d_w(-\frac{5}{12}+\frac{5}{2}d_A^2-9f_Ad_A+\frac{9}{2}f_A^2)\\
&- \frac{1}{2\sqrt{3}}f_w(-\frac{5}{4}+\frac{3}{2}d_A^2-9f_Ad_A+\frac{27}{2}f_A^2)\\
\rho(\Lambda^0_0)^\eta &= -\frac{1}{2\sqrt{3}}d_w(-\frac{3}{8}+\frac{1}{2}d_A^2-\frac{3}{2}d_Af_A)-\frac{1}{2\sqrt{3}}f_w(-\frac{9}{8}+\frac{3}{2}d_A^2-\frac{9}{2}f_Ad_A)\\
\rho(\Xi^0_0)^\pi &= -\frac{1}{2\sqrt{3}}d_w(\frac{7}{24}-\frac{9}{2}d_A^2+\frac{9}{2}d_Af_A)+\frac{1}{2\sqrt{3}}f_w(\frac{7}{8}+\frac{9}{2}d_A^2-\frac{9}{2}d_Af_A)\\
\rho(\Xi^0_0)^K &= -\frac{1}{2\sqrt{3}}d_w(-\frac{5}{12}+\frac{5}{2}d_A^2+9d_Af_A+\frac{9}{2}f_A^2)\\
&+ \frac{1}{2\sqrt{3}}f_w(-\frac{5}{4}+\frac{3}{2}d_A^2+9d_Af_A+\frac{27}{2}f_A^2)\\
\rho(\Xi^0_0)^\eta &= -\frac{1}{2\sqrt{3}}d_w(-\frac{3}{8}+\frac{1}{2}d_A^2+\frac{3}{2}d_Af_A)+\frac{1}{2\sqrt{3}}f_w(-\frac{9}{8}+\frac{3}{2}d_A^2+\frac{9}{2}d_Af_A)\\
\rho(\Sigma^+_0)^\pi &= \sqrt{\frac{1}{2}}d_w(\frac{7}{24}+3f_A^2+\frac{5}{2}d_Af_A-\frac{1}{2}d_A^2)\\
&- \frac{1}{\sqrt{2}}f_w(\frac{7}{24}+3f_A^2+\frac{9}{2}d_Af_A+\frac{3}{2}d_A^2)\\
\rho(\Sigma^+_0)^K &= \frac{1}{\sqrt{2}}d_w(-\frac{5}{12}-\frac{1}{2}d_A^2+d_Af_A+\frac{3}{2}f_A^2)\\
&- \frac{1}{\sqrt{2}}f_w(-\frac{5}{12}+\frac{3}{2}d_A^2+3d_Af_A+\frac{3}{2}f_A^2)\\
\rho(\Sigma^+_0)^\eta &= \frac{1}{\sqrt{2}}d_w(-\frac{3}{8}-\frac{1}{2}d_A^2+\frac{3}{2}d_Af_A)-\frac{1}{\sqrt{2}}f_w(-\frac{3}{8}-\frac{1}{2}d_A^2+\frac{3}{2}d_Af_A)\\
\rho(\Sigma^+_+)^\pi &= \rho(\Sigma^+_+)^K=\rho(\Sigma^+_+)^\eta=0 \qquad (60)
\end{aligned}$$

The correction to the Lee-Sugawara relation is found

$$\begin{aligned}
&\sqrt{3}A(\Sigma^+_0)-2A(\Xi^-_-)-A(\Lambda^0_-)=-\sqrt{\frac{2}{3}}\frac{1}{16\pi F_\pi^2}\\
&\times[m_K^2\ln m_K^2(d_w(\frac{9}{2}d_A^2+3d_Af_A+\frac{9}{2}f_A^2)+f_w(\frac{3}{2}d_A^2-9d_Af_A-\frac{9}{2}f_A^2))
\end{aligned}$$

$$
\begin{aligned}
&+m_\eta^2 \ln m_\eta^2 (d_w(\frac{3}{2}d_A^2 - \frac{3}{2}d_A f_A) + f_w(-\frac{3}{2}d_A^2 - \frac{9}{2}d_A f_A)) \\
&+m_\pi^2 \ln m_\pi^2 (d_w(-6d_A^2 - \frac{3}{2}d_A f_A - \frac{9}{2}f_A^2) + f_w(\frac{27}{2}d_A f_A + \frac{9}{2}f_A^2))] \\
&\approx -6.4 \times 10^{-7} \qquad (61)
\end{aligned}
$$

When analysed using the physical values of the masses, we uncover the problems described in the introduction.

A very similar analysis obtains as was describe in the situation for the axial currents. In the cutoff formalism the nonanalytic pieces proportional to $m_P^2 \ln m_P^2$ are simply replaced by the function $J(m_P^2)$. Again, the chiral consistency of of the renormalization program can be verified by noting that for small meson masses the component quadratic in Λ—

$$
\delta A(\Upsilon_j^i) = \frac{\Lambda^2}{16\pi^2 F_\pi^2}[\rho(\Upsilon_j^i)^k - \frac{1}{2}\zeta(\Upsilon_j^i)(\lambda_i^k + \lambda_j^k)] \qquad (62)
$$

can be absorbed into renormalized values of the lowest order couplings f_w, d_w via

$$
\begin{aligned}
d_w^r &= d_w - \frac{1}{2}[d_w(1 + 13d_A^2 + 9f_A^2) + 18 f_w d_A f_A]\frac{\Lambda^2}{16\pi^2 F_\pi^2} \\
f_w^r &= f_w - \frac{1}{2}[f_w(1 + 5d_A^2 + 9f_A^2) + 10 d_w d_A f_A]\frac{\Lambda^2}{16\pi^2 F_\pi^2} \qquad (63)
\end{aligned}
$$

Once this renormalization is accomplished, we exactly recover the usual chiral analysis.

In the case of a physically realistic masses, we again use the same mass-independent renormalization to define the residual integral $\tilde{J}(m)$. The shift in s-wave amplitudes is then given by

$$
\delta A(\Upsilon_j^i) = \sum_k [\rho(\Upsilon_j^i)^k - \frac{1}{2}\zeta(\Upsilon_j^i)(\lambda_i^k + \lambda_j^k)]\frac{\tilde{J}(m_k^2)}{16\pi^2 F_\pi^2} \qquad (64)
$$

The numerical results are compared with those of dimensional regularization in Table 3 and it is clear that once again the results are dominated by the lowest order SU(3) forms—there no longer exist large chiral corrections.

	dim.	Λ=300	Λ=400	Λ=500	Λ=600
$A(\Lambda^0_0)$	-3.57	-0.62	-0.95	-1.30	-1.65
$A(\Xi^0_0)$	1.96	0.36	0.54	0.73	0.92
$A(\Sigma^+_0)$	-1.57	-0.26	-0.41	-0.56	-0.72

Table 3: Given are the nonanalytic contributions to s-wave semileptonic hyperon decay amplitudes in dimensional regularization and for various values of the cutoff parameter Λ in MeV.

8 Magnetic moments

The final case considered here is that of magnetic moments. The lowest order parameterization is given in Eq. 8. The leading nonanalytic chiral corrections are linear in m_P and were first calculated by Caldi and Pagels. They have the form

$$\delta\mu_i = \frac{M_0}{8\pi F_\pi^2}\sum_j \sigma_i^j m_j \qquad (65)$$

with

$$\begin{aligned}
\sigma_p^\pi &= -(f_A+d_A)^2, \quad \sigma_p^K = -\frac{2}{3}(d_A^2+3f_A^2) \\
\sigma_n^\pi &= (d_A+f_A)^2, \quad \sigma_n^K = -(d_A-f_A)^2 \\
\sigma_\Lambda^\pi &= 0, \quad \sigma_\Lambda^K = 2f_Ad_A \\
\sigma_{\Sigma^+}^\pi &= -\frac{2}{3}(d_A^2+3f_A^2), \quad \sigma_{\Sigma^+}^K = -(d_A+f_A)^2 \\
\sigma_{\Sigma_0}^\pi &= 0, \quad \sigma_{\Sigma^0}^K = -2d_Af_A \\
\sigma_{\Sigma^-}^\pi &= \frac{2}{3}(d_A^2+3f_A^2), \quad \sigma_{\Sigma^-}^K = (d_A-f_A)^2 \\
\sigma_{\Lambda\Sigma}^\pi &= -\frac{4}{\sqrt{3}}d_Af_A, \quad \sigma_{\Lambda\Sigma}^K = -\frac{2}{\sqrt{3}}d_Af_A \\
\sigma_{\Xi^-}^\pi &= (d_A-f_A)^2, \quad \sigma_{\Xi^-}^K = \frac{2}{3}(d_A^2+3f_A^2) \\
\sigma_{\Xi^0}^\pi &= -(d_A-f_A)^2, \quad \sigma_{\Xi^0}^K = (d_A+f_A)^2
\end{aligned} \qquad (66)$$

In this analysis, all Feynman integrals are given by the linear form called $K(m)$ in Section 3. The general result appropriate for a cutoff regularization is obtained by replacing the nonanalytic dependence m_P by $K(m_P)$. We can then verify that the leading term in Λ can be absorbed into the renormalization of the chiral parameters, leading to an identical analysis for small values of m_P. In this case, examination of the term in the magnetic moment shift linear in Λ—

$$\delta\mu_i^\Lambda = -\frac{M_0\Lambda}{24\pi F_\pi^2}\sum_j \sigma_i^j \tag{67}$$

shows that it be absorbed into renormalizations of the lowest order parameters f_μ, d_μ via

$$\begin{aligned} d_\mu^r &= d_\mu + \frac{M_0\Lambda}{4\pi F_\pi^2} d_A f_A \\ f_\mu^r &= f_\mu + \frac{M_0\Lambda}{24\pi F_\pi^2}(\frac{5}{3}d_A^2 + 3f_A^2) \end{aligned} \tag{68}$$

Since f_μ, d_μ are determined empirically, the analysis is then identical to that of the dimensionally regularized case.

On the other hand with the use of a physically realistic cutoff and meson masses, the magnetic moment shifts cam be obtained by using the mass independent renormalization given by

$$\tilde{K}(m) = K(m) + \frac{1}{3}\Lambda \tag{69}$$

The shifts in the magnetic moments are given by

$$\delta\mu_i = \frac{M_0}{8\pi F_\pi^2}\sum_j \sigma_i^j \tilde{K}(m_j) \tag{70}$$

The numerical results for this form for reasonable values of the cutoff are compared with those from dimensional regularization in Table 4. Again the chiral corrections are no longer out of control.

9 Summary

We have seen above that a significant component of the poor convergence found in previous calculations in SU(3) baryon chiral perturbation theory is due to the inclusion of large and spurious short-distance contributions

	dim.	Λ=300	Λ=400	Λ=500	Λ=600
μ_p	0.76	0.22	0.27	0.31	0.34
μ_n	-0.22	-0.12	-0.14	-0.15	-0.16
μ_Λ	-0.43	-0.08	-0.11	-0.13	-0.15
μ_{Σ^+}	1.05	0.24	0.30	0.36	0.40
μ_{Σ^0}	0.44	0.08	0.11	0.13	0.15
μ_{Σ^-}	-0.18	-0.08	-0.09	-0.10	-0.11
$\mu_{\Sigma\Lambda}$	0.39	0.12	0.14	0.16	0.18
μ_{Ξ^-}	-0.52	-0.10	-0.13	-0.16	-0.18
μ_{Ξ^0}	-0.90	-0.17	-0.22	-0.26	-0.30

Table 4: Given are the nonanalytic contributions to magnetic moments in dimensional regularization and for various values of the cutoff parameter Λ in MeV.

when loop processes are regularized dimensionally. The use of a momentum space cutoff keeps only the long distance portion of the loops and leads to an improved behavior. Indeed although we have formulated our discussion in terms of merely a different sort of regularization procedure within the general framework of chiral perturbation theory, it is interesting to note that our results are quite consistent with the sort of SU(3) breaking effects found in chiral confinement models such as the cloudy bag, when the effects of kaon and/or eta loops are isolated[16].

We might ask why baryon chiral perturbation theory has this problem while mesonic chiral theories do not. Most applications in mesons work perfectly well using dimensional regularization. At first sight one might argue that the separation scale in baryons corresponds to lower energies because the physical size of baryons is larger than mesons. While this is a true statement, it does not really answer the question, since the baryon problem surfaces entirely within the point particle theory. For some reason, given the same meson masses, the loop corrections are larger in the baryonic point particle theory compared to a mesonic point particle theory. This feature can perhaps be blamed on the baryon propagator in the loop integral which, being linear in the momentum, suppresses high momentum contributions less than a corresponding quadratic mesonic propagator. However, the existence of the problem is beyond doubt, given the troubles discussed in the introduction. Fortunately, we do not as a consequence have to abandon all such chiral calculations—a revised regularization scheme seems capable of resolving the problem.

The simplicity that underlies baryon physics is more evident when chiral loops are calculated with a long-distance regularization. In this context, we hope that baryon chiral perturbation theory will become more phenomenologically useful. One can hopefully now use the chiral calculations in order to provide a model independent description of the very long distance physics, and this can be a welcome addition to our techniques for describing the low energy phenomenology of baryons.

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