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'SECOND BEST' CONGESTION TAXES IN TRANSPORTATION SYSTEMS

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The optimal policy prescription in response to congestion on a traffic network involves taxes levied so as to increase the private costs of vehicle use by the amount of the costs imposed on other users of the system. However, technical and political constraints may make this taxation policy infeasible. Using assumptions on the effect of taxation on the level and structure of demand for transportation services, this paper provides guidelines for taxation (and possibly subsidization) in a multi-mode traffic system in which such constraints are effective.

SINCE LIPSEY AND LANCASTER'S ARTICLE on the general theory of the second best [**8**], the difficulties in formulating economic policy aimed at a partial or piecemeal elimination of distortions to the equi–marginal conditions characterizing Pareto optimal solutions have been well understood. While the "second best" problem calls for analyses which explicitly take into account the interdependence of a sector or activity with other sectors or activities in the economy subject to "distortions" that are exogenously given[2] and while theoretical work has succeeded in formulating some general principles for policy formulation in such an environment,[3] the interactions between sectors operating through product or factor markets are in general so difficult to define that the step from the theoretical results to their empirical application has rarely been attempted.

However, for problems less general than policy formulation in a distorted general equilibrium economic system, there is greater hope for taking "second best" theory into account in a meaningful way. The analysis of a multi-mode congested trnsportation system is a case in point. In such a system "second best" considerations should not be ignored because of the quantitative importance of distortions to the optimal conditions resulting from congestion and related externalities, the technical and political difficulties in introducing optimal taxation, and because the interactions between travel activities by mode are more susceptible to empirical estimation than the interactions encountered in the usual general equilibrium economic system.

Due to several papers in this journal, a good deal has been learned about the situation where optimal taxes equal to the marginal costs imposed on others by vehicle use are not feasible. With a specific example, Levy-Lambert [**7**] solves for the optimal tax on one route when it is impossible to levy a tax on another route.

[1] This paper is based on research undertaken while I was a Visiting Professor at Thammasart University. I am indebted to Borwornsri Somboonpanya, Ammar Siamwalla, Bruce Hamilton, Herb Mohring and Carl Christ for comments on this work and to the Rockefeller Foundation for financial support.

[2] On this point, see the discussion in Harberger [**5**].

[3] See Bertrand and Vanek [**1**], Dixit [**3**], Davis and Whinston [**2**], Green [**4**], and Hatta [**6**].

Marchand [**9**] generalizes this result by developing a general equilibrium model which shows that in the two route problem, the price facing the vehicle user on the taxable route should fall short of the marginal costs of his vehicle use if this shifts use from a route where taxes cannot be levied. Sherman [**11**] modifies the Marchand model to allow interdependence in congestion by two alternative modes of travel, buses and automobiles, to study the problem when automobiles are not subject to direct taxes[4] and in particular to investigate the conditions under which the second best direct tax policy implies a subsidy to buses.

In this paper, a general equilibrium model is designed to analyze second best taxation. The main contribution is in the derivation of guidelines for second best tax policies which provide useful reference points when all of the parameters required for exact calculation of the taxes are not known. The derivation of these guidelines also helps clarify the factors underlying congestion taxation in a second best environment.

The model on which this paper is based leading to the derivation of an equation system we use as a starting point in obtaining our results is set out in the Appendix. This avoids working through some tedious derivations resulting in a relationship with a straightforward interpretation which is consistent with findings previously established in the literature.[5]

The model as set out in the Appendix analyzes tax policies consistent with maximizing a community welfare function whose arguments are the use of n types of transportation services, consumption of non-transportation commodities and services, leisure by different types of labor, and pollution caused by congestion. Transportation services, a maintenance service for vehicles and roads, and non-transportation services and commodities are produced subject to a production possibility set with variable inputs consisting of the production times of different types of labor. Consumers maximize welfare subject to budget constraints and producers maximize profits in perfectly competitive markets. The laissez-faire operation of the economy is not optimal, however, since use of the transportation modes creates congestion costs for others in the community by creating pollution, causing travel time delays that reduce leisure time or productive time for others, or increasing vehicle operating and road maintenance costs borne by others. These congestion costs imposed on others are not taken into account by consumers of transportation services who are affected only by the prices of these services and the congestion costs they bear themselves. The optimal solution is achieved when there are no distortions on non-transportation sectors in the economy and when the prices of transportation services are increased by taxes until the tax inclusive price faced by consumers is equated to the marginal cost of providing these services inclusive of the congestion costs imposed on others.

[4] Sherman [**11**] does investigate the case where automobiles can be indirectly taxed through an input tax.

[5] Equation set (1) with which we start our analysis is, for instance, a genealized version of relation (37) in the paper by Marchand [**9**].

The first order optimality conditions when no constraints operate in setting rates are given in matrix form in (1):[6]

$$\begin{bmatrix} -\dfrac{\partial V_1}{\partial t_1} & -\dfrac{\partial V_2}{\partial t_1} & \cdots & -\dfrac{\partial V_n}{\partial t_1} \\ -\dfrac{\partial V_1}{\partial t_2} & -\dfrac{\partial V_2}{\partial t_2} & \cdots & -\dfrac{\partial V_n}{\partial t_2} \\ \vdots & \vdots & & \\ -\dfrac{\partial V}{\partial t_n} & -\dfrac{\partial V_2}{\partial t_n} & \cdots & -\dfrac{\partial V_n}{\partial t_n} \end{bmatrix} \begin{bmatrix} (t_1 - s_1) \\ (t_2 - s_2) \\ \vdots \\ (t_n - s_n) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{1}$$

where s_h represents the costs at the margin born by others due to use of the h type vehicle, and t_h is the tax applied to the use of a unit of the h-type transportation mode, V_h. Each equation in (1) is an optimality condition that holds for a small change in the tax rate for one of the n different vehicle types. The comment made above concerning the first best solution is easily confirmed from equation set (1). If there are no distortions in non-transportation sectors of the economy and if transportation service tax rates are set equal to the congestion costs borne by others (i.e., $t_h = s_h$ for all h) each of the n equalities in equation set (1) hold and the first order conditions for maximizing welfare are satisfied.

It should be noted that we have not yet defined the units of transportation services denoted by V_h. The relationship holds whether the units of measure are passenger miles, vehicle miles, passenger trips or vehicle trips as long as the prices and costs borne by others are made consistent with the choice of units of measure. Thus, if V_h measures passenger miles in h type vehicles, s_h would be the costs borne by others of a marginal increase in passenger miles travelled by the h type mode and t_h would be the specific tax (or subsidy if negative) that is paid per passenger mile travelled. The alternative assumption we use is that there is an identical pattern of use for each type of vehicle on the road network and an identical amount of congestion created by this use of each type of vehicle, although patterns of use and type of congestion differ between vehicle types. Thus, we seek an optimal set of taxes or subsidies to be imposed on the different vehicle types with V_h interpreted as units of the h type vehicle and s_h as the costs imposed on others of an extra V_h type vehicle using the road network.

1. ONE MODE NOT FULLY TAXED

While (1) defines a well-known result on optimal taxation, it can also be used to provide guidelines on setting taxes when some optimal unconstrained taxes are not feasible. Our first case is in the context of a system where the tax on one of the vehicle types, say V_n, is constrained below marginal cost imposed on others, s_n. Relevant examples may arise when it is technically difficult to levy a congestion fee

[6] This is equation (A14) in the Appendix with the f's, the price distortions for non-transportation goods, set equal to zero.

on a particular vehicle type, when the appropriate tax rate is so high that possibilities of evasion make it administratively infeasible, or when political interest groups can block the imposition of a congestion fee on a certain vehicle. Congestion taxes on automobiles, for instance, may be subject to some or all of these constraints.

The case where one tax rate (say t_n) is fixed at a level below the appropriate optimal level in the unconstrained problem is considered with respect to equation set (2):

$$\text{(2)} \quad \begin{bmatrix} -\frac{\partial V_1}{\partial t_1} & -\frac{\partial V_2}{\partial t_1} & \cdots & -\frac{\partial V_{n-1}}{\partial t_1} \\ -\frac{\partial V_1}{\partial t_2} & -\frac{\partial V_2}{\partial t_2} & \cdots & -\frac{\partial V_{n-1}}{\partial t_{n-1}} \\ \vdots & \vdots & & \vdots \\ -\frac{\partial V_1}{\partial t_{n-1}} & -\frac{\partial V_2}{\partial t_{n-1}} & \cdots & -\frac{\partial V_{n-1}}{\partial t_{n-1}} \end{bmatrix} \begin{bmatrix} -D_1 \\ -D_2 \\ \vdots \\ -D_{n-1} \end{bmatrix} = \begin{bmatrix} -D_n \frac{\partial V_n}{\partial t_1} \\ -D_n \frac{\partial V_n}{\partial t_2} \\ \vdots \\ -D_n \frac{\partial V_n}{\partial t_{n-1}} \end{bmatrix}$$

where the equation from (1) derived by varying t_n has been eliminated and where the difference between the tax rate and the marginal costs borne by others $(s_h - t_h)$ has been denoted by D_h. Since $\partial V_n/\partial t_h$ (for $h = 1, \ldots, n-1$) is not in general equal to zero and since D_n is positive, setting the congestion fees for the other vehicle types equal to the corresponding marginal costs imposed on others will not satisfy the first order conditions for maximizing welfare. This conforms to the theory of the second best wherein given distortions to the equi-marginal conditions in the system leads to the result that the unconstrained equi-marginal conditions should not be maintained elsewhere.[7] Guidelines on setting taxes for vehicles not subject to constraints requires interactions within the system

[7] See Lipsey and Lancaster [**8**]. Sherman [**11**] has made use of the Marchand [**9**] model for a purpose quite similar to what is attempted in this paper. To put our work in perspective and to illustrate the comment that relation set (1) would hold as well if V_h referred to passenger miles as long as s_h were similarly interpreted as the effect on congestion costs borne by others of a marginal increase in passenger miles travelled by mode h, it is useful to briefly consider the Sherman problem in the context of our model. Sherman deals with two alternatives, buses and automobiles, in an environment where no tax is imposed on an automobile passenger mile. Letting vehicle 1 be the automobile and vehicle 2 be the bus, and setting $t_1 = 0$, we consider the first order condition for maximum welfare and a variation in t_2 when there are no other modes of transportation;

$$\text{(1')} \qquad s_1 \frac{\partial V_1}{\partial t_2} + (t_2 - s_2) \frac{\partial V_2}{\partial t_2}.$$

The congestion costs borne by others will differ for a bus passenger mile and an automobile passenger mile but only by the negligible amount given by the difference in the congestion cost borne by a bus passenger mile consumer and an automobile passenger mile consumer. Ignoring this difference, s_1 and s_2 are respectively $\partial C/\partial V_1$ and $\partial C/\partial V_2$ where C is total congestion costs borne by others. Solving for the optimal congestion fee t_2 we obtain

$$\text{(2')} \qquad t_2 = \left(\frac{\partial C}{\partial V_2} \frac{\partial V_2}{\partial t_2} + \frac{\partial C}{\partial V_1} \frac{\partial V_1}{\partial t_2} \right) \bigg/ \frac{\partial V_2}{\partial t_2}.$$

described by the relationship between the partial derivatives in equation set (2) to be specified.

Our approach focuses attention on aggregate passenger miles travelled in the road network. Relation (3) defines the equality between total passenger miles (T) and passenger miles summed over the n different vehicle types:

$$\text{(3)} \qquad \sum_{h=1}^{n} q_h V_h = T$$

where q_h is the assumed invariant number of passenger mile per h type vehicle. Differentiating with respect to t_a yields equation set (4):

$$\text{(4)} \qquad \sum_{h=1}^{n} q_h \frac{\partial V_h}{\partial t_a} = \frac{\partial T}{\partial t_a} \qquad (a = 1 \ldots n-1).$$

Substituting (4) into (2) and rearranging terms, we obtain (5):

$$\text{(5)} \quad \begin{bmatrix} -q_1 \frac{\partial V_1}{\partial t_1} & -q_2 \frac{\partial V_2}{\partial t_1} & \cdots & -q_{n-1} \frac{\partial V_{n-1}}{\partial t_1} \\ -q_1 \frac{\partial V_1}{\partial t_2} & -q_2 \frac{\partial V_2}{\partial t_2} & \cdots & -q_{n-1} \frac{\partial V_{n-1}}{\partial t_2} \\ \vdots & \vdots & & \vdots \\ -q_1 \frac{\partial V_1}{\partial t_{n-1}} & -q_2 \frac{\partial V_2}{\partial t_{n-1}} & \cdots & -q_{n-1} \frac{\partial V_{n-1}}{\partial t_{n-1}} \end{bmatrix} \begin{bmatrix} \left(\frac{D_n}{q_n} - \frac{D_1}{q_1}\right) \\ \left(\frac{D_n}{q_n} - \frac{D_2}{q_2}\right) \\ \vdots \\ \left(\frac{D_n}{q_n} - \frac{D_{n-1}}{q_{n-1}}\right) \end{bmatrix} = \begin{bmatrix} -\frac{D_n}{q_n} \frac{\partial T}{\partial t_1} \\ -\frac{D_n}{q_n} \frac{\partial T}{\partial t_2} \\ \vdots \\ -\frac{D_n}{q_n} \frac{\partial T}{\partial t_{n-1}} \end{bmatrix}.$$

Sherman was concerned with conditions under which this second best policy forced by $t_1 = 0$ would imply a negative t_2, a subsidy to a bus passenger mile. Since $\partial V_2/\partial t_2$ can be expected to be negative, a subsidy will be required if the numerator of the right-hand side of (2′) is positive. Defining the elasticities of congestion costs with respect to automobile and bus passenger miles as ξ_1 and ξ_2 respectively and the elasticity of demand for vehicle h passenger miles with respect to a change in the tax rate t_1 and η_1^L, this will occur if

$$\frac{\xi_2}{\xi_1} < \frac{\eta_2^1}{\eta_2^2}.$$

This is the basic condition obtained by Sherman; the only differences turning on the specifications of how congestion costs enter the respective models and how income effects of tax changes are compensated for. In this paper, use is made of equation set (1) in combination with assumptions on how demands for transportation services are related to derive additional guidelines on second best taxation in the same spirit as the Levy-Lambert, Marchand, and Sherman work.

We will consider two cases; that in which the total number of passenger miles is not affected by congestion fees ($\partial T/\partial t_a = 0$) for all $a = 1 \ldots n-1$) and that in which imposing higher congestion fees reduces the total number of passenger miles ($\partial T/\partial t_a \leq 0$ for all $a = 1 \ldots n-1$). The case where total passenger miles is fixed and the tax system is used to affect the choice of modes by which this travel is carried out results in a very simple principle for setting the taxes. The rule is to *equalize the distortions between congestion fees and marginal costs imposed on others per passenger mile across all modes.* If the congestion fees are levied in this way $D_n/q_n = D_h/q_h$ for all h by the tax rule and $\partial T/\partial t_a = 0$ by assumption, and therefore all $n-1$ of the optimality equations in (5) will be satisfied. If estimates are available for both the congestion effects of a marginal increase in the number of vehicles of each type and for the number of passenger miles obtained per vehicle, this rule defines the appropriate congestion fees for all vehicle types other than the one subject to the constraint.

The result just derived is of importance in understanding congestion arguments for subsidizing certain modes of travel. Since the marginal congestion costs are all positive, optimal taxation requires positive tax rates on all travel modes.[8] But where the optimal tax is not feasible for one of the modes, subsidization of other modes may be required. Consider the implications for mass transport modes of the case where the unconstrained optimal tax cannot be levied on, say, automobiles. Since the rate on automobiles is below the corresponding marginal cost borne by others, our guidelines show that the fee on all other modes will also be below their respective marginal costs imposed on others. The characteristic of mass transport modes that suggest subsidies may be appropriate is that such modes account for a greater number of passenger miles per vehicle and our guidelines call for the equalization of the difference between the marginal costs imposed on others and the tax rate *per* passenger mile.[9] Manipulation of the guidelines give a simple rule for subsidies on any h mode,

$$(6) \qquad t_h < 0 \quad \text{iff} \quad \frac{s_n - t_n}{q_n} > \frac{s_h}{q_h};$$

that is, a subsidy should be given to the h mode if and only if the distortion per passenger mile on the mode where political or administrative constraints on the congestion fee are effective is greater than the marginal social cost imposed on others per passenger mile. Thus, subsidies are more likely given the congestion characteristics of any vehicle with higher passenger miles per vehicle.

While the constant passenger miles assumption with taxes simply affecting the mix of modes is a useful reference point yielding clear-cut guidelines, the

[8] This result must be tempered by a realization that income distribution has not been explicitly introduced in our model. There may be valid arguments for subsidizing public transportation as a means of redistributing income from the rich who tend to use private transportation to the poor who tend to use mass transportation if more efficient redistribution programs are not available. However, these arguments should be conceptually separated from problems of congestion.

[9] A mass transportation mode, such as a bus, may be expected to have higher congestion effects per vehicle but not to the extent required to cancel its advantage with respect to passenger load.

alternative case in which higher fees reduce passenger miles travelled is an important extension. We therefore consider the case where $\partial T/\partial t_a \leqslant 0$ for all $a = 1 \ldots n-1$. In order to derive clear-cut guidelines for this case, it is necessary to further restrict the partial derivatives defining interactions between travel demands and tax rates. It is assumed that

$$(7) \qquad \frac{\partial V_j}{\partial t_a} > 0 \quad \text{for} \quad a = 1 \ldots n-1, a \neq j;$$

that is, all 'cross' derivatives between the tax rates and vehicle demands are positive. Ruling out complementarity between travel modes in this sense implies that the off diagonal elements in the matrix on the left-hand side of (5) are negative. Since $D_n < 0$ and $\partial T/\partial t_a \leqslant 0$ for all a, the elements in the column vector on the R.H.S. of (5) are non-negative. For equations systems with these restrictions, the solution will be characterized by non-negative elements in the column vector on the left-hand side of (5) if and only if all upper left corner principal minors of the determinant of the left-hand side matrix are positive.[10] With this condition satisfied, we have

$$\frac{D_h}{q_h} = \frac{s_h - t_h}{q_h} \leqslant \frac{D_n}{q_n} = \frac{s_n - t_n}{q_n} \quad \text{for all} \quad h = 1 \ldots n-1;$$

the difference between the tax rates and the costs imposed on others by a marginal increase in the use of the h-type vehicle, calculated on a per passenger mile basis, is less than or eqal to the distortion on the vehicle where political constraints limit the tax rate.

The fulfillment of the upper left corner principal minors all positive condition is easily interpreted in a three vehicle model; the weighted derivatives for vehicle demand with respect to their 'own' congestion fees are negative and dominate cross derivatives with the weights equal to passenger miles per vehicle, i.e., $q_1(\partial V_1/\partial t_1) q_2(\partial V_2/\partial t_2) > q_2(\partial V_2/\partial t_1) q_1(\partial V_1/\partial t_2)$. In the n-mode model, the conditions still turn on the dominance of the weighted "own" derivatives in that the positive diagonal elements must be greater in absolute value than the sum of the negative weighted cross derivatives in the same row or column.[11] Given the assumptions on the effects of the tax rates on total number of passenger miles travelled $(\partial T/\partial t_a) < 0$ for all $a = 1 \ldots n-1$) and on the cross derivatives in (7), the restriction on the principal minors is necessarily satisfied. This can be seen by using this assumption and (4) to obtain

$$(9) \qquad -q_a \frac{\partial V_a}{\partial t_a} > \sum_h q_h \frac{\partial V_h}{\partial t_h} - \frac{\partial T}{\partial t_a}, \qquad h = 1 \ldots n-1; \qquad h \neq a,$$

so that not only are "own derivatives" positive, but the positive diagonal elements in the left-hand side matrix are greater than the sum of the negative off diagonal elements in the same row in the left-hand side matrix. The Brauer-Solow row sum

[10] For a proof see Nikaido [**10**, Ch. 1].

[11] This is the Brauer-Solow row sum condition. For a discussion see Nikaido [**10**, Ch. 1].

criteria is satisfied which is equivalent to the upper left corner principal minors all positive condition.[12]

The interpretation of the above results is straightforward. If total passenger miles are not affected by tax rate changes, it maximizes net benefits resulting from these trips if these distortions (the excess of costs over benefits) are equalized at the margin. Otherwise, gains could be obtained by using taxes to switch travel from modes with high distortions to modes with low distortions. If, however, total passenger miles can be reduced, congestion fees should be raised to some extent on the vehicles where the constraint is not effective because this reduces the use of services where consumption benefits are below the costs of providing the services.

2. MULTIPLE CONSTRAINTS EFFECTIVE

It has been assumed in Section I that the exogenous constraint affects only one mode. A more realistic situation may involve effective constraints on several modes which keep tax rates below their unconstrained optimal levels. We therefore consider the case where transportation modes are classified between those on which tax rates can be freely adjusted (denoted as modes 1 to j) and those on which constraints are effective (denoted as modes $j+1$ to n). Without loss of generality among the constrained modes, let mode n be subject to the greatest distortion between marginal costs imposed on others and feasible tax rate per passenger mile with both tax rates less than the unconstrained optimal level ($D_n/q_n > D_h/q_n > D_{n-1}/q_{n-1} > 0$ for all $h = j+1, \ldots, n-1$).

The tax rate constraints eliminate the equations derived by varying the $j+1$ to n tax rates in (1). Use of the remaining j equations and (4) allows the first order conditions for an optimal solution to be written as

$$\begin{bmatrix} -q_1\dfrac{\partial V_1}{\partial t_2} & -q_2\dfrac{\partial V_2}{\partial t_1} & \cdots & -q_j\dfrac{\partial V_j}{\partial t_1} \\ -q_1\dfrac{\partial V_1}{\partial t_2} & -q_2\dfrac{\partial V_2}{\partial t_2} & \cdots & -q_j\dfrac{\partial V_j}{\partial t_2} \\ \vdots & \vdots & & \vdots \\ -q_1\dfrac{\partial V_1}{\partial t_j} & -q_2\dfrac{\partial V_2}{\partial t_j} & \cdots & -q_j\dfrac{\partial V_j}{\partial t_j} \end{bmatrix} \begin{bmatrix} \dfrac{D_n}{q_n}-\dfrac{D_1}{q_1} \\ \dfrac{D_n}{q_n}-\dfrac{D_2}{q_2} \\ \vdots \\ \dfrac{D_n}{q_n}-\dfrac{D_j}{q_j} \end{bmatrix} \tag{10}$$

$$= \begin{bmatrix} \sum_{h=j+1}^{n-1}\left(\dfrac{D_n}{q_n}-\dfrac{D_h}{q_h}\right) & q_h\dfrac{\partial V_h}{\partial t_1}-\dfrac{\partial T}{\partial t_1} \\ \sum_{h=j+1}^{n-1}\left(\dfrac{D_n}{q_n}-\dfrac{D_h}{q_h}\right) & q_h\dfrac{\partial V_h}{\partial t_2}-\dfrac{\partial T}{\partial t_2} \\ \vdots & \vdots \\ \sum_{h=j+1}^{n-1}\left(\dfrac{D_n}{q_n}-\dfrac{D_h}{q_h}\right) & q_h\dfrac{\partial V_h}{\partial t_q}-\dfrac{\partial T}{\partial t_q} \end{bmatrix}$$

[12] For proof, see Nikaido [**10**, Ch. 1].

or

$$(11)\quad \begin{bmatrix} -q_1\dfrac{\partial V_1}{\partial t_1} & -q_2\dfrac{\partial V_1}{\partial t_1} & \cdots & -q_j\dfrac{\partial V_j}{\partial t_1} \\ -q_1\dfrac{\partial V_1}{\partial t_1} & -q_2\dfrac{\partial V_2}{\partial t_2} & \cdots & -q_j\dfrac{\partial V_j}{\partial t_2} \\ \vdots & \vdots & & \vdots \\ -q_1\dfrac{\partial V_1}{\partial t_j} & -q_2\dfrac{\partial V_2}{\partial t_i} & \cdots & -q_j\dfrac{\partial V_j}{\partial t_j} \end{bmatrix} \begin{bmatrix} \dfrac{D_1}{q_1}-\dfrac{D_{n-1}}{q_{n-1}} \\ \dfrac{D_2}{q_2}-\dfrac{D_{n-1}}{q_{n-1}} \\ \vdots \\ \dfrac{D_j}{q_j}-\dfrac{D_{n-1}}{q_{n-1}} \end{bmatrix} = \begin{bmatrix} \sum\limits_{\substack{h=j+1\\ h\neq n-1}}^{n} \left(\dfrac{D_h}{q_h}-\dfrac{D_{n-1}}{q_{n-1}}\right) \quad q_h\dfrac{\partial V_h}{\partial t_1}+\dfrac{\partial T}{\partial t_1} \\ \sum\limits_{\substack{h=j+1\\ h\neq n-1}}^{n} \left(\dfrac{D_h}{q_h}-\dfrac{D_{n-1}}{q_{n-1}}\right) \quad q_h\dfrac{\partial V_h}{\partial t_2}+\dfrac{\partial T}{\partial t_2} \\ \vdots \\ \sum\limits_{\substack{h=j+1\\ h\neq n-1}}^{n} \left(\dfrac{D_h}{q_h}-\dfrac{D_{n-1}}{q_{n-1}}\right) \quad q_h\dfrac{\partial V_h}{\partial t_j}+\dfrac{\partial T}{\partial t_j} \end{bmatrix}$$

where the distortions between tax rates and marginal costs imposed on others per passenger mile for all modes where tax rates may be changed relative to the greatest and smallest of the distortions for the other modes have been isolated in the left-hand column vector. We again assume that $\partial T/\partial t_a \leqslant 0$ for all $a = 1 \ldots j$ to rule out complementarity between modes. As in the previous analysis, this implies that the Brauer-Solow row sum criteria are satisfied and the upper left corner principal minors of (10) and (11) are positive.

If total passenger miles in the system are constant ($\partial T/\partial t_a = 0$ for all $a = 1 \ldots j$), the elements of the column vectors on the left-hand side of (10) and (11) are non-negative and the previously used theorem again results in a solution with non-negative elements in the column vectors on the left-hand side of (10) and (11); that is,

$$(12)\quad \frac{D_n}{q_n} \geqslant \frac{D_a}{q_a} \geqslant \frac{D_{n-1}}{q_{n-1}} \quad \text{for all} \quad a = 1 \ldots j.$$

The distortion per passenger mile on the modes where constraints on the congestion fees are not effective should be squeezed within the range of the exogenously defined distortions per passenger mile on the vehicles where the constraints are effective. The interpretation of (12) is straightforward. Consider the case where the tax rate on one of the modes not subject to an effective constraint results in the highest distortion per passenger mile among all modes. Then increasing the congestion fee on this mode would shift travel to other modes where the excess of costs over benefits is smaller. A welfare gain could be obtained in this way until all distortions were equal to or below D_n/q_n. A similar argument applies for distortions per passenger mile below D_{n-1}/q_{n-1} with reductions in tax rates attracting travel demand from modes where excess of costs over benefits are relatively high.

When increases in congestion fees reduce total passenger miles in the system ($\partial T/\partial t_a \leqslant 0$ for all $a = 1 \ldots j$), the elements in the right-hand side column vector of (10) are still non-negative and the theorem on the solution values for the elements in the left-hand side column vector can be used to give

$$(13) \qquad \frac{D_a}{q_a} = \frac{s_a - t_a}{q_a} \geqslant \frac{s_n - t_n}{q_n} = \frac{D_n}{t_n} \qquad \text{for} \quad h = 1 \ldots j;$$

that is, the tax rate on all of the modes where the constraints are not effective should never be set lower than the level required to equalize the distortion between the tax and marginal costs imposed on others per passenger mile with the distortion per passenger mile on the model with the largest fixed distortions as in the previous case with fixed total passenger miles. However, with $\partial T/\partial t_a \leqslant 0$ for all $a = 1 \ldots j$, the elements in the column vector on the right-hand side of (11) may or may not be non-negative. The theorem leading to non-negative solution values for the elements in the left-hand side column vector in (11) cannot be applied and the first order conditions may or may not require a smaller distortion per passenger mile in absolute value on modes where the constraints are not operative than the smallest distortion per passenger mile found on modes where the constraints are effective, i.e., $D_{n-1}/q_{n-1} \gtreqless D_a/q_a$ for all $a = 1 \ldots j$. This results because tax increases beyond the point which equalizes the distortion for any particular mode to the minimum distortion on the modes where tax rates are constrained may be justified by the resulting reduction in passenger miles travelled in a network where marginal costs (inclusive of those imposed on others) exceed the marginal benefits resulting from the trips.

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APPENDIX

In this appendix, the optimality conditions used in the text are derived in a simple general equilibrium framework. Some generalizations are then discussed.

Assume a social welfare function W:

$$(\text{A1}) \qquad W = W(U_1(v_1, x_1, L_1); U_2(v_2, x_2, L_2)),$$

the arguments of which are the utility levels of two individuals U_1 and U_2, which are functions of consumption of a non-transportation good, x, vehicle services, v, and leisure L. It is assumed that all producers' prices are fixed but that taxes may vary so that demands for the two goods are functions of these tax rates. Leisure time of each person is a function of vehicle use by *both* individuals because of congestion.[13]

[13] The allocation of time between leisure and work could be made to depend directly on the tax rates. To simplify the expressions, we assume this effect is indirect through the effect on vehicle use.

The first order condition for an optimal tax on vehicle use, t, is obtained by setting the partial derivatives of W with respect to the tax rate equal to zero;

$$\text{(A2)}\quad \frac{\partial W}{\partial t}=\frac{\partial W}{\partial U_1}\left[\frac{\partial U_1}{\partial v_1}\frac{\partial v_1}{\partial t}+\frac{\partial U_1}{\partial x_1}\frac{\partial x_1}{\partial t}+\frac{\partial U_1}{\partial L_1}\frac{\partial L_1}{\partial v_1}\frac{\partial v_1}{\partial t}+\frac{\partial U_1}{\partial L_1}\frac{\partial L_1}{\partial v_2}\frac{\partial v_2}{\partial t}\right]$$
$$+\frac{\partial W}{\partial U_2}\left[\frac{\partial U_1}{\partial v_2}\frac{\partial v_2}{\partial t}+\frac{\partial U_2}{\partial x_2}\frac{\partial x_2}{\partial t}+\frac{\partial U_2}{\partial L_2}\frac{\partial L_2}{\partial v_2}\frac{\partial v_2}{\partial t}+\frac{\partial U_2}{\partial L_2}\frac{\partial L_2}{\partial v_1}\frac{\partial v_1}{\partial t}\right]=0$$

Equations (A3)–(A6) are assumed to be satisfied;

$$\text{(A3)}\quad \frac{\partial U_j/\partial v_j}{\lambda_j}=(p+t)-\frac{(\partial U_j/\partial L_j)(\partial L_j/\partial v_j)}{\lambda_j}-w\frac{\partial H_j}{\partial v_j},\qquad j=1,2,$$

$$\text{(A4)}\quad \frac{\partial U_j/\partial x_j}{\lambda_j}=(p_x+f),\qquad j=1,2,$$

$$\text{(A5)}\quad \frac{\partial U_j/\partial L_j}{\lambda_j}=w,\qquad j=1,2,$$

$$\text{(A6)}\quad (p_x+f)(x_1+x_2)+(p+t)(v_1+v_2)=w(H_1+H_2)+f(x_1+x_2)+t(v_1+v_2)$$

where λ_j is the marginal utility of income to the jth individual, and H_j is the work time of jth individual, and w is the constant wage rate.[14] Equations (A3)–(A5) are the equi-marginal optimality conditions for consumers.[15] (A3) indicates that each consumer equates the marginal benefits of vehicle use to the price of the vehicle service inclusive of tax, $p+t$, and the time costs involved in vehicle use in either reduced leisure or reduced work time. (A4) indicates that each consumer equates the marginal benefits of increased consumption of the non-transportation good to the price of the vehicle service comprised of the producer price p_x and the tax f. (A5) indicates that the consumer equates the marginal benefits of leisure to the wage rate. (A6) indicates that the value at prices inclusive of tax of the consumption bundle for the economy equals the value of wage payments on work time plus transfers to consumers of tax proceeds. Writing this constraint in aggregate terms reflects our lack of concern with the transfer process and the resulting impact on the distribution of the consumption bundle. Noting individual demands sum to output levels, we also assume

$$\text{(A7)}\quad p_x(dx_1+dx_2)+p(dv_1+dv_2)-w(dH_1+dH_2)=0;$$

producer price weighted changes in outputs and inputs sum to zero reflecting the satisfaction of the equi-marginal optimality conditions for producers.

Marginal costs imposed on others by vehicle use are defined in (A8):

$$\text{(A8)}\quad \begin{aligned}-s_{12}&=w\frac{\partial L_2}{\partial v_1}+w\frac{\partial H_2}{\partial v_1},\\ -s_{21}&=w\frac{\partial L_1}{\partial v_2}+w\frac{\partial H_1}{\partial v_2},\end{aligned}$$

where s_{ij} is the time costs imposed through congestion on individual j by increased use of transportation service by i, where use has been made of (A5). We assume $(\partial L_2/\partial v_1)+(\partial H_2/\partial v_1)=(\partial L_1/\partial v_2)+(\partial H_1/\partial v_2)$ so that the congestion effect of increased vehicle use is not differentiated by vehicle operator, $(s_{12}=s_{21}=s)$. We also assume that the marginal utility of income is identical between consumers $(\lambda_1=\lambda_2)$ and that the social welfare function gives equal weight to consumers $(\partial W/\partial U_1=\partial W/\partial U_2=1)$. (A3)–(A5) and (A7)–(A8) may then be used in (A2) to obtain

$$\text{(A9)}\quad f\frac{\partial X}{\partial t}+(t-s)\frac{\partial V}{\partial t}=0,$$

[14] The simplifying assumption of a constant wage rate may be justified by an underlying constant opportunity costs production possibility set.

[15] These may be obtained by maximizing individual welfare functions for i subject to a budget constraint with respect to choice of consumption of x and v by i.

where $X=\Sigma_{i=1}^{2} x_i$ and $V=\Sigma_{i=1}^{2} v_i$. This is the first order condition for maximizing welfare requiring zero taxes on non-transportation goods and taxes equal to the marginal costs imposed on others for the transportation services.

The above model can be generalized in several ways. In addition to congestion costs in the form of time delays, account should also be taken of the costs imposed on others by increased vehicle use in the form of noise and air pollution, higher maintenance costs on vehicles travelling in congested networks, and higher network maintenance costs.[16] The dimensions of the model can be expanded to include n types of transportation services, m non-transportation commodities, and e types of labor. The model then consists of

$$W=W\Big(\sum_j U^j(\cdot)\Big), \tag{A10}$$

$$U^j=U_j\Big(\sum_h v_h^j, \sum_i x_i^j, \pi, N, \sum_e L_e^j\Big) \qquad (h=1\ldots,n;\, i=1\ldots m;\, e=1\ldots r); \tag{A11}$$

$$\sum_j\sum_i p_i x_i^j+\sum_j\sum_h p_h v_h^j+\sum_j\sum_h p_h^y y_h^j+p_z Z=\sum_e\sum_j w_R H_e^j, \tag{A12}$$

where the social welfare function in (A10) is a function of the utility levels of the j individuals, and these utilities are functions, as defined in (A11), of use of n types of vehicles, m non-transportation commodities, air pollution π, noise pollution N, and leisure classified by r types of labor. The budget constraint for the economy is given in (A12) where p_h^y is the price of a unit of maintenance on the h vehicle, y_h^j is the amount of maintenance required on the h vehicle used by the jth consumer, p_z is the price of the transportation network maintenance service Z, w_e is the wage rate of the eth type labor, and H_e is the work time of the eth type labor. All producers' prices are held fixed.[17] Demands for non-transportation commodities, transportation services, and the allocation of time between work, leisure, and travel are functions of all tax rates which alter consumers' prices. Leisure, work time, travel time, maintenance services on vehicles and the network, air and noise pollution are all functions of vehicle use. Producer and consumers satisfy equi-marginal optimality conditions, but consumers of transportation services ignore the cost through time delays, increased air and noise pollution, and higher maintenance costs on vehicles and the transportation network that are imposed on others by the consumption of transportation services. Letting the summation of these costs imposed on others by the jth consumer's use of transportation services of the h vehicle be denoted as s_{jh}, setting partial derivatives of social welfare with respect to each of the n tax rates on vehicles equal to zero while assuming marginal utilities of income identical between consumers and giving equal weight in the social welfare to the welfare of all consumers, leads to the first order condition for the tax rate on the ath vehicle in (A13):

$$\sum_j\sum_h (t_h-s_{jh})\frac{\partial v_h^j}{\partial t_a}+\sum_j\sum_i f_i\frac{\partial x_i^j}{\partial t_a}=0. \tag{A13}$$

Again assuming that s_{jh} is identical for all consumers, which means that the costs created for others are independent of who is actually using the services of the h-type vehicle, (A13) may be written as

$$\sum_h (t_h-s_h)\frac{\partial V_h}{\partial t_a}+\sum_i f_i\frac{\partial X_i}{\partial t_a}=0 \qquad (a=1\ldots n), \tag{A14}$$

which is the starting point of our analysis in the text as relation (1) is (A14) written in matrix form with the assumption that there are zero distortions in the non-transportation sectors of the economy.

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[16] On certain models, network maintenance costs may be reflected in prices charged users. However, for free access roads this is not the case.

[17] Again an assumption of constant opportunity costs along the production possibility surface may be used to justify this simplification.

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