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Sequential time-step generation companies decisions in oligopolistic electricity market

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Abstract

This paper studies the production decisions of generation companies (GENCOs) which are fully engaged in oligopolistic electricity markets. The model presented is based upon the static equilibrium model solved sequentially in time. By decomposing the problem in time, each time-step is solved independently using a Cournot-like market model. The time dimension is divided into discrete, 1-h time-steps. The model also incorporates the effects of technical and temporal constraints such as time on/off and ramp up/down. Since GENCOs tend toward repetitive decision-making, they can more easily learn from the market. The concept of forward expectations and the lessons derived from the market are introduced, and several numerical examples are provided.

Keywords: Cournot model; Electricity markets; Oligopolistic competition

1. Introduction

In a market-driven environment, a power generating utility solves the self-unit commitment problem to obtain an optimal bidding strategy [1]. Ideally, its optimal policy is designed to reap the maximum expected profit. In reality, however, the environment in which decisions (and decision-making policies) are made is often defined by the operational and technical constraints of the utility's generating units, its short-term financial requirements, or other restrictions.

The easiest way to model the dynamic behavior of market players is to replicate static snapshot of single periods [2]. The single-period models then provide the basis for multi-period models. In the single-period Cournot model each firm wants to maximize profits by deciding its optimal decision output. In the multi-period extension of the Cournot model, each firm wants to maximize its discount profits by selecting the optimum output levels for each time period [2,3].

Most of the work applied to the electricity market analysis reported in the literature covered a single period. At the beginning most of these models were constructed as single-node generation-only models [4]. Later, basic representations and linear dc transmission network were introduced for modeling spatiality [5,24–26]. Recently, ac network representation has been incorporated in a non-linear programming problem in order to systematically study for the impacts of network constraints on the market equilibrium [6].

Since GENCOs operate in a sequential-period market where, in each period, simultaneous output decisions are made, in most market scenarios, it may not be enough to maximize gain in the current and next period. Therefore, the GENCOs will seek to maximize total gain over the next several periods. However, not knowing (or being unaware of) their competitors' future output decisions will make it difficult for any one GENCO to predict its rivals' behavior [7,8]. Faced with this difficulty, a GENCO may adjust its own output expectation of the current period according to both the output of the last period and the expected output in the next subsequent period. In addition, each GENCO will probably rely upon other information it gathers over time, especially the data which will most likely influence its present choice. In other words, when the same bidder plays the same opponents multiple times, we would expect that the bidding agents will adjust their own behaviors to maximize their profits [8]. A procedure to identify multi-period equilibria in an

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electricity market is important for market regulators who may use it for market monitoring [9]. A multi-period equilibrium in a pool-based electricity market that may include minimum profit constraints for online generating units is analyzed in Ref. [23]. An oligopoly with spatially dispersed generators and consumers and with multi-period demand is modeled in Ref. [4]. A dynamic sequential framework by using DESS is reported in Ref. [10]; that analysis focuses on the dynamics within a single period.

We can also expect that forward expectations will accompany the learning process [11,12]. This integration is crucial for two reasons: forward expectations teach a GENCO how its current stock valuation is affected (since stocks are the physical link between successive periods, and the valuation will transform expectations about future trading into desires to exchange current goods), and they are based on *available information*, i.e., the stream of past and present price-quantity signals [13]. In today's competitive, volatile markets, accurate modeling of both the operational and temporal constraints of all of its generating units may give a GENCO the "edge" over its competition. Conjectural variation method has been widely applied to estimate the strategic behavior in game-theoretical contexts in terms of imperfect information [14]. A conjectural variation-based learning model that can be used by a GENCO to improve its bidding performance is reported in Ref. [15]. Each firm learns and dynamically regulates its conjectures based upon the reactions of its rivals to its bidding according to the available information published in the electricity market. Unfortunately, these conjectural variation models have been criticized for the drawbacks of logical consistency and the possibility of abundant equilibria. The existence and uniqueness of consistent conjectural variation equilibrium in electricity markets is investigated in Ref. [16].

Even what appears to be an insignificant constraint can quickly alter a GENCOs market strategies [17]. For example, the strategic use of ramp rates beyond elastic limits in generation dispatch has been investigated in Ref. [18], because they incur ramping costs and also widen the possible range of energy delivery. A detailed formulation to model the power trajectories followed by a thermal unit during start-up and shut-down processes, as well as the ramping limitations when increasing or decreasing power is reported in Ref. [19].

The Cournot model still does not analyze significant electricity market issues including intertemporal considerations. In Ref. [20] intertemporal decisions related with maintenance decisions are reported. In an electricity market with only a few major competing GENCOs, maintenance plays a critical role that goes beyond traditional least-cost analysis. In this document the authors extend the previous work reported in Ref. [12]. A rigorous formulation of the ramping constraints reported in Ref. [19] has been implemented to analyze the effect of intertemporal constraints on a GENCOs decision-making process. The learning aspect, represented by forward expectations, is compared with the Cournot model without learning. A sensitivity analysis is performed to observe how the solution of the short-term equilibrium problem varies with the generation cost parameters, the demand parameters, and the adjusting coefficients.

The paper is organized as follows: Section 2 describes the electricity spot market model. Section 3 presents a set of numerical examples to illustrate the points at hand. Section 4 presents a parameter dependency analysis. Finally, our conclusions are given in Section 5.

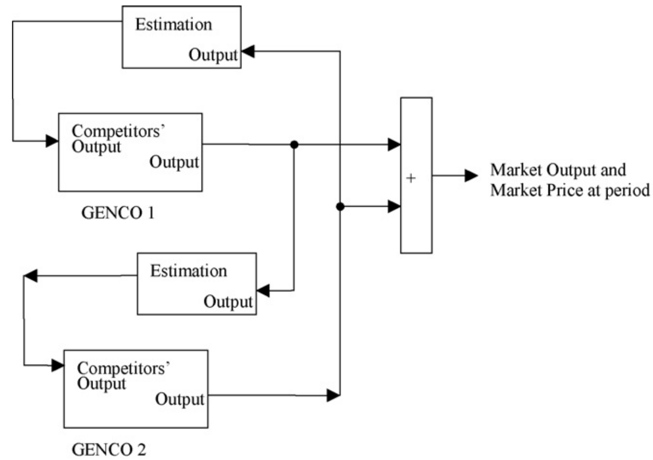

Fig. 1. Electricity spot market model.

2. Electricity market model

In this paper we consider a spot market operated on an hourly basis where each time-step is solved individually using the Cournot market model. A representation of this electric market is shown in Fig. 1.

Since GENCOs tend to make repetitive decisions, it is expected that they will learn from the market [22]. For each time period, GENCOs must form an expectation of their rivals' output in the subsequent period in order to determine their own corresponding profit-maximizing quantity for period $k+1$, and so on. The sequential decision-making process of GENCO 1 is depicted in Fig. 2.

Consider the inverse linear market-demand function at period k given by

$$P(Q) = a - bQ(k) \tag{1}$$

where $Q(k) = \sum_{i=1}^{n} q_i(k)$ and a and b are the market-demand function parameters.

We assume that a GENCO knows the inverse demand function, and that it must estimate the demand when it does not know

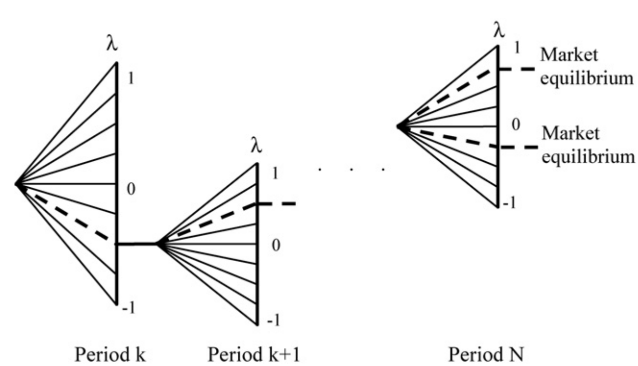

Fig. 2. Sequential decision-making for GENCO 1.

the actual demand function. Its optimization program is to maximize the expected profit from its generation assets, energy and reserve, subject to operational constraints, over time. Mathematically, this can be expressed as

$$\max_{q_i(k)} \pi_i(k) = P(q_i(k) + \hat{q}_j(k))q_i(k) - C_i(q_i(k)) \quad (2)$$

where $\hat{q}_j(k) = \lambda_j q_j(k-1) + (1-\lambda_j)q_j(k)$, $\hat{q}_j(k)$ represents GENCO j's expectation of the decisions made by GENCO i, $q_j\,(k-1)$ is GENCOs j's decision output at period $(k-1)$, λ_j is the adjustment coefficient for GENCO j, and $\lambda_2 \in [-1 < \lambda_2 \le 1]$.

Subject to the following constraints:

Ramp-up constraints: From one time instant to the next the unit cannot increase its output above a maximum increment; this yields:

$$q_i(k+1) - q_i(k) \le Z_i \quad \forall k = 1, \ldots, K \quad (3)$$

where Z_i is the maximum power ramp-up increment of unit i.

Ramp-down constraints: A unit cannot decrease its output power above a maximum power decrement. Therefore

$$q_i(k) - q_i(k+1) \le W_i \quad \forall k = 1, \ldots, K \quad (4)$$

where W_i is the maximum power ramp-down decrement of unit i.

Unit capacity constraint: Any unit at any time should operate within operational limits, then:

$$q_i^{\min} \le q_i \le q_i^{\max} \quad \forall i = 1, \ldots, n \quad (5)$$

where $q_i^{\min}$ and $q_i^{\max}$ are the lower and upper generation limit, respectively, of unit i.

State transition constraints: The length of time the unit has been off or online.

$$x_{ik} = \begin{cases} \min(t_i^{\text{on}}, x_{k+1} + 1) & \text{if } u_{ik} = 1 \\ \max(t_i^{\text{off}}, x_{k+1} - 1) & \text{if } u_{ik} = 0 \end{cases} \quad (6)$$

where x_{ik} is a state variable indicating the length of time that unit i has been up or down at period k, and u_{ik} is a binary decision variable indicating whether unit i at period k is up or down.

Unit status constraint: The unit can be either on or off, then:

$$u_{ik} = \begin{cases} 1 & \text{if } 1 \le x_{i,k-1} < t_i^{\text{on}} \\ 0 & \text{if } -1 \ge x_{i,k-1} > -t_i^{\text{off}} \end{cases} \quad (7)$$

The GENCO i production cost function is given by

$$C_i(q_i(k)) = d_i + e_i q_i(k) + f_i q_i^2(k) \quad \forall i = 1, \ldots 2 \quad (8)$$

where d_i, e_i, and f_i are the production cost factors.

Temporarily ignoring operational and temporal constraints and solving the problem as if they did not exist, then:

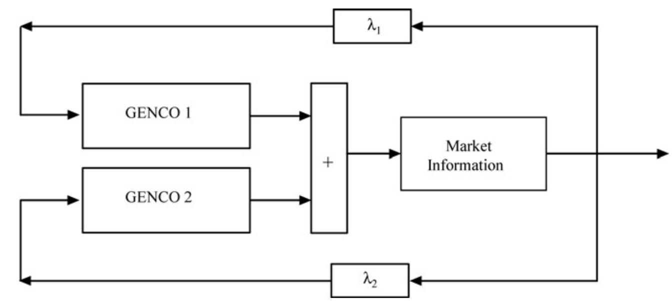

Fig. 3. Two-GENCO electricity market equivalent.

$$\max_{q_i(k)} \pi_i(k) = P(q_i(k) + \hat{q}_j(k))q_i(k) - d_i - e_i q_i(k) - f_i q_i^2(k) \quad (9)$$

The first-order condition is

$$\frac{\partial \pi_i}{\partial q_i(k)} = a - 2bq_i(k) - b\lambda_j q_j(k-1) + b(1-\lambda_j)q_j(k) - e_i - 2f_i q_i(k) = 0 \quad (10)$$

For the two players, in matrix form we have

$$\begin{bmatrix} 2(b+f_1) & b(1-\lambda_2) \\ b(1-\lambda_1) & 2(b+f_2) \end{bmatrix} \begin{bmatrix} q_1(k) \\ q_2(k) \end{bmatrix} = \begin{bmatrix} a - e_1 - b\lambda_2 q_2(k-1) \\ a - e_2 - b\lambda_1 q_1(k-1) \end{bmatrix} \quad (11)$$

A representation of this electric market is shown in Fig. 3.

Solving for $q_1(k)$ and $q_2(k)$ yields:

$$q_1(k) = \frac{2(b+f_2)(a - e_1 - b\lambda_2 q_2(k-1)) - b(1-\lambda_2)(a - e_2 - b\lambda_1 q_1(k-1))}{4(b+f_1)(b+f_2) - b^2(1-\lambda_1)(1-\lambda_2)} \quad (12)$$

$$q_2(k) = \frac{2(b+f_1)(a - e_2 - b\lambda_1 q_1(k-1)) - b(1-\lambda_1)(a - e_1 - b\lambda_2 q_2(k-1))}{4(b+f_1)(b+f_2) - b^2(1-\lambda_1)(1-\lambda_2)} \quad (13)$$

If the GENCOs do not know the inverse demand function, they must estimate the demand. Assume that GENCO i's estimate is $P(Q) = a_i - b_i Q(k)$, $i = 1, \ldots, 2$. Then, the system becomes:

$$\begin{bmatrix} 2(b_1+f_1) & b_1(1-\lambda_2) \\ b_2(1-\lambda_1) & 2(b_2+f_2) \end{bmatrix} \begin{bmatrix} q_1(k) \\ q_2(k) \end{bmatrix} = \begin{bmatrix} a_1 - e_1 - b_1\lambda_2 q_2(k-1) \\ a_2 - e_2 - b_2\lambda_1 q_1(k-1) \end{bmatrix} \quad (14)$$

2.1. Generation upper limits

If GENCO 1 has a capacity constraint, its profit-maximization decision becomes:

$$\max \pi_1(k) = P(Q(k))q_1(k) - C_1(q_1(k)) \quad \text{S. to } q \le q_1^{\max} \quad (15)$$

We construct the new function:

$$L = P(Q(k))q_1(k) - c_1(q_1(k)) - \mu(q_1(k) - q_1^{\max}) \quad (16)$$

where μ is a Lagrange multiplier.

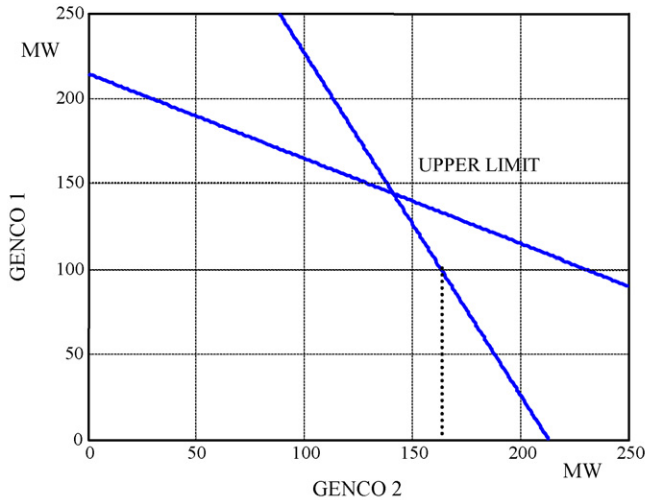

Fig. 4. Effect of capacity limit in market equilibrium.

The first-order conditions are

$$\frac{\partial L}{\partial q_1(k)} = a - e_1 - 2(b + f_1)q_1(k) - bq_2(k) - \mu = 0 \quad (17)$$

$$\frac{\partial L}{\partial \mu} = q_1(k) - q_1^{\max}(k) = 0 \quad (18)$$

In the two-player market model, the resulting set of equations is

$$\begin{bmatrix} 2(b+f_1) & b(1-\lambda_2) & -1 \\ b(1-\lambda_1) & 2(b+f_2) & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1(k) \\ q_2(k) \\ \mu \end{bmatrix} = \begin{bmatrix} a - e_1 - b\lambda_2 q_2(k-1) \\ a - e_2 - b\lambda_1 q_1(k-1) \\ q_1^{\max}(k) \end{bmatrix} \quad (19)$$

A similar procedure is applied when the lower limit is bounded.

The intersection of the two reaction functions, Eqs. (12) and (13), determines the market equilibrium in the Cournot model. This equilibrium represents a Nash equilibrium if each GENCO believes the other will not change output regardless of what its competitor does. The standard Nash equilibrium is also reached even when maximum generation limit is reached (which we can observe in a situation where one GENCO reaches its maximum limit under the assumption that its competitor lacks complete information). If this was not the case, the other GENCO will exercise market power.

Fig. 4 portrays the reaction functions for the two GENCOs at specific period. Here we observe that the upper generating limit of any unit is not reached given that such limits are above the market equilibrium, $q_1 = 141.53$ and $q_2 = 143.83$. If a generating upper limit is reached, the new market equilibrium is determined at the intersection point between the reaction function and the generating unit's upper limit. Therefore, the limit will restrict the pure Cournot equilibrium.

From Fig. 4, we also observe that the upper limit will never be reached under demand and cost production parameters: if the upper limit is 100 MW instead of 150 MW, the new equilibrium is $q_1 = 100$ and $q_2 = 164.53$, as shown.

2.2. Time on/off and ramp up/down constraints

The increment or decrement of the generation level of a unit over any two successive online periods (excluding start-up and shut-down periods) is bounded by the ramp-up (RU) and ramp-down (RD) limits, respectively as shown in Fig. 5. Temporal constraints and ramp up/down are incorporated in our model from Ref. [19].

3. Numerical examples

This section presents three numerical examples of the model described above. In each case the Cournot model is executed twice: without and with learning. The production cost data shown in Table 1 has been taken from Ref. [1] and modified.

The expected demand function parameters for each period of the day-ahead market are listed in Table 2. The same demand function is retained for the three cases.

The forward expectation adjusting factors for each period of the day-ahead market are listed in Table 3 (obtaining the adjusting coefficients is an important topic, but beyond the scope of this paper). These parameters must be estimated for each GENCO; they can be found utilizing several methods (e.g., data mining, neural nets, and forecasting approaches) [21].

Case A. In this case, operational and temporal constraints are omitted. The market equilibrium is found for each trading period

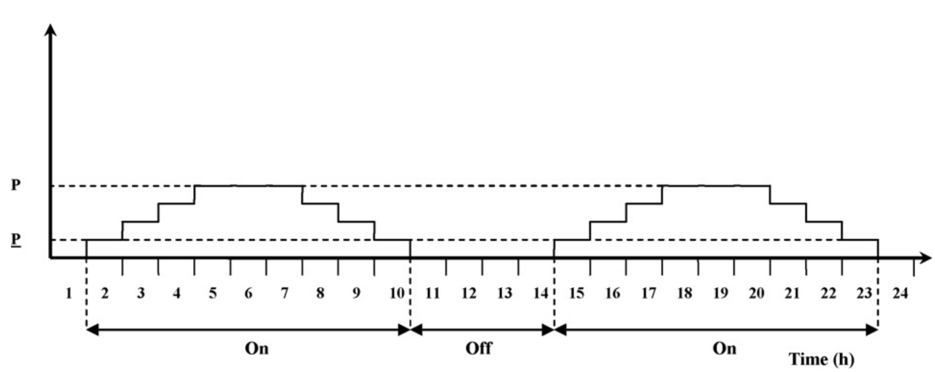

Fig. 5. Illustration of ramp up/down and maximum/minimum constraints.

Table 1
Producers' data

GENCO	d_i (\$)	e_i (\$/MW)	f_i (\$/MW^2)	$q_i^{\min}$ (MW)	$q_i^{\max}$ (MW)	t_i^{on} (h)	t_i^{off} (h)	Ramp up$_i^{\max}$ (MW)	Ramp down$_i^{\max}$ (MW)
1	820	9.023	0.00113	0	150	16	6	40	30
2	400	7.654	0.00160	0	300	12	4	50	30

Table 2
Expected demand function parameters for the day-ahead market

Period	*a*	*b*
1	185	0.42
2	190	0.35
3	210	0.46
4	120	0.34
5	130	0.40
6	140	0.62
7	195	0.34
8	150	0.20
9	180	0.37
10	240	0.42
11	230	0.99
12	160	0.28
13	148	0.22
14	330	0.5
15	135	0.25
16	180	0.43
17	168	0.35
18	160	0.36
19	198	0.49
20	175	0.30
21	190	0.48
22	140	0.60
23	150	0.52
24	130	0.20

Table 3
Forward expectation adjusting coefficient

Period	λ_1	λ_2
1	0.0	1.0
2	−1.0	1.0
3	0.0	0.0
4	−1.0	0.9
5	−0.9	1.0
6	−0.3	0.3
7	−0.8	0.5
8	0.7	0.5
9	0.6	1.0
10	0.8	0.7
11	−0.7	−0.4
12	1.0	1.0
13	0.3	0.2
14	0.8	−0.8
15	−0.9	−0.9
16	−0.4	0.4
17	0.0	−1.0
18	0.0	0.7
19	0.9	0.2
20	0.4	1.0
21	−0.3	−0.1
22	0.0	0.7
23	−0.4	1.0
24	−0.6	0.8

individually. The expected market supply and the expected outputs of the two GENCOs for each period of the day-ahead are reported in Table 4 and graphically depicted in Fig. 6.

By comparing columns 2 and 3 with columns 4 and 5 in Table 4, we observe that each GENCOs contribution to the market is the same when both adjusting coefficients equal 0 (this occurs at period 3). Hence this case represents the traditional Cournot equilibrium with two players and a linear demand function. The equilibrium is more competitive when both coefficients are positive; the opposite occurs when both coefficients are negative [17]. Each market equilibrium is a Nash equilibrium since neither GENCO will change its output if the other does not change, given the current information.

From Fig. 6(a) we can observe that the GENCOs' outputs differ slightly. The differences between their outputs are due only to different production costs. We see that GENCO 1 is more costly and therefore its output is lower. However, when learning is introduced, the outputs of the two GENCOs differ because of production costs *and* because of the adjusting factor involved in each one's decisions as shown in Fig. 6(b).

The market price for each period as displayed in Fig. 7 is determined by the total market generation.

Table 4
Expected GENCOs' outputs: Case A

Period	No learning		Learning	
	GENCO 1	GENCO 2	GENCO 1	GENCO 2
1	138.51	141.60	209.21	106.32
2	170.95	174.62	205.05	159.69
3	144.58	147.40	144.58	147.40
4	107.40	111.26	89.33	147.82
5	99.62	102.92	77.20	119.55
6	69.66	71.81	59.89	79.28
7	180.88	184.64	230.30	91.98
8	232.44	238.69	270.24	233.79
9	152.71	156.21	113.98	128.75
10	182.14	185.18	198.02	210.81
11	73.93	75.27	65.87	125.51
12	177.97	182.54	206.43	238.43
13	208.30	214.04	205.35	215.35
14	212.98	215.51	209.90	218.87
15	166.01	171.14	193.04	165.23
16	131.42	134.44	118.98	155.44
17	150.01	153.71	151.73	152.85
18	138.45	142.05	134.37	144.09
19	127.57	130.23	127.40	127.19
20	182.77	187.03	212.63	189.13
21	124.67	127.39	118.22	144.75
22	71.98	74.20	45.31	87.51
23	89.46	92.00	91.70	81.62
24	199.14	205.47	256.29	127.83

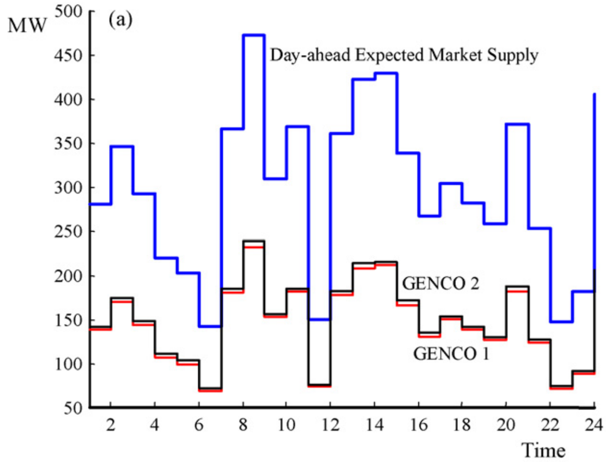

Fig. 6. GENCOs' expected outputs (a) without learning and (b) with learning: Case A.

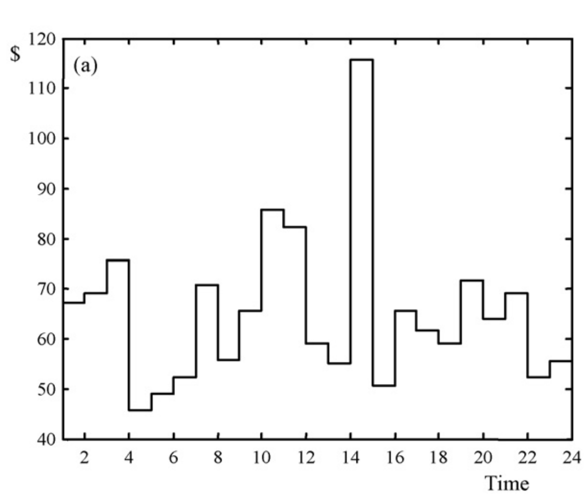

Fig. 7. Market-clearing prices (a) without learning and (b) with learning: Case A.

From Fig. 7(a), the lowest market price occurs at period 5 and at period 13 in Fig. 7(b). In the first case, it is due only to the market demand and production cost parameters. In the second case, the adjusting factors play an important role such that the market equilibrium reaches the perfect competitive outcome in that specific period.

Fig. 8 shows the profits for each GENCO at each period. Fig. 8(a) shows that profits are quite similar (the differences

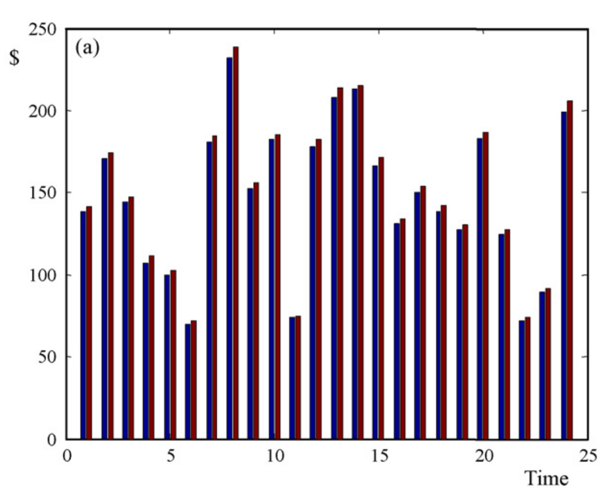

Fig. 8. Profits per period per GENCO (a) without and (b) with learning: Case A.

Table 5
Total revenues, total costs and net profits: Case A

GENCO	Total revenue ($)	Total cost ($)	Net profit ($)
1	232,080	52,207	179,873
2	237,410	38,257	199,153
1	231,610	54,149	177,461
2	220,000	38,094	181,906

occur because the GENCOs' outputs differ slightly). However, profits vary more when the learning effect is considered. Moreover, in some cases (i.e., periods 1, 2, 7, 8, and 24), GENCO 1's profits are higher due to the adjusting factors.

Table 5 summarizes the total revenues, total costs, and net profits over the 24 periods.

From Table 5 we observe that net profits are higher for both GENCOs when learning is not included. This indicates that the traditional Cournot outcome is even greater because the coefficients selected were not the optimum values.

Case B. In this case, maximum/minimum on/off times and operational limits are considered. The new expected market supply and the new expected GENCOs' outputs for each period of the day-ahead are presented in Table 6.

Table 6
Expected GENCOs' outputs: Case B

Period	No learning		Learning	
	GENCO 1 (MW)	GENCO 2 (MW)	GENCO 1 (MW)	GENCO 2 (MW)
1	138.51	141.60	150.00	135.87
2	150.00	185.07	150.00	214.61
3	144.58	147.40	144.58	147.40
4	107.40	111.26	89.33	147.82
5	99.62	102.92	77.20	119.55
6	69.66	71.81	59.89	79.28
7	150.00	200.04	150.00	164.08
8	150.00	279.75	150.00	251.75
9	150.00	157.56	113.98	128.75
10	150.00	201.22	150.00	215.60
11	73.93	75.27	65.87	125.51
12	150.00	196.49	150.00	238.43
13	150.00	0.00	150.00	0.00
14	150.00	0.00	150.00	0.00
15	150.00	0.00	150.00	0.00
16	150.00	0.00	150.00	0.00
17	0.00	228.54	0.00	228.54
18	0.00	211.12	0.00	211.12
19	0.00	193.91	0.00	193.91
20	0.00	278.17	0.00	278.17
21	0.00	189.63	0.00	189.63
22	0.00	110.14	0.00	110.14
23	89.46	92.00	80.40	80.47
24	150.00	229.95	150.00	209.15

Here we can see that GENCO 1 reaches its upper limit of generation in several periods and that market equilibrium is found for each period even when GENCO 1 reaches its upper limit. Table 6 also shows that there is one shut-down for each GENCO. Each time that a GENCO goes "off," the market supply becomes the GENCOs online output. Maximum up and minimum down times are met throughout the timespan. The remaining operational constraints are satisfied.

By comparing Table 4 with Table 6, we observe that the GENCOs' outputs differ only for the periods in which the upper limit is reached, in addition to the shut-down periods. A graphic representation of the two outputs with and without the learning effect is shown in Fig. 9.

Fig. 9 shows that each time a GENCO is off, the market supply becomes the GENCOs online output. In addition, we observe that GENCO 1 reaches its upper generating limit in several periods even when GENCO 2 is off.

During those periods when only one unit is on and it reaches its upper limit, the learning aspect affects the market equilibrium. The market equilibrium is still a Nash equilibrium. The capacity-constrained price game potentially will appear if the players become informed (Figs. 10 and 11).

Similar to Table 5, the net profits are higher when learning is not considered. However, GENCOs 2 profits increase substantially while GENCOs 1 profits decrease. Changes in profits occur because the units went off for several periods. During periods

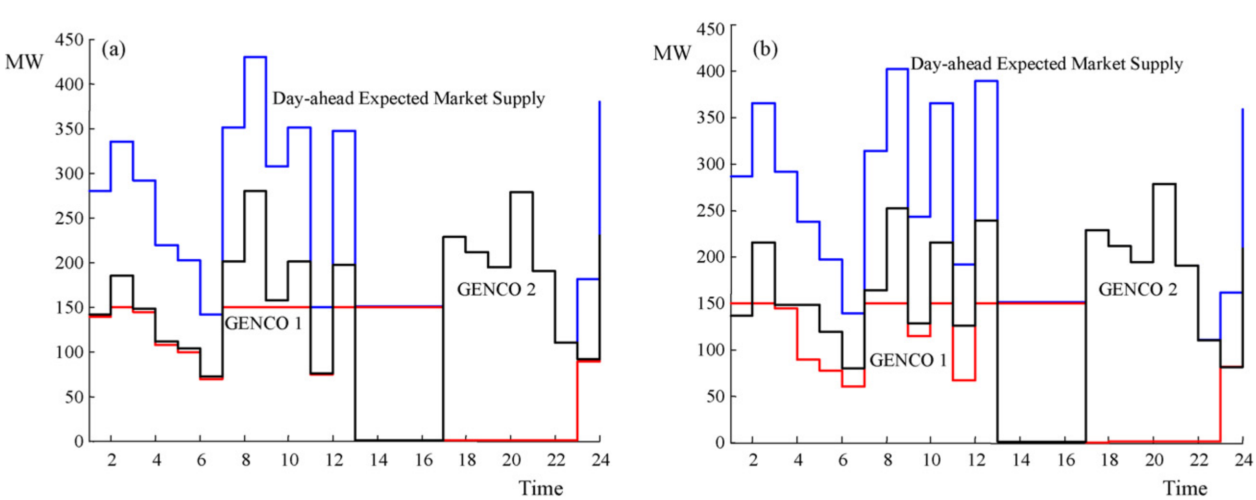

Fig. 9. GENCOs' expected outputs (a) without learning and (b) with learning: Case B.

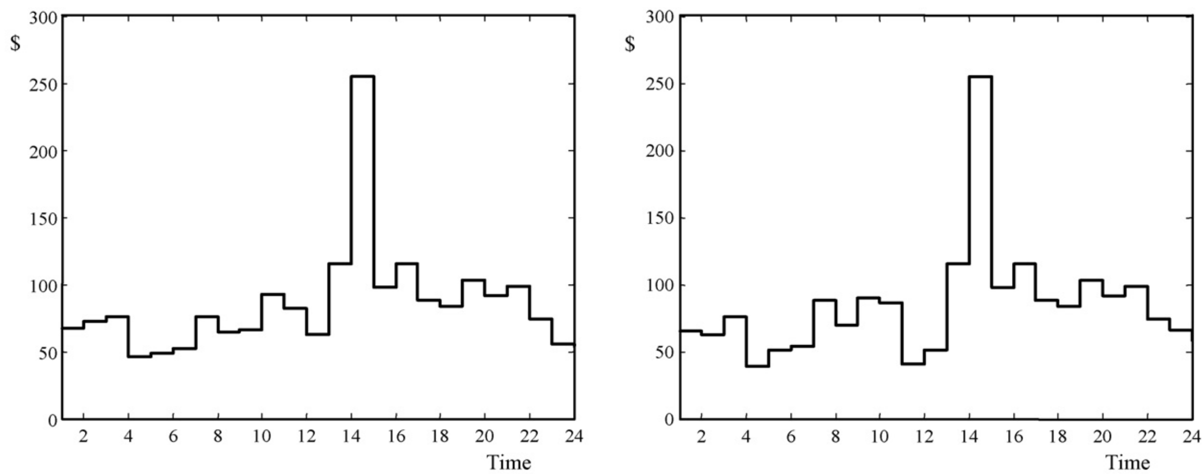

Fig. 10. Market-clearing prices (a) without learning and (b) with learning: Case B.

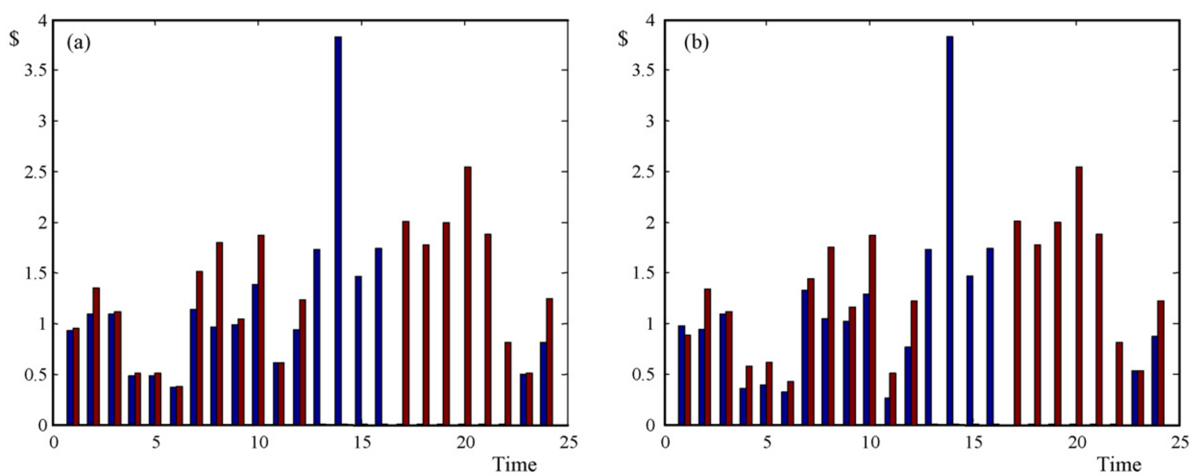

Fig. 11. Profits per period per GENCO (a) without and (b) with learning: Case B.

when only one unit is supplying the market, a GENCOs profits at day's end are higher than when all of its units are online for all the periods (Table 7).

Case C. This case accounts for temporal and operational constraints. The expected GENCOs' outputs for the day-ahead are presented in Table 8.

Here we observe that the commitment schedule differs from the two previous cases. There is one shut-down for each GENCO. In addition, ramp-up and ramp-down constraints occur (seen in the GENCOs' outputs). In Case B above, once the unit reached its maximum time online, it goes off (this also occurs when it reaches its maximum offline time). However, in Case C, before the unit goes off, the ramp-down constraint begins working so that the unit decreases its output for several periods before it finally goes off.

Table 7
Total revenues, total costs and net profits: Case B

GENCO	Total revenue	Total cost	Net profit
1	205,480	41,463	164,017
2	256,490	36,698	219,792
1	199,530	40,614	158,916
2	256,840	37,221	219,619

As seen in Fig. 12, the commitment schedule differs with respect to the two previous cases. There is one shut-down for each GENCO. The market-clearing price as displayed in Fig. 13 differs considerably due to the ramp-up and ramp-down constraints (Fig. 14, Table 9).

In the situation depicted, GENCO 2 makes the highest profits with and without learning. Moreover, GENCO 2's profits are higher when the learning aspect is considered via the use of adjusting factors. However, the inclusion of ramp-up and ramp-down reduces its profits with respect to Case B.

Table 10 summarizes total expected revenues, total expected costs, and net expected profits for each GENCO for each case.

Table 10 reveals that net profits differ from case to case. In all cases, GENCO 2 earns higher profits, with Case B resulting in the most favorable conditions. The table also shows that the benefits differ with the incorporation of additional constraints, operative generation limits, and ramping constraints.

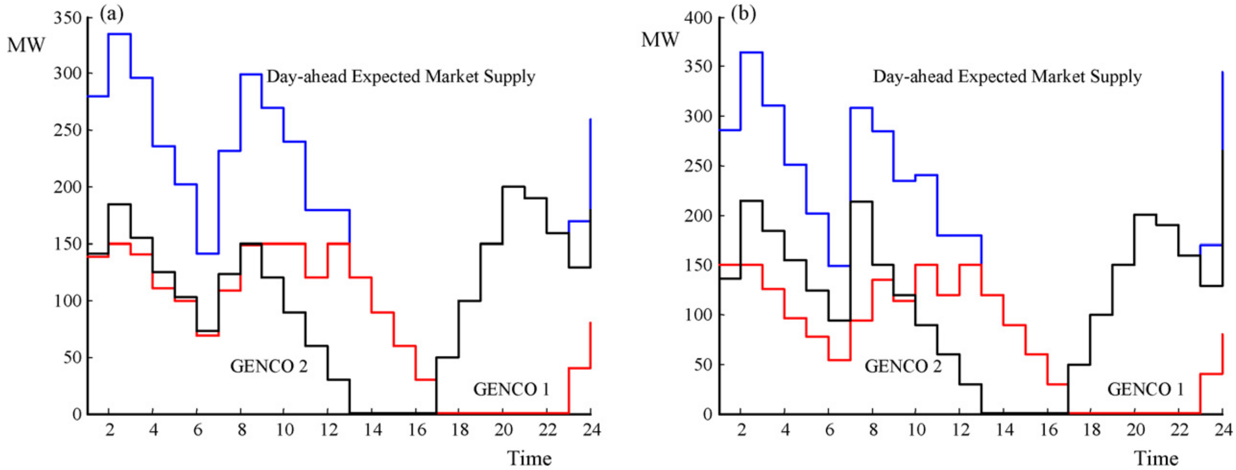

Fig. 12. GENCOs' expected outputs (a) without learning and (b) with learning: Case A.

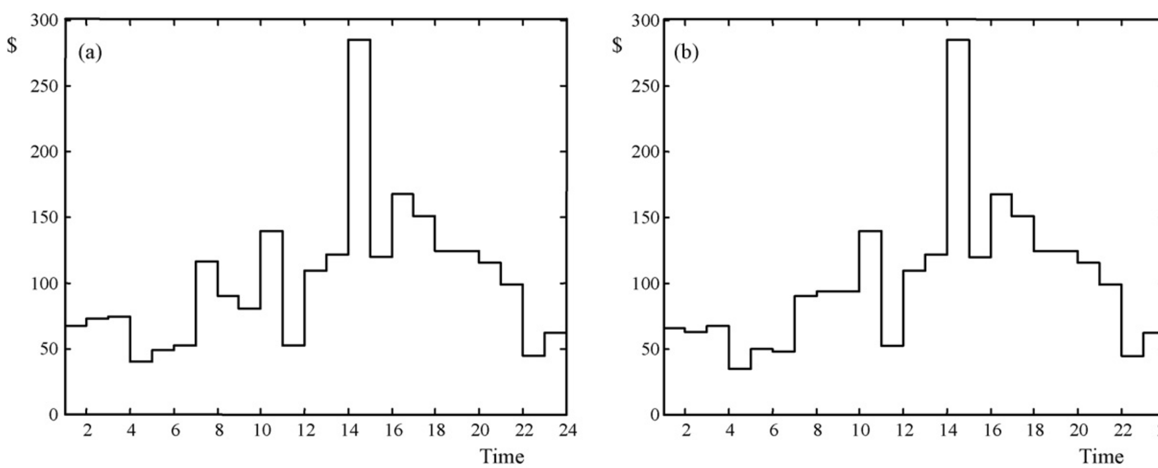

Fig. 13. Market-clearing prices (a) without learning and (b) with learning: Case C.

4. Parameter dependency

A different choice of parameters will influence market outcomes. Market equilibrium depends on all system parameters except fixed-cost parameters.

Adjusting factors assume a key role in the determination of market equilibrium since they modify the reaction functions. By changing the adjusting factors, we can find a factible region. The factible region is determined by the extreme maximum values reached by the adjusting factors. For instance, when both factors

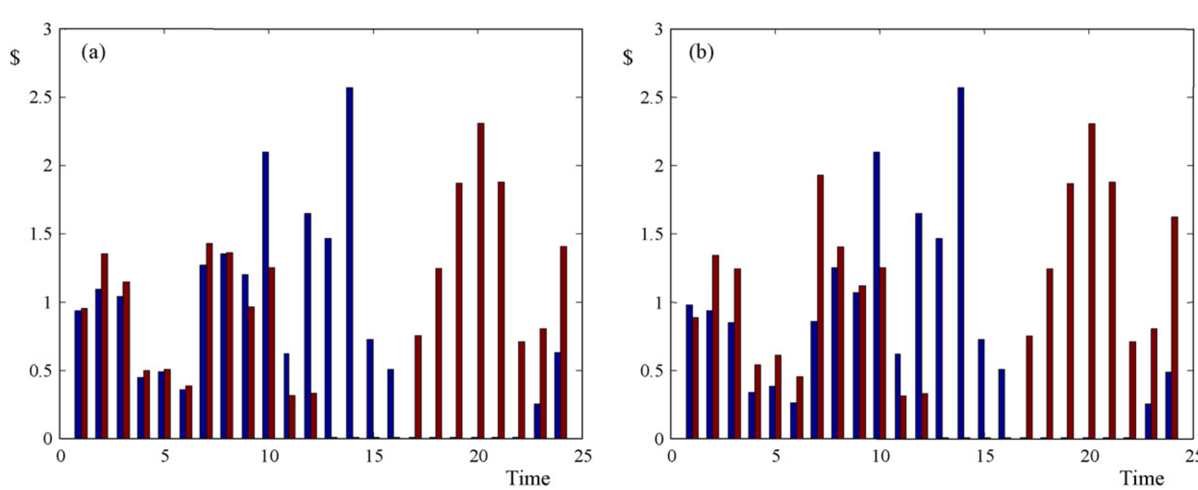

Fig. 14. Profits per period per GENCO (a) without and (b) with learning: Case C.

Table 8
Expected GENCOs' outputs: Case C

Period	No learning		Learning	
	GENCO 1 (MW)	GENCO 2 (MW)	GENCO 1 (MW)	GENCO 2 (MW)
1	138.51	141.60	150.00	135.87
2	150.00	185.07	150.00	214.61
3	140.74	155.07	125.99	184.61
4	110.74	125.07	95.99	154.61
5	99.62	102.92	77.20	124.61
6	69.11	72.92	54.53	94.61
7	109.11	122.92	94.53	213.89
8	149.11	150.00	134.53	150.00
9	150.00	120.00	113.98	120.00
10	150.00	90.00	150.00	90.00
11	120.00	60.00	120.00	60.00
12	150.00	30.00	150.00	30.00
13	120.00	0.00	120.00	0.00
14	90.00	0.00	90.00	0.00
15	60.00	0.00	60.00	0.00
16	30.00	0.00	30.00	0.00
17	0.00	50.00	0.00	50.00
18	0.00	100.00	0.00	100.00
19	0.00	150.00	0.00	150.00
20	0.00	200.00	0.00	200.00
21	0.00	189.63	0.00	189.63
22	0.00	159.63	0.00	159.63
23	40.00	129.63	40.00	129.63
24	80.00	179.63	80.00	264.92

Table 9
Producers' revenues, costs and profits: Case C

GENCO	Total revenues ($)	Total costs ($)	Net profits ($)
1	186,360	37,608	148,752
2	214,070	29,419	184,651
1	172,330	36,494	135,836
2	225,600	31,903	193,697

equal 1, it represents the maximum market quantity which is in essence the Bertrand outcome. The intersection of reaction functions still determines the market equilibrium. On the other hand, when both factors approach −1, the lower market quantity bound is established. Graphically this factible region is represented by the shadowed area depicted in Fig. 15. We note that any combination of adjusting factors will fall within the factible region.

Table 10
Producers' revenues, costs and profits

GENCO	Total revenues ($)	Total costs ($)	Net profits ($)	Case
1	232,080	52,207	179,873	A
2	237,410	38,257	199,153	
1	231,610	54,149	177,461	
2	220,000	38,094	181,906	
1	205,480	41,463	164,017	B
2	256,490	36,698	219,792	
1	199,530	40,614	158,916	
2	256,840	37,221	219,619	
1	186,360	37,608	148,752	C
2	214,070	29,419	184,651	
1	172,330	36,494	135,836	
2	225,600	31,903	193,697	

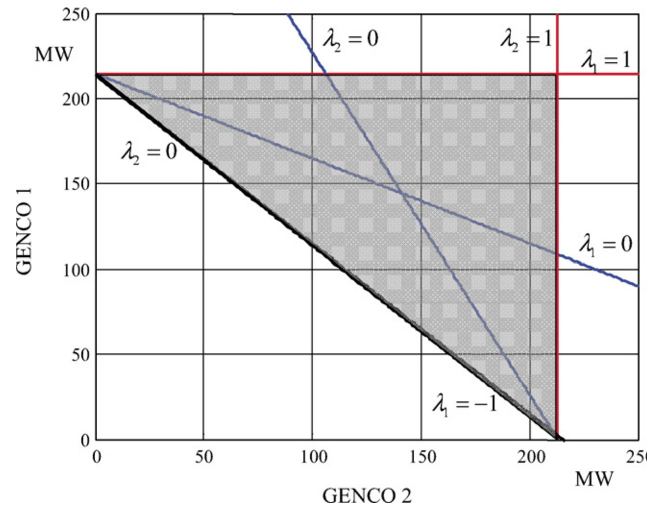

Fig. 15. Equilibrium market factible region.

It is well known that changes in market-demand function parameters will increase or decrease the factible region [ ]. For instance market demand "shifts up" when increasing parameter a and keeping everything else constant. Consequently, the factible region increases. If parameter b decreases, and everything else is kept constant, the market demand also shifts up and therefore the factible region increases.

5. Conclusions

This paper studies the production decisions of GENCOs in an oligopolistic electricity market solved by sequential market equilibriums. The formulation of sequential market equilibriums is represented by an independent linear set of equations with unique solutions when temporal constraints are omitted. Operational and temporal constraints have been included in the model. Once the temporal constraints are considered, the independent time-steps solutions are coordinated by the supervision of the maximum/minimum on/off time constraints.

The model elaborated in this paper was reduced to a two-player model to facilitate the analysis and make it relatively easy to identify the results derived a priori. The model can be extended to an n-player model in a single-node. Under this condition, the problem can be reduced to a two-player model. To reduce a two-player model we can use a composite of the generation production cost curves, and reduce our own generation units and the rival units to one composite unit. The Cournot game results if all the adjusting coefficients equal zero, $\lambda = 0$. When all of the GENCOs' adjusting coefficients are equal to 1, the market equilibrium moves to the Bertrand outcome; monopoly is reached when they tend to −1.

The solution of the short-term equilibrium problem varies with the generation cost parameters, the demand parameters, and the adjusting coefficients. A numerical example that illustrates the impact of the ramping process shows that the benefits will differ with the incorporation of ramping constraints.

Modeling the repetition of static snapshot with learning effect in the decision-making process is an alternative method to analyze the dynamic behavior of the market players. We incorporated learning by using forward expectations. In the examples given, these coefficients are assumed to be known. However, they must be estimated for each GENCO utilizing methods such as data mining, neural nets, and forecasting.

The issue of transmission network effect merits further research. Currently, we are applying it to our model and will report the results in further publications.

Appendix A. List of symbols

a, b	market-demand parameters
$C_i(q_i(k))$	production cost function of GENCO i
d_i, e_i, f_i	coefficients of production cost function $C_i(q_i(k))$
i	index for the number of GENCOs
k	index for the number of time intervals (h)
$P(Q)$	inverse linear market demand at period k
$q_i(k)$	output from player i at period k
$\hat{q}_j(k)$	GENCO j's expectation of the decisions made by GENCO i at period k
$q_i^{\min}$	minimum output of the GENCO i
$q_i^{\max}$	maximum output of the GENCO i
$Q(k)$	total market output at period k
t_i^{off}	minimum time off of the GENCO i
t_i^{on}	maximum time on of the GENCO i
u_{ik}	binary decision variable indicating whether the unit i at period k is up or down
W_i	maximum power ramp-down decrement of unit i
x_{ik}	state variable indicating the length of time that the unit i has been up or down at period k
Z_i	maximum power ramp-up increment of unit i

Greek symbols

λ_j	adjustment coefficient for GENCO j
μ	Lagrange multiplier
$\pi_i(k)$	profit of GENCO i at period k

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