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Multi-objective mean–variance–skewness model for generation portfolio allocation in electricity markets

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ABSTRACT

This paper proposes an approach for generation portfolio allocation based on mean–variance–skewness (MVS) model which is an extension of the classical mean–variance (MV) portfolio theory, to deal with assets whose return distribution is non-normal. The MVS model allocates portfolios optimally by considering the maximization of both the expected return and skewness of portfolio return while simultaneously minimizing the risk. Since, it is competing and conflicting non-smooth multi-objective optimization problem, this paper employed a multi-objective particle swarm optimization (MOPSO) based meta-heuristic technique to provide Pareto-optimal solution in a single simulation run. Using a case study of the PJM electricity market, the performance of the MVS portfolio theory based method and the classical MV method is compared. It has been found that the MVS portfolio theory based method can provide significantly better portfolios in the situation where non-normally distributed assets exist for trading.

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1. Introduction

Based on trading protocols, the competitive electricity markets (EMs) essentially consist of energy market (day-ahead, hour-ahead, and real-time balancing market) and several contractual instruments, such as forward and future contracts [1]. Forward and future contracts are similar, but future contracts are exclusively of financial type while forward contracts comprise the physical delivery of the energy. In competitive environment, generation companies (GenCos) are required to devise their own strategies on how to optimally allocate their generation capacities to the different markets for profit maximization. Moreover, while deriving the profit based generation strategies, the GenCos are confronted with volatile electricity prices and other uncertainties like congestion in transmission lines, unscheduled generating unit outages, etc. Therefore, while making the trading decision, GenCos' objective is not only to maximize its profit, but also to manage the associated risks and this problem can be viewed as a portfolio optimization.

In the last decade, the comprehensive studies [2,3] on various aspects of risk assessment and management for GenCos in competitive electricity markets have been conducted. Value at Risk (VaR) has been applied to risk assessment in electricity markets [4,5]. For hedging the spot price risks for market participants, different forward contracts with their valuation are proposed in [6–8]. In EMs, statistical studies of hedging strategies using financial instruments have been demonstrated in [9,10]. Moreover, some research papers [11–13] have also discussed the problem of allocating the generation capacities between the spot market and various contracts. Majority of aforementioned works for electricity portfolio optimization have employed the standard portfolio optimization approach, i.e., mean–variance (MV) formulation [14] which is precisely a first step of portfolio management. The MV model is a bi-criteria optimization problem where a rational portfolio choice is based on trade-off between risk and return.

However, the standard MV model is based on the assumption that each asset's return follows a normal distribution, so that asset returns can be portrayed only by their first (mean) and second (variance) central moments of distributions. But, substantial number of studies in finance sector [15–20] argued that the higher moments cannot be neglected unless there are reasons to believe that the asset returns are symmetrically distributed around the mean. Moreover, they point out the importance of skewness in the portfolio management. On the other end, empirical studies [21–23] in competitive electricity markets provide evidence indicating that, because of high volatility, spot price as well as return series exhibit statistically significant levels of positive skewness. To support this argument, a detail analysis of historical return of the spot market and bilateral contracts in PJM electricity market is presented in this paper. This study shows that because of high volatility in spot price, it follows the positively skewed distribution and therefore, GenCos returns do not exactly follow the normal distribution.

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Looking to the above issues in electricity portfolio managements, this paper is mainly contributing the followings:

- Using mean–variance–skewness (MVS) model, which is an extension of the classical MV portfolio theory, this paper proposed an approach for generation portfolio allocation considering the maximization of both the expected return and skewness while simultaneously minimizing the risk.
- The MVS portfolio theory is competing and conflicting non-smooth three objectives optimization problem. Third central moment is non-concave function and hence, it looks difficult to solve the resulting MVS portfolio optimization problem. Therefore, unlike single objective optimization method being used in the portfolio literature [11,12], this paper proposed a multi-objective particle swarm optimization (MOPSO) based meta-heuristic method to provide Pareto frontier in single run.

This paper is organized as follows. Section 2 provides a brief review of MVS portfolio framework followed by single and multi-objective portfolio optimization formulation. The brief concept of multi-objective optimization along with Pareto-optimal front and MOPSO are presented in Section 3. The proposed MVS based generation allocation modeling is derived in Section 4 and a case study of the PJM electricity market is given in Section 5 to demonstrate the effectiveness of the proposed method. Finally conclusions are drawn in Section 6.

2. Mean–variance–skewness Portfolio framework

A prerequisite to use the mean–variance (MV) framework is either the relevant distribution of asset returns is normally distributed or the utility function is approximated by only the first two moments. As a results MV approach does not take into account the higher moments in order to describe the investor's assessment of the probability distribution. The first moment represents the expected returns. The second and higher central moments characterize the uncertainty associated to the returns. Investors prefer to maximize the odd portfolio moments and to minimize the even ones. All the even moments measure dispersion (thus, volatility) which is undesirable due to increase in the uncertainty of returns. On the other hand, the odd moments express measures of asymmetry and it can be seen as a way to decrease the extreme values on the loss side and increase them on the gains. For example, maximizing positive skewness (positive skewness refers to a right-handed, elongated tail for the density function) may decrease the probability of having negative returns. As a result, investors are in favor of including skewness in portfolio selection problem because it seems that the combinations that result are more accurate and these give the investors a broader idea of how they can benefit from a portfolio.

The mean–variance–skewness (MVS) model first proposed by Konno and Suzuki [18] is a direct extension of the classical mean–variance (MV) portfolio model. The MVS model is most appropriate choice to the situation where the skewness of the return of assets plays significant role in choosing an optimal portfolio. The general MVS model for portfolio selection problem with $N (N \geq 2)$ risky assets can be described as follows. Let $\boldsymbol{w}_p$ and $\boldsymbol{R}$ denote, respectively, the $(N \times 1)$ vector of proportionate weight and expected returns of the N risky assets in the portfolio p. $\boldsymbol{\Omega}$ and $\boldsymbol{\Lambda}$ represent the non-singular $(N \times N)$ variance–covariance matrix and the $(N \times N^2)$ skewness–coskewness matrix of the N risky asset returns, respectively. The first (mean), second (variance), and third (skewness) central moments, respectively, of the return of a given portfolio p are given by:

$$E(r_p) = \sum_{i=1}^{N} w_{pi} E(r_i) = \boldsymbol{w}'_p \boldsymbol{R} \quad (1)$$

$$\sigma^2(r_p) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{pi} w_{pj} \sigma_{ij} = \boldsymbol{w}'_p \boldsymbol{\Omega} \boldsymbol{w}_p = \sum_{i=1}^{N} w_{pi}^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_{pi} w_{pj} \sigma_{ij} \quad (2)$$

$$s^3(r_p) = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} w_{pi} w_{pj} w_{pk} s_{ijk} = \boldsymbol{w}'_p \boldsymbol{\Lambda} (\boldsymbol{w}_p \otimes \boldsymbol{w}_p)$$

$$s^3(r_p) = \sum_{i=1}^{N} w_{pi}^3 s_i^3 + 3 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_{pi}^2 w_{pj} s_{iij} + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \sum_{\substack{k=1 \\ k \neq j \\ k \neq i}}^{N} w_{pi} w_{pj} w_{pk} s_{ijk} \quad (3)$$

where w_{pi} and r_i represent the weight of asset i in the portfolio p and the return on the asset i, respectively. σ_i^2 and σ_{ij} represent the variance of return on the asset i and covariance between the returns of assets i and j, respectively. s_i^3 and s_{ijk} represent skewness of the return of asset i and coskewness between the returns of assets i, j, and k, respectively. The sign $\otimes$ stands for the Kronecker symbol product.

The MVS model is competing and conflicting multi-objective optimization problem. An optimal portfolio should maximize both the expected return and skewness while minimizing the risk associated with the return (i.e., variance) simultaneously, as stated below.

$$(\text{Prob}_1) \begin{cases} \text{maximize} f_1(w_p) = \left[E(r_p)\right] \\ \text{minimize} f_2(w_p) = \left[\sigma^2(r_p)\right] \\ \text{maximize} f_3(w_p) = \left[s^3(r_p)\right] \\ \text{subject to} \sum_{i=1}^{N} w_{pi} = 1, \quad w_{pi} \geq 0 \end{cases} \quad (4)$$

2.1. Single objective optimization formulation

Most of the traditional algorithms reformulate a given multi-objective optimization problem into a single objective-function with the help of weighting factors. Using this approach, classical MV portfolio optimization problem in [11,12] has been solved using quadratic programming with help of risk aversion factor. Similarly, a single objective-function of the above multi-objective programming problem (e.g. Prob_1) can be formed by combining the three objective functions and then the same can be optimized by assigning relative weights to represent the importance of each individual function as given below.

$$(\text{Prob}_2) \begin{cases} \text{minimize} f(w_p) = \left[-\beta_1 \left[E(r_p)\right] + \beta_2 \left[\sigma^2(r_p)\right] - \beta_3 \left[s^3(r_p)\right]\right] \\ \text{subject to} \sum_{i=1}^{N} w_{pi} = 1, \quad w_{pi} \geq 0 \end{cases} \quad (5)$$

where β_1, β_2 and β_3 represent investor's relative preference for expected return, risk and skewness, respectively. The Prob_2 is a constrained nonlinear programming problem and generally solved using nonlinear programming techniques. Classical optimization methods like goal programming [19] and linear programming [20], have been used to solve the above problem. However, in order to make this method working, an apriori assumption of the relative importance of each objective has to be incorporated. This makes the solution to be guided in a given direction based on the judgment of the investor.

2.2. Multi-objective optimization formulation

In order to prevent subjectivity coming into the solution space of traditional single objective optimization formulation, the concept of Pareto dominance has been introduced. According to this principle, instead of giving an absolute (scalar) value to a solution, a partial order is defined based on dominance. A solution is said to dominate another solution when it is better on one objective, and not worse on all the other objectives. For this class of formulation, weighting factor is not required and thus, no a-priori information on the problem is needed. The objective space to be optimized constitutes the mutually conflicting objective functions as given bellow.

$$(\text{Prob}_3)\begin{cases}\text{minimize } F(w_p) = \left[-\left[E(r_p)\right], \left[\sigma^2(r_p)\right], -\left[s^3(r_p)\right]\right] \\ \text{subject to } \sum_{i=1}^{N} w_{pi} = 1, \quad w_{pi} \geq 0\end{cases} \quad (6)$$

To address the limitations of the conventional optimization methods when generating the Pareto-optimal front, the meta-heuristic optimization algorithms have been successfully applied to the multi-objective optimization problems. One class of meta-heuristic optimization techniques, multi-objective particle swarm optimization (MOPSO) proposed by Coello and Lechuga [24] allows the PSO algorithm to be able to deal with multi-objective optimization problems. Because MOPSO allows concurrent exploration of different points of the Pareto front, they can generate multiple solutions in a single run. The optimization can be performed without a-priori information about objectives' relative importance. This paper proposed MOPSO to efficiently solve the problem (Prob_3) for MVS portfolio optimization.

3. Multi-objective particle swarm optimization

Brief concept of multi-objective optimization followed by multi-objective PSO (MOPSO) is presented in this section.

3.1. Multi-objective optimization (MOO) concept and Pareto front

Most of the real-world problems employ the simultaneous optimization of several objective functions, which are often conflicting in nature and equally important. In general, multi-objective minimization problem with n decision variables and m objective functions ($f_i : \Re^n \to \Re, \quad i = 1, 2, \ldots, m$) associated with constraints can be stated as [24,25]:

$$\text{minimize } \boldsymbol{F}(\vec{x}) = \left[f_1(\vec{x}), f_2(\vec{x}), \ \ldots, f_m(\vec{x})\right] \quad (7)$$

Subject to

$$g_i(\vec{x}) \leq 0; \quad i = 1, 2, \ldots, k \quad (8)$$

$$h_j(\vec{x}) = 0; \quad j = 1, 2, \ldots, p \quad (9)$$

where $\vec{x} = [x_1, x_2, \ \ldots, x_n] \in X$ is vector of decision variables, X is the decision space and $g_i, h_j : \Re^n \to \Re$ are the constraint functions of the problem. The solution to MOO problems does not consist of a single solution (as in global optimization); rather aim is to determine from among the set $\boldsymbol{F}(\vec{x})$ of all vectors which satisfy (8) and (9) for the particular set of values $\vec{x}^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]$ which yield best compromise solutions among all the objective functions. The most commonly used notion of optimality adopted in MOO is the so-called Pareto-optimality, which can be explained with the concept of dominance relation [25]. The vector $\vec{x}^*$ corresponding to the solution included in the Pareto-optimal set are said to be nondominated (by other solutions) and for a given Pareto-optimal set, the corresponding objective function values in the objective space are called the Pareto front. Concepts of the Pareto-optimal front, nondominated and dominated solutions are further explained in Fig. 1. The axes on Fig. 1 (f_1 and f_2) are two objective functions. Possible solutions for minimization are presented in the f_1–f_2 plane. In general, there are two major goals to achieve while solving MOO problem: (1) to find a set of solutions as close as possible to the true Pareto-optimal front and (2) to achieve a well-distributed set of solutions, as a results we can assure a good set of trade-off solutions among the objectives.

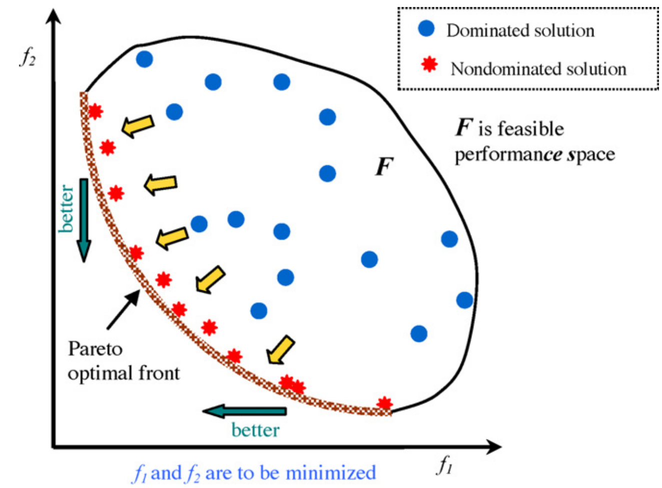

Fig. 1. Pareto-optimality, nondominated and dominated solutions: bi-objective case.

3.2. Multi-objective PSO (MOPSO)

MOPSO is an extensive version of the standard PSO to handle multiple objectives by redefining global and local best individuals in order to obtain a Pareto-front of optimal solutions. In the standard PSO [26], the global best particle is determined easily by selecting the particle which has the best position. The velocity and position of each particle in single-objective PSO can be modified by [26]

$$v_{id}^{k+1} = w \times v_{id}^k + c_1 R_1 (pbest_{id} - x_{id}^k) + c_2 R_2 (gbest_d - x_{id}^k) \quad (10)$$

$$x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \quad (11)$$

where $d = 1, 2, \ldots, D, i = 1, 2, \ldots, S$ and S is the size of the swarm. c_1 and c_2 are two positive acceleration coefficients which keep balance between the particle's individual and social behavior. R_1 and R_2 are uniformly distributed random numbers in [0,1] added in the model to introduce stochastic nature. The inertia weight of the particle, w, is suitably selected to control the exploration properties of the algorithm.

Since MOO problems have a set of Pareto-optimal solutions, each particle of the population should use Pareto-optimal solutions to select one of its global best particles. Therefore, choosing the global best and local best to guide the swarm particles becomes nontrivial task in multi-objective domain. Another consideration is the method of maintaining and keeping the Pareto-optimal solutions. Typically, these solutions are stored in an archive-like database which is updated after each iteration in order to maintain a pure set of nondominated solutions. The MOPSO algorithm with the use of crowding distance mechanism together with mutation operator, called "MOPSO-CD [27]", has been used in this paper. The mutation operator is used to improve the exploration capability of the MOPSO. The flow chart of MOPSO-CD algorithm is depicted in Fig. 2 and its brief explanation is given in Appendix A.

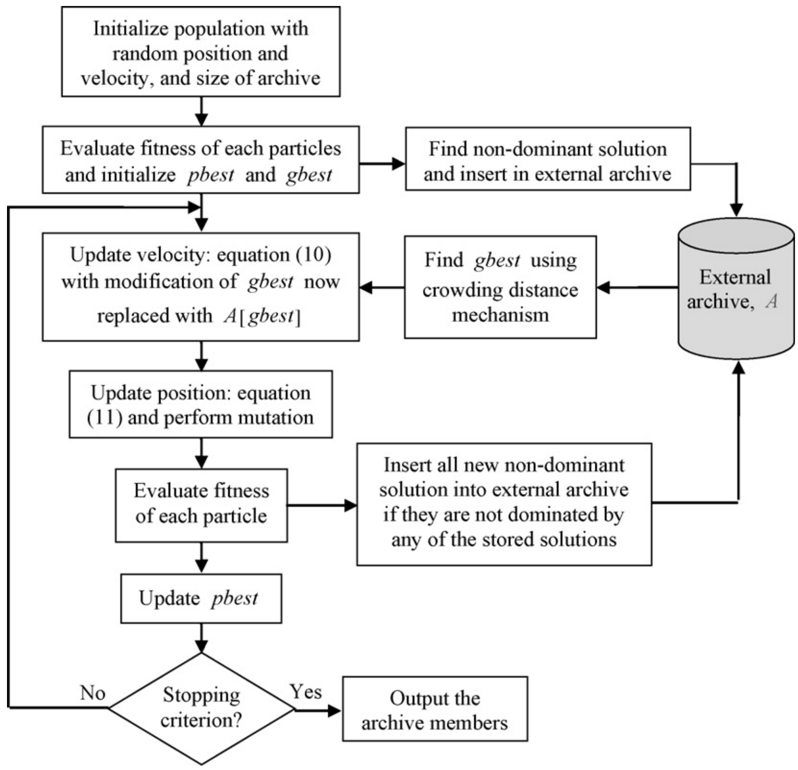

Fig. 2. Flow chart of MOPSO-CD algorithm.

4. Electricity portfolio allocation problem and its MVS modeling

In competitive electricity markets, for a GenCo, a number of different assets are available for constructing a portfolio. For instance, an asset for a generator can be the power sold through the spot energy market, the power sold through forward contracts, or the futures contracts hold for risk hedging against spot price volatility. GenCos make extensive use of spot energy market to sell their output power on an hourly or half-hourly basis. Therefore, the spot electricity price has significant impact on GenCos' profits. Moreover, it can also influence the prices of other financial instruments, such as future and option contracts. Therefore, spot price fluctuation is a major and most important risk source for the GenCos. Although the trading quantity and price are decided in advance in the physical forward contract, the main risk in such contract is the risk associated with the congestion charges. The congestion charge between any two locations is the product of the spot price difference between these two locations and the transmitted energy (quantity in MWh) [11]. For instant, locational marginal prices (LMPs) in case of PJM electricity market are highly volatile, therefore, only local contracts signed with local customers are risk-free trades and non-local contracts signed with non-local customers are risky trades due to the uncertainty in congestion charges. There are financial instruments, such as financial transmission rights (FTRs), to deal with risk associated with the congestion charges. Inclusion of FTRs in presented model is beyond the scope of this paper.

This paper mainly considers risky assets: spot energy market and the bilateral non-local contracts. However, the proposed approach can be extended for other assets like future contracts, option contracts, etc. Similar to works in [3] and [11], it is assumed that there are n areas or pricing zones in an electricity market. A GenCo is located in Area 1 and other areas are labeled from Area 2 to Area n. To simplify, suppose that the GenCo could sign one bilateral contract with other each area's customers at fixed energy price. Hence, the GenCo has n potential transactions during the planning (or decision period), i.e., one risky transaction traded in the spot market and $n-1$ risky non-local bilateral contracts. Now, question is how to optimally allocate these GenCos' assets in a decision period. To answer this question, the mathematical MVS model (which discussed in previous section) for generation portfolio optimization has been derived in this section.

Unlike, in the financial markets of electricity markets, the total cost of generation depends on the cost functions of individual generators and the amount of their generator power outputs. Assume that the production cost of a generator in the trading interval is quadratic function of generator output power, P_G, $pc(P_G) = (a + bP_G + cP_G^2)$, where a, b, c are generating unit cost coefficients. According to [12,13], the return of each asset in a decision period is defined as (total revenue − total cost)/total cost. Assume that there are T trading intervals in a decision period. The decision period could be a day, a week, a month, a year or several years, etc. The mathematical derivations of returns characteristic up to second central moments (mean and variance), presented in this paper, have been quoted from [11]. The main contribution of this paper is to incorporate third central moment (skewness) while obtain the generation portfolio and that is mathematically derived in this section. The following nomenclatures will be used when deriving the returns and its associated distribution characteristics of each asset.

t, i	index of the trading interval and trading area
E and $\lambda_{i,t}^S$	expectation and tth trading interval's spot energy price of Area i
$\lambda_{i,t}^B$	tth trading interval's contract price signed with customers of area i
γ	congestion charge factor ($0 \le \gamma \le 1$)
r_i	return on ith trade; $i = 1$ represents trading in spot energy market, and $i = 2{\sim}n$ denotes bilateral contract with the ith area's customers
$E(r_i)$, $\sigma^2(r_i)$, $s^3(r_i)$	expected return, variance, and skewness of return on ith trade

4.1. Return model of spot energy market

According to the previous definition of the return, the return characteristics of the trading in spot energy market during the decision period can be calculated using following equations.

$$E(r_1)=\frac{\sum_{t=1}^{T}E(\lambda_{1,t}^S)P_{G,t}-\sum_{t=1}^{T}(a+bP_{G,t}+cP_{G,t}^2)}{\sum_{t=1}^{T}(a+bP_{G,t}+cP_{G,t}^2)}$$

$$E(r_1)=M\sum_{t=1}^{T}E(\lambda_{1,t}^S)P_{G,t}-1,\quad \text{where } M=\frac{1}{\sum_{t=1}^{T}(a+bP_{G,t}+cP_{G,t}^2)} \tag{12}$$

$$\sigma^2(r_1)=M^2\sum_{t=1}^{T}P_{G,t}^2\sigma^2(\lambda_{1,t}^S) \tag{13}$$

$$s^3(r_1)=M^3\sum_{t=1}^{T}P_{G,t}^3 s^3(\lambda_{1,t}^S) \tag{14}$$

4.2. Return model of non-local contracts

$$E(r_i)=M\sum_{t=1}^{T}P_{G,t}[\lambda_{i,t}^B-\gamma[E(\lambda_{i,t}^S)-E(\lambda_{1,t}^S)]]-1;\quad i=2\sim n \tag{15}$$

$$\sigma^2(r_i)=M^2\sum_{t=1}^{T}P_{G,t}^2\gamma^2[\sigma^2(\lambda_{1,t}^S)+\sigma^2(\lambda_{i,t}^S)-2\,Cov(\lambda_{1,t}^S,\lambda_{i,t}^S)];\quad i=2\sim n \tag{16}$$

$$s^3(r_i)=M^3\sum_{t=1}^{T}P_{G,t}^3\gamma^3[s^3(\lambda_{1,t}^S)-s^3(\lambda_{i,t}^S)-3\text{Coskew}(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{i,t}^S)+3\text{Coskew}(\lambda_{1,t}^S,\lambda_{i,t}^S,\lambda_{i,t}^S)];\quad i=2\sim n \tag{17}$$

4.3. Covariance between risky trades

$$\sigma_{ij}=\text{Cov}(r_i,r_j)=M^2\sum_{t=1}^{T}P_{G,t}^2\gamma^2[\sigma^2(\lambda_{1,t}^S)-\text{Cov}(\lambda_{1,t}^S,\lambda_{i,t}^S)-\text{Cov}(\lambda_{1,t}^S,\lambda_{j,t}^S)+\text{Cov}(\lambda_{i,t}^S,\lambda_{j,t}^S)];\quad i,j=2\sim n \tag{18}$$

$$\sigma_{1j}=\text{Cov}(r_1,r_j)=M^2\sum_{t=1}^{T}P_{G,t}^2\gamma[\sigma^2(\lambda_{1,t}^S)-\text{Cov}(\lambda_{1,t}^S,\lambda_{j,t}^S)];\quad j=2\sim n \tag{19}$$

4.4. Coskewness between risky trades

$$s_{1jk}=\text{Coskew}(r_1,r_j,r_k)=M^3\sum_{t=1}^{T}P_{G,t}^3\gamma^2[s^3(\lambda_{1,t}^S)-\text{Coskew}(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{j,t}^S)-\text{Coskew}(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{k,t}^S)+\text{Coskew}(\lambda_{1,t}^S,\lambda_{j,t}^S,\lambda_{k,t}^S)];\quad j,k=2\sim n \tag{20}$$

$$s_{ijk}=\text{Coskew}(r_j,r_j,r_k)=M^3\sum_{t=1}^{T}P_{G,t}^3\gamma^3[s^3(\lambda_{1,t}^S)+\text{Coskew}(\lambda_{1,t}^S,\lambda_{j,t}^S,\lambda_{j,t}^S)-\text{Coskew}(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{k,t}^S)-\text{Coskew}(\lambda_{j,t}^S,\lambda_{j,t}^S,\lambda_{k,t}^S)-2(\text{Coskew}(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{j,t}^S)-\text{Coskew}(\lambda_{1,t}^S,\lambda_{j,t}^S,\lambda_{k,t}^S))];\quad j,k=2\sim n \tag{21}$$

$$s_{11k}=\text{Coskew}(r_1,r_1,r_k)=M^3\sum_{t=1}^{T}P_{G,t}^3\gamma[s^3(\lambda_{1,t}^S)-\text{Coskew}(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{k,t}^S)];\quad k=2\sim n \tag{22}$$

In the above equations, the estimation of $E(\lambda_{i,t}^S)$, $\sigma^2(\lambda_{i,t}^S)$, $\text{Cov}(\lambda_{i,t}^S,\lambda_{j,t}^S)$, $s^3(\lambda_{i,t}^S)$ and $\text{Coskew}(\lambda_{i,t}^S,\lambda_{j,t}^S,\lambda_{k,t}^S)$ is a spot-price forecasting problem, which is an applied area of research field. In this paper, they are simply estimated based on historical data according to the statistical method. To model the daily and seasonal periodicity, the sample of the spot energy price in the tth trading interval consists of historical data in similar hours and months.

5. Case study

To demonstrate the effectiveness of the mean–variance–skewness (MVS) portfolio theory based proposed model for generation portfolio allocation, this paper considers the case study of PJM electricity market [28]. The PJM energy market consists of two-settlements (day ahead and real-time) spot market with locational marginal price (LMP) which are calculated at each bus in every 5 min. LMP reflects the value of the energy at the specific location at a given time. For this study, let us consider three pricing zones viz., PECO, PEPCO, and PENELEC zones in PJM markets and only risky assets for trading. Suppose a GenCo owing a 455 MW generation unit with a quadratic cost function of $pc(P_G)=1000+16.19P_G+0.00048P_G^2$, is located in PECO control zone (Area 1). This GenCo has three trading settlements to sell electricity: (1) trading in spot energy market, (2) bilateral contract with customer in PEPCO zone (Area 2), and (3) bilateral contract with customer in PENELE zone (Area 3).

Assume that on 31st July 2007, a GenCo in PECO zone is seeking for optimal portfolio allocation solution, i.e., determining the optimal trading amount (or trading ratio) of each market for the decision period of month of August 2007. The GenCo will rely its decision on the historical PJM market data from the month of August 1999–2006, which are published on the website of PJM electricity market [28]. The trading interval is one hour, so there are 744 (=24 × 31) trading intervals in the decision period. First, based on historical data and the return models derived in Section 4, the returns distribution of each asset (i.e., trading in spot energy market and through bilateral contracts) are tested for normality. Then, multi-objective optimal portfolio allocation with skewness (described in Section 2) is determined with the MOPSO based algorithm.

5.1. Testing for normality of return distribution

As a first step, it is required to test whether the GenCo's returns follow a normal distribution or not. Since the equations derived in Section 4 shows that the returns of the electricity assets are function of the spot energy price in the decision period, the empirical analysis initially examines the normality of spot energy prices. Then, the return distribution characteristics are tested for normality. These

Table 1
Distribution tests results.

	Jarque-Bera test statistic (critical value: 5.9915)	Lilliefors test statistic (cutoff value: 0.0886)
Best	0.3572	0.0348
Median	22.6078	0.1741
Worst	726.6603	0.3980
Return in spot energy market	14.5339	0.1747

tests provide the ground work for constructing an optimal portfolio with inclusion of skewness in MV portfolio model. The normality of returns is tested using Jarque-Bera [29] and Lilliefors [30] numerical methods. Both the Jarque-Bera and Lilliefors test methods are performed at 5% level of significance and results are presented in Table 1. Since there are 744 spot prices in this example, Table 1 presents only the best, the median, and the worst (approximation to normal distribution) from the 744 samples, which have the minimal, median, and maximal Lilliefors test statistic, respectively. If the Jarque-Bera and Lilliefors test statistic is greater than their respective critical values, then the null hypothesis of normality can be rejected (at 5% level of significance). Test results presented in Table 1 show that only in case of the best spot energy prices, the null hypothesis of normality cannot be rejected. The null hypothesis of normality is rejected in case of both the median and worst spot energy prices. Table 1 also shows the normality test results for the return in the spot energy market. Since both the Jarque-Bera and Lilliefors test statistic values are significantly higher than their respective critical values, the null hypothesis of normality is rejected in case of the return in the spot energy market as well.

These asymmetry and tail characteristics can also be more intuitively viewed through the frequency distribution of the spot energy market return, as depicted in Fig. 3. It shows that the distribution is skewed to right from the peak near value of 2.5. Coefficient of skewness (equal to third central moment (skewness)/(standard deviation)^3), for spot energy market is found to be 0.4083 (a symmetrical distribution has zero value). The distribution characteristics for the other electricity assets (returns from bilateral contracts in this numerical study) are also tested and it was found that they also show positive skewness. Two cases with different set of contract prices (viz, case 1: $\lambda_2^B = 51.2$ US$/MWh, $\lambda_3^B = 41.0$ US$/MWh and case 2: $\lambda_2^B = 52.5$ US$/MWh and $\lambda_3^B = 42.2$ US$/MWh) for costumer in PEPCO and PENELE pricing zones, respectively, are considered in this study. The estimated return, variance–covariance (second central moment), and skewness–coskewness (third central moment) matrices for case 1 are shown in Tables 2–4, respectively, for each asset's return. Both statistical analysis and graphic illustration show that the non-normality is significant; therefore, there is a need to utilize the information about the skewness to obtain the optimal portfolio in the electricity markets.

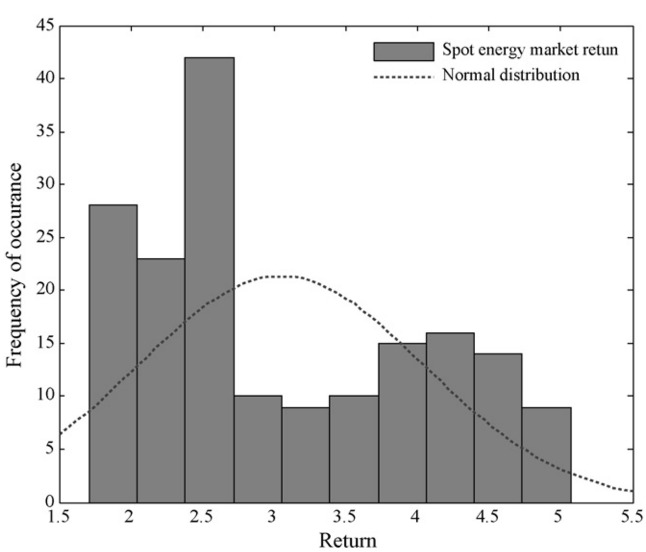

Fig. 3. Frequency distributions of the spot energy market return.

Table 2
Estimated returns.

Spot energy market	Bilateral contract-1	Bilateral contract-2
1.80	1.54	1.60

Table 3
Variance–covariance matrix of the distribution of returns.

	Spot energy market	Bilateral contract-1	Bilateral contract-2
Spot energy market	0.0148	0.0021	0.0058
Bilateral contract-1	0.0021	0.0031	0.0015
Bilateral contract-2	0.0058	0.0015	0.0037

Table 4
Skewness–coskewness matrix of the distribution of returns.

	s_{ijk} ($\times 10^{-3}$)		
	Spot energy market	Bilateral contract-1	Bilateral contract-2
$k = 1$ (Spot energy market)			
Spot energy market	0.4794	0.1335	0.2152
Bilateral contract-1	0.1335	0.0782	0.0846
Bilateral contract-2	0.2152	0.0846	0.1314
$k = 2$ (Bilateral contract-1)			
Spot energy market	0.1335	0.0782	0.0846
Bilateral contract-1	0.0782	0.0376	0.0526
Bilateral contract-2	0.0846	0.0526	0.0561
$k = 3$ (Bilateral contract-2)			
Spot energy market	0.2152	0.0846	0.1314
Bilateral contract-1	0.0846	0.0526	0.0561
Bilateral contract-2	0.1314	0.0561	0.0812

5.2. *Solving the multi-objective portfolio problem*

To efficiently solve the multi-objective MVS portfolio theory based model for electricity portfolio selection without a-priori information about objectives' relative importance, this paper proposed MOPSO algorithm for portfolio optimization. Before applying MOPSO to solve portfolio problem, representation of a particle must be formulated. The quality of an individual string of the population is found using fitness evaluation function as given by equation (6). Based on fitness values, *pbest* and *gbest* are initialized. Then, algorithm follows the computational steps as depicted in Fig. 2. The main constraint, $\sum_{i=1}^{N} w_{pi} = 1$, is handled through the normalization of each w_{pi} dividing by their sum. The tuning parameters of

Table 5
MOPSO parameters.

Parameter	Values
Population size	200
External archive size	200
Mutation rate (P_{MUT})	0.5
Initial inertia weight ($w_{\max}$)	0.9
Final inertia weight ($w_{\min}$)	0.4
Maximum iterations	500
Acceleration constants (c_1 and c_2)	2.0

Table 6
Pareto-optimal solutions.

Bilateral contact prices	Portfolio model	Energy allocation proportion			Expected return
		Spot energy market	Bilateral contract-1	Bilateral contract-2	
$\lambda_2^B = 51.2$ US$/MWh	MV	0.4303	0.4182	0.1515	1.6621
$\lambda_3^B = 41.0$ US$/MWh	MVS	1.00	0.00	0.00	1.8007
$\lambda_2^B = 52.5$ US$/MWh	MV	0.3877	0.3385	0.2738	1.6991
$\lambda_3^B = 42.2$ US$/MWh	MVS	0.9838	0.0162	0.00	1.7976

MOPSO algorithm are selected through experiment and presented in Table 5.

MOPSO is first applied for MV theory based electricity portfolio selection, where only first two central moments (i.e., expected returns and risk) are optimized. Then, since distribution of GenCos' returns are non-normal and exhibits positive skewness; the MVS portfolio model is used to utilize the information about the skewness (third central moments) and optimized using MOSPO to obtain the optimal generation portfolio. The distribution of Pareto-optimal set over the MV and MVS efficient frontiers for case 1 are shown in Figs. 4 and 5, respectively. It can be seen that the MOPSO technique preserves the diversity of the non-dominated solutions over the efficient frontier and solves effectively the multi-objective portfolio problem in a single run. Once, a set of Pareto-optimal solutions is obtained through MOPSO, GenCo needs to select one optimum solution, which satisfies the respective goals. For this, the fuzzy membership functions [31] that represent the goals of each objective function are used. The best trade-off solutions for MV and MVS frontier are depicted on their efficient frontier in Figs. 4 and 5, respectively, and correspondingly electricity portfolio allocations are presented in Table 6.

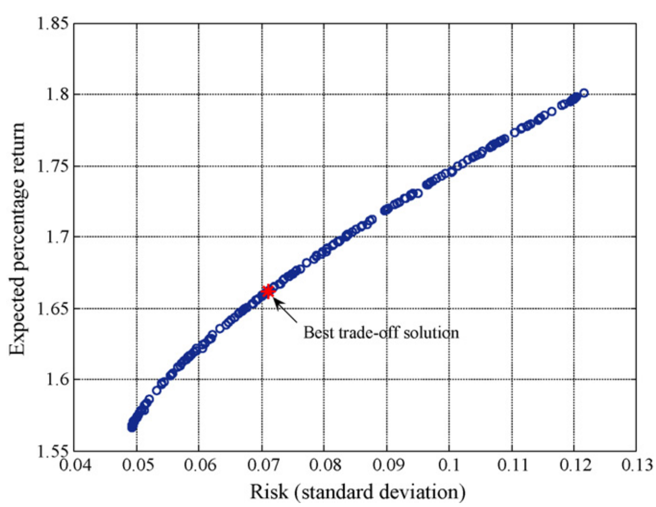

Fig. 4. MV portfolio frontier.

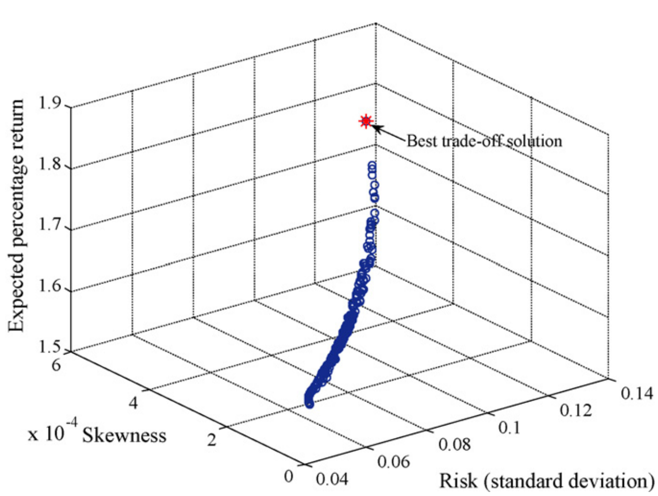

Fig. 5. MVS portfolio frontier: three objectives space.

Simulation results presented in Table 6 indicate that in MV framework GenCo participates in spot and contract market (contract 1) with almost equal share of 43.03% and 41.82% respectively, whereas its share in contract 2 is only 15.15%. This can be justified by looking to the returns distribution characteristics in Tables 2 and 3. Although the expected return in contract 1 is less than that in contract 2, GenCo's share is higher in contract 1 than in contract 2 because contract 1 is associated with low risk. It can be seen from Figs. 4 and 5 that when skewness is considered, the optimal portfolio is pushed further up on the efficient frontier signifying GenCo can get a higher return if it includes skewness in its decision making process. The corresponding asset proportions are presented in Table 6 and it is observed that GenCo gets higher return in comparison to MV based portfolio allocation (refer last column in Table 6). This suggests that the MV optimal criteria will lead to sub-optimal portfolios in the presence of skewness. Moreover, to observe the effect of bilateral contract prices on portfolio allocations, case 2 has also been analyzed. The simulation results based on MV and MVS portfolio model are presented in Table 6. These results show that the optimal proportion to be allocated to the bilateral contract, is increasing with the increase of bilateral contract price.

6. Conclusions

The simulation results presented in this paper supports the view that electricity assets have significant non-normal return characteristics and because of this, GenCo should consider all the first three central moments (mean, variance, and skewness) of the return distribution while deriving the generation portfolio allocation. Therefore, the mean–variance–skewness (MVS) model utilizes the multi-objective particle swarm optimization (MOPSO) as optimization tool is proposed in this paper. The performance of MVS model is compared with the classical mean–variance (MV) model using a case study of the PJM electricity market. Simulation results reveal that the MVS model can provide better portfolios in comparison to the MV model particularly when assets have non-normal return characteristics.

Appendix A.

Particle swarm optimization (PSO), due to its fast convergence for solving single-objective optimization problems [26], has been extended for solving multi-objective optimization (MOO) problems, called "multi-objective PSO" (MOPSO) [24]. When dealing with MOO problem, every particle represents one potential solution and all the particles in the swarm can search for the different parts of the Pareto-optimal front simultaneously. Therefore, MOPSO cannot use equation (10) (in Section 3.2) in a straight forward manner for *pbest* and *gbest*. As a result, selection of the local and global best guide for each particle in the swarm is a crucial step in MOPSO algorithm. Indeed, in the recent years some other techniques have been incorporated into multi-objective-PSO algorithms in order to make better convergence to the true Pareto-optimal front as well as to achieve a well-distributed Pareto-optimal set of solutions [32].

This paper presents an implementation of MOPSO-CD [27] that incorporating the mechanism of crowding distance computation in the global best selection, for MVS generation portfolio allocation and therefore, it briefly explained here. In MOPSO-CD, all the non-dominated solutions found along the search process are stored in a bounded external archive. Whenever this archive is full, it will be truncated using crowding distance mechanism. This promotes to implicitly maintain the diversity among the non-dominated solutions stored in the external archive since those solutions which are most crowded area are most likely to be replaced by a new solution. The mutation operator of MOPSO boosts the exploration capability and thereby preventing premature convergence due to existing local Pareto-fronts in some optimization problems (that means helps to improve the global search). At the end of the execution, all the particles stored in the external archive give us an approximation of the true Pareto-optimal front. The pseudo-code of MOPSO-CD is given below and its block diagram shown in Fig. 2. More details of this algorithm and pseudo-code for computing crowding distance can be found in [27].

```
Pseudo-code for MOPSO-CD
Initialize swarm and size of external archive
Evaluate objective functions
Store pbests
Store non-dominated particles in external archive
Iter = 0;
while (Iter < Iter_max); where Iter_max is the maximum number of iterations
  Compute the crowding distances in external archive
  Select the global best guide (using crowding distances)
  Update velocity and positions of the particles
  if (Iter < (P_MUT × Iter_max)); where P_MUT is the probability of mutation
    Perform mutation
  end if
  Evaluate objective functions
  Update external archive
  Update pbests
  Iter = Iter + 1
end while
Report results (external archive)
```

References

[1] M. Shahidehpour, H. Yamin, Z. Li, Market Operations in Electric Power Systems: Forecasting, Scheduling, and Risk Management, Wiley-Interscience, 2002.
[2] T.S. Chung, S.H. Zhang, C.W. Yu, K.P. Wong, Electricity market risk management using forward contracts with bilateral options, Proc. IEE Gen. Trans. Dist. 150 (5) (2003) 588–594.
[3] M. Liu, F.F. Wu, Managing price risk in multi market environment, IEEE Trans. Power Syst. 21 (4) (2006) 1512–1519.
[4] R. Dahlgren, C.C. Liu, J. Lawarree, Risk assessment in energy trading, IEEE Trans. Power Syst. 18 (2) (2003) 503–511.
[5] M. Denton, A. Palmer, R. Masiello, P. Skantze, Managing market risk in energy, IEEE Trans. Power Syst. 18 (2) (2003) 494–502.
[6] R. Bjorgan, C.C. Liu, J. Lawarree, Financial risk management in a competitive electricity market, IEEE Trans. Power Syst. 14 (4) (1999) 1285–1291.
[7] R. Bjorgan, H. Song, C.C. Liu, R. Dahlgren, Pricing flexible electricity contracts, IEEE Trans. Power Syst. 15 (2) (2000) 477–482.
[8] R.J. Kaye, H.R. Outhred, C.H. Bannister, Forward contracts for the operation of an electricity industry under spot pricing, IEEE Trans. Power Syst. 5 (1) (1990) 46–52.
[9] E. Tanlapco, J. Lawarrée, C.C. Liu, Hedging with futures contracts in a deregulated electricity industry, IEEE Trans. Power Syst. 17 (3) (2002) 577–582.
[10] R.A. Collins, The economics of electricity hedging and a proposed modification for the futures contract for electricity, IEEE Trans. Power Syst. 17 (2002) 100–107.
[11] M. Liu, F.F. Wu, Y. Ni, Portfolio optimization in electricity markets, Electr. Power Syst. Res. 77 (2007) 1000–1009.
[12] D. Feng, D. Gan, J. Zhong, Y. Ni, Supplier asset allocation in a pool-based electricity market, IEEE Trans. Power Syst. 22 (3) (2007) 1129–1138.
[13] X. Yin, Z.Y. Dong, T.K. Shaha, Optimal portfolio selection for generators in the electricity market, Proceedings of the IEEE PES General Meeting 2008, Pittsburgh, PA, USA.
[14] H. Markowitz, Portfolio selection, J. Finance 7 (1952) 77–91.
[15] P. Samuelson, The fundamental approximation of theorem of portfolio analysis in terms of means, variance and higher moments, Rev. Econ. Stud. 37 (4) (1970) 537–542.
[16] J.C. Singleton, J. Wingender, Skewness persistence in common stock returns, J. Financial Quant. Anal. 21 (3) (1986) p.335–341.
[17] P. Chunhachinda, K. Dandapani, S. Hamid, A.J. Prakash, Portfolio selection and skewness: evidence from international stock markets, J. Bank. Finance 21 (2) (1997) 143–167.
[18] H. Konno, K. Suzuki, A mean–variance–skewness optimization model, J. Oper. Res. Jpn. 38 (1995) 137–187.
[19] T. Lai, Portfolio selection with skewness: a multiple-objective approach, Rev. Quant. Finance Account. 1 (1991) 293–305.
[20] S.C. Liu, S.Y. Wang, W.H. Qiu, A mean–variance–skewness model for portfolio selection with transaction costs, Int. J. Syst. Sci. 34 (4) (2003) 255–262.
[21] E. Eberlein, G. Stahl, Both sides of the fence: a statistical and regulatory view of electricity risk, Energy Power Risk Manage. 8 (6) (2003) 34–38.
[22] A. Eydeland, K. Wolyniec, Energy and Power Risk Management, Wiley Publications, 2003.
[23] F.A. Longstaff, A.W. Wang, Electricity forward prices: a high-frequency empirical analysis, J. Finance 10 (2) (2002).
[24] C.A.C. Coello, M.S. Lechuga, MOPSO: a proposal for multiple objective particle swarm optimization, in: Proceedings of the IEEE conf. on Evolutionary Computation (CEC2002), vol. 2, 2002, pp. 1051–1056.
[25] S.S. Rao, Engineering Optimization Theory and Practice, 3rd ed., Wiley Publications, 1996.
[26] J. Kennedy, R. Eberhart, Particle swarm optimization, Proc. IEEE Conf. Neural Networks (1995) 1942–1948.
[27] C.R. Raqual, P.C. Naval, An effective use of crowding distance in multiobjective particle swarm optimization, Proceedings of the GECCO'05, 2005, Washington, DC, USA.
[28] http://www.pjm.com, accessed on February 2009.
[29] G.G. Judge, R.C. Hill, W.E. Griffiths, H. Lutkepohl, T.-C. Lee, Introduction to the Theory and Practice of Econometrics, Wiley Publications, New York, 1988.
[30] W.J. Conover, Practical Nonparametric Statistics, Wiley Publications, New York, 1980.
[31] M. Sakawa, Fuzzy Sets and Interactive Multi-Objective Optimization, Plenum Press, London, 1993.
[32] M.R. -Sierra, C.A.C. Coello, Multi-objective particle swarm optimizers: a survey of the state-of-the-art, Int. J. Comput. Intell. Res. 2 (3) (2006) 287–308.