

# Simulation of producers behaviour in the electricity market by evolutionary games

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## Abstract

Simulation of the electricity market participant's behaviour is important for producers and consumers to determine their bidding strategies and for regulating the market rules. In literature, for this aim a lot of papers suggest to use the well-known theory of non-cooperative games and the concept of Nash equilibrium. Unfortunately they cannot be applied in an easy way when a multi-players game has to be considered to simulate the operation of the electricity market. In this paper, the authors suggest to use the new theory of evolutionary games and the concept of near Nash equilibrium to simulate the electricity market in the presence of more than two producers. In particular, an opportune genetic algorithm has been developed; from the results reported in the paper, it is clear that this algorithm can be usefully utilised.

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## 1. Introduction

Market simulators are increasingly used to replicate the behaviour of electricity market participants (producers and consumers). These have to reproduce the actual electricity market operation as closely as possible. Thus, their design must to take into account all of the electricity market rules and peculiarities.

The electricity market is organised in two sessions: the day-ahead market and the reserve market. In particular, in an auction base day-ahead market, the market operator processes the bid information provided by the producers and consumers and aggregates this information creating hourly offer and demand curves, respectively. Once the bids are submitted, a market clearing algorithm matches the production and demand curves producing a series of hourly equilibrium prices and accepted quantities.

Identifying the electricity market equilibrium is an objective both for market participants and for regulators: for participants because the market equilibrium shows long-term bidding strategies of their rivals; for regulators because in this way the market

power monitoring and corrective measures implementing are possible.

To achieve this objective, it is necessary to design a market simulator that is: (1) to create a realistic electricity market model, (2) to simulate how participants generate their bids and, so, (3) to identify admissible market equilibrium. Step 1 is performed by means of a sophisticated optimisation technique that selects producers and consumers bids; step 2 is performed by opportune consumer–producer bids linear stepwise model and step 3 is implemented using the concepts of non-cooperative games and Nash equilibrium (NE) [1,2].

However, there are some difficulties in designing a rigorous market simulator applicable to the electricity market. One of them is due to transmission system constraints that complicate the market clearing mechanism and cause the income functions to be non-differentiable and non-concave. Another difficulty is to identify NE when three or more players participate in the same market as in the electricity one [3].

Several approaches can be used to overcome the above mentioned difficulties. Among them the attention is focused on mathematical and evolutionary programming.

The mathematical programming approach uses a numerical framework such as: the linear complementarity problem; mathematical programming with equilibrium constraints; or a conventional optimisation technique. This approach can find the

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NE of a multiplayer game that has differentiable and concave incomes. Then, when transmission constraints or population capacity bounds are present this approach has difficulties in determining NE because the income functions can be non-differentiable and non-concave.

On the contrary, the evolutionary programming approach requires neither differentiability nor concavity of the income functions, so it can be used in an effective way [4,5]. In the context of evolutionary programming, the evolutionary game theory is based on the player's evolutionary learning. Besides, Riechmann [6] showed that the canonical genetic algorithm (GA) based on the economic learning processes corresponded to evolutionary games and noted that the learning process results in "near Nash equilibrium".

From the above considerations, in the paper, a GA is designed to simulate a multiplayer electricity market with transmission system constraints, in order to analyze the behaviour of the producers in a real life market.

A continuous strategy for the producers is assumed and, for the sake of simplicity, no consumers' competition is considered. The multi-zonal market framework is adopted for the market clearing algorithm to take into account transmission system constraints such as the Italian power exchange (PX).

In the first part of the paper, the basic characteristics of the adopted market model and the concepts of evolutionary games are illustrated. Then the implemented GA evolutionary game for electricity market simulation is reported in detail.

In this context, the numerical experiment is performed in order to demonstrate how the proposed market simulator works well to simulate the behaviour of the producers.

## 2. Market model

In this section the market participants and power exchange (PX) models that will be used for the simulator, are presented.

### 2.1. Producer model

As well known, production cost function is a quadratic function:

$$C_T(QV) = c_0 + c_1QV + c_2QV^2 \quad (1)$$

where QV is the produced energy in MWh.

From (1) it is possible to consider the following marginal cost function:

$$\lambda = \frac{\partial C_T}{\partial QV} = c_1 + 2c_2QV \quad (2)$$

The utility may increase its production up to its maximum capacity  $P_{\max}$  and construct its trading price curves either from the marginal cost or from the incremental cost curve. The incremental cost curve will provide a utility with the minimum price to receive from the sale of energy.

Since marginal costs are linear functions, producers' bids are also considered a linear function and then the bid price is

$$PV_j^k = m\lambda \text{ with } m > 1 \text{ for } j\text{th producer in area-}k \quad (3)$$

Then, the decision variable is  $m$  which denotes the bid curve slope.

It is worth to underline that more complex strategies can be adopted without any difficulty inside the model f.i. taking into account ramping limit, etc.

Being the basic aim of the paper to propose a simulation technique of the competitive environment the simplest but effective producer model is adopted.

### 2.2. Consumer model

Consumers are modelled in a simple way because the main purpose of the electricity market simulator proposed in this paper is to analyze the behaviour of producers; then each consumer bid is considered a fixed couple of price-quantity values.

### 2.3. Market clearing algorithm

A network-constrained single period auction to minimize energy price is used to clear the market. It results in an optimisation problem. The power system is divided into several areas connected by interconnection lines: if no congestion is detected on the interconnection lines, a unique energy price is determined for the system; otherwise the market is split in two or more zones corresponding to the area where the congestion is detected and each sub-market is cleared and then equilibrium price is valued for each zone.

The complete formulation of the optimisation problem, denoted as PX model, is as follows:

$$\max \sum_{k,i} QA_i^k PA_i^k - \sum_{k,j} QV_j^k PV_j^k \quad (4)$$

s.v.

$$0 \leq QV_j^k \leq \overline{QV_j^k} \quad \forall j, k \quad (5)$$

$$0 \leq QA_i^k \leq \overline{QA_i^k} \quad \forall i, k \quad (6)$$

$$\sum_{k,j} QV_j^k - \sum_{k,i} QA_i^k = 0 \quad (7)$$

$$TR_h = \sum_k S_h^k QV_j^k \quad \forall h \quad (8)$$

$$\text{MIN}F_h \leq TR_h \leq \text{MAX}F_h \quad (9)$$

where (4) indicates the energy transactions; (5) and (6) are the bounds of energy quantities of producers and consumers bids; (7) defines the energy balance in the system neglecting losses; (8) and (9) concern the energy flowing through the interconnections and its limit.

It is worth to underline that this model is adopted by PX in Italy [7]. The optimisation problem is linear and so it can be solved by the well known methods and algorithms.

If  $QV_j^k$  is accepted  $QV_j^k = \overline{QV_j^k}$  this means that the Lagrange multiplier  $\rho_j^k$  related to the upper limit constraint of type (5) on  $QV_j^k$  is different from zero. As well known a Lagrange multiplier is the increment of the objective function in consequence

of a unitary relaxation of the related constraint. In this case, considering  $\rho_j^k$ , the unitary relaxation means that 1 MWh of the  $j$  producer in zone  $k$  is sold. In this case the increment of the objective function is the difference between the price to pay energy (zonal equilibrium price  $P^{*k}$ ) and the price offered by the seller ( $PV_j^k$ ).

From this consideration and how described in [7] the zonal equilibrium price is given by

$$P^{*k} = \rho_j^k + PV_j^k \quad (10)$$

Obviously, this is true for each generator  $j$  in a zone  $k$ :  $\rho_j^k + PV_j^k$  in equal to  $P^{*k}$  for each  $k$ . It is worth to underline that, in presence of congested network the Lagrange multipliers related to transmission constraints are, in general, different from zero as the constraints are met and so they affect zonal prices.

It is clear that:

- Every consumer bid with bid price above  $P^{*k}$  is completely accepted.
- Every producer bid with bid price below  $P^{*k}$  is accepted.
- Bids with a price equal to  $P^{*k}$  can be partially accepted.

### 3. Genetic algorithm as evolutionary game

In this section, using the results reported in [6,8], it will be demonstrated that a canonical GA can be considered an evolutionary game. According to this aim three assumptions will be discussed in detail: (a) every GA is an evolutionary game; (b) in GA learning processes, populations tend to converge towards a near Nash equilibrium; (c) a concept of evolutionary superiority and evolutionary stability exist.

#### 3.1. Canonical genetic algorithm and evolutionary games

The economic market process can be reproduced by an Economic GA. A canonical GA whose individuals are economic agents is called Economic GA. The fitness related to each individual is its economic success in the market against the competitors, then, in economic GAs the fitness related to an economic agent does not only depend on its own strategy, but also on the strategies of all other agents involved in the model. So the following proposition holds:

**Proposition 1** (Economic GA). *An economic GA is a GA with a state-dependent function.*

In the economic market process, every economic agent produces its strategy by a process of learning. There are two kinds of learning: *social*, learning by interaction within a single economic agents population, examples of social learning are learning by imitation and learning by communication; *individual*, learning by experiment. They are variety generating processes. In the canonical GA these processes are reproduction that can be interpreted as learning by imitation, crossover as learning by communication, and mutation as learning by experiment.

The market plays the role of variety restricting process, i.e. only the “best” strategies survive. In canonical GA, there is

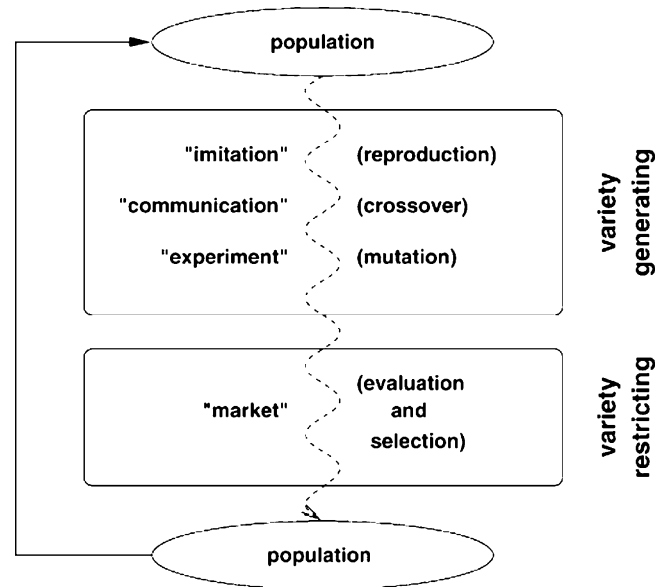


Fig. 1. Similitude between GA and market process.

one restricting process which is the genetic operator of selection. GA selection decreases the number of different economic strategies within the population. It firstly evaluates the economic success of each strategy in the market and selects strategies to be part of the next population. The used selection operator (often called roulette-wheel selection) consists of repeatedly drawing the strategies from the pool of the old population to be reproduced in the next one, on the basis of its relative fitness which is the ratio of its market success to the sum of the market success of all strategies in the population.

Fig. 1 shows the similitude between the economic market process and canonical GA process.

**Proposition 2** (GA-evolutionary games). *Every simple, one-population, economic genetic algorithm is an evolutionary game.*

In order to provide some evidence for this proposition, a definition is needed that clearly states what an evolutionary game is. This paper makes use of the definition by Friedman [8], who gives three characteristics for an evolutionary game:

**Definition 1** (Evolutionary game; Friedman). *An evolutionary game is a dynamic model of strategic interaction with the following characteristics:*

- higher income strategies tend over time to displace lower income strategies;
- there is inertia;
- players do not intentionally influence other players' future actions.

Riechmann [8] demonstrates how GA matches the characteristics (a), (b) and (c). In a few words GA learning models describe a repeated economic game. Imagine an economic genetic algorithm using a population of  $M$  genetic individuals with the length of each individual's bit string of  $L$ . Each individual represents

the strategy of the economic agent. Due to the binary coding of genetic individuals, this means that each genetic individual represents one out of  $N = 2^L$  different values constituting the set of available strategies  $S$ . This means that the GA is able to deal with every economic strategy in the set of all available strategies  $S$ . Thus, the GA can be interpreted as a repeated symmetric one population  $M$  person game with up to  $N$  pure strategies. But, compared to normal evolutionary games, within most economic GA learning models, the rules of the game are different. Whereas in evolutionary games most of the time a strategy is repeatedly paired with single competing strategies, in genetic algorithm learning, each strategy plays against the whole aggregate rest of the population. Instead, every economic agent aims to find a strategy belonging to  $S$  performing as well as possible relative to its environment, which is completely determined by the current population and its objective function ( $R$ ).

At a first glance, it is the structure of genetic algorithms and evolutionary models that suggests a close relationship between GAs and evolutionary game theory: both face the central structure of a population of economic agents interacting within some well-defined economic environment and aiming to optimize individual behaviour.

### 3.2. Genetic population as near Nash equilibrium

It is known, that every economic agent aims to find the best performing strategy  $i$  with respect to the objective function  $R$  and given the strategies of the rest of his population  $n$ . This means that every economic agent faces the problem:

$$\max_{i \in S} R(i) \quad (11)$$

where  $S$  is the set of the possible strategies.

This immediately leads to the concept of Nash equilibrium. A Nash strategy of an economic agent is defined as the best strategy *given the strategies of the competitors*, and it is exactly what every economic agent, alias genetic individual, is trying to achieve.

Although selection and reproduction tends to drive the population towards Nash equilibrium, mutation (learning by experiment) prevents the population from fully reaching such equilibrium. Obviously, a mutation probability value greater than zero is necessary to cover a wider search region. It can be so concluded that genetic population represents a state which is not a real Nash equilibrium point but it is “not far from it”. This state will be called “near Nash equilibrium”.

Then, the following proposition can be stated.

**Proposition 3** (GA populations-near Nash equilibrium). *In every simple one—population, economic genetic algorithm, the population tends over time to move to a Nash equilibrium without fully reaching it.*

### 3.3. Evolutionary stability and superiority of genetic population

In the evolutionary game it is important to define the concept of evolutionary stability. The evolutionarily stable strategies

are based on the notion that invading strategies are somehow rejected or eliminated from the population.

GAs present a clear concept of rejection: every strategy will be exposed to a test, which has been described as a one-against-the-rest game in the previous sections. Then the strategy will be reproduced or rejected with a probability depending on its performance (i.e. market performance) in this game. Indeed, GA reproduction has two main features: it selects due to performance and it selects due to probability; a bad strategy will be rejected in this way almost surely although not with probability one.

Thus, a refined concept of evolutionary stability is needed for genetic algorithms: a genetic population is evolutionarily stable if the process of the genetic algorithm rejects an invasion by one or more strategies from the genetic population.

In GA the concept of evolutionary stability is replaced by the concept of an evolutionarily superior population. More formally, a genetic population  $\bar{n}$  will be called evolutionarily superior to population  $\bar{m}$  if it exhibits two characteristics:

1. Every strategy contained within population  $\bar{n}$  maintains or increases its fitness value with respect to the one in the basic population  $\bar{m}$ , while at least one strategy increases its fitness in  $\bar{n}$  than in  $\bar{m}$ .
2. The invading strategies  $k \in \{\bar{m}/\bar{n}\}$  are the worst performing strategies contained in  $\bar{m}$ , so that they will be most surely rejected.

This definition of evolutionary superiority induces a partial ordering on the space of genetic populations, which resembles the concept of Pareto superiority.

## 4. Market simulator

The main objective of this section is to show how it is possible to simulate the electricity market by GA-evolutionary games.

### 4.1. Representation

The main problem is to represent the decision variables as genetic individual. As said each participant, in particular producer, submits a couple of quantity and price; so two variables have to be considered. For the sake of simplicity only the price is considered as a decision variable in competition; this assumption is realistic as it is reasonable to suppose that the quantity of energy is strictly related to technical limits.

The price is an integer number expressing the unitary price in euro per MWh easily convertible in binary string of opportune length for crossover and mutation operation and represents the individual of genetic population.

An individual represents the price offered by a producer and so the population represents the set of prices offered by the producers operating in same market. The evolution of this individual in the population represents the “natural” competitive game in the market.

Obviously, the price may vary inside realistic minimum and maximum values, in this case the possible producers’

prices; note that in such a way a continuous strategy is adopted.

#### 4.2. Initialization

The initial population is randomly valuated; for each producer a price is chosen randomly between the minimum and maximum values above defined.

#### 4.3. Fitness evaluation

In the paper the attention is focused on the dynamic of bid price; the quantity offered by each producer is assumed constant. The fitness is calculated in two steps, as follows:

- Step 1. Using the individuals of the population as producers' price  $PV_i^k$ , and the fixed energy quantity  $\overline{QV}_j^k$ , run the PX model to clear the market and find the equilibrium price  $P^*$ .
- Step 2. The fitness of each individual  $i$  is calculated as follows:

$$f_i = P^{*k}QV_i^k - C_T(QV_i^k) \quad \forall i, k \quad (12)$$

where  $QV_i^k$  is the accepted quantity according to the clearing market model depicted in Section 2.3. Note that the fitness of each individual is related to the value of the other individual: it is "state dependent".

#### 4.4. Selection and competition

The problem of selecting the parents in offspring population is important for evolution. Parents are selected according to their fitness. In this paper a roulette rule selection method is used. Imagine a roulette wheel where all the individuals of the population are placed; the size of the section in the roulette wheel is proportional to the value of the fitness function of every individual—the bigger the value, the larger the section. A marble is thrown in the roulette wheel and where it stops the individual is selected. Clearly, the individuals with a higher fitness value will be selected more often.

This process can be described by the following algorithm:

1. **[Sum]** Calculate the sum of all individual fitness in the population—sum  $S$ .
2. **[Select]** Generate a random number from the interval  $(0, S) - r$ .
3. **[Loop]** Go through the population and sum the fitness from  $0$ —sum  $s$ . When the sum  $s$  is greater than  $r$ , stop and return the individual to where you are.

Of course, Step 1 is performed only once for each population.

#### 4.5. Creation of offspring

In this paper a single point crossover is used between two individual selected using the procedure illustrated in Section 4.4. The selected individuals are converted in binary strings of opportune length compatible with the required precision. One crossover point is selected, the binary string from the beginning of the individual to the crossover point is copied from the first

parent, and the rest is copied from the other parent. Naturally, the individual that has the best fitness value is reproduced as it is in the next population.

After the crossover, mutation is performed inverting the selected bits with a given probability.

#### 4.6. Iterative algorithm and termination criteria

To simulate the competition, several populations have to be performed; nevertheless not all the populations are a solution of the competition: only a population satisfying the condition of evolutionary superiority with respect to a previous solution is detected as a solution to the evolutionary game. This population represents the set of best strategies of the market participants. The diversity of populations is so based on the concept of evolutionary superiority given in Section 3.3.

Having in mind to simulate the behaviour of the producers over a long period, the proposed algorithm will be stopped when the predefined maximum number of populations is reached.

The iterative algorithm that simulates the electricity market is summarized as follows:

- (a) Perform the initialisation procedure illustrated in Section 4.2, and then obtain the population that is conventionally fixed as problem solution.
- (b) Fitness evaluation.
- (c.1) Solve the PX model.
- (c.2) Calculate the income of each producer and assign the relative value as fitness to each individual.
- (d) If the new population is evolutionarily superior with respect to the actual problem solution then the solution is updated otherwise not.
- (e) Apply the selection, competition and creation of offspring; then obtain the new population.
- (f) If the predefined maximum number of populations is reached then go to (g) else repeat steps (b)–(f).
- (g) Obtain the final solution.

The above algorithm is illustrated in Fig. 2.

### 5. A numerical example

The aim of this numerical example is to show how the proposed market simulator works well to simulate the behaviour of the producers. In particular, the simulator is used for a 10-player day-ahead energy market simulation to investigate how the price changes when multiple producers are in competition. Moreover, the number of ten producers is a realistic case [1].

The following simplifying hypotheses have been made:

1. the total power required by the consumers is  $\overline{QA} = 11,000$  MWh;
2. only one zone is considered;
3.  $\overline{QA} = \sum_{i=1}^{10} \overline{QV}_i$ ;
4.  $PV_i \in [5, 30]$ ,  $i = 1, \dots, 10$

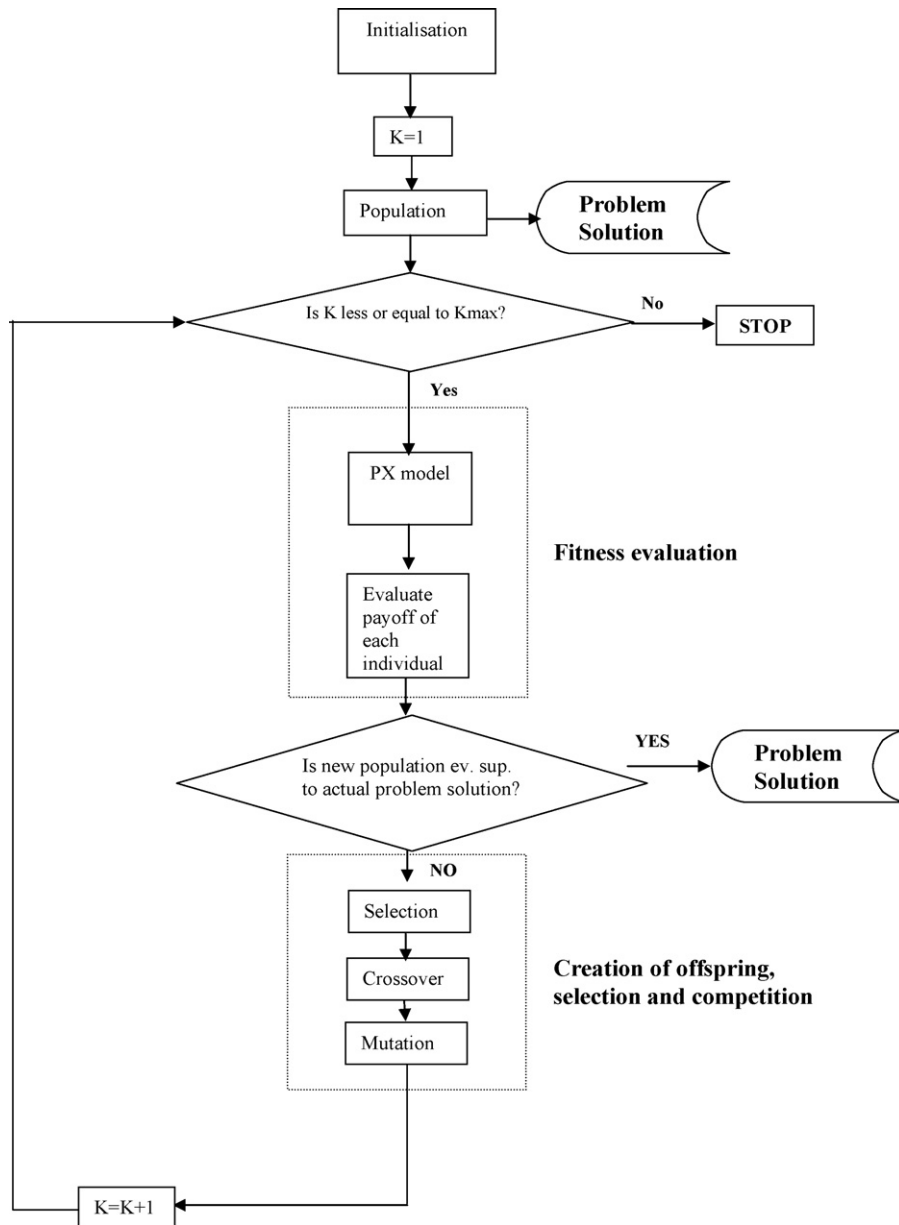


Fig. 2. Flow chart of the algorithm.

For the case considered in the paper according to point 3 all the producers bids will be selected so the producers quantities are fixed, i.e. production cost are fixed so maximize the fitness is translated in maximizing the total income.

The consumer price bid is 35 €/MWh; note that is higher than the producer price bid upper bound so the consumers will be in any case satisfied. In Table 1 the bounds of the power offered by producers are reported. For sake of simplicity only one zone is considered.

According to (3) the hypothesis 3 implies that  $m$  is a continuous variable with opportune bounds in consequence of the marginal cost of each producer.

A random population has been chosen as the initial evolutionary superior population for simulation and it is reported in Table 2; its clearing price is 13 €/MWh.

Table 1  
Quantity of energy of the producers (MWh)

$\overline{QV}_1$	1500
$\overline{QV}_2$	1200
$\overline{QV}_3$	1300
$\overline{QV}_4$	1000
$\overline{QV}_5$	800
$\overline{QV}_6$	1050
$\overline{QV}_7$	700
$\overline{QV}_8$	900
$\overline{QV}_9$	1700
$\overline{QV}_{10}$	950

Table 2  
Initial population

Producer	Income (€)	Price (€/MWh)
1	19500.00	11.00
2	15600.00	6.00
3	16900.00	9.00
4	13000.00	12.00
5	10400.00	5.00
6	14300.00	7.00
7	4550.00	13.00
8	11700.00	10.00
9	23400.00	8.00
10	13650.00	5.00

After 64 populations an evolutionary superior population is detected and is reported in Table 3 and a clearing price equal to 27 €/MWh is reached.

Comparing the initial solution and the solution obtained at 64th population, it is clear that the income is higher for each individual; moreover, the strategy has been modified by the producers that have a worst income in the initial solution. Note that the clearing price is increased by 107.7% and so the increase in income is up more than 100% for several individuals. Besides it can be underlined that the 9th individual, having the best fitness in initial population, is reproduced in the analyzed population.

After 425 populations another evolutionary superior solution is detected with a clearing price equal to 29 €/MWh. The population is reported in comparison with the solution at the 64th population in Table 4.

It is clear that the income increases slowly with respect to the last solution. This means we are close to a reasonable equilibrium point. Note that the 3rd individual determines the equilibrium price in both populations.

After 654 populations another evolutionary superior solution is detected with a clearing price equal to 30 €/MWh. The population is reported in comparison with the solution at the 425th population in Table 5.

Note that also in this case the income increase is below 4%, and then we are very near to the final solution.

Indeed also after 1500 populations no evolutionarily superior solution is detected so we can conclude that no producer has convenience in changing his strategy: a “near Nash equilibrium” is detected; after 654 generations the price cap is reached and each competitor has maximized his income.

*Comments:* As expected in the presence of an oligopoly the clearing market price increases: a kind of collusion exists (see Fig. 3). The income is strictly dependent on the state of the population. It can be noted that the 9th individual (in bold in the tables) by his income increase for the price strategy has the same price strategy as the other individuals in the population.

Looking at Fig. 4, it can be noted that the income increase, for each producer, with different slope in a first stage then they increase in the same way. That is an equilibrium of power market is reached. It is confirmed by Figs. 5–8 that report how the total income is divided in percent by the producers. Moreover, it can be seen that such values are very close to those that represent,

Table 3  
Evolutionary superior population at 64th population compared with the initial one

Producer	Payoff (€) at 64th population	Payoff (€) at 0th population	Payoff (€) improvement (%)	Strategy/price (€/MWh) at 64th population	Strategy/price (€/MWh) at 0th population
1	40500.00	19500.00	+107.7	12.00000	11.00000
2	32400.00	15600.00	+107.7	8.00000	6.00000
3	25650.00	16900.00	+51.77	27.00000	9.00000
4	27000.00	13000.00	+107.7	7.00000	12.00000
5	21600.00	10400.00	+107.7	13.00000	5.00000
6	29700.00	14300.00	+107.7	14.00000	7.00000
7	18900.00	4550.00	+315.4	8.00000	13.00000
8	24300.00	11700.00	+107.7	18.00000	10.00000
<b>9</b>	<b>48600.00</b>	<b>23400.00</b>	<b>+107.7</b>	<b>8.00000</b>	<b>8.00000</b>
10	28350.00	13650.00	+107.7	23.00000	5.00000

Table 4  
Evolutionary superior population at 425th population compared with 64th population

Producer	Payoff (€) at 425th population	Payoff (€) at 64th population	Payoff (€) improvement (%)	Strategy/price (€/MWh) at 425th population	Strategy/price (€/MWh) at 64th population
1	43500.00	40500.00	+7.41	24.00000	12.00000
2	34800.00	32400.00	+7.41	27.00000	8.00000
3	27550.00	25650.00	+7.41	29.00000	27.00000
4	29000.00	27000.00	+7.41	6.00000	7.00000
5	23200.00	21600.00	+7.41	8.00000	13.00000
6	31900.00	29700.00	+7.41	19.00000	14.00000
7	20300.00	18900.00	+7.41	5.00000	8.00000
8	26100.00	24300.00	+7.41	12.00000	18.00000
<b>9</b>	<b>52200.00</b>	<b>48600.00</b>	<b>+7.41</b>	<b>8.00000</b>	<b>8.00000</b>
10	30450.00	28350.00	+7.41	26.00000	23.00000

Table 5  
Evolutionary superior population at 654th population compared with 425th population

Producer	Payoff (€) at 654th population	Payoff (€) at 425th population	Payoff (€) improvement (%)	Strategy/price (€/MWh) at 654th population	Strategy/price (€/MWh) at 425th population
1	45000.00	43500.00	+3.45	23.00000	24.00000
2	36000.00	34800.00	+3.45	23.00000	27.00000
3	28500.00	27550.00	+3.45	30.00000	29.00000
4	30000.00	29000.00	+3.45	10.00000	6.000000
5	24000.00	23200.00	+3.45	22.00000	8.000000
6	33000.00	31900.00	+3.45	10.00000	19.00000
7	21000.00	20300.00	+3.45	19.00000	5.000000
8	27000.00	26100.00	+3.45	27.00000	12.00000
<b>9</b>	<b>54000.00</b>	<b>52200.00</b>	<b>+3.45</b>	<b>8.000000</b>	<b>8.000000</b>
10	31500.00	30450.00	+3.45	27.00000	26.00000

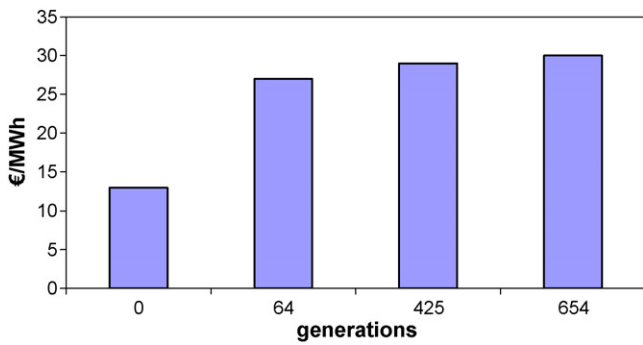


Fig. 3. Clearing price over generations.

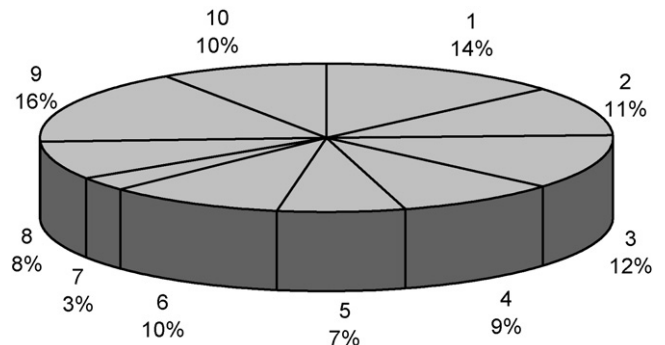


Fig. 6. Total income repartition among producers at iteration 64.

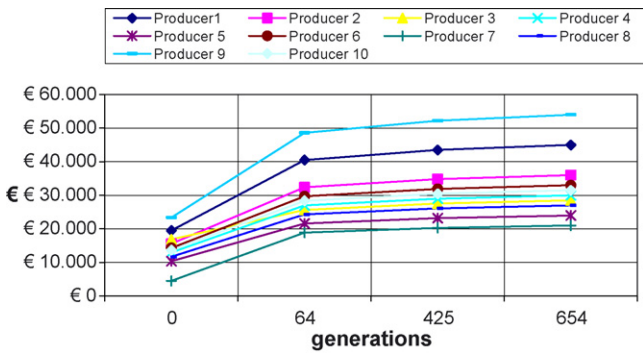


Fig. 4. Income over generations.

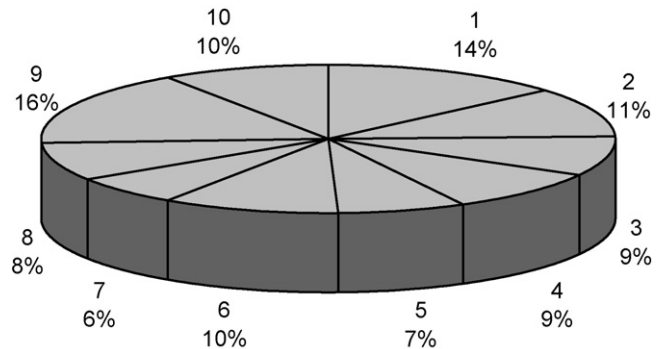


Fig. 7. Total income repartition among producers at iteration 425.

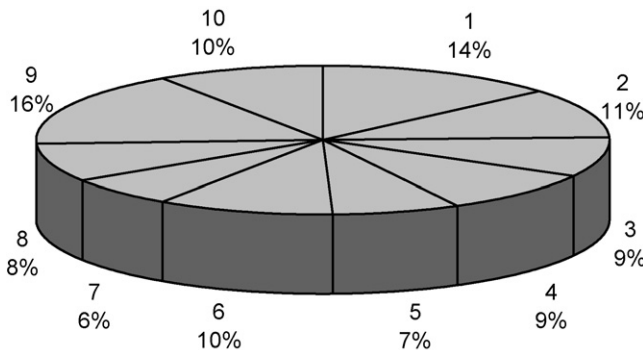


Fig. 5. Total income repartition among producers at iteration 0.

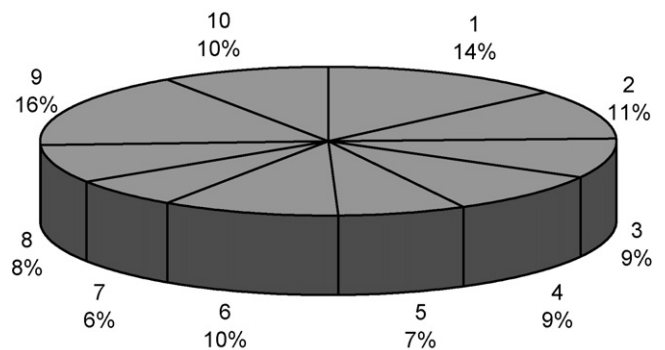


Fig. 8. Total income repartition among producers at iteration 654.



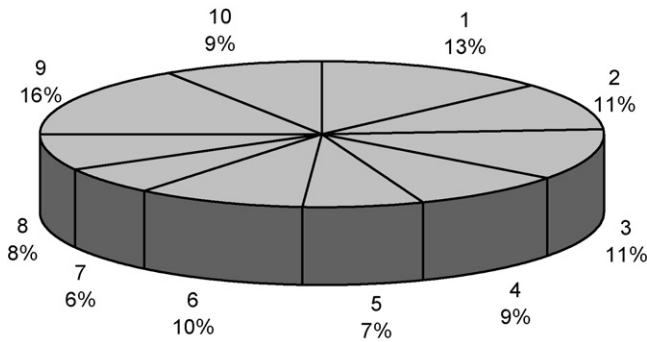


Fig. 9. Energy offered by producers.

in percent respect to the total, the quantity of energy offered by the producer (Fig. 9).

### 6. Conclusion

In the paper a GA-evolutionary game is proposed to simulate the behaviour of two or more producers operating in the same electricity market. The GA can be used to forecast, over a long period, the electricity price and how the competition can influence it. A future aim of the authors is to implement evolutionary games among producers and consumers taking into account also interzonal congestions, an elastic demand and so to use the proposed market simulator for more realistic cases. The basic idea is to perform competition between two populations: producers and consumers and using the clearing market model to evaluate the economic success of the competitors.

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### Appendix A. List of symbols

$C_T(QV_i^k)$  is the cost function for individual  $i$

- $f_i$  fitness of each individual  $i$
- $MAXF_h, MINF_h$  maximum and minimum power flowing rate on interconnection  $h$ .
- NTR is the number of interconnections among areas
- $P^*$  energy national price
- $P^{*k}$  is the market cleared price in area  $k$
- $PA_i^k$  buyer  $i$  in zone  $k$  price
- $PV_j^k$  seller/buyer bid price
- $QA_i^k$  buyer  $i$  in zone  $k$  bought power
- $QA_i^k$  buyer  $i$  in zone  $k$  offered power
- $QV_j^k$  seller  $i$  in zone  $k$  sold power
- $QV_j^k$  seller  $i$  in zone  $k$  required power
- $R$  objective function
- $S_h^k$  sensitivity coefficient of power flowing on the interconnection  $h$
- Subscript  $i$  denotes buyers
- Subscript  $j$  denotes sellers
- Superscript  $k$  denotes the market zone
- $TR_h$  power flowing on interconnection  $h$  ( $h = 1, \dots, NTR$ )
- $QVN^k = \sum_j QV_j^k - \sum_i QA_i^k$  is the power injection in area  $k$

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