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Free Choice and the theory of Scalar Implicatures*
 Danny Fox, MIT,

This paper will be concerned with the conjunctive interpretation of a family of disjunctive constructions. The relevant conjunctive interpretation, sometimes referred to as a “free choice effect,” (FC) is attested when a disjunctive sentence is embedded under an existential modal operator. I will provide evidence that the relevant generalization extends (with some caveats) to all constructions in which a disjunctive sentence appears under the scope of an existential quantifier, as well as to seemingly unrelated constructions in which conjunction appears under the scope of negation and a universal quantifier.

Alonso-Ovalle (2005), following Kratzer and Shimoyama (2002), has presented important evidence arguing that free choice effects should be derived by the system that accounts for Scalar Implicatures (SIs). However, we will see that deriving a free choice implicature is not a simple matter within standard approaches to implicature computation. More specifically, FC directly contradicts neo-Gricean attempts to deal with another observation about disjunction due to Chierchia (2004): Sauerland 2004, Spector 2005. In response to this predicament, I will argue for a system that derives SIs within the linguistic system, though in a somewhat different manner from Chierchia (2004). Specifically, I will argue for a covert exhaustivity operator with meaning somewhat akin to that of *only* (in the spirit of Chierchia (2004), but more directly following suggestions by Groenendijk and Stokhof (1984), Krifka (1995), Landman (1998), and van Rooy (2002)). We will see that all of our observations about FC, as well as Chierchia’s observations about disjunction, follow from a novel (though fairly natural) approach to the meaning of the exhaustivity operator.

It is often claimed that the neo-Gricean account of SIs follows from basic truisms about the nature of communication. However, as is well known, one assumption is crucial, and far from trivial, namely the assumption that Grice’s Maxim of Quantity should be stated with reference to a formally defined set of alternatives. There is clearly no escape from formally defined alternatives. However, if the perspective argued for here is correct, access to these alternatives should be limited to grammar. A quantity maxim which is not contaminated by syntactic stipulations (together with appropriately placed syntactic stipulations, i.e., within grammar) derives better empirical results.

* This work is very much inspired by earlier proposals of Kratzer and Shimoyama, Alonso-Ovalle, Gennaro Chierchia, in particular by Kai von Stechow’s class presentation of Kratzer and Shimoyama. Special thanks go to Gennaro Chierchia, Kai von Stechow, and Irene Heim. I’ve also benefited from discussions with Luis Alonso-Ovalle, Jon Gajewski, Nathan Klindinst, Ezra Keshet, Angelika Kratzer, Fred Landsman, Philippe Schlenker, Benjamin Spector, and from comments and questions at the 2005 LSA summer institute (MIT and Harvard), and at colloquia at UMass Amherst, SUNY Stony Brook, Tel-Aviv University, and Tuebingen University.

1. Some Background on Scalar Implicatures

Consider a simple disjunctive sentence such as that in (1). When we hear such a sentence we draw a variety of inferences.

(1) Sue talked to John or Fred.

First, we conclude that (if the speaker is correct) Sue talked to John or to Fred, a conclusion, in and of itself, consistent with the possibility that Sue talked to both (*Basic Inference*). However, we typically also conclude (again assuming that the speaker's utterance is correct) that this latter possibility was not attested (*Scalar Implicature, SI*). Finally, we infer that the speaker's beliefs don't determine which person (i.e. John or Fred) Sue talked to (*Ignorance Inferences*).¹

(2) Inferences we draw from (1):

- a. *Basic Inference*:
Sue talked to John or Fred (or both).
- b. *Scalar Implicature, SI*:
Sue didn't talk to both John and Fred.
- c. *Ignorance Inferences*:
The speaker doesn't know that Sue talked to John.
The speaker doesn't know that Sue talked to Fred.

The nature of the inferences in (2)a and (2)c seems rather straightforward. The Basic Inference, (2)a, is derived quite directly from the basic meaning of the sentence. The Ignorance Inferences, (2)c, are not as direct, but, nevertheless, receive a fairly natural explanation. They are derived straightforwardly from a general reasoning process about the belief states of speakers, along lines outlined by Grice (1975). The source of the inference in (2)b, SI, is, at least in my opinion, less obvious. The standard, neo-Gricean, approach captures this inference by enriching the set of assumptions that enter into the derivation of Ignorance Inferences, while various competing proposals attribute the inference to a particular enrichment of the basic meaning.

Before we see what is at stake, let's start with a formulation of what might be uncontroversial, namely the account of (2)c.² The basic idea is that communicative principles require speakers to contribute as much as possible to the conversational enterprise. This idea is further elaborated when it is assumed that the goal of certain speech acts is to convey information, and that if all information is to be relevant, more is

¹ Following Gazdar (1979) and Sauerland (2004), I will sometimes use the verb *know* to describe Ignorance Inferences. This choice is problematic because of factivity inferences associated with *know*, which are clearly inaccurate. However, it's not clear that there is a better choice: *believe* is problematic because of neg-raising. When I will find factivity particularly disturbing (or neg-raising sufficiently innocuous), I will favor belief-talk. My choices will, however, be far from systematic. The reader should bear all of this in mind and ignore factivity inferences associated with *know*, as well as the neg-raising property of *believe*.

² Although I think that there is agreement that (2)c ought to be derived from principles of communication, there have been conflicting proposals concerning the precise derivation. As we will see below, the complications could be argued to follow from the neo-Gricean perspective on SIs. See our discussion in p. 6 of Gazdar (1979) and Sauerland (2004).

better. So, assume that two sentences are true and both contribute information that is completely relevant to the topic of conversation. If one contains more information than the other (i.e., is logically stronger),³ use of the more informative one would constitute a greater contribution:

(3) **Maxim of Quantity (basic version)**: If S_1 and S_2 are both relevant to the topic of conversation and S_1 is more informative than S_2 , if the speaker believes that both are true, the speaker should utter S_1 rather than S_2 .

Typically, when (1) is uttered, the information conveyed by each of the disjuncts is relevant. Furthermore, each disjunct is more informative than the entire disjunction. (p entails p or q , but not vice versa.) The fact that the speaker, s , uttered the entire disjunction rather than just a disjunct, therefore, calls for an explanation. If we, the people who interpret the utterance, assume that s obeys the Maxim of Quantity, we conclude, for each disjunct, p , that it is false to claim that s believes that p is true, or if we keep to our convention of using the verb *know* instead of *believe* (see note 1), we can state this as a conclusion that s does not know that p is true.

If we assume that s believes that her utterance of the disjunction is correct, we derive the Ignorance Inferences. But one logical property of the situation is worth focusing on. When we conclude that s does not believe that p is true, that is, in principle, consistent with two different states of affairs. s might believe that p is false, or, alternatively, she might have no (conclusive) opinion. The reason we infer the latter is that the former would be inconsistent with our other inferences. Under normal circumstances, we infer that s believes that her utterance of p or q is true (Maxim of Quality). If we were to assume that s believes that p is false, we would have to conclude that she believes that q is true. But that would conflict with our inference about q (based on the Maxim of Quantity). Hence we must conclude, for each disjunct, that the speaker has no opinion as to whether or not it is true.

1.1. The Symmetry Problem⁴

Consider now whether we could extend this line reasoning to account for the SI in (2)b. Since we've already concluded that the speaker does not know that p is true and that the speaker does not know that q is true, it follows that the speaker does not know that the conjunction p and q is true. This, again, is consistent with two different states of affairs. s might believe that p and q is false, or, alternatively, she might have no (conclusive) opinion. If this time we could exclude the latter possibility, we would derive the SI.

The problem is that basically the same line of reasoning we've employed above leads us exactly to the opposite conclusion, namely to the exclusion of the possibility that s believes that p and q is false. The idea is fairly simple. The information that p and q is false, if true, would be relevant to the topic of conversation, hence the fact that s did not

³ The relevant notion of informativity for a pragmatic account should probably be that of contextual-strength (i.e. logical strength given contextual presuppositions). For the consequence of this distinction to the theory of SIs, with a potential argument against (neo-)Gricean accounts, see Fox (2004), and Magri (2005).

⁴ The discussion in this section relies heavily on the introduction to pragmatics taught by Kai von Stechow and Irene Heim at MIT.

provide us with this information calls for an explanation.⁵ Once again, the natural explanation is that *s* did not have the information, i.e., that *s* did not know that *p and q* is false. In other words, instead of an SI, we derive, once again, an Ignorance Inference: we conclude that *s* does not know that *p and q* is true, and (exactly by the same type of reasoning) that *s* does not know that *p and q* is false, i.e., we conclude that *s* does not know whether or not *p and q* is true.

As far as I know, a version of this problem was first noticed in Kroch (1972), and stated in its most general form in class notes of Kai von Fintel and Irene Heim. To appreciate the problem in its full generality, consider a general schema for deriving SIs in response to *s*'s utterance of *p* (of, say, *I have 3 children*). We start by considering a more informative relevant utterance, *p'* (say, *I have 4 children*), and reason that if *p'* were true, and if *s* knew that *p'* were true, the Maxim of Quantity would have forced *s* to utter *p'* instead of *p*. We then might reason that it is plausible to assume that *s* knows whether or not *p'* is true (say, that it is reasonable to assume that *s* knows how many children she has), and hence that *s* knows that *p'* is false.

The problem, however, is that there is always an equally relevant more informative utterance than *p*, namely *p and not p'* (in our case, *I have exactly 3 children*), call it *p''*. By the same reasoning process, if *p''* were true, and if *s* knew that *p''* were true, the communicative principles would have forced *s* to utter *p''* instead of *p*.⁶ Furthermore, if *s* knows the truth value of *p*, and of *p'*, then *s* knows the truth value of *p''*. So, the same reasoning process leads to the conclusion that *s* knows that *p''* is false. The assumption that the speaker knows whether or not *p'* is true, thus, leads to a contradiction and must, therefore, be dropped.

This problem was dubbed the symmetry problem in class notes of Kai von Fintel and Irene Heim. Whenever *p* is uttered, and fails to settle the truth value of a relevant proposition, *q*, there will be two symmetrical ways of settling it, leading necessarily to an Ignorance Inference. Stated somewhat differently, *p* is equivalent to the following disjunction $(p \wedge q) \vee (p \wedge \neg q)$, and therefore should lead to Ignorance Inferences parallel to those stated in (2)c.

1.2. The Neo-Gricean Approach

The neo-Griceans respond to this problem by a revision of the Maxim of Quantity. Specifically, they suggest that the maxim doesn't require speakers to utter the most informative proposition that is relevant to the topic of conversation, but is more limited in scope. The Maxim merely requires speakers to choose the most informative relevant proposition *from a formally defined set of alternatives*. It does not require speakers to consider *all* relevant propositions.

⁵ The assumptions made here about "relevance" are the following:

1. If *p* and *q* are both relevant, so is "*p and q*".
2. if *p* is relevant, so is "*not p*". (To say that *p* is relevant is to say that the question *Is p true or false?* relevant.)

⁶ It is sometimes suggested that Grice's Maxim of Manner (M) could be used to explain *s*'s avoidance of *p*. Such a suggestion requires an ordering of linguistic expressions by which *p'* would be more optimal than *p''* from M's perspective. For arguments against obvious orderings (various measures of complexity), see Matsumoto (1995), as well as Fox and Hackl (2005).

The common way to work this out, pioneered by Larry Horn (1972), starts out with the postulation of certain sets of lexical items, Scalar Items, and sets of alternatives to which the scalar items belong, which we will call Horn-Sets:⁷

(4) Examples of Horn-Sets

- a. {*or, and*}
- b. {*some, all*}
- c. {*one, two, three, ...*}
- d. {*can, must*}

These sets of lexical alternatives determine the set of (Horn) alternatives for a sentence by a simple algorithm. The set of alternatives for S , $\text{Alt}(S)$, is defined as the set of sentences that one can derive from S by successive replacement of Scalar Items with members of their Horn-Set.⁸

(5) $\text{Alt}(S) = \{S' : S' \text{ is derivable from } S \text{ by successive replacement of scalar items with members of their Horn-Set}\}$

The Maxim of Quantity can now be stated as follows:

(6) **Maxim of Quantity (Neo-Gricean version):** If S_1 and S_2 are both relevant to the topic of conversation, S_1 is more informative than S_2 , and $S_1 \in \text{Alt}(S_2)$, then, if the speaker believes that both are true, the speaker should prefer S_1 to S_2 .

Consider the sentence in (1). The postulated scalar item in this sentence is disjunction, for which conjunction is lexically specified as the only alternative, (4)a. (1), thus, has just one alternative (other than (1) itself):

(7) $\text{Alt}(1) = \{(1), \text{Sue talked to John and Fred}\}$

When s utters (1), his addressee, h (for hearer), typically concludes (on the assumption that s obeys the revised Maxim of Quantity) that s does not know that the conjunctive sentence in $\text{Alt}(1)$ is true, since this alternative sentence is more informative than s 's utterance, and is typically relevant. If h assumes, further, that s has an opinion as to whether or not the conjunctive sentence is true, h would conclude that s believes that it is false. The neo-Griceans, thus, attribute a general tendency to addressees, namely the tendency to assume that speakers are opinionated. I state Sauerland's formulation of this assumption in (8).

(8) **Opinionated Speaker (OS):** When a speaker, s , utters a sentence, S , the addressee, h , assumes, for every sentence $S' \in \text{Alt}(S)$, that s 's beliefs determine the truth value of S' , unless this assumption about S' leads to the conclusion that s 's beliefs are contradictory.

⁷ Usually called Horn-Scales for bad reasons, as discussed in Sauerland 2004.

⁸ This definition, which comes from Sauerland (2004), is implicit in much earlier work, and is of course very similar to the definition of alternative sets in Rooth (1985).

Under the basic version of the Maxim of Quantity in (3), B-MQ, there was no way to maintain the assumption that the speaker is opinionated about any relevant sentence S' (not entailed by S). To repeat, B-MQ derived the symmetric results (a) that the speaker does not know that S and S' is true, and (b) that the speaker does not know that S and not S' is true. This, together with the assumption that the speaker knows that S is true (Quality), derived the conclusion that the speaker is not opinionated about S' .

By contrast, under the Neo-Gricean version of the Maxim of Quantity in (6), NG-MQ, the assumption that the speaker is opinionated about various sentences (not entailed by S) is innocuous. NG-MQ does not always derive the inference that the speaker does not know that S and S' is true. It derives such an inference only when S and S' (or some equivalent sentence) is a member of $\text{Alt}(S)$. Under such circumstances, the speaker could be opinionated about S' as long as $\text{Alt}(S)$ does not have S and not S' as a member (nor some equivalent sentence). If S and not S' is not a member of $\text{Alt}(S)$, NG-MQ does not derive the inference that the speaker does not know that S and not S' is true, and the assumption that the speaker believes that S and S' is false could be made consistently.

To summarize, assume that a speaker s utters the sentence in (1), *Sue talked to John or Bill*. The addressee, h , assumes that s obeys the Maxim of Quality as well as the revised Maxim of Quantity, NG-MQ. Based on this assumption, h reasons in the following way:

1. Given NG-MQ, there is no $X \in \text{Alt}(1)$, such that X is logically stronger than (1), and s thinks that X is true.
2. $\text{Alt}(1)$ contains the conjunctive sentence *Sue talked to John and Bill*, which is logically stronger than s 's utterance. Hence, given 1, it's not the case that s thinks that this conjunctive sentence is true.
3. Given OS, the default assumption is that s has an opinion as to whether *Sue talked to John and Bill* is true or false. Given 2 (the conclusion that it's not the case that s thinks that the sentences is true), we can conclude that s thinks that it is false.

So, by modifying the set of assumptions that derive Ignorance Inferences (replacing B-MQ with NG-MQ) one can account for the SI in (2)b. I would like at this point to discuss a possible alternative that keeps B-MQ in tact but instead enriches the set of syntactic representations available for (1). But it is worth pointing out first that, as things stand right now, our account of the Ignorance Inferences in (2)c is in jeopardy. Specifically, it is incompatible with NG-MQ and our assumption in (4)a about the Horn-Set for disjunction. The account was crucially dependent on the assumption that the Maxim of Quantity would prefer the utterance of a disjunct to the utterance of a disjunction, an assumption incompatible with the way $\text{Alt}(1)$ is defined on the basis of (4)a. One might respond to this problem with an independent (pragmatic) account for (2)c (Gazdar 1979) or by enriching the Horn-Set for disjunction (Sauerland 2004). The latter will be discussed in greater detail in section 4.

1.3. An Alternative Syntactic Approach

The alternative syntactic approach that I would like to defend is guided by the intuition that a principle of language use (such as the Maxim of Quantity) should not be sensitive to the formal (and somewhat arbitrary) definition of Alt(S).⁹ If this intuition is correct, B-MQ is to be preferred to NG-MQ.

However, as pointed out in 1.1., B-MQ derives Ignorance Inferences that contradict attested SIs. Therefore, if B-MQ is correct, something else is needed to derive SIs. More specifically, SIs must be derived from the basic meaning of the relevant sentences; otherwise the symmetry situation would yield unwanted Ignorance Inferences. Following proposals by Groenendijk and Stokhof (1984), Krifka (1995), Landman (1998), van Rooy (2002), and to some extent Chierchia (2004), I would like to suggest that the syntax of natural language has a covert operator which is optionally appended to sentences, and that this operator is responsible for SIs.¹⁰

The guiding observation is that there is a systematic way to state the SI of a sentence, using the focus sensitive operator *only*. Consider the sentence in (9), which has the SI that John didn't buy 4 houses.

(9) John bought three houses.

This SI could be stated explicitly using the focus sensitive particle *only* in association with the numeral expression *three*.

(10) John only bought THREE houses.

This observation extends to all SIs; SIs can always be stated explicitly with the focus sensitive particle *only*, as long as the relevant scalar items bear pitch accent:

(11) a. John did some of the homework.
 b. Implicature: John only did SOME of the homework.
For all of the alternatives to 'some', d,
if the proposition that John did d of the homework is true,
then it is entailed by the proposition that John did some of the homework.

(12) a. John talked to Mary or Sue.
 b. Implicature: John only talked to Mary OR Sue.
For all of the alternatives to 'or', con,
if the proposition that John talked to Mary con Sue is true
then it is entailed by the proposition that John talked to Mary or Sue.

⁹ This consideration would be weakened significantly if one could make sense of Alt(S) from the perspective of a general theory of language use. For efforts along these lines, See Spector (2005).

¹⁰ For a collection of some of the arguments in favor of this covert operator, see Fox (2004) and Chierchia (2005).

Sentence that generates SIs usually contain scalar items,¹¹ and in such cases it is always possible to state the SIs explicitly, by appending the operator *only* to the sentence and placing focal accent on the relevant scalar item:

- (13) The *only* implicature generalization (OIG): A sentence, *S*, as a default, licenses the inference/implicature that (the speaker believes) *onlyS'*, where *S'* is a modification of *S* with focus on scalar items.

From the neo-Gricean perspective discussed in 1.2., it is pretty clear why (9)-(12) should obey the OIG. As is commonly assumed, and as indicated by the italic paraphrases, the role of *only* is to eliminate alternatives. Furthermore, when focus is placed on scalar items, the relevant alternatives are precisely the Horn-alternatives that NG-MQ refers to.

However, the OIG suggests yet another possibility, namely that B-MQ is the right conversational maxim and SIs are derived within the grammar, namely by of a covert exhaustivity operator with a meaning somewhat a kin to that of *only*. Assume, for the moment, the semantics for *only* suggested by the paraphrases in (9)-(12). Specifically, assume that *only* combines with a sentence (the prejacent), *p*, and a set of alternatives, *A*, (determined by focus). The result of this combination is a sentence which presupposes that *p* is true and furthermore asserts that every true member of *A* is already entailed by *p* (i.e., that all non-weaker alternatives, all “real alternatives”, are false):

- (14) $[[\text{only}]] (A_{\langle \text{st}, \text{t} \rangle})(p_{\text{st}}) = \lambda w: p(w) = 1. \forall q \in \text{NW}(p, A): q(w) = 0$ ¹²
 $\text{NW}(p, A) = \{q \in A: p \text{ does not entail } q\}$

The exhaustivity operator, *exh*, should mean the same, with one small modification. While with *only* the requirement that the prejacent be true is a presupposition, with *exh* this requirement should be part of the assertive component:

- (15) $[[\text{Exh}]] (A_{\langle \text{st}, \text{t} \rangle})(p_{\text{st}})(w) \Leftrightarrow p(w) \ \& \ \forall q \in \text{NW}(p, A): \neg q(w)$

Assume that natural language has *exh* as a covert operator. Assume, further, that this operator can append to a sentence, *S*, thereby yielding a stronger sentence $S^+ = \text{Exh}(\text{Alt}(S))(S)$.¹³ It is easy to see that such a representation would derive both the “basic meanings” and the SIs of the sentences in (9)-(12).

This would allow us to keep to the non-stipulative quantity maxim (B-MQ). The cost lies, of course, in the stipulation of *exh*. There is a clear trade-off here, one that suggests that no decision will be justifiable on a-priori grounds. The goal of this paper is to

¹¹ This statement is not always true, nor is it predicted to be. Scalar Items generate alternatives, but alternatives could be specified in other ways as well: by pitch-accent or by an explicit question. See the discussion of (59) below, as well as footnotes 29, 32, and 46.

¹² $\lambda \chi: \psi(\chi). \phi$ is a function defined only for objects of which ψ is true (convention from Heim and Kratzer 1998).

¹³ This assumption is most natural if $\text{Alt}(S)$ is the focus value of *S*, which could follow if scalar items are assumed to be inherently focused, see Krifka (1995) for a possible implementation.

provide an empirical argument in favor of a theory in which B-MQ is at the heart of pragmatic reasoning and *exh* is responsible for SIs. If such a theory is correct, we might think of *exh* as a syntactic device designed (“by a super-engineer”) to facilitate communication in a pragmatic universe governed by B-MQ.

Every conversational situation, C , can be characterized by a set of sentences that are relevant at C , Q_C (for question). An utterance of S at C will be associated with a set of Ignorance Inferences determined by the set of sentences $Q_{S,C} \subseteq Q_C$, whose truth value is not determined by S . B-MQ will derive the set of Ignorance Inferences that correspond to $Q_{S,C}$, i.e., $I\text{-INF}(S,C) = \{\neg K_s \varphi : \varphi \in Q_{S,C}\}$. Furthermore, if φ is relevant to the topic of conversation, it seems that same would be true of $\neg\varphi$ (see note 5). Hence $I\text{-INF}(S,C) = \{\neg K_s \varphi : \varphi \in Q_{S,C}\} = \{\neg K_s \varphi \text{ and } \neg K_s \neg\varphi : \varphi \in Q_{S,C}\}$

Sometimes the set of Ignorance Inferences will be implausible and this would motivate a new parse of the linguistic stimuli, one that involves an exhaustive operator on top of S , i.e., S^+ . If no further stipulations are added, the procedure should be able to apply recursively leading to S^{++} , S^{+++} , etc. As we will see, this possibility will empirically distinguish our syntactic perspective from the Neo-Gricean alternative.

In the next section, I will present the core empirical phenomena that I will use to motivate *exh*, namely the conjunctive interpretation of disjunction under existential modal constructions (free choice, FC). Furthermore, I will present an argument due to Kratzer and Shimoyama (2002) (K&S) and Alonso-Ovalle (2005), that FC should be derived by the system that derives SIs. In section 3, I will present evidence that FC arises in additional circumstances: when disjunction is embedded under certain other existential quantifiers and when conjunction is embedded under negation and universal quantifiers (under the sequence $\neg\forall$). In section 4, I will discuss a relevant observation about disjunction due to Chierchia (204), and in section 5 I will present the Neo-Gricean response to Chierchia’s observation (Sauerland 2004) and its failure to predict FC phenomena. Finally, in section 6-10, I will propose a resolution based on recursive exhaustification which extends to the phenomena discussed in sections 2 and 3.

2. The Problem of Free Choice Permission

Consider the sentence in (16) when uttered by someone who’s understood to be an authority on the relevant rules and regulations, for example, a parent who is accustomed to specifying limits pertaining to the consumption of sweets.

(16) You’re allowed to eat the cake or the ice-cream.

In such a context, (17) would be an immediate inference (Kamp 1973). This inference, sometimes referred to as an inference of *free choice permission*, is not an expected entailment of obvious candidates for the logical form of (16).

(17) You’re allowed to eat the cake *and* you are allowed to eat the ice-cream.

For example, (17) is not entailed by (18).¹⁴

- (18) Plausible LF for (17):
 allowed [[you eat the cake] or [you eat the ice-cream]]

What (18) states is that the relevant rules do not prohibit the disjunctive sentence (the complement of *allowed*), or, in the terms of possible world semantics, that there is a world consistent with the rules in which one of the disjuncts is true. (18) is thus equivalent to (19), which is clearly weaker than (17).

- (19) You are allowed to eat the cake *or* you are allowed to eat the ice-cream.

The problem is to understand how a disjunctive LF such as (18) can be strengthened to yield the conjunctive inference in (17). In other words, we need to understand how a sentence that should receive the modal logic formalization in (20)a – which is equivalent to (20)b – justifies the FC inference in (20)c.

- (20) a. $\diamond(p \vee q)$
 b. $\diamond p \vee \diamond q$ $(a \equiv b)$
 c. Free Choice: $\diamond p \wedge \diamond q$

2.1. Downward entailing operators, evidence that Free Choice is an Implicature

K&S studied FC effects that arise when certain indefinite expressions are embedded under existential modals, and presented a fairly strong argument that the effect should be derived by the system that yields SIs.¹⁵ The argument, which has been elaborated and extended to disjunctive constructions by Alonso-Ovalle (2005), is based on the observation that there are no traces of FC in certain downward entailing contexts.

Consider the sentence in (21). If FC were to follow from the basic meaning of (16), we would expect (21) to have a fairly weak meaning; it should be able to assert that no one is both allowed to eat the cake and allowed to eat the ice-cream, (21)'a. We would thus predict (21) to be true in a situation in which everyone is allowed to eat one of the two desserts but no one has free-choice, i.e., in situations in which no one is allowed to decide which of the two deserts to eat. Such an interpretation, if available, is extremely dispreferred.¹⁶ To derive the natural interpretation, (21)'b, we must factor out whatever is responsible for FC.

¹⁴ In (18) the subject is reconstructed into both disjuncts, and *to* is omitted. This is done for expository purposes and, of course, does not affect meaning.

¹⁵ Although this paper focuses on disjunction, the basic proposal can derive FC for the relevant indefinites as long as we assume (along the lines of Chierchia (2005)) that the relevant indefinites have the following alternatives: $ALT(irgendein NP) = \{ irgendein NP': NP' \subseteq NP \} \cup \{ all NP': NP' \subseteq NP \}$.

¹⁶ I think that the interpretation is available in contexts that, more generally, allow for “intrusive” implicatures of the relevant sort (See Recanati 2003, Levinson 2000, Horn 1989, Cohen 1971, among others):

- i. None of my students did SOME of the homework, They all did ALL of it.
- ii. No one is allowed to eat the cake OR the ice-cream. Everyone will be told what to eat.

(21) No one is allowed to eat the cake or the ice-cream.

- (21)' a. *negation of FC: $\neg\exists x[\diamond P(x) \wedge \diamond Q(x)]$
 b. negation of standard meaning: $\neg\exists x\diamond(P(x) \vee Q(x))$

As pointed out by Alonso-Ovalle (following K&S), the natural interpretation, (21)'b, is expected if FC were to be derived as an SI. Although it is not yet clear how to derive FC as an SI, it is clear that if a derivation were available for the basic case, it would, nevertheless, not be available (at least not necessarily) for (21). This is seen most clearly under the neo-Gricean approach to SIs. Under this approach, an SI is derived as a pragmatic strengthening of the basic meaning of a sentence. The meaning in (21)'a is *weaker* than the basic meaning in (21)'b, and, therefore, cannot be derived along neo-Gricean lines.

Under the syntactic alternative, the preference for (21)'a would be stated as a preference for stronger interpretation (See Chierchia 2004, 2005). More specifically, assume, contrary to what you might think at this point, that an exhaustive operator can somehow derive the basic FC effect. We might then suggest that *exh* can only be introduced if the overall result is a stronger proposition. This could be motivated by the observation that as propositions get stronger fewer Ignorance Inferences are derived by B-MQ. We might, thus, suggest that the introduction of *exh* is subject to an economy condition related to its functional motivation, namely to the elimination of Ignorance Inferences. (I.e., a sentence with *exh* must lead to fewer Ignorance Inferences than its counterpart without *exh*, see footnote 37.) Alternatively, we might suggest, in line with the neo-Griceans, that *exh* must be introduced in matrix position.¹⁷

Be that as it may, it is reasonable to assume that the preference for (21)'b would be predicted if FC could be derived as an SI, but not otherwise. Quite independently of particular proposals, it is well known that SIs tend to disappear in downward entailing environments (Gazdar 1979, Chierchia 2004). The fact that FC appears to share this property with SIs seems to be a good incentive to search for a theory that would derive the effect as an SI.

2.2. Deriving an FC implicature: the nature of the problem

The projection properties of FC seem to suggest that the effect should be derived as an SI. But how could we derive such a result? K&S made a very interesting proposal which is the basis for the proposal that I will make in section 6-10. But this will require quite a

However, Kamp (1973) points out that it is easier for FC to “intrude” into the antecedent of a conditional: *If you are allowed to eat the cake or the ice-cream, you are pretty lucky*. If K&S are correct, the FC interpretation of the antecedent would require an analysis involving an embedded implicature, as pointed out by Alonso-Ovalle, but without the “meta-linguistic” feel that is sometimes associated with such intrusion. Unfortunately, I have nothing interesting to add.

¹⁷ The latter possibility seems less plausible given the accumulation of evidence for “intrusive” or “embedded” implicatures (at least in non-downward entailing environments), see Chierchia 2004. The discussion in footnote 16 is of course, problematic, and more so from the Neo-Gricean perspective.

bit of ground work. At this point, it is worth understanding what one might say in order to derive SIs based on the neo-Gricean maxim of quantity (NG-MQ).

Quite generally, suppose that φ is the basic meaning of a sentence, S , and that our goal is to derive a stronger meaning, φ' , based on NG-MQ. The result could be achieved if we proposed a set of alternatives of the following sort: $ALT(S) := \{S, S \ \& \ not \ S_{\varphi'}\}$, where the meaning of $S_{\varphi'}$ is φ' . If s was to utter S , the addressee, h , would conclude, based on NG-MQ, that s does not believe that *not* $S_{\varphi'}$ is true. Furthermore, based on the assumption that s is an opinionated speaker, h , would conclude that s believes that $S_{\varphi'}$ is true.

More specifically, suppose that the alternatives of the sentence in (16), repeated below, are the sentences in (22). This is slightly different from the general scheme for deriving implicatures characterized above, but the basic idea is the same.¹⁸

- (16) You're allowed to eat the cake or the ice-cream.
- (22) Alternatives needed to derive FC for (16) based on NG-MQ:
- a. You are allowed to eat the cake or the ice-cream.
 - b. You are allowed to eat the cake but you are not allowed to the ice-cream
 - c. You are allowed to eat the ice cream but you are not allowed to eat the cake.

Based on NG-MQ, we would now derive the SI that (22)b and (22)c are both false, which, together with (22)a, yields the FC inference. To see this, assume (22)a is true. Now assume that one of the conjuncts in (17) is false, say that you cannot eat the ice-cream. From this it follows that (22)b is true, contrary to the SI.

But of course this is not intended as a serious proposal. It follows from a general algorithm that allows us to derive, on a case by case basis, any SI that we would like to, and, hence, does not explain the particular SIs that are actualized (See Szaebo 2003). The obvious way to turn this into a serious proposal is to show that the alternatives in (22) are needed on independent grounds.

K&S, and in particular Alonso-Ovalle, propose a more natural set of alternatives, namely the one in (23).

- (23) Alternatives proposed by K&S/Alonso-Ovalle:
- a. You are allowed to eat the cake or the ice-cream.
 - b. You are allowed to eat the cake.
 - c. You are allowed to eat the ice cream.

The set of alternatives in (23), in contrast to the one in (22), is consistent with a general constraint on alternatives proposed in Matsumoto (1995).¹⁹ Furthermore, as we will see in section 4, there is independent evidence for the type of Horn-Sets that would derive (a

¹⁸ It would instantiate the scheme if b and c were replaced by a single alternative, namely the disjunction of the two.

¹⁹ Matsumoto argues that lexical items can be members of the same Horn-Set only if they denote functions of the same monotonicity. \vee is upward monotone with respect to both arguments, but $\wedge \neg$ is downward monotone with respect to its right-hand argument. Skipping ahead to Sauerland's Horn-set, L and R are upward (as well as downward) monotone with respect to their immaterial arguments.

super-set of) the alternatives in (23). The problem, however, is that NG-MQ can not derive the FC effect on the basis of (23). In fact, as we will see in greater detail in section 4.2., it derives Ignorance Inferences that directly conflict with FC.

K&S suggest, however, that FC should be derived from (23) based on a novel principle, which they call anti-exhaustivity. When h interprets s's utterance of (23)a, s needs to understand why it is that s preferred this sentence to any of the alternatives. The standard Neo-Gricean reasoning, which relates to the basic meaning of the alternatives, would lead to the conclusion that s does not know/believe that any of the alternatives is true. K&S, however, suggest that h might reason based on the strong meaning (basic meaning + implicatures) of the alternatives. Specifically, K&S suggest that h would attribute the choice of s to the belief that the strong meanings of (23)b and (23)c are both false. Furthermore they assume that the strong meaning of (23)b and c is the basic meaning of (22)b and c respectively.

As pointed out by Aloni and van Rooy (2005), this line of reasoning raises a question pertaining to simple disjunctive sentences, such as (1). We would like to understand why such sentences don't receive a conjunctive interpretation via an anti-exhaustivity inference of the sort outlined above. If each disjunct is an alternative to a disjunctive sentence, why doesn't the speaker infer that the exhaustive implicature of each disjunct is false?

K&S provide an answer this question by postulating a covert modal operator for any disjunctive sentence. I will not go over this proposal and the way it might address Aloni and van Rooy's objection. I would like, instead, to raise another challenge to K&S's basic idea. I think it is important to try to understand how the anti-exhaustive inference fits within a general pragmatic system that derives Ignorance Inferences (as well as SIs). Specifically, I think it is important to understand why NG-MQ does not lead to the Ignorance Inferences in (24) (see section 4.2. for details).

- (24) Predicted inferences of (16), based on (23) and NG-MQ:
- a. s doesn't know whether or not you can eat the cake.
 - b. s doesn't know whether or not you can eat the ice cream.

This is a challenge that this paper attempts to meet. The idea, in a nut-shell, is to eliminate NG-MQ in favor of the non-stipulative alternative B-MQ. However, understanding how this is to work requires the introduction of a proposal made in Sauerland (2004), which would be extracted from its neo-Gricean setting in order to meet our goals. But before I get there, I would like to introduce an additional challenge. Specifically, I would like to present a few other surprising inferences that are intuitively similar to FC, and should, most likely, be derived by the same system.

3. Other Free Choice Inferences

In this section we will see effects that are very similar in nature to FC, but arise in somewhat different syntactic contexts. These effects will argue for a fairly general explanation of the basic phenomenon, one that is not limited to modal environments or to disjunction.

3.1. FC Under negation and universal modals

Consider the sentence in (25) when uttered by someone who is understood to be an authority on the relevant rules and regulations, for example, a parent who is accustomed to assigning after-dinner chores.

(25) You are not required to both clear the table and do the dishes.

In such a context, (26) would normally be inferred by the addressee.

(26) You are not required to clear the table and you are not required to do the dishes.

This inference seems very similar to the FC inference drawn in (17) based on (16). To see the similarity, notice that the basic meaning of (25) is predicted to be equivalent to the disjunctive statement that you are allowed to either avoid clearing the table or evade doing the dishes, (27)a, and that (26) is equivalent to the conjunction of two possibility statements. (*You are allowed to avoid clearing the table and you are allowed to avoid doing the dishes*, (27)b)

(27)a. Standard Meaning of (25)

$$\neg\Box(p \wedge q) \equiv \Diamond\neg(p \wedge q) \equiv \Diamond(\neg p \vee \neg q) \equiv \Diamond(\neg p) \vee \Diamond(\neg q)$$

b. Free Choice Inference

$$\Diamond(\neg p) \wedge \Diamond(\neg q)$$

Just as in (16), the basic meaning does not explain the inference, and the gap is formally identical. Once again, we have to understand how a sentence that is equivalent to a disjunctive construction can be strengthened to something equivalent to the corresponding conjunction.

3.2. More generally under existential quantifiers

In the basic FC permission sentence in (16), disjunction appears in the scope of the existential modal *allowed*. Furthermore, as is well-known, FC extends to all constructions in which *or* is in the scope of an existential modal:

- (28) a. The book might be on the desk or in the drawer.
 (= The book might be on the desk and it might be in the drawer)
 b. He is a very talented man. He can climb Mt. Everest or ski the Matterhorn.
 (=He can climb Mt. Everest and he can ski the Matterhorn.)

What has not been discussed in any systematic way is that this type of conjunctive interpretation extends also to some non-modal constructions:²⁰

- (29) a. There is beer in the fridge or the ice-bucket.

²⁰While working on this paper, I have learned about two new papers about FC that make this same observation: Klindinst (2005) and Eckardt (this volume).

- (= There is beer in the fridge and there is beer in the ice-bucket.)
- b. Most people walk to the park, but some people take the highway or the scenic route. (Irene Heim, pc attributed to Regine Eckardt, pc)
(= Some people take the highway and some people take the scenic route.)
- c. This course is very difficult. In the past, some students waited 3 semesters to complete it or never finished it at all. (Irene Heim, pc)
(= Some students waited 3 semesters to complete the course and some students never finished it at all.)

It is thus tempting to suggest that conjunctive interpretations for disjunction are available whenever disjunction is in the scope of an existential quantifier (with the domain of quantification, worlds or individuals, immaterial). However, there are limitations:

- (30) a. There is a bottle of beer in the fridge or the ice-bucket.
(\neq There is a bottle of beer in the fridge and there is a bottle of beer in the ice-bucket.)
- c. Someone took the highway or the scenic route.
(\neq Someone took the highway and someone took the scenic route.)
- d. This course is very difficult. In the past, some student waited 3 semesters to complete it or never finished it at all.
(\neq some student waited 3 semesters to complete the course and some student never finished it at all.)

As pointed out in Klindinst (2005), the relevant factor seems to be number marking on the indefinite. We might, therefore, suggest the following generalization:

- (31) Existential FC: A sentence of the form $\exists x [P(x) \vee Q(x)]$ can lead to the FC inference, $\exists x P(x) \wedge \exists x Q(x)$, as long as the existential quantifier, $\exists x$, is not marked by singular morphology.

3.3. More generally, under negation and universal quantifiers

In (25) we saw an FC effect arising when conjunction is under the scope of negation and a universal modal (under the sequence, $\neg\Box$). As illustrated in (32), and stated in (33), the effect arises also when \Box is replaced by an ordinary universal quantifier:

- (32) We didn't give every student of ours both a stipend and a tuition waiver.
1. basic meaning: $\neg\forall x [P(x) \wedge Q(x)] \equiv$
 $\exists x \neg [P(x) \wedge Q(x)] \equiv$
 $\exists x [\neg P(x) \vee \neg Q(x)] \equiv$
 $\exists x \neg P(x) \vee \exists x \neg Q(x)$
 2. Free Choice: $\exists x \neg P(x) \wedge \exists x \neg Q(x)$

- (33) Conjunctive FC: A sentence of the form $\neg\forall x [P(x) \wedge Q(x)]$ can lead to the FC inference $\exists x \neg P(x) \wedge \exists x \neg Q(x)$.

In section 7-10 we will provide an account of our two generalizations ((31) and (33)) within a general theory of SIs that we will introduce in section 6, based on a discussion, in section 4-5, of Sauerland's approach to SIs. But before we move on, it is important to rule out an alternative explanation of (25) and (32), in terms of wide scope conjunction. To understand the concern, focus on (25). One might think that this sentence has a logical form in which conjunction takes wide scope over the sequence $\neg\Box$. If such a logical form were available, the inference in (26) would follow straightforwardly from the basic meaning, and would thus be unrelated to the FC effects that are distributed according to (31).

But wide scope conjunction is not a probable explanation. One argument against such an explanation is based on the sentences in (34). If conjunction could take scope over the sequence $\neg\Box$ in (25) (and over $\neg\forall$ in (32)), we would expect it to be able to outscope negation in (34), an expectation that is not born out.²¹

- (34) a. I didn't talk to both John and Bill.
b. We didn't give both a stipend and a tuition waiver to every student.

What I think we learn from (34) is that conjunction can appear to outscope negation only when a universal quantifier intervenes.²² This is expected if conjunction never outscores negation, and the generalization in (33) is real.

Another argument against wide scope conjunction comes from an additional inference we draw from sentences such as (25) and (32). In both cases we draw the inference that the alternative sentence with disjunction instead of conjunction is false. That is, we would tend to draw (35)a as an inference from (25), and (35)b from (32). These inferences are not expected if conjunction receives wide scope, but, as we will see later on, are expected if the phenomenon is derived along with other FC effects.

- (35) a. You are required to clear the table or do the dishes.
b. We gave every student of ours a stipend or a tuition waiver.

4. Chierchia's Puzzle

The account of FC that I will develop will be based on a modification of a proposal made in Sauerland (2004) to deal with a puzzle discovered in Chierchia (2004).²³ To understand the puzzle in greater detail, consider first (36) and its implicature that (36)' is false.

- (36) John did some of homework.

²¹ If the distributor *both* is omitted the resulting interpretation is equivalent to wide scope conjunction. The correct account relies most likely on a "homogeneity" presupposition (Fodor 1976, Gajewski 2005).

²² In order to account for the difference between (32) and (34)b, we would also have to say that inversion of the surface scope of conjunction and universal quantification is impossible in (34)b, a consequence of Scope Economy, a principle I've argued for in Fox (2000).

²³ See also Lee (1995), and Simons (2002).

(36)' John did all of the homework.

As outlined in 1.2., this implicature can be derived by NG-MQ under the assumption that *some* and *all* are members of the same Horn-Set, (4)b,²⁴ from which it follows that (36)' is an alternative to (36). NG-MQ, together with the assumption of an opinionated speaker, lead to the conclusion that the speaker believes that (36)' is false.

Consider next what happens when (36) is embedded as one of two disjuncts:

(37) John did the reading or some of homework.

This type of embedding was presented by Chierchia (2004) as a challenge to the neo-Gricean derivation of implicatures. As Chierchia points out, (37)' should be an alternative to (37), and it would therefore seem that (with the assumption that the speaker is opinionated) we should derive the implicature that (the speaker believes that) (37)' is false.

(37)' John did the reading or all of homework.

This implicature, however, is clearly too strong. If a disjunctive sentence is false, then each of the disjuncts is false. When (37) is uttered, we do derive the inference that the second disjunct of (37)' is false. However, we clearly do not derive a similar inference for the first disjunct (which is also the first disjunct of (36)).

Chierchia's challenge for the Neo-Griceans is to avoid the implicature that the first disjunct of (36) is false while at the same time to derive the implicature that the stronger alternative to the second disjunct is false:

- (38) Let U be an utterance of *p or q* where *q* has a stronger alternative, *q'*.
- a. Problem 1: to avoid the implicature of $\neg p$
 - b. Problem 2: to derive the implicature of $\neg q'$

Chierchia provides an account for the relevant generalization based on a recursive definition of strengthened meanings. I will not discuss his account, since I can't figure out how to extend it to FC. Instead, I will discuss the neo-Gricean alternative, which also fails to account for FC, but, which can, nevertheless, be modified in order to provide a syntactic (non-Gricean) alternative that successfully extends to FC.²⁵

5. Sauerland's Proposal²⁶

As pointed out at the end of section 1.2., the Horn-Set for disjunction in (4)a (*{or, and}*) cannot account for the Ignorance Inferences that are attested when a simple disjunctive

²⁴ and that the set does not include the "symmetric alternative" to *all*, *some but not all*.

²⁵ Chierchia himself developed an account of FC which is quite similar to the account proposed here and is to some extent independent of his recursive procedure for implicature computation. Specifically, his account, like mine, is based on operators that apply to a prejacent and a set of alternatives. However, the crucial operator for him is an "anti-exhaustivity" operator, distinct from what might be responsible for implicature computation.

²⁶ Benjamin Spector made the same proposal, in a somewhat different (more generalized) format. A related proposal can be found in Lee (1995).

sentence such as (1) is uttered. Sauerland suggests a remedy for this problem which also resolves Chierchia's puzzle.

To derive the appropriate Ignorance Inferences for (1), Sauerland suggests that the alternatives for a disjunctive statement include each of the disjuncts in addition to the corresponding conjunction:

$$(39) \quad \text{Alt}(p \vee q) = p \vee q \begin{array}{c} \text{p} \\ \diagdown \quad \diagup \\ \text{p} \wedge q \\ \diagup \quad \diagdown \\ \text{q} \end{array}$$

These alternatives, which are plotted to represent logical strength,²⁷ derive (based on NG-MQ) the following inferences with respect to a speaker, *s*, who utters *p or q*, inferences which Sauerland calls Primary (or weak) Implicatures, PIs:

- (40) PIs for *p or q* (based on NG-MQ)
- a. *s* does not believe that *p* is true.
 - b. *s* does not believe that *q* is true.
 - c. *s* does not believe that *p and q* is true. Already follows from both a and b.

Given that *s* is assumed to believe that her utterance of *p or q* is true (Quality), we derive the Ignorance Inferences discussed in section 1, that is, for each disjunct, we derive the inference that the speaker does not know whether or not it is true. To derive SIs, the principle of an Opinionated Speaker is employed, (8):

- (8) **Opinionated Speaker (OS)**: When a speaker, *s*, utters a sentence, *S*, the addressee, *h*, assumes, for every sentence $S' \in \text{Alt}(S)$, that *s*'s beliefs determine the truth value of *S'*, unless this assumption would lead to the conclusion that *s*'s beliefs are contradictory.

This principle asks us to scan the set of alternatives that are stronger than *S*, and to identify those for which the assumption that the speaker is opinionated is consistent with our prior inferences based on Quality and NG-MQ. For each such alternative, the speaker is assumed to be opinionated, and given the relevant PI, a stronger inference is derived, namely that the speaker believes that the relevant alternative is false, an inference which Sauerland calls a Secondary Implicature (an SI, conveniently).

As mentioned above, (40)a,b together with Quality, lead to ignorance with respect to *p* and to *q*. Hence, *p and q* is the only alternative for which the assumption that the speaker is opinionated is consistent with prior inferences. Therefore, only one SI is derived based on OS, namely the inference that the speaker believes that *p and q* is false:

- (41) SI for *p or q* (based on OS)
s believes that *p and q* is false.

Sauerland, thus, derives the following definition for the two relevant sets of implicatures:

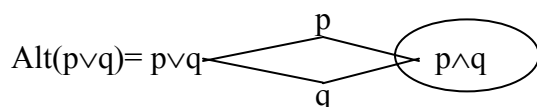
- (42) When a speaker *s* utters a sentence *A*, the following implicatures are derived:

²⁷ If *x* is to the left of *y* with a connecting line, then *x* is weaker than *y*.

- a. PIs = $\{\neg B_s(A'): A' \in \text{ALT}(A) \text{ and } A' \text{ is stronger than } A\}$
 b. SIs = $\{B_s(\neg A'): A' \in \text{ALT}(A), A' \text{ is stronger than } A, \text{ and } B_s(A) \wedge \text{PI} \wedge B_s(\neg A') \text{ is not contradictory}\}$

Based on these definitions, a PI is derived for every alternative stronger than the assertion and an SI for a subset of the stronger alternatives for which an Ignorance Inference hasn't already been derived (based on NG-MQ and Quality).²⁸

(43) Implicatures for $p \vee q$:



- a. PIs: $\neg B_s(p), \neg B_s(q)$ The rest, $\neg B_s(p \wedge q)$, follows
 b. SI: $B_s \neg(p \wedge q)$

Sauerland shows that this rather principled approach solves Chierchia's puzzle once the lexical alternatives that derive the sentential alternatives in (39) are specified. The basic intuition is fairly straightforward. An utterance of p or q derives Ignorance Inferences that are inconsistent with the assumption that the speaker is opinionated about p , thereby solving problem (38)a. Problem (38)b is solved as well, but seeing this requires precision about the relevant lexical alternatives and the way they determine sentential alternatives for complex disjunctions, such as (37).

The starting point is the observation that in order to derive (39) the alternatives for disjunction must contain two lexical entries that are never attested:²⁹

(44) Horn-Set(*or*) = $\{or, L, R, and\}$, where $pLq = p$ and $pRq = q$.

These four alternatives, when combined with the alternatives for *some* ($\{some, all\}$), yield 8 alternatives to (37), based on (5) above:³⁰

(45) $\text{Alt}(r \text{ or } sh) =$ a. $r \vee sh$

²⁸ From now on, I will circle those alternatives for which an SI can be derived consistently with Quality and NG-MQ.

²⁹ Spector (2003, 2005) suggests a different perspective. Specifically, he suggests that alternative sets are defined as the closure under \wedge and \vee of the set of positive answers to a given question. The Sauerland alternatives for *John talked to Mary or Bill* would, thus, be derived (along with other useful alternatives) if the relevant question was *who did John talk to?*

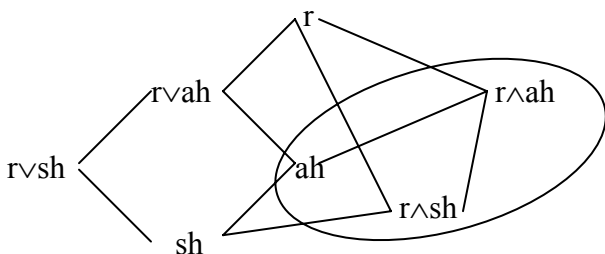
What FC teaches us, if my proposal is correct, is that there is no closure under \wedge . Some of what I say could work if Sauerland's alternatives were replaced by basic answers to a Hamblin-question closed under \vee . (Conjunction in unembedded cases will be part of the basic Hamblin denotation, derived from quantification over pluralities). One would still have to make sense of second layers of exhaustivity (see section 11.2 note 46).

³⁰ $r := \text{John did the reading}; sh := \text{John did some of the homework}; ah := \text{John did all of the homework}$.

- b. $r \text{ L } sh \equiv r$
- c. $r \text{ R } sh \equiv sh$
- d. $r \wedge sh$
- e. $r \vee ah$
- f. $r \text{ L } ah \equiv r$
- g. $r \text{ R } ah \equiv ah$
- h. $r \wedge ah$

To see what PIs and SIs are derived, it is useful to plot the alternatives in a way that indicates relative strength. But it is already easy to see how the two problems in (38) are solved. To repeat, problem (38)a is solved based on the observation that the speaker cannot believe that r is false if a PI ensures that she does not believe that sh is true and Quality ensures that she believes that r or sh is true. Problem (38)b is solved based on the observation that ah is a member of the alternative set (alternative g), and that an SI can be derived for this alternative (consistent with prior inferences):

- (46) Implicatures for $r \vee sh$:
 $ALT(r \vee sh) =$



$$PI = \neg B_s(r \vee ah), \neg B_s(sh),$$

$$SI = B_s(\neg ah), B_s(\neg(r \wedge sh))$$

(the rest follow)
 (the rest, $B_s \neg(r \wedge ah)$, follows)

5.1. Advantage of Sauerland's Proposal: embedding under universal quantifiers

The Horn-Set in (44) plays two independent roles for Sauerland. It provides NG-MQ with the alternatives needed to derive the Ignorance Inferences for $p \vee q$. These inferences explain (within the Neo-Gricean paradigm) the lack of certain SIs when scalar items are embedded within one of the disjuncts (problem (38)a). Furthermore, given (5), we can generate alternatives for complex disjunctive sentences (e.g., q' for the sentence in (38)) that derive otherwise surprising SIs (problem (38)b).

However, the system makes a further prediction. Specifically, it predicts that in certain contexts the two basic alternatives p and q will generate SIs rather than Ignorance Inferences. The relevant case involves embedding of disjunction under an upward

monotone operator O such that $O(p \vee q)$ does not entail the disjunctive sentence $O(p) \vee O(q)$. For such an operator, the following is not contradictory.

$$(47) \quad O(p \vee q) \wedge \neg O(p) \wedge \neg O(q) \wedge \neg O(p \wedge q)$$

Hence, if s utters $O(p \vee q)$, an SI would be generated for each of the stronger alternatives ($O(p)$, $O(q)$, and $O(p \wedge q)$).

Evidence that this prediction is correct comes from (48) and (49), which naturally yield the implicatures in (a) and (b).³¹

(48) You're required to talk to Mary or Sue.

Implicatures:

- a. You're not required to talk to Mary.
- b. You're not required to talk to Sue.

(49) Every friend of mine has a boy friend or a girl friend.

Implicatures:

- a. It's not true that every friend of mine has a boy friend.
- b. It's not true that every friend of mine has a girl friend.

These facts follow straightforwardly from the Sauerland scale:

$$(50) \quad \text{Alt}(\forall x(P(x) \vee Q(x)) = \forall x(P(x) \vee Q(x)) \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} \quad \forall x(P(x) \wedge Q(x))$$

PIs = $\neg B_s(\forall x P(x))$, $\neg B_s(\forall x Q(x))$ (the rest, $\neg B_s \forall x(P(x) \wedge Q(x))$, follows)

SIs = $B_s(\neg \forall x P(x))$, $B_s(\neg \forall x Q(x))$ (the rest, $B_s \neg \forall x(P(x) \wedge Q(x))$, follows)

5.2. But...what about FC?

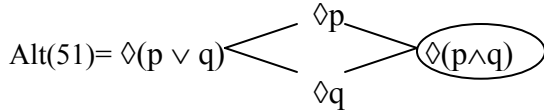
Sauerland's system makes yet another prediction about disjunction embedding, a prediction which is in direct conflict with FC. If disjunction is embedded under an upward monotone operator O such that $O(p \vee q)$ entails the disjunctive sentence $O(p) \vee O(q)$, the system predicts Ignorance Inferences with respect to $O(p)$ and $O(q)$. The reasoning is exactly identical to the basic case of unembedded disjunction: there is no way to assume that the speaker is opinionated about one of the alternatives $O(p)$ and $O(q)$ without contradicting the Primary Implicature that the speaker does not know that the other disjunct is true (given Quality).

This does not seem to be the correct prediction for existential modals and plural existential DPs (generalization (31)). These operators, under their basic meaning, are both

³¹ In Fox (2003) I pointed out this prediction, but was not sure about the empirical facts. I was convinced by conversations with Benjamin Spector and the discussion in Sauerland (2005).

commutative with respect to disjunction ($\diamond(p \vee q) \equiv (\diamond p \vee \diamond q)$; $\exists x(P(x) \vee Q(x)) \equiv \exists xP(x) \vee \exists xQ(x)$). Hence Ignorance Inferences are predicted.

(51) You may eat the cake or the ice-cream.



$$\text{PIs} = \neg \mathbf{B}_s(\diamond p), \neg \mathbf{B}_s(\diamond q), \neg \mathbf{B}_s \diamond(p \wedge q)$$

$$\text{SIs} = \mathbf{B}_s \neg \diamond(p \wedge q)$$

We've seen good arguments that FC should be derived as an implicature. However, under Sauerland's system we derive Primary Implicatures (in bold) that contradict FC.

The situation is quite interesting. The alternatives that K&S and Alonso-Ovalle appeal to in order to derive FC derive contradictory Ignorance Inferences in Sauerland's system. If K&S are right, Sauerland's system needs to change. However, K&S's insight, if correct, needs to be embedded in a general system for implicature computation, one that can account for Chierchia's puzzle, as well as for the emergence of Ignorance Inferences.

6. An alternative Perspective

There are two problems with Sauerland's system. On the one hand, it derives Ignorance Inferences that directly contradict the attested FC effect. On the other hand, it does not provide the basis for anti-exhaustivity, which, if K&S are correct, is at the heart of FC. I will argue that the first problem teaches us that, contrary to the neo-Gricean assumption, Primary Implicatures do not serve the foundation for the computation of SIs. Instead, SIs are derived in the syntactic/semantic component via an exhaustive operator, as suggested in section 1.3. Once a semantic representation is chosen, Ignorance Inferences are computed by the pragmatic system, based on the non-stipulative maxim of quantity (B-MQ). Without an exhaustive operator, incorrect Ignorance Inferences are computed in FC environments. However, once we modify the meaning of *exh*, based on Sauerland's insights, the inferences can be avoided by a sequence of two exhaustive operators, which yield, in effect, anti-exhaustivity, thereby solving the second problem. Furthermore, it turns out that FC is predicted in all the environments discussed in section 3.

6.1. A new lexical entry for the exhaustive operator

Let's start by reviewing our lexical entry for *only* and *exh* from section 1.3. These entries (14) and (15), repeated below) derive strong meanings that are in most cases equivalent to the basic meaning conjoined with the SIs derived by the neo-Gricean system.

$$(52)\text{a. } [[\text{only}]] (A_{\langle \text{st}, \text{t} \rangle})(p_{\text{st}}) = \lambda w: p(w) = 1. \forall q \in \text{NW}(p, A): q(w) = 0$$

$$\text{NW}(p, A) = \{q \in A: p \text{ does not entail } q\}$$

$$\text{b. } [[\text{Exh}]] (A_{\langle \text{st}, \text{t} \rangle})(p_{\text{st}})(w) \Leftrightarrow p(w) \ \& \ \forall q \in \text{NW}(p, A): \neg q(w)$$

However, predictions are sometimes different when the alternatives are not totally ordered by entailment. In particular, for the sets of alternatives that Sauerland has postulated, the lexical entries in (52) can derive contradictory results. To see this, consider the following dialogue:

- (53) A: John talked to Mary or Sue.
 B: Do you think he might have spoken to both of them?
 A: No, he only spoke to Mary OR Sue.

Under (52)a, A's final sentence should presuppose that the prejacent, *John spoke to Mary or Sue*, is true and that this is not the case for any of the (non-weaker) alternatives. Thus, if Sauerland is right about the lexical alternatives for disjunction, the two alternatives in (54) would both have to be false for the utterance to be true, which would, of course, contradict the presupposition.

- (54) a. John talked to Mary.
 b. John talked to Sue.

This is a wrong result, which means that if Sauerland is right about the alternatives for disjunction, the lexical entries in (52) probably need to be revised.³² A revision of this sort is also needed based on much older observations due to Groenendijk and Stokhof (1984):

- (55) a. Who did John talk to?
 Only Mary or SUE
 b. Who did John talk to?
 Only Some GIRL
 Groenendijk and Stokhof (1984)

Let's focus on (55)a. If (52)a is correct, the answer to the question should assert that every alternative not entailed by the prejacent, *John talked to Mary or Sue*, is false. This time the set of alternatives consists (most likely) of every proposition of the form *John talked to x* based on the denotation of the question, and the fact that the whole DP *Mary or Sue* is focused. In other words, the answer to the question in (55)a should entail the propositions that John didn't talk to Mary and that he didn't talk to Sue, which should contradict the presupposed prejacent.

Groenendijk and Stokhof (1984), who noticed the problem, suggested a modification to the standard lexical entry for *only*, which was accommodated in Spector (2005) to the syntax we are assuming (based on van Rooy and Schultz (2003)):

- (56)a. $[[\text{only}]](A_{\langle \text{st}, \text{t} \rangle})(p_{\text{st}}) = \lambda w: p(w) = 1. \text{Minimal}(w)(A)(p)$
 b. $[[\text{Exh}]](A_{\langle \text{st}, \text{t} \rangle})(p_{\text{st}})(w) \Leftrightarrow p(w) \ \& \ \text{Minimal}(w)(A)(p)$

³² Gennaro Chierchia (p.c.) points out that in the dialogue in (53) *or* might be receiving contrastive focus with conjunction, with the other alternatives (*L* and *R*) inactive. This possibility will not be helpful in explaining the avoidance of a contradiction in Groenendijk and Stokhof's examples in (55).

$$\text{Minimal}(w)(A)(p) \Leftrightarrow \neg \exists w' p(w')=1 (A_{w'} \subset A_w)$$

$$A_w = \{p \in A: p(w)=1\}$$

As pointed out by Spector (again, based on van Rooy and Schultz), this lexical entry can solve Chierchia's problem. However, it yields results that contradict FC (see note 41). For this reason, I would like to suggest an alternative, one that is linked in a very direct way to Sauerland's proposal.

What we learn from Groenendijk and Stokhof is that there is something in the meaning of *only* "designed" to avoid contradictions: *only* takes a set of alternatives A and a prejacent p , and attempts to exclude as many propositions from A in a way that would be consistent with the requirement that the prejacent be true. I would like to suggest that the basic algorithm is Sauerland's, i.e., that propositions from A are excluded as long as their exclusion does not lead (given p) to the inclusion of some other proposition in A :

$$(57)a. \text{[[only]] } (A_{\langle st, \triangleright \rangle})(p_{st}) = \lambda w: p(w)=1.$$

$$\forall q \in \text{NW}(p, A) [q \text{ is innocently excludable given } A \rightarrow q(w)=0]$$

$$b. \text{[[Exh]] } (A_{\langle st, \triangleright \rangle})(p_{st})(w) \Leftrightarrow p(w) \ \& \ \forall q \in \text{NW}(p, A)$$

$$[q \text{ is innocently excludable given } A \rightarrow \neg q(w)]$$

q is innocently excludable given A if $\neg \exists q' \in \text{NW}(p, A) [p \wedge \neg q \Rightarrow q']$

To see how this is supposed to work, consider an utterance of the disjunction p or q . Consider first what happens without an exhaustive operator, under the basic syntactic representation. Under such a representation, the sentence would assert that the disjunction is true and would be consistent with the truth of the conjunction (inclusive *or*). By B-MQ this would yield a variety of Ignorance Inferences, which might be implausible in a particular context, and if so, would motivate the introduction of an exhaustive operator, $\text{Exh}(\text{Alt}(p \text{ or } q))(p \text{ or } q)$.

Under this alternative parse the sentence would assert that the prejacent p or q is true and that every innocently excludable alternative is false. Assuming the Sauerland alternatives, we derive the simple ExOR meaning. None of the disjuncts is innocently excludable, since the exclusion of one will lead to the inclusion of the other, given the prejacent. Once again, we will circle the innocently excludable alternatives.

$$(58) \text{Alt}(p \vee q) = p \vee q \begin{array}{c} p \\ \diagdown \quad \diagup \\ \quad \quad \quad p \wedge q \\ \diagup \quad \diagdown \\ q \end{array}$$

Excluding p will necessarily include q while excluding q will necessarily include p .

$p \wedge q$ is thus the only proposition in $\text{NW}(p \vee q, \text{Alt}(p \vee q))$ that can be innocently excluded given the set of alternatives in (58). Thus, it is the only proposition that is excluded and the derived meaning is the familiar ExOR.

Before moving to FC, I would like to show how the lexical entries in (57)b replicates Sauerland results. But even before that, I would like to point out that Sauerland's algorithm is not totally contradiction free, and that his assumptions should therefore be

modified slightly. This modification would motivate a corresponding modification in (57).

Consider the question answer pair in (59) from Groenendijk and Stokhof. Assume that the alternatives for A is the Hamblin denotation of Q, $\text{Alt}((59)A)$, in (60).

- (59) Q: Who did Fred talk to?
A: Some GIRL

$$(60) \quad \text{Alt}((59)A) = \{\text{that Fred talked to } x: x \text{ is a person or a set of people}\}$$

Assume also that there are more than two girls in the domain of quantification. If all these assumptions are correct, it would be possible (by Sauerland's algorithm) to introduce an SI of the form $B_s \neg \varphi$, for every φ in $\text{Alt}((59)A)$. Each SI of this sort is consistent with the set of PIs and Quality. However, once all the SIs are collected, the result is contradictory. The problem extends to the lexical entry for *exh* in (57) (and for *only*, if we look back at (55b). Every member of $\text{Alt}((59)A)$ is innocently excludable. Hence, if we were to append *exh* to (59)A, the result would be contradictory.

One way to deal with this problem is to assume that the set of alternatives is always closed under disjunction (see Spector 2005, as well as footnote 29). An alternative, which is available when *exh* is assumed, is to eliminate additional elements from the set of innocently excludable propositions for a prejacent, p, given a set of alternatives A, I-E(p,A):

- (61)a. $[[\text{only}]] (A_{\langle \text{st}, \text{t} \rangle})(p_{\text{st}}) = \lambda w: p(w) = 1.$
 $\quad \quad \quad \forall q \in \text{I-E}(p,A) \rightarrow q(w) = 0$
 b. $[[\text{Exh}]] (A_{\langle \text{st}, \text{t} \rangle})(p_{\text{st}})(w) \Leftrightarrow p(w) \ \& \ \forall q \in \text{I-E}(p,A) \rightarrow \neg q(w)$

$$\text{I-E}(p,A) = \cap \{A' \subseteq A: A' \text{ is a maximal set in } A, \text{ s.t., } A'^{\neg} \cup \{p\} \text{ is consistent}\}$$

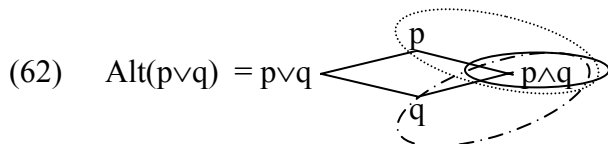
$$A^{\neg} = \{\neg p: p \in A\}$$

To see if a proposition q is innocently excludable, we must look at every maximal set of propositions in A such that its exclusion is consistent with the prejacent. Every such set could be excluded consistently as long as nothing else in A is excluded. Hence the only propositions that could be excluded non-arbitrarily are those that are in every one of these sets (the innocently excludable alternatives). Every proposition which is not in every such set would be an arbitrary exclusion, since the choice to exclude it, will force us to include a proposition from one of the other maximal exclusions (if the result is to be consistent), and the choice between alternative exclusion appears arbitrary.³³

To see what results is derived by this lexical entry, it is probably best to go through the various cases we've discussed. Let's first see how we would exhaustify $p \vee q$ given

³³ As pointed out to me by Angelika Kratzer and Fred Landman, the proposed mechanism for exhaustification is reminiscent of what is needed for counterfactuals in the premise semantics developed by Veltman (1977) and Kratzer (1981). In particular, the set of propositions that can be added as premises to a counterfactual antecedent p is $\cap \{A \subseteq C: A \text{ is a maximal set in } C, \text{ s.t., } A \cup \{p\} \text{ is consistent}\}$ where C is the set of all true propositions.

the Sauerland alternatives. The first step would be to identify the maximal consistent exclusions given the prejacent $p \vee q$. If p is excluded, q must be true and vice versa. Hence, one maximal exclusion is $\{p, p \wedge q\}$, and the other is $\{q, p \wedge q\}$. The intersection is $p \wedge q$, hence, $\text{Exh}(\text{Alt}(p \vee q))(p \vee q) = (p \vee q) \wedge \neg(p \wedge q) = p \nabla q$.

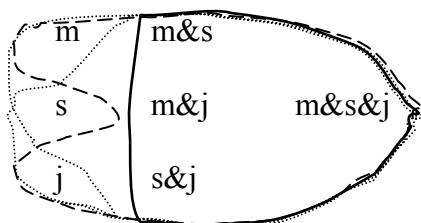


We circle (with dotted-lines) the maximal exclusions consistent with the prejacent, and we circle the intersection, the set of innocently excludable alternatives, with a completed line.

$$\text{Exh}(\text{Alt}(p \vee q))(p \vee q) = (p \vee q) \wedge \neg(p \wedge q) = p \nabla q.$$

Consider now the exhaustification of (59)A under the assumption that the set of alternatives is the Hamblin-denotation of the question, (60). Assume that there are three girls in the domain of quantification, Mary, Sue, and Jane, and that there are no non-girls.³⁴ Every maximal exclusion will include every member of the Hamblin-set but one of the following: (m) *Fred talked to Mary*, (s) *Fred talked to Sue*, and (j) *Fred talked to Jane*. So the intersection of all-maximal exclusions, the set of innocently-excludable alternatives, is the set of propositions of the form *Fred talked to X*, where X is a plurality of girls:

(63) $\text{Alt}((59)A) = \{\text{Fred talked to } x: x \text{ a person or a set of people}\} =$



$$\begin{aligned} \text{Exh}(\text{Alt}((59)A))((59)A) &= \text{Fred talked to some girl} \wedge \neg(m \& s) \wedge \neg(m \& j) \wedge \neg(s \& j) \\ &= \text{Fred talked to exactly one girl.} \end{aligned}$$

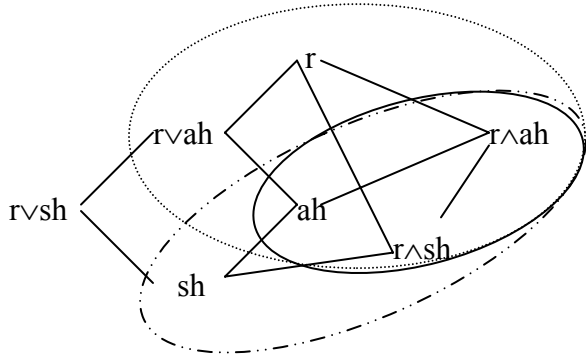
6.2. Replicating Sauerland's results

Consider again Chierchia's sentence $r \vee sh$ and its Sauerland alternatives. To see which alternative can be innocently excluded we have to identify the maximal (consistent)

³⁴ Without non-girls, the answer is somewhat strange. That's probably because questions presuppose that at least one answer (in the Hamblin sense) is true, and, thus, without non-girls, the answer just repeats the presupposition. Adding non-girls is thus crucial, but, it is trivial to see that it will not affect the result, in any interesting way; propositions related to non-girls will be excluded and things will be more difficult to draw, but other than that, it's all the same.

exclusions. The set of innocently excludable alternative is the intersection. The reader can consult the diagram in (64) to see that Sauerland's results are replicated.

(64) $\text{Alt}(r \vee sh) =$



$ah, r \wedge sh, r \wedge ah$ are the proposition in $\text{Alt}(ss \vee b)$ that can be innocently excluded given the set of alternatives:

$$\text{Exh}(\text{Alt}(r \vee sh))(r \vee sh) = (r \vee sh) \wedge \neg ah \wedge \neg (r \wedge sh)$$

Consider next embedding under universal quantifiers. As discussed in section 5.1., such embedding allows for the consistent exclusion of all the Sauerland-alternatives (other than the prejacent). Hence, there is only one maximal exclusion, which is excluded by *exh*.³⁵

(65) You're required to talk to Mary or Sue.

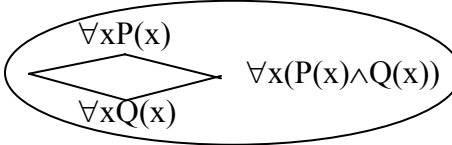
Implicatures:

- a. You're not required to talk to Mary.
- b. You're not required to talk to Sue.

(66) Every friend of mine has a boy friend or a girl friend.

- a. It's not true that every friend of mine has a boy friend.
- b. It's not true that every friend of mine has a girl friend.

³⁵In conversation with Gennaro Chierchia, we've noticed that things are a little more complicated. As things stand right now, $\text{Alt}(\forall x(P(x) \vee Q(x)))$ contains additional members: $\exists x(P(x) \vee Q(x))$, $\exists x(P(x))$, and $\exists x(Q(x))$. The latter two make it impossible to innocently exclude $\forall xP(x)$ and $\forall xQ(x)$. There are various simple ways to correct for this problem. The obvious thing that comes to mind is to define $\text{Alt}(S)$ so that it includes only stronger sentences than S . However, this would be a problematic move given data that is not discussed in this paper. Here's another possibility: $\text{Alt}(S)$ is the smallest set, s.t. (a) $S \in \text{Alt}(S)$, and (b) If $S' \in \text{Alt}(S)$ and S'' can be derived from S' by replacement of a single scalar item with an alternative, and S' does not entail S'' , $S'' \in \text{Alt}(S)$.

$$(67) \quad \text{Alt}(\forall x(P(x) \vee Q(x))) = \forall x(P(x) \vee Q(x))$$


$$\text{Exh}(\text{Alt}(\forall x(P(x) \vee Q(x))))(\forall x(P(x) \vee Q(x))) = \forall x(P(x) \vee Q(x)) \wedge \neg \forall x P(x) \wedge \neg \forall x Q(x)$$

So the exhaustive operator as defined in (61)b, based on what's needed for *only*, (61)a, derives the same results as Sauerland's system (with the exception of cases such as (59) for which Sauerland's system can derive contradictory implicatures). This is not surprising. The set of innocently excludable proposition is (modulo (59)) precisely the set of propositions for which Sauerland's system yields SIs – for which SIs can be introduced innocently.

However, there is an important architectural difference between the two systems, one that relates to the division of labor between syntax/semantics and pragmatics. Under Sauerland's neo-Gricean system, NG-MQ (and the PIs that it generates) is the underlying source of SIs. Under the syntactic alternative that we are considering, SIs have a syntactic source, and can serve to avoid Ignorance Inferences, which are computed post syntactically based on B-MQ. This architectural difference has empirical ramifications for FC. We've already seen that Sauerland's system predicts Ignorance Inferences that contradict FC. We will see that under our syntactic alternative, the problem can be avoided by recursive exhaustification.

7. Recursive Exhaustification and FC³⁶

Suppose that *exh* is a covert operator which can append to any sentence. It is reasonable to assume that, in parsing (or producing) a sentence, *exh* will be used whenever the result fairs better than its counter-part without *exh*. One way in which a sentence with *exh* would be better than its *exh*-less counterpart is if the latter generates implausible Ignorance Inferences based on B-MQ. We thus predict the following recursive parsing strategy:

- (68) *Recursive Parsing-Strategy*: If a sentence S has an undesirable Ignorance Inference, parse it as $\text{Exh}(\text{Alt}(S))(S)$.³⁷

³⁶ The core idea was developed during conversations with Ezra Keshet.

³⁷ This should be modified to allow introduction of *exh* in a non-matrix position.

- (i) *Recursive Parsing-Strategy*: If a sentence S has an undesirable Ignorance Inference, try to append *exh* to some constituent X in S. I.e., modify the parse [s...X...] as follows: [s...Exh(Alt(X))(X)...].

We could also incorporate an economy condition of the sort alluded to in section 2.1.:

- (ii) Condition on *exh*-insertion: *exh* can be appended to a constituent X, only if the resulting sentence generates fewer Ignorance Inferences (based on B-MQ)

7.1. Simple Disjunction:

Consider the disjunctive sentence in (69)

(69) I ate the cake or the ice-cream.

If this sentence is parsed without an exhaustive operator, B-MQ will generate the Ignorance Inference that the speaker doesn't know what she ate (only that it included the cake or the ice-cream or both). This inference might seem implausible, and the hearer might therefore prefer the following parse, where C is the set of Sauerland-alternatives to the disjunctive sentence.

(70) $\text{Exh}(C)(\text{I ate the cake or the ice-cream})$

As we've seen already, the meaning of (70) is the ExOR meaning of (69). This meaning will now generate (given B-MQ) the Ignorance Inference that the speaker doesn't know what she ate (only that, whatever it was, it included the cake or the ice-cream but not both). This, again, might seem implausible, and the hearer might employ the parsing strategy, again:

(71) $\text{Exh}(C')[\text{Exh}(C)(\text{I ate the cake or the ice-cream})]$
 where $C' = \text{Alt}[\text{Exh}(C)(\text{I ate the cake or the ice-cream})] = \{\text{Exh}(C)(p) : p \in C\}$ ³⁸

However, (71) ends up equivalent to (70). And further application of the parsing strategy is not helpful either. It will, thus, follow that there is no way to avoid the (sometimes undesirable) Ignorance Inference. To see this, we need to compute the set of alternatives, C' :

(72) $C' = \{1. \text{Exh}(C)(p \vee q), 2. \text{Exh}(C)(p), 3. \text{Exh}(C)(q), 4. \text{Exh}(C)(p \wedge q)\}$

$$\begin{aligned} 1. \text{Exh}(C)(p \vee q) &= (p \vee q) \wedge \neg(p \wedge q) \\ &= (p \wedge \neg q) \vee (q \wedge \neg p) \end{aligned}$$

$$2. \text{Exh}(C)(p) = p \wedge \neg q$$

$$3. \text{Exh}(C)(q) = q \wedge \neg p$$

$$4. \text{Exh}(C)(p \wedge q) = p \wedge q \quad (\text{can be ignored since already excluded by the prejacent})$$

$$\text{Exh}(C)(p \vee q) = \text{Exh}(C)(p) \vee \text{Exh}(C)(q)$$

Two simple observations are worth making. The first alternative, the prejacent of (71), is equivalent to the disjunction of the second and third alternative, and the fourth alternative is already excluded by the prejacent, and hence can be ignored. The relevant alternatives are thus the following:

³⁸By the algorithm in (5).

$$(73) \quad C' = \text{Alt}(\text{Exh}(p \vee q)) = \text{Exh}(C)(p) \vee \text{Exh}(C)(q)$$

$$\text{Exh}(C')[\text{Exh}(C)(p \vee q)] = \text{Exh}(C)(p \vee q) = (p \vee q) \wedge \neg(p \wedge q)$$

If 2 is excluded, 3 must be true, and vice versa. Hence, meaning does not change with a second level of exhaustification, nor will it change when *exh* is appended yet another time.³⁹ There is thus no way to avoid what might be an undesirable Ignorance Inference.

7.2. The Basic Free Choice Effect

Consider now (74).

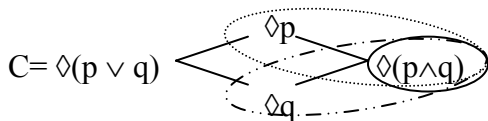
(74) You may eat the cake or the ice-cream.

Without an exhaustive operator, this sentence will generate the Ignorance Inference that the speaker doesn't know what one is allowed to eat (only that the allowed things include the cake or the ice-cream or both). This might seem implausible, and the hearer might opt for another parse:

(75) $\text{Exh}(C)(\text{You may eat the cake or the ice-cream})$

Given the Sauerland alternatives for disjunction, the set of alternatives, *C*, would be the following:⁴⁰

(76) $\text{Alt}(74)$



Notice $\diamond(p \vee q) \Leftrightarrow \diamond p \vee \diamond q$ but (crucially)
 $\diamond(p \wedge q) \not\Leftrightarrow \diamond p \wedge \diamond q$

³⁹The theorem in 1 is completely trivial, and the one in 2 (due to Benjamin Spector, p.c.) is less so:
 1. Let *C* be a set of alternatives, *S_i*, such that for each *i* exhaustification is trivial (i.e., $\text{Exh}(C)(S_i) \Leftrightarrow S_i$), then for each *i*, 2nd exhaustification is trivial (i.e., $\text{Exh}(C')(\text{Exh}(C)(S_i)) \Leftrightarrow \text{Exh}(C)(S_i)$, where $C' = \{\text{Exh}(C)(S) : S \in C\}$)
 2. due to Spector: Let *C* be a set of finite alternatives, *S_i*, then there is an $n \in \mathbb{N}$, s.t. $\forall m > n$,
 $\text{Exh}^n(C)(S_i) = \text{Exh}^m(C)(S_i)$
 $\text{Exh}^n(C)(S_i) := \text{Exh}(C')\text{Exh}^{n-1}(C)(S_i)$,
 where $C' = \{\text{Exh}^{n-1}(C)(S) : S \in C\}$, and $\text{Exh}^1(C)(S) = \text{Exh}(C)(S)$.

⁴⁰ The set of alternatives is actually larger, including a variant of each alternative in *C* with a universal modal replacing the existential modal. This does not affect our results as the reader can verify. See the appendix, as well as (84) and (85) where a parallel computation is carried out with the full set of alternatives.

$\diamond(p \wedge q)$ is the only proposition in $\text{Alt}(\diamond(p \vee q))$ that can be innocently excluded given the set of alternatives (excluding $\diamond p$ will necessarily include $\diamond q$ while excluding $\diamond q$ will necessarily include $\diamond p$). Hence, the meaning of (75) in our modal logic formalization is $\diamond(p \vee q) \wedge \neg \diamond(p \wedge q)$.

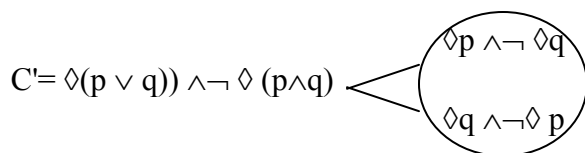
Crucially (75) is consistent with the free choice possibility, $\diamond p \wedge \diamond q$, though it, of course, does not assert free choice.⁴¹ This new meaning will now generate the Ignorance Inference that the speaker doesn't know what one is allowed to eat (only that the allowed things include the cake or the ice-cream but not both). This might seem implausible, and the hearer might employ the parsing strategy again:

(77) $\text{Exh}(C')[\text{Exh}(C)(\text{You may eat the cake or the ice-cream})]$
 where $C' = \{\text{Exh}(C)(p) : p \in C\}$

This time, second exhaustification has consequences. To see this, we need to compute the meanings of the various alternatives:

(78) $C' = \{1. \text{Exh}(C)(\diamond(p \vee q)), 2. \text{Exh}(C)(\diamond p), 3. \text{Exh}(C)(\diamond q), 4. \text{Exh}(C)(\diamond(p \wedge q))\}$

1. $\text{Exh}(C)(\diamond(p \vee q)) = \diamond(p \vee q) \wedge \neg \diamond(p \wedge q)$, crucially
 $\neq (\diamond p \wedge \neg \diamond q) \vee (\diamond q \wedge \neg \diamond p)$
2. $\text{Exh}(C)(\diamond p) = \diamond p \wedge \neg \diamond q$
3. $\text{Exh}(C)(\diamond q) = \diamond q \wedge \neg \diamond p$
4. $\text{Exh}(C)(\diamond(p \wedge q)) = \diamond(p \wedge q)$ (can be ignored since already excluded by the prejacent)



There are now two propositions in C' that can be innocently (and non-trivially) excluded. (Excluding $\text{Exh}(C)(\diamond p)$ will not necessarily include $\text{Exh}(C)(\diamond q)$, and excluding $\text{Exh}(C)(\diamond q)$ will not necessarily include $\text{Exh}(C)(\diamond p)$.)

Hence,

(79) $\text{Exc}(C')(\text{Exh}(C)(\diamond(p \vee q))) = \diamond(p \vee q) \wedge \neg \diamond(p \wedge q)$ and
 $\neg(\diamond p \wedge \neg \diamond q)$ and
 $\neg(\diamond q \wedge \neg \diamond p)$

⁴¹ This exemplifies the difference between the lexical entry we are considering and the Groenendijk and Stokhof-type alternative in (56). Under (56), (75) would express a stronger proposition $\diamond(p \vee q) \wedge \neg(\diamond p \wedge \diamond q)$, which will be inconsistent with FC.

$$= \diamond(p) \wedge \diamond(q) \quad \text{and} \\ \neg \diamond(p \wedge q)$$

We thus derive the FC effect for cases in which disjunction is embedded under existential modals.⁴²

8. Other Existential Quantifiers

The key to the distinction between disjunction embedded under an existential modal and unembedded disjunction is that in the latter case the strongest alternative $\diamond(p \wedge q)$ is stronger than the conjunction of the two other alternatives $\diamond p$ and $\diamond q$. Hence, the first layer of exhaustification is consistent with the later exclusion of $Exh(C)\diamond p$ and $Exh(C)\diamond q$.⁴³ This answers Aloni and van Rooy's (2005) objection to Kratzer and Shimoyama (section 2.3.), and extends to account for embedding under existential quantifiers:

- (80)
- a. There is beer in the fridge or the ice-bucket.
 - b. People sometimes take the highway or the scenic route (Irene Heim, pc attributing Regine Eckardt, pc)
 - c. This course is very difficult. Last year, some students waited 3 semesters to complete it or never finished it at all. (Irene Heim, pc)

Here, too, first exhaustification will be fairly weak $\exists x(Px \vee Qx) \wedge \neg \exists x(Px \wedge Qx)$ consistent with later exclusion of $Exh(C)\exists xPx$ and $Exh(C)\exists xQx$, the cumulative effect of which entails $\exists xPx \wedge \exists xQx$.

9. Singular Indefinites

At the moment the system makes wrong predictions for embedding under singular indefinites:

- (81)
- a. There is a bottle of beer in the fridge or the ice-bucket.
 - b. This course is very difficult. Some student waited 3 semesters to complete it or never finished it at all.

Right now, an FC effect is expected for this case as well. However, the expectation changes once an independently needed difference between plural and singular indefinites

⁴² As pointed out in Simons (2005), $\diamond(p \vee q)$ sometimes yields FC without the inference that $\diamond(p \wedge q)$ is false. A solution to this problem will be discussed in section 11.1.

⁴³ The following is easily verifiable:

Let $C = \{w, s, n, e\}$ be a diamond set of alternatives going stronger from w to e (w is weaker than s , n , and e ; s and n are logically independent and weaker than e), where w entails $(s \vee n)$. With such alternatives, 2nd exhaustification of w is vacuous ($Exh^2(C)(w) \Leftrightarrow Exh(C)(w)$) iff $e \Leftrightarrow s \wedge n$. Furthermore, when 2nd exhaustification of w is not vacuous, $Exh^2(C)(w) \Leftrightarrow s \wedge n \wedge \neg e$.

is factored in. Consider the sentences in (82). These sentences have the indicated implicature that the alternative assertion involving quantification over plural individuals is false:

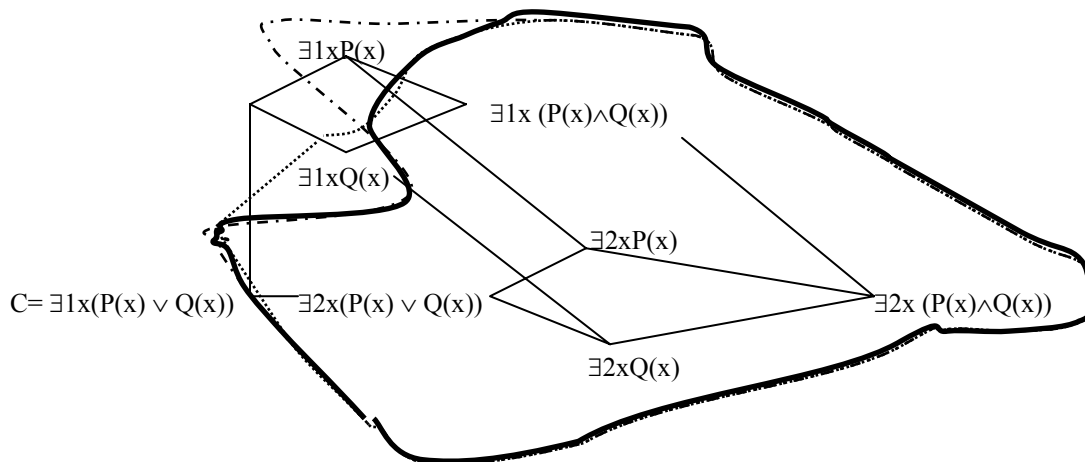
- (82) a. There is a bottle of beer in the fridge.
Implicature: there aren't two bottles of beer in the fridge.
 b. Some student talked to Mary
Implicature: It's not true that two students talked to Mary.

This implicature leads to the conclusion that a singular indefinite is a scalar item, with a plural (or dual) indefinite as an alternative:

$$(83) \quad \text{Horn-Set}(\text{Some NP-sing}) = \{ \text{Some NP-sing (henceforth } \exists 1), \\ \text{two NPs (henceforth } \exists 2) \}$$

With this Horn-Set, *exh* would, of course, derive the correct implicature for (82). But, interestingly, we also explain the lack of FC in (81). To see this consider (81)a, and the set of alternatives derived by (5), (84). The alternative-set includes alternatives of the sort we've considered in (76) (upper diamond of (84)). But it also includes alternatives generated by replacing $\exists 1$ with $\exists 2$ (lower diamond of (84)).

(84) Alt((82)a)



The set of innocently-excludable alternatives contains $\exists 1x(P(x) \wedge Q(x))$ as well as $\exists 2x(P(x) \wedge Q(x))$. Hence the exhaustification of (82)a is the following:⁴⁴

$$(85) \quad \text{Exh}(C)(\exists 1x(P(x) \vee Q(x))) = \exists 1x(P(x) \vee Q(x)) \ \& \\ \neg \exists 1x(P(x) \wedge Q(x)) \ \& \\ \neg \exists 2x(P(x) \vee Q(x))$$

⁴⁴This explanation, however, depends on the assumption that the alternatives for $\exists 1$ cannot be inactive when *exh* associates with \vee . An assumption of this sort was argued for on independent grounds in Chierchia (2005).

$$\Rightarrow \neg(\exists!xP(x) \wedge \exists!x Q(x))$$

This representation already contradicts the FC effect, a situation which, of course, cannot change by further exhaustification. We thus derive the generalization in (31)

It is of course important to return to our computation of basic FC and make sure that nothing changes when universal quantifiers are introduced as alternatives to existential quantifiers (4)b,d (see footnote 40). I leave this as a task for the interested reader, though an equivalent computation will be carried out in (87) and (88), below, and the appendix will contain a theorem that will make all of our results transparent with fewer computations.

10. Other FC effects

Multiple exhaustification also accounts for the generalization in (33), i.e., it generates FC effects for the sequences $\neg \square \wedge$ and $\neg \forall \wedge$ (introduced in 3.1 and 3.3.) I will illustrate this for $\neg \square \wedge$, and allow the reader to verify that nothing changes when \square is replaced with \forall .

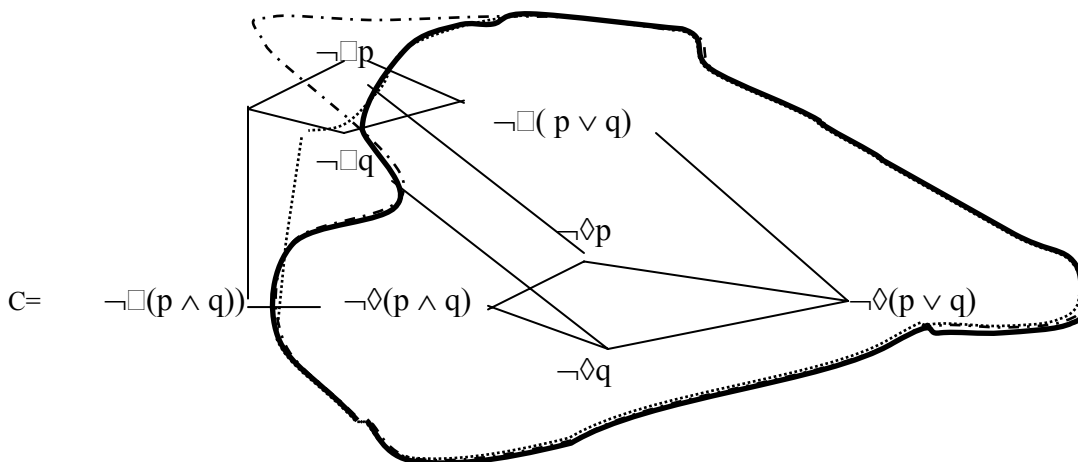
Consider (25), repeated below as (86), with its FC inference, which (to repeat) is not predicted by the basic meaning.

(86) You are not required to both clear the table and do the dishes.

1. basic meaning: $\neg \square(p \wedge q) \equiv \diamond \neg(p \wedge q) \equiv \diamond(\neg p \vee \neg q) \equiv \diamond(\neg p) \vee \diamond(\neg q)$
2. Free Choice: $\diamond(\neg p) \wedge \diamond(\neg q)$

Once again, FC will follow after two layers of exhaustivity are computed. Let's start with the first layer, which we compute based on the alternatives generated by Sauerland's Horn-Set $\{\wedge, L, R, \vee\}$ and the traditional Horn-Set $\{\diamond, \square\}$, (5)d.

(87) Alt((86))



$$\text{Exh}(C)(\neg\Box(p \wedge q)) = \neg\Box(p \wedge q) \ \& \ \Box(p \vee q) \ \& \ \Diamond(p \wedge q)$$

If we decide to add another layer of exhaustification, we get the following result:

(88)

$$C' = \neg\Box(p \wedge q) \wedge \Box(p \vee q) \wedge \Diamond(p \wedge q)$$

The diagram shows the expression $\neg\Box(p \wedge q) \wedge \Box(p \vee q) \wedge \Diamond(p \wedge q)$ on the left. Two lines branch from the $\Diamond(p \wedge q)$ part to a circle containing two disjunctive forms: $\neg\Box p \wedge \Box q \wedge \Diamond(p \wedge q)$ and $\neg\Box q \wedge \Box p \wedge \Diamond(p \wedge q)$.

$$\text{Exh}(C')[\text{Exh}(C)(\neg\Box(p \wedge q))] = \neg\Box(p \wedge q) \ \& \ \Box(p \vee q) \ \& \ \Diamond(p \wedge q) \ \& \ \neg(\neg\Box p \wedge \Box q) \ \& \ \neg(\neg\Box q \wedge \Box p)$$

This yields the FC effect, based on the following equivalences:

$$\begin{aligned} \neg\Box(p \wedge q) &\equiv \Diamond\neg p \vee \Diamond\neg q \\ \neg(\neg\Box p \wedge \Box q) &\equiv \neg(\Diamond\neg p \wedge \neg\Diamond\neg q) \\ \neg(\neg\Box q \wedge \Box p) &\equiv \neg(\Diamond\neg q \wedge \neg\Diamond\neg p) \end{aligned}$$

11. Remaining Issues

In section 6-10 we have seen how our two generalizations about the distribution of FC ((31) and (33)) can be derived based on recursive exhaustification under a Sauerland-inspired meaning for *exh*. But before concluding, I would like to discuss two apparent predictions of the account which are somewhat problematic.

11.1. $\neg\Diamond(p \wedge q)$

The lack of a conjunctive interpretation for $p \vee q$ was derived in section 7 on the basis of the observation that the first layer of exhaustification excludes $p \wedge q$, an exclusion which cannot be overridden at the second level of exhaustification. The situation changes in FC environments, by the introduction of appropriate operators. When $p \vee q$ is embedded under an existential quantifier, e.g. $\Diamond(p \vee q)$, the first level of exhaustification excludes $\Diamond(p \wedge q)$, a relatively weak exclusion, i.e. consistent with $\Diamond p \wedge \Diamond q$. Hence it is possible (at the second level of exhaustification) to innocently exclude the exhaustive interpretation of $\Diamond p$ and of $\Diamond q$.

This proposal makes a clear prediction, or at least so it seems. Specifically, it predicts that FC will always be accompanied by the anti-conjunctive inference $\neg\Diamond(p \wedge q)$.

However, Simons (2005) claims that this prediction is false. Specifically, she presents (89) as a sentence that can produce FC while lacking an anti-conjunctive inference.

- (89) Jane may sing or dance. (Simons 2005)
Possible Reading:
Jane may sing and Jane may dance, compatible with permission to do both.

I think I agree with Simons judgments, and the speakers I've consulted with also seem to agree. Interestingly, people seem to have a different feelings when asked about the FC effect that arises for the sequence $\rightarrow\forall\wedge$. Consider, again, (86).

- (86) You are not required to both clear the table and do the dishes.

It seems quite hard to get rid of the inference that you are required to either clear the table or do the dishes. More specifically, although judgments of this sort or notoriously difficult, there doesn't seem to be an interpretation which involves FC (i.e. entails that each chore is such that you are allowed to avoid it), which does not, at the same time, entail that at least one of the chores is required.⁴⁵

I will try to explain these facts on the assumption that in (89) each of the disjuncts could be exhaustified separately. Assume, as we have assumed above, that the alternatives for *Exh* could be determined (at least in the absence of scalar items) based on focus. (89) might now receive the following parse, where *C'* and *C''* are determined based on scalar items (as outlined above), and *C*₁, *C*₂ are determined based on the focus value of the relevant prejacent .

- (89)' $\text{Exh}(C'')(\text{Exh}(C')(\diamond(\text{Exh}(C_1)(\text{Jane sing}) \text{ or } \text{Exh}(C_2)(\text{Jane dance}))))$.

Assume, further, that *sing* and *dance* are focused so that *C*₁ = *C*₂. To simplify the exposition (but with no loss of generality) let's assume that *C*_{1,2} has only two members $\{p = \text{Jane sing}, q = \text{Jane dance}\}$, with the following result:

$$\begin{aligned} p! &:= \text{Exh}(C_1)(p) = p \wedge \neg q \\ q! &:= \text{Exh}(C_2)(q) = q \wedge \neg p \end{aligned}$$

As we see in (90), we derive Simons' reading.

- (90) $\text{Exh}(C'')(\text{Exh}(C')(\diamond(p! \vee q!))) = \diamond(p! \vee q!) \wedge \neg \diamond(p! \wedge q!) \wedge \diamond(p!) \wedge \diamond(q!) = \diamond(p! \vee q!) \wedge \diamond(p!) \wedge \diamond(q!)$

11.2. The Scope of Disjunction

The analysis of FC crucially depends on the assumption that in the relevant sentences disjunction receives narrow scope relative to the relevant existential quantifier. This assumption is corroborated by the contrast in (91).

⁴⁵ Note that *both* is crucial. See note 21.

- (91) a. We may either eat the cake or the ice-cream. (+FC)
 b. Either we may eat the cake or the ice-cream. (-FC)

Larson (1985) has pointed out that *either* marks the scope of disjunction in constructions such as (91). The fact that FC is present only for (91)a thus corroborates the scopal assumptions made in this paper.

However, as pointed out in Zimmerman (2002), FC seems to be available in (92).

- (92) You may eat the cake or you may eat the ice-cream.

I leave this as an unresolved problem, noting that the behavior with indefinites is different. (See Simons 2005, Alonso-Ovalle (2005), and Klindinst (2005) for relevant discussion.)⁴⁶

- (93) a. Some students waited 3 semester to complete this course or never finished it at all. (+FC)
 b. Some students either waited 3 semester to complete this course or never finished it at all. (+FC)
 c. (Either) Some students waited 3 semester to complete this course or some students never finished it at all. (-FC)

⁴⁶ FC depends on the nature of the alternatives (e.g., E must be stronger than the conjunction of N and S, see note 43, or the appendix for a more general statement). The correlation with scope is predicted on the basis of the algorithm that determines alternatives, (5): at every level of exhaustification, alternatives are determined on the basis of the structure of the prejacent. If (92) turns out to be indicative (rather than (91) and (93)), i.e., if it turns out that FC is possible even when disjunction has scope over the relevant existential quantifies, it would be possible to capture the facts with a relatively simple modification of the system I've proposed.

In this paper I've assumed that Alt(S) is determined either contextually or by the algorithm in (5), where the algorithm was crucial for the recursive step (see note 38). However, we could define a special rule for recursive exhaustification which would allow the rule to apply even when alternatives are contextually determined. Suppose that a sentence S is uttered in a context in which Q is the salient set of alternatives. If S were to be exhaustified (relative to Q), the syntactic representation would be Exh(Q)(S). We could now define the 2nd layer of Exhaustification, as follows: $\text{Exh}^2(\text{S}) := \text{Exh}(\text{C})[\text{Exh}(\text{Q})(\text{S})]$, where $\text{C} = \{\text{Exh}(\text{Q})(\varphi) : \varphi \in \text{Q}\}$. Now (92) could receive an FC interpretation if Q could be the set of sentences of the form *You can eat x*, where x denotes a singular or plural individuals (perhaps with closure under disjunction, see note 29).

Conclusion

In this paper I've argued that Free Choice effects arise in two seemingly unrelated contexts: when disjunction is embedded under non-singular existential quantifiers, (31), and when conjunction is embedded under a universal quantifier which is, itself, c-commanded by negation, (33). Both FC effects follow from a method for exhaustification inspired by Sauerland's solution for Chierchia's puzzle, a method in which the notion of an innocently excludable alternative plays a central role.

However, the proposal can work only if Sauerland's basic idea is removed from its Neo-Gricean setting. The reason for this is rather plain. Under the Neo-Gricean assumptions, SIs are derived as strengthenings of inferences that follow from NG-MQ, a maxim which would derive Ignorance Inferences based on the symmetric alternatives generated by disjunction. Hence the Neo-Gricean assumptions derive Ignorance Inferences that conflict with the empirically attested Free Choice effects.

A necessary conclusion, given the alternatives for disjunction, is that NG-MQ cannot be correct, and that there can be no primary implicatures which are computed on the basis of the relevant alternatives. The conclusion, itself, is an immediate consequence of a system in which pragmatic reasoning is based on *all* relevant alternative assertions (B-MQ), i.e. a pragmatic system which can only derive Ignorance Inferences.

If B-MQ is correct, SIs must be derived within grammar, as argued for on independent grounds in Chierchia (2004) and Fox (2004). The grammatical mechanism needed for FC seems to be an exhaustive operator, along the lines of Groenendijk and Stokhof (1984), and Krifka (1995), which can apply recursively to a single linguistic expression (based on a Sauerland-inspired lexical entry).

If this is correct, it might be useful to ask questions about possible external/functional motivations for *exh*. I hinted at the possibility that *exh* is needed to solve a communication problem that will arise very often in a pragmatic universe governed by B-MQ.

Appendix

In this appendix, I prove a rather simple theorem which should allow the reader to understand the results described in this paper with fewer computations. I define an FC interpretation which I call AnEx (for Anti-exhaustivity), and prove that this interpretation, if consistent, is the result of the 2nd layer of exhaustification.

Let:

C be a set of propositions with $p \in C$.

$I = I-E(p, C) \neq \emptyset$

$I' = (C \setminus I \setminus \{p\}) \neq \emptyset$

$AnEx = \bigcap \{ \neg Exh_C(q) : q \in I' \} \cap Exh_C(p)$

Claim: If $AnEx \neq \emptyset$ (is consistent), $Exh^2_C(p) = AnEx$.

Proof:

$\text{Exh}_C(p)$ entails $\neg q$, for all $q \in I$ (by definition of Exh)

Hence, $\text{Exh}_C(p)$ entails $\neg \text{Exh}_C(q)$, for all $q \in I$ ($\neg q$ entails $\neg \text{Exh}_C(q)$)

Hence, AnEx entails $\neg \text{Exh}_C(q)$, for all $q \in I$ (AnEx has $\text{Exh}_C(p)$ as a conjunct)

Hence, $\text{AnEx} = \bigcap \{ \neg \text{Exh}_C(q) : q \in I \} \cap \{ \neg \text{Exh}_C(q) : q \in I \} \cap \text{Exh}_C(p)$
 $= \bigcap \{ \neg \text{Exh}_C(q) : q \in C \setminus \{p\} \} \cap \text{Exh}_C(p)$.

Hence,

If AnEx is consistent, $I\text{-E}(\text{Exh}_C(p), C') = C' \setminus \{\text{Exh}_C(p)\}$ (where $C' := \{\text{Exh}_C(q) : q \in C\}$)

Hence,

$\text{Exh}^2_C(p) = \bigcap \{ \neg \text{Exh}_C(q) : q \in C' \setminus \{\text{Exh}_C(p)\} \} \cap \text{Exh}_C(p) = \text{AnEx}$ (by definition of exh)

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