文章编号: 1001-0920 (2012) 06-0855-06

块控非线性系统自适应神经网络控制

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摘 要: 针对一类含有非匹配不确定性的块控型多输入多输出非线性系统, 提出一种基于反演技术和 RBF 神经网络的控制系统设计方案. 通过引入一种改进型的 Lyapunov 函数, 避免了控制矩阵未知情况下可能出现的奇异问题. 在控制系统设计过程中, 充分应用鲁棒自适应控制技术, 解决了多输入多输出结构不确定性所带来的设计难题, 得到了系统所有状态量将全局指数收敛至原点附近一个邻域的结论. 最后的仿真结果表明了设计方案的正确性.

关键词: 不确定性；块控标准型；自适应控制；神经网络

中图分类号: TP271 **文献标识码:** A

Adaptive neural controller design for a class of block nonlinear systems

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Abstract: For a class of multi-input multi-output(MIMO) block nonlinear systems with mismatched uncertainties, an adaptive controller design scheme using backstepping and RBF neural networks is proposed. By introducing a modified Lyapunov function, control singularity problem brought by unknown control matrices is avoided. By using of robust adaptive control technique, many difficulties brought by MIMO uncertainties are solved. The conclusion is obtained that all states variables are bounded and will exponentially converge to a neighborhood of the origin globally. Finally, simulation results are given to show the correctness of proposed scheme.

Key words: uncertainties；block structure；adaptive control；neural networks

1 引 言

非线性系统的自适应控制问题在近几十年已引起人们的广泛关注, 并取得了一些重要成果. 文献 [1-3] 提出了一些基于神经网络逼近的控制方法, 为含有不确定性非线性系统的控制提供了一种新思路. 随后, 结合反演技术, 这种设计思想被应用于解决一些严格反馈型不确定非线性系统的控制问题[4-6]. 目前, 该技术已推广到非仿射系统中[7-8].

相对于一般的严格反馈系统而言, 块控非线性系统结构复杂且难以控制, 相关研究较少[9-12]. 文献 [10-11] 基于块控制、滑模控制、高增益鲁棒控制以及高阶神经网络等技术, 分别对两种不确定块控非线性系统的控制问题进行了研究, 但这些设计方法取得的稳定性是局部的. 文献 [12] 应用神经网络自适应控制技术解决了一类二阶块控非线性系统的控制问题, 但是研究的前提是系统的标称模型已知, 比较依赖于系统模型.

本文的研究对象是一类含有非匹配不确定性的块控型多输入多输出非线性系统, 该系统的不确定性是完全未知的, 且每个子系统都是多维的. 对于这种维数高且含有高度不确定性的块控非线性系统, 目前还没有有效的方法能够对其进行控制. 本文将利用神经网络自适应控制技术, 同时结合矩阵变换技巧, 尝试解决该控制难题.

2 系统描述

本文考虑的块控非线性系统数学模型描述如下:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + b_i(\bar{x}_i)x_{i+1},\ 1 \leqslant i \leqslant n-1; \\ \dot{x}_n = f_n(\bar{x}) + b_n(\bar{x})u; \\ y = x_1. \end{cases} \tag{1}$$

其中: $\bar{x} = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \cdots, x_n^{\mathrm{T}}]^{\mathrm{T}}(x_i \in R^{n_i}, i = 1, 2, \cdots,$

收稿日期: 2010-12-02；修回日期: 2011-06-02.

基金项目: 国家自然科学基金项目(61004002).

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n) 为系统的状态量, 且 $n_1 = n_2 = \cdots = n_n = m$ 为各子系统维数, $\bar{x}_i = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \cdots, x_i^{\mathrm{T}}]^{\mathrm{T}}$, $i < n$; $u \in R^{n_n}$, $y \in R^{n_1}$ 分别为控制量和输出量; 假设 $f_i(\cdot)$, $b_i(\cdot)$ 是具有相应维数的未知平滑函数矩阵. 系统的设计难度来自 $f_i(\cdot)$ 和 $b_i(\cdot)$ 的不确定性. 文中 $\|\cdot\|$ 代表矢量的 2-范数和矩阵的 Frobenius- 范数, $\lambda_{\max}(A)$ 代表方阵 A 的最大特征根.

控制目标为: 给定期望轨迹 y_d, 设计自适应控制器 u, 使得系统 (1) 的输出 y 能够跟踪上 y_d.

在进行具体的设计之前, 首先给出如下的基本假设:

假设 1 对于任意的 $\bar{x}_i \in R^{n_1+n_2+\cdots+n_i}$, 存在一个常数 $b_{i0} > 0$ 和一个已知的光滑函数 $\bar{b}_i(\bar{x}_i)$ 使得 $\bar{b}_i(\bar{x}_i) \geqslant \|b_i(\bar{x}_i)\| \geqslant b_{i0}$ 成立.

假设 2 对于函数矢量[13] $h: \Omega \to R^p$, 给定任意的 $\sigma > 0$, 总存在一个高斯基函数矢量 $B: R^m \to R^l$ 和一个最优权重矩阵 $W^* \in R^{l\times p}$ 使得 $\|h(x) - W^{*\mathrm{T}}B(x)\| \leqslant \sigma, \forall x \in \Omega$. 其中: Ω 是属于 R^m 的一个紧子集, 且 $\Delta h(x) = h(x) - W^{*\mathrm{T}}B(x)$ 称为网络重构误差. 权重误差定义为 $\tilde{W} = \hat{W} - W^*$.

假设 3 $b_i(\bar{x}_i)$ 可逆.

定义 $\beta_i(\bar{x}_i) = b_i^{-1}(\bar{x}_i)\bar{b}_i(\bar{x}_i)$, $i = 1, 2, \cdots, n$. 其中 $b_i^{-1}(\bar{x}_i)$ 为 $b_i(\bar{x}_i)$ 的逆.

3 控制器设计及稳定性分析

首先, 通过一个引理的证明来说明控制系统的设计思路, 并通过推导得到系统的一些重要性质.

定义 $z_1 = x_1 - y_d$, 于是第 1 个子系统的误差状态方程为

$$\dot{z}_1 = f_1(x_1) - \dot{y}_d + b_1(x_1)u_1. \tag{2}$$

此时, 先不考虑其他子系统, 则 u_1 为控制量.

假设 4 $\int_0^1 \theta\beta_1(\theta z_1 + y_d)\mathrm{d}\theta$ 正定, 这对于很多系统是满足的.

引理 1 对于系统 (2), 在假设 1∼假设 4 的前提下, 如果选择直接反馈控制量为

$$u_1^* = \frac{1}{\bar{b}_1(x_1)}[-k_1(t)z_1 - h_1(Z_1)], \tag{3}$$

则系统跟踪误差将指数收敛至 0. 其中

$$h_1(Z_1) = \beta_1(x_1)f_1(x_1) - \int_0^1 \beta_1(\theta z_1 + y_d)\mathrm{d}\theta\dot{y}_d + (A_1^{\mathrm{T}} - A_1)\dot{z}_1 + \varDelta_1\dot{z}_1 + \varDelta_1'\dot{y}_d,$$

$Z_1 = [x_1^{\mathrm{T}},\ \dot{y}_d^{\mathrm{T}},\ y_d^{\mathrm{T}}]^{\mathrm{T}}$, $k_1(t) > 0$ 为设计增益函数. A_1, $\varDelta_1$ 和 $\varDelta_1'$ 的定义将在下文给出.

证明 选取 Lyapunov 函数

$$V_0 = z_1^{\mathrm{T}}\int_0^1 \theta\beta_1(\theta z_1 + y_d)\mathrm{d}\theta z_1. \tag{4}$$

其中: $z_1 = [z_{11}, z_{12}, \cdots, z_{1m}]^{\mathrm{T}}$, $y_d = [y_{d1}, y_{d2}, \cdots, y_{dm}]^{\mathrm{T}}$.

由假设 4 可知 $V_0 > 0$, 对 V_0 求导可得

$$\dot{V}_0 = \frac{\partial V_0}{\partial z_1}\dot{z}_1 + \frac{\partial V_0}{\partial y_d}\dot{y}_d. \tag{5}$$

对式 (5) 右侧各项逐项展开. 首先, 第 1 项对应的求偏导项满足

$$\frac{\partial V_0}{\partial z_1} = \Big[\frac{\partial V_0}{\partial z_{11}}\ \ \frac{\partial V_0}{\partial z_{12}}\ \ \cdots\ \ \frac{\partial V_0}{\partial z_{1m}}\Big]. \tag{6}$$

现以状态量为二维的情况为例推导其各项的求导结果. 令 $z_1 = [z_{11}, z_{12}]^{\mathrm{T}}$, 假设 $A_1 = \int_0^1 \theta\beta_1(\theta z_1 + y_d)\mathrm{d}\theta$, 则有

$$A_1 = \begin{bmatrix} \int_0^1 \theta\beta_{11}^1(\theta z_1 + y_d)\mathrm{d}\theta & \int_0^1 \theta\beta_{12}^1(\theta z_1 + y_d)\mathrm{d}\theta \\ \int_0^1 \theta\beta_{21}^1(\theta z_1 + y_d)\mathrm{d}\theta & \int_0^1 \theta\beta_{22}^1(\theta z_1 + y_d)\mathrm{d}\theta \end{bmatrix}. \tag{7}$$

其中: 矩阵各元素的上标表示子系统编号, 下标表示矩阵各元素的编号. 从而 $V_0 = z_1^{\mathrm{T}}A_1z_1$. 进一步, 得

$$\frac{\partial V_0}{\partial z_1} = \Big[\frac{\partial V_0}{\partial z_{11}}\ \ \frac{\partial V_0}{\partial z_{12}}\Big] = z_1^{\mathrm{T}}A_1 + z_1^{\mathrm{T}}A_1^{\mathrm{T}} + z_1^{\mathrm{T}}\Big[\frac{\partial A_1}{\partial z_{11}}z_1\ \ \frac{\partial A_1}{\partial z_{12}}z_1\Big]. \tag{8}$$

本文中, 积分型矩阵 $\int_0^1 \theta\beta_1(\theta z_1 + y_d)\mathrm{d}\theta$ 中的每一个元素均为关于向量 $[\theta z_{11} + y_{d1}, \theta z_{12} + y_{d2}]^{\mathrm{T}}$ 的函数. 定义 $\gamma_1 = \theta z_{11} + y_{d1}$, $\gamma_2 = \theta z_{12} + y_{d2}$, 则有

$$\frac{\partial A_1}{\partial z_{11}}z_1 = \frac{\partial\Big[\int_0^1 \theta\beta_1(\theta z_1 + y_d)\mathrm{d}\theta\Big]}{\partial z_{11}}\begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix} = \begin{bmatrix} \int_0^1 \theta^2\frac{\partial\beta_{11}^1}{\partial\gamma_1}\mathrm{d}\theta z_{11} + \int_0^1 \theta^2\frac{\partial\beta_{12}^1}{\partial\gamma_1}\mathrm{d}\theta z_{12} \\ \int_0^1 \theta^2\frac{\partial\beta_{21}^1}{\partial\gamma_1}\mathrm{d}\theta z_{11} + \int_0^1 \theta^2\frac{\partial\beta_{22}^1}{\partial\gamma_1}\mathrm{d}\theta z_{12} \end{bmatrix}. \tag{9}$$

本文中积分变量为 θ, 则 z_{11}, z_{12}, y_{d1}, y_{d2} 均可看作常量, 从而有

$$\begin{cases} \mathrm{d}\theta z_{11} = \mathrm{d}(\theta z_{11} + y_{d1}) = \mathrm{d}\gamma_1, \\ \mathrm{d}\theta z_{12} = \mathrm{d}(\theta z_{12} + y_{d2}) = \mathrm{d}\gamma_2. \end{cases} \tag{10}$$

将式 (10) 代入 (9), 有

$$\frac{\partial A_1}{\partial z_{11}}z_1 = \begin{bmatrix} \int_0^1 \theta^2\frac{\partial\beta_{11}^1}{\partial\gamma_1}\mathrm{d}\gamma_1 + \int_0^1 \theta^2\frac{\partial\beta_{12}^1}{\partial\gamma_1}\mathrm{d}\gamma_2 \\ \int_0^1 \theta^2\frac{\partial\beta_{21}^1}{\partial\gamma_1}\mathrm{d}\gamma_1 + \int_0^1 \theta^2\frac{\partial\beta_{22}^1}{\partial\gamma_1}\mathrm{d}\gamma_2 \end{bmatrix},$$

同理可得

$$\frac{\partial A_1}{\partial z_{12}}z_1 = \begin{bmatrix} \int_0^1 \theta^2\frac{\partial\beta_{11}^1}{\partial\gamma_2}\mathrm{d}\gamma_1 + \int_0^1 \theta^2\frac{\partial\beta_{12}^1}{\partial\gamma_2}\mathrm{d}\gamma_2 \\ \int_0^1 \theta^2\frac{\partial\beta_{21}^1}{\partial\gamma_2}\mathrm{d}\gamma_1 + \int_0^1 \theta^2\frac{\partial\beta_{22}^1}{\partial\gamma_2}\mathrm{d}\gamma_2 \end{bmatrix}.$$

综合上面的推导,可得

$$\Big[\frac{\partial A_1}{\partial z_{11}}z_1\ \ \frac{\partial A_1}{\partial z_{12}}z_1\Big] =$$

$$\begin{bmatrix} \int_0^1 \theta^2 \frac{\partial \beta_{11}^1}{\partial \gamma_1} \mathrm{d}\gamma_1 + \int_0^1 \theta^2 \frac{\partial \beta_{12}^1}{\partial \gamma_1} \mathrm{d}\gamma_2 \\ \int_0^1 \theta^2 \frac{\partial \beta_{21}^1}{\partial \gamma_1} \mathrm{d}\gamma_1 + \int_0^1 \theta^2 \frac{\partial \beta_{22}^1}{\partial \gamma_1} \mathrm{d}\gamma_2 \end{bmatrix} \to$$
$$\leftarrow \begin{bmatrix} \int_0^1 \theta^2 \frac{\partial \beta_{11}^1}{\partial \gamma_2} \mathrm{d}\gamma_1 + \int_0^1 \theta^2 \frac{\partial \beta_{12}^1}{\partial \gamma_2} \mathrm{d}\gamma_2 \\ \int_0^1 \theta^2 \frac{\partial \beta_{21}^1}{\partial \gamma_2} \mathrm{d}\gamma_1 + \int_0^1 \theta^2 \frac{\partial \beta_{22}^1}{\partial \gamma_2} \mathrm{d}\gamma_2 \end{bmatrix}. \tag{11}$$

考虑到$\mathrm{d}\beta_{ij}^1 = \dfrac{\partial \beta_{ij}^1}{\partial \gamma_1}\mathrm{d}\gamma_1 + \dfrac{\partial \beta_{ij}^1}{\partial \gamma_2}\mathrm{d}\gamma_2$, 有

$$\int_0^1 \theta^2 \frac{\partial \beta_{ij}^1}{\partial \gamma_1} \mathrm{d}\gamma_1 + \int_0^1 \theta^2 \frac{\partial \beta_{ij}^1}{\partial \gamma_2} \mathrm{d}\gamma_2 = \int_0^1 \theta^2 \mathrm{d}\beta_{ij}^1. \tag{12}$$

利用式 (12) 对 (11) 进行等效变换, 得

$$\left[\frac{\partial A_1}{\partial z_{11}} z_1 \ \ \frac{\partial A_1}{\partial z_{12}} z_1\right] = \begin{bmatrix} \int_0^1 \theta^2 \mathrm{d}\beta_{11}^1 & \int_0^1 \theta^2 \mathrm{d}\beta_{12}^1 \\ \int_0^1 \theta^2 \mathrm{d}\beta_{21}^1 & \int_0^1 \theta^2 \mathrm{d}\beta_{22}^1, \end{bmatrix} + \varDelta_1, \tag{13}$$

其中

$$\varDelta_1 = \begin{bmatrix} \int_0^1 \theta^2 \frac{\partial \beta_{12}^1}{\partial \gamma_1} \mathrm{d}\gamma_1 - \int_0^1 \theta^2 \frac{\partial \beta_{11}^1}{\partial \gamma_2} \mathrm{d}\gamma_2 \\ \int_0^1 \theta^2 \frac{\partial \beta_{22}^1}{\partial \gamma_1} \mathrm{d}\gamma_2 + \int_0^1 \theta^2 \frac{\partial \beta_{21}^1}{\partial \gamma_2} \mathrm{d}\gamma_2 \end{bmatrix} \to$$
$$\leftarrow \begin{bmatrix} \int_0^1 \theta^2 \frac{\partial \beta_{11}^1}{\partial \gamma_2} \mathrm{d}\gamma_1 - \int_0^1 \theta^2 \frac{\partial \beta_{12}^1}{\partial \gamma_1} \mathrm{d}\gamma_1 \\ \int_0^1 \theta^2 \frac{\partial \beta_{21}^1}{\partial \gamma_2} \mathrm{d}\gamma_1 - \int_0^1 \theta^2 \frac{\partial \beta_{22}^1}{\partial \gamma_1} \mathrm{d}\gamma_1 \end{bmatrix}.$$

由分部积分法, 得

$$\int_0^1 \theta^2 \mathrm{d}\beta_{ij}^1 =$$
$$\theta^2 \beta_{ij}(\theta z_1 + y_d)|_0^1 - 2\int_0^1 \theta \beta_{ij}^1(\theta z_1 + y_d)\mathrm{d}\theta =$$
$$\beta_{ij}^1(x_1) - 2\int_0^1 \theta \beta_{ij}^1(\theta z_1 + y_d)\mathrm{d}\theta.$$

则式 (13) 可进一步转化为

$$\left[\frac{\partial A_1}{\partial z_{11}} z_1 \ \ \frac{\partial A_1}{\partial z_{12}} z_1\right] = \beta_1(x_1) - 2A_1 + \varDelta_1. \tag{14}$$

将式 (14) 代入 (8) 并整理,可得

$$\frac{\partial V_0}{\partial z_1}\dot{z}_1 = z_1^{\mathrm{T}}\beta_1(x_1)\dot{z}_1 + z_1^{\mathrm{T}}(A_1^{\mathrm{T}} - A_1)\dot{z}_1 + z_1^{\mathrm{T}}\varDelta_1\dot{z}_1. \tag{15}$$

当 z_1 为 n 维向量时, 所得到的展开式 (15) 不变, 只是余式 $\varDelta_1$ 是 n 维的, 其表达式为

$$\varDelta_1 = \begin{bmatrix} \delta_{11}^1 & \delta_{12}^1 & \cdots & \delta_{1m}^1 \\ \delta_{21}^1 & \delta_{22}^1 & \cdots & \delta_{2m}^1 \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{m1}^1 & \delta_{m2}^1 & \cdots & \delta_{mm}^1 \end{bmatrix}.$$

其中

$$\delta_{jq}^1 = \sum_{k=1,k\neq q}^{m} \int_0^1 \theta^2 \frac{\partial \beta_{jk}^1}{\partial \gamma_q} \mathrm{d}\gamma_k - \sum_{k=1,k\neq q}^{m} \int_0^1 \theta^2 \frac{\partial \beta_{jq}^1}{\partial \gamma_q} \mathrm{d}\gamma_k,$$
$$j = 1, 2, \cdots, m, \ \ q = 1, 2, \cdots, m,$$

γ_k 的表达式可根据式 (10) 类推.

同时, 对式 (5) 的第 2 项进行类似的变换, 可得

$$\frac{\partial V_0}{\partial y_d}\dot{y}_d = z_1^{\mathrm{T}}\beta_1(x_1)\dot{y}_d - z_1^{\mathrm{T}}\int_0^1 \beta_1(\theta z_1 + y_d)\mathrm{d}\theta\dot{y}_d + z_1^{\mathrm{T}}\varDelta_{11}'\dot{y}_d. \tag{16}$$

其中

$$\varDelta_1' = \begin{bmatrix} \delta_{11}^{1}{}' & \delta_{12}^{1}{}' & \cdots & \delta_{1m}^{1}{}' \\ \delta_{21}^{1}{}' & \delta_{22}^{1}{}' & \cdots & \delta_{2m}^{1}{}' \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{m1}^{1}{}' & \delta_{m2}^{1}{}' & \cdots & \delta_{mm}^{1}{}' \end{bmatrix},$$

$$\delta_{jq}^{1}{}' = \sum_{k=1,k\neq q}^{m} \int_0^1 \theta \frac{\partial \beta_{jk}^1}{\partial \gamma_q} \mathrm{d}\gamma_k - \sum_{k=1,k\neq q}^{m} \int_0^1 \theta \frac{\partial \beta_{jq}^1}{\partial \gamma_k} \mathrm{d}\gamma_k.$$

综上所述, 可得

$$\dot{V}_0 =$$
$$z_1^{\mathrm{T}}\beta_1(x_1)(\dot{z}_1 + \dot{y}_d) - z_1^{\mathrm{T}}\int_0^1 \beta_1(\theta z_1 + y_d)\mathrm{d}\theta\dot{y}_d +$$
$$z_1^{\mathrm{T}}(A_1^{\mathrm{T}} - A_1)\dot{z}_1 + z_1^{\mathrm{T}}\varDelta_1\dot{z}_1 + z_1^{\mathrm{T}}\varDelta_1'\dot{y}_d. \tag{17}$$

将式 (2) 和控制器 (3) 代入 (17), 同时考虑到定义 $\beta_i(\bar{x}_i) = b_i^{-1}(\bar{x}_i)\bar{b}_i(\bar{x}_i)$, 则有

$$\dot{V}_0 = z_1^{\mathrm{T}}[-k_1(t)z_1 - h_1(Z_1) + h_1(Z_1)] =$$
$$-k_1(t)\|z_1\|^2. \tag{18}$$

由此引理 1 得证. □

引理 1 成立的前提是 $h_1(Z_1)$ 已知, 然而该函数显然不可获得, 只能用神经网络进行逼近. 下面给出具体的设计步骤及稳定性分析.

Step 1 定义误差状态量 $z_2 = x_2 - \alpha_1$, α_1 为虚拟控制量, 则第 1 个子系统的状态方程为

$$\dot{z}_1 = f_1(x_1) + b_1(x_1)(z_2 + \alpha_1) - \dot{y}_d. \tag{19}$$

设计虚拟控制量为

$$\alpha_1 = \frac{1}{\bar{b}_1(x_1)}[-k_1(t)z_1 - \hat{W}_1^{\mathrm{T}}B_1(Z_1)], \tag{20}$$

其中 $\hat{W}_1^{\mathrm{T}}B_1(Z_1)$ 用于逼近 $h_1(Z_1)$. 函数 $k_1(t)$ 设计为

$$k_1(t) = 1 + \int_0^1 \theta\bar{b}_1(\theta z_1 + y_d)\mathrm{d}\theta. \tag{21}$$

RBF 神经网络权重矩阵调节律设计为

$$\dot{\hat{W}}_1 = \varGamma_{W_1}B_1(Z_1)z_1^{\mathrm{T}} - \varGamma_{W_1}\delta_{W_1}\hat{W}_1. \tag{22}$$

选取 Lyapunov 函数

$$V_1 = V_0 + \frac{1}{2}\mathrm{tr}(\tilde{W}_1^{\mathrm{T}}\varGamma_{W_1}^{-1}\tilde{W}_1). \tag{23}$$

其中: $\varGamma_{W_1} = \varGamma_{W_1}^{\mathrm{T}} > 0$, $\delta_{W_1} > 0$ 为设计参数.

对式 (23) 求导, 可得

$$\dot{V}_1 = \dot{V}_0 + \mathrm{tr}(\tilde{W}_1^{\mathrm{T}}\varGamma_{W_1}^{-1}\dot{\hat{W}}_1). \tag{24}$$

对式 (24) 右侧两项分别展开. 对于第 1 项, 由引理 1 可知

$$\dot{V}_0 = z_1^{\mathrm{T}}[\bar{b}_1(x_1)(z_2 + \alpha_1) + h_1(Z_1)]. \tag{25}$$

将式 (20)∼(22) 代入 (25), 得

$$\dot{V}_0 = z_1^{\mathrm{T}}[-k_1(t)z_1 - \hat{W}_1^{\mathrm{T}}B_1(Z_1) + h_1(Z_1)] + z_1^{\mathrm{T}}\bar{b}_1(x_1)z_2 = -\|z_1\|^2 - z_1^{\mathrm{T}}\int_0^1 \theta\bar{b}_1(\theta z_1 + y_d)\mathrm{d}\theta z_1 - z_1^{\mathrm{T}}\tilde{W}_1^{\mathrm{T}}B_1(Z_1) + z_1^{\mathrm{T}}\Delta h_1(Z_1) + z_1^{\mathrm{T}}\bar{b}_1(x_1)z_2. \tag{26}$$

由假设2知, 存在相应维数的神经网络使得 $h_1(Z_1) = W_1^{*\mathrm{T}}B_1(Z_1) + \Delta h_1(Z_1)$, $\|\Delta h_1(Z_1)\| \leqslant \sigma_1$, 其中 $\sigma_1 > 0$. 所设计的参数自适应律 (22) 即用于逼近最优权重矩阵 W_1^*.

利用矩阵性质 $z_1^{\mathrm{T}}\tilde{W}_1^{\mathrm{T}}B_1(Z_1) = \mathrm{tr}[\tilde{W}_1^{\mathrm{T}}B_1(Z_1)z_1^{\mathrm{T}}]$, 将所设计的自适应律 (22) 代入式 (24), 同时考虑到 (26), 经整理最终可得

$$\dot{V}_1 = -\|z_1\|^2 - z_1^{\mathrm{T}}\int_0^1 \theta\bar{b}_1(\theta z_1 + y_d)\mathrm{d}\theta z_1 - \delta_{W_1}\mathrm{tr}[\tilde{W}_1^{\mathrm{T}}\hat{W}_1] + z_1^{\mathrm{T}}\Delta h_1(Z_1) + z_1^{\mathrm{T}}\bar{b}_1(x_1)z_2. \tag{27}$$

利用性质 $2\mathrm{tr}(\tilde{W}_1^{\mathrm{T}}\hat{W}_1) \geqslant \|W_1\|^2 - \|W_1^*\|^2$, $\sigma_1\|z_1\| \leqslant -\dfrac{1}{4}\|z_1\|^2 + \sigma_1^2$, 同时考虑到 $\beta_i(\bar{x}_i) = b_i^{-1}(\bar{x}_i)\bar{b}_i(\bar{x}_i)$, 可得如下表达式:

$$V_0 \leqslant \lambda_{\max}\{b_1^{-1}(\theta z_1 + y_d)\}z_1^{\mathrm{T}}\int_0^1 \theta\bar{b}_1(\theta z_1 + y_d)\mathrm{d}\theta z_1, \tag{28}$$

$$\mathrm{tr}(\tilde{W}_1^{\mathrm{T}}\varGamma_{W_1}^{-1}\tilde{W}_1) \leqslant \lambda_{\max}\{\varGamma_{W_1}^{-1}\}\mathrm{tr}(\tilde{W}_1^{\mathrm{T}}\tilde{W}_1) \leqslant \lambda_{\max}\{\varGamma_{W_1}^{-1}\}\|\tilde{W}_1\|^2, \tag{29}$$

$$z_1^{\mathrm{T}}\Delta h_1(Z_1) \leqslant \sigma_1\|z_1\| \leqslant \frac{1}{4}\|z_1\|^2 + \sigma_1^2. \tag{30}$$

将式 (28)~(30) 代入 (27) 并对其右侧进行放大, 可得

$$\dot{V}_1 \leqslant -\frac{V_0}{\lambda_{\max}\{b_1^{-1}(\theta z_1 + y_d)\}} - \frac{1}{2}\delta_{W_1}\|\tilde{W}_1\|^2 + z_1^{\mathrm{T}}\bar{b}_1(x_1)z_2 + \frac{1}{2}\delta_{W_1}\|W_1^*\|^2 + \sigma_1^2 \leqslant -\lambda_1 V_1 + c_1 + z_1^{\mathrm{T}}\bar{b}_1(x_1)z_2. \tag{31}$$

其中

$$\lambda_1 = \min\Big\{\frac{1}{\lambda_{\max}\{b_1^{-1}(\theta z_1 + y_d)\}}, \frac{\delta_{W_1}}{\lambda_{\max}\{\varGamma_{W_1}^{-1}\}}\Big\},$$

$$c_1 = \sigma_1^2 + \frac{1}{2}\delta_{W_1}\|W_1^*\|^2.$$

Step 2 当 $i = 2$ 时, 定义 $z_3 = x_3 - \alpha_2$, α_2 为虚拟控制量, 则误差状态方程为

$$\dot{z}_2 = f_2(\bar{x}_2) + b_2(\bar{x}_2)(z_3 + \alpha_2) - \dot{\alpha}_1. \tag{32}$$

假设 5 $\int_0^1 \theta\beta_i(\bar{x}_{i-1}, \theta_{z_i} + \alpha_{i-1})\mathrm{d}\theta$ 正定, $i = 1, 2, \cdots, n$, z_i 和 α_{i-1} 将在下面定义.

选取 Lyapunov 函数为

$$V_2 = V_1 + z_2^{\mathrm{T}}\int_0^1 \theta\beta_2(\bar{x}_1, \theta_{z_2} + \alpha_1)\mathrm{d}\theta_{z_2} + \frac{1}{2}\mathrm{tr}(\tilde{W}_2^{\mathrm{T}}\varGamma_{W_2}^{-1}\tilde{W}_2). \tag{33}$$

其中: $\varGamma_{W_2} = \varGamma_{W_2}^{\mathrm{T}} > 0$, $\tilde{W}_2$ 将在下面定义. 对 V_2 求导, 可得

$$\dot{V}_2 = \dot{V}_1 + z_2^{\mathrm{T}}\beta_2(\bar{x}_2)\dot{z}_2 + \mathrm{tr}(\tilde{W}_2^{\mathrm{T}}\varGamma_{W_2}^{-1}\dot{\hat{W}}_2) + z_2^{\mathrm{T}}\frac{\partial}{\partial x_1}\int_0^1 \theta[\beta_2(x_1, \theta z_2 + \alpha_1)]\mathrm{d}\theta z_2\dot{x}_1 + z_2^{\mathrm{T}}\frac{\partial}{\partial \alpha_1}\int_0^1 \theta[\beta_2(x_1, \theta z_2 + \alpha_1)]\mathrm{d}\theta z_2\dot{\alpha}_1 + z_2^{\mathrm{T}}\varDelta_2\dot{z}_2 + z_2^{\mathrm{T}}(A_2^{\mathrm{T}} - A_2)\dot{z}_2 = \dot{V}_1 + \mathrm{tr}(\tilde{W}_2^{\mathrm{T}}\varGamma_{W_2}^{-1}\dot{\hat{W}}_2) + z_2^{\mathrm{T}}[\bar{b}_2(\bar{x}_2)(z_3 + \alpha_2) + h_2(Z_2)]. \tag{34}$$

其中

$$A_2 = \int_0^1 \theta\beta_2(x_1, \theta z_2 + \alpha_1)\mathrm{d}\theta,$$

$$Z_2 = [\bar{x}_2^{\mathrm{T}}, \alpha_1^{\mathrm{T}}, \dot{\alpha}_1^{\mathrm{T}}]^{\mathrm{T}},$$

$$h_2(Z_2) = \beta_2(\bar{x}_2)(f_2(\bar{x}_2) - \dot{\alpha}_1) + (A_2^{\mathrm{T}} - A_2)\dot{z}_2 + \varDelta_2\dot{z}_2 + \frac{\partial}{\partial x_1}\int_0^1 \theta[\beta_2(x_1, \theta z_2 + \alpha_1)]\mathrm{d}\theta z_2\dot{x}_1 + \frac{\partial}{\partial \alpha_1}\int_0^1 \theta[\beta_2(x_1, \theta z_2 + \alpha_1)]\mathrm{d}\theta z_2\dot{\alpha}_1.$$

$\varDelta_2$ 的表达式可通过与 Step 1 类似的方法推导获得, 限于篇幅, 这里不给出具体表达式.

与 Step 1 类似, 存在相应的神经网络使得 $h_2(Z_2) = W_2^{*\mathrm{T}}B_2(Z_2) + \Delta h_2(Z_2)$, 且 $\|\Delta h_2(Z_2)\| \leqslant \sigma_2, \sigma_2 > 0$, 其中 W_2^* 为理想权重矩阵. 设计虚拟控制量为

$$\alpha_2 = \frac{-1}{\bar{b}_2(\bar{x}_2)}[\bar{b}_1(x_1)z_1 + k_2(t)z_2 + \hat{W}_2^{\mathrm{T}}B_2(Z_2)]. \tag{35}$$

其中: $\hat{W}_2$ 为 W_2^* 的估计值, 权重误差 $\tilde{W}_2 = \hat{W}_2 - W_2^*$.

设计增益函数和神经网络权重矩阵调节律为

$$k_2(t) = 1 + \int_0^1 \theta\bar{b}_2(\bar{x}_1, \theta z_2 + \alpha_1)\mathrm{d}\theta, \tag{36}$$

$$\dot{\hat{W}}_2 = \varGamma_{W_2}B_2(Z_2)z_2 - \delta_{W_2}\varGamma_{W_2}\hat{W}_2, \tag{37}$$

其中 $\delta_{W_2} > 0$ 为设计参数.

将所设计的控制系统 (35) ~ (37) 代入式 (34), 同时考虑到 (31), 采用类似 Step 1 的推导方法, 并归纳整理, 可得

$$\dot{V}_2 \leqslant -\lambda_1 V_1 - \frac{z_2^{\mathrm{T}}\int_0^1 \theta\beta_2(x_1, \theta z_2 + \alpha_1)\mathrm{d}\theta z_2}{\lambda_{\max}\{b_2^{-1}(x_1, \theta z_2 + \alpha_1)\}} - \frac{\delta_{W_2}}{2}\|\tilde{W}_2\|^2 + z_2^{\mathrm{T}}\bar{b}_2(\bar{x}_2)z_3 + \frac{\delta_{W_2}}{2}\|W_2^*\|^2 + \sigma_2^2 + c_1 \leqslant -\lambda_2 V_2 + c_2 + z_2^{\mathrm{T}}\bar{b}_2(\bar{x}_2)z_3. \tag{38}$$

其中

$$\lambda_2 = \min\Big\{\lambda_1, \frac{1}{\lambda_{\max}\{b_2^{-1}(x_1, \theta z_2 + \alpha_1)\}}, \frac{\delta_{W_2}}{\lambda_{\max}\{\varGamma_{W_2}^{-1}\}}\Big\},$$

$$c_2 = \frac{\delta_{W_2}}{2}\|W_2^*\|^2 + \sigma_2^2 + c_1.$$

按照上面的方法进行设计, 直至 Step n.

Step n 　定义 $z_n = x_n - \alpha_{n-1}$, 有

$$\dot{z}_n = f_n(\bar{x}) + b_n(\bar{x})u - \dot{\alpha}_{n-1}. \tag{39}$$

选取 Lyapunov 函数

$$V_n = V_{n-1} + z_n^{\mathrm{T}} \int_0^1 \theta\beta_n(\bar{x}_{n-1}, \theta z_n + \alpha_{n-1})\mathrm{d}\theta z_n + \frac{1}{2}\mathrm{tr}(\tilde{W}_n^{\mathrm{T}} \Gamma_{W_n}^{-1} \tilde{W}_n), \tag{40}$$

其中 $\Gamma_{W_n} = \Gamma_{W_n}^{\mathrm{T}} > 0$. 对式 (40) 求导, 得

$$\begin{aligned}\dot{V}_n = {} & \dot{V}_{n-1} + z_n^{\mathrm{T}}\beta_n(\bar{x})\dot{z}_n + \mathrm{tr}(\tilde{W}_n^{\mathrm{T}}\Gamma_{W_n}^{-1}\dot{\hat{W}}_n) + \\ & z_n^{\mathrm{T}}\frac{\partial\Big(\int_0^1 \theta[\beta_n(\bar{x}_{n-1}, \theta z_n + \alpha_{n-1})]\mathrm{d}\theta z_n\Big)}{\partial \bar{x}_{n-1}}\dot{\bar{x}}_{n-1} + \\ & z_n^{\mathrm{T}}\frac{\partial\Big(\int_0^1 \theta[\beta_n(\bar{x}_{n-1}, \theta z_n + \alpha_{n-1})]\mathrm{d}\theta z_n\Big)}{\partial \alpha_{n-1}}\dot{\alpha}_{n-1} + \\ & z_n^{\mathrm{T}}\Delta_n\dot{z}_n + z_n^{\mathrm{T}}(A_n^{\mathrm{T}} - A_n)\dot{z}_n = \\ & \dot{V}_{n-1} + z_n^{\mathrm{T}}[\bar{b}_n(\bar{x}_n)(z_n + \alpha_{n-1}) + h_n(Z_n)] + \\ & \mathrm{tr}(\tilde{W}_n^{\mathrm{T}}\Gamma_{W_n}^{-1}\dot{\hat{W}}_n).\end{aligned} \tag{41}$$

其中

$$A_n = \int_0^1 \theta\beta_n(\bar{x}_{n-1}, \theta z_n + \alpha_{n-1})\mathrm{d}\theta,$$

$$Z_n = [\bar{x}, \alpha_{n-1}^{\mathrm{T}}, \dot{\alpha}_{n-1}^{\mathrm{T}}]^{\mathrm{T}},$$

Δ_n 的定义与前面类似, $h_n(Z_n)$ 定义为

$$\begin{aligned} & h_n(Z_n) = \\ & \beta_n(\bar{x})(f_n(\bar{x}) - \dot{\alpha}_{n-1}) + z_n^{\mathrm{T}}\Delta_n\dot{z}_n + \\ & \frac{\partial\Big(\int_0^1 \theta[\beta_n(\bar{x}_{n-1}, \theta z_n + \alpha_{n-1})]\mathrm{d}\theta z_n\Big)}{\partial \bar{x}_{n-1}}\dot{\bar{x}}_{n-1} + \\ & \frac{\partial\Big(\int_0^1 \theta[\beta_n(\bar{x}_{n-1}, \theta z_n + \alpha_{n-1})]\mathrm{d}\theta z_n\Big)}{\partial \alpha_{n-1}}\dot{\alpha}_{n-1} + \\ & z_n^{\mathrm{T}}(A_n^{\mathrm{T}} - A_n)\dot{z}_n.\end{aligned}$$

同样, 存在神经网络使得 $h_n(Z_n) = W_n^{*\mathrm{T}}B_n(Z_n) + \Delta h_n(Z_n)$ 且 $\|\Delta h_n(Z_n)\| \leqslant \sigma_n$, $\sigma_n > 0$, 其中 W_n^* 为理想权重矩阵. 设计控制量为

$$u = \frac{-1}{\bar{b}_n(\bar{x})}[\bar{b}_{n-1}(\bar{x}_{n-1})z_{n-1} + k_n(t)z_n + \hat{W}_n^{\mathrm{T}}B_n(Z_n)]. \tag{42}$$

其中: $\hat{W}_n$ 为 W_n^* 的估计值, 定义 $\tilde{W}_n = \hat{W}_n - W_n^*$.

设计增益函数和神经网络权重矩阵调节律为

$$k_n(t) = 1 + \int_0^1 \theta\bar{b}_n(\bar{x}_{n-1}, \theta z_n + \alpha_{n-1})\mathrm{d}\theta, \tag{43}$$

$$\dot{\hat{W}}_n = \Gamma_{W_n}B_n(Z_n)z_n - \delta_{W_n}\Gamma_{W_n}\hat{W}_n, \tag{44}$$

其中 $\delta_{W_n} > 0$ 为设计参数. 代入所设计的控制系统, 可以证明

$$\dot{V}_n \leqslant -\lambda_n V_n + c_n. \tag{45}$$

其中

$$\lambda_n = \min\Big\{\lambda_{n-1}, \frac{1}{\lambda_{\max}\{b_n^{-1}(x_{n-1}, \theta z_n + \alpha_{n-1})\}}, \frac{\delta_{W_n}}{\lambda_{\max}\{\Gamma_{W_n}^{-1}\}}\Big\},$$

$$c_n = \frac{\delta_{W_n}}{2}\|W_n^*\|^2 + \sigma_n^2 + c_{n-1}.$$

通过上述稳定性分析, 可以得到如下结论:

定理 1 　考虑系统 (1), 在假设 1∼假设 5 的前提下, 设计形如式 (20) 和 (35) 的虚拟控制量及形如 (42) 的控制量, 采用 RBF 神经网络参数调节律 (22), (37) 和 (44), 则闭环系统中所有信号均有界且全局指数收敛至原点附近的一个邻域.

4 　仿真分析

为验证本文算法的有效性, 对满足假设条件的如下"块控标准型"系统进行仿真研究:

$$\begin{cases} \dot{x}_{11} = x_{21},\ \dot{x}_{12} = x_{22}, \\ \dot{x}_{21} = x_{21}x_{22} + (1 + x_{11}^2)u_1, \\ \dot{x}_{22} = x_{22} + x_{21}x_{22} + (1 + x_{12}^2)u_2, \\ y = x_1. \end{cases}$$

其中: $x_1 = [x_{11} \ \ x_{12}]^{\mathrm{T}}$, $x_2 = [x_{21} \ \ x_{22}]^{\mathrm{T}}$ 为系统状态量; $u = [u_1 \ \ u_2]^{\mathrm{T}}$ 为系统控制量; y 为系统输出量. 仿真过程中, 选择初始状态量为 $x_1 = x_2 = [0, 0]^{\mathrm{T}}$, 并设 $\bar{b}_i(\bar{x}_i) = 1$. 这实际上是放宽了假设 1 的条件. 选取 $\Gamma_{W_1} = \Gamma_{W_2} = \mathrm{diag}\{1.0\}$, $\delta_{W_1} = \delta_{W_2} = 0.01$.

图 1∼图 3 为跟踪期望输出轨迹 $y_d^1 = [1 \ \ 1]^{\mathrm{T}}$ 的仿真结果. 其中: 图 1 和图 2 分别为 x_{11} 和 x_{12} 跟踪期望轨迹的情况, 实线为命令曲线, 虚线为跟踪曲线; 图 3 为控制量 u 的仿真结果. 图 4∼图 6 为跟踪期望输出轨迹 $y_d^2 = [\sin(0.5t) + \sin(0.25t) \ \ \cos(0.5t)]^{\mathrm{T}}$ 的仿真结果. 其中: 图 4 和图 5 分别为 x_{11} 和 x_{12} 跟踪期望轨迹的情况, 图 6 为控制量 u 的仿真结果.

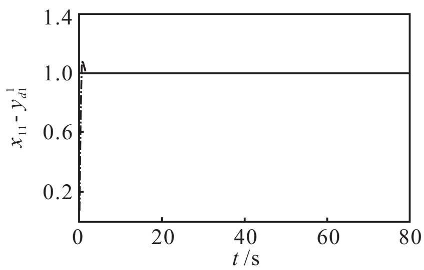

图 1 　x_{11} 跟踪期望轨迹 y_{d1}^1 的情况

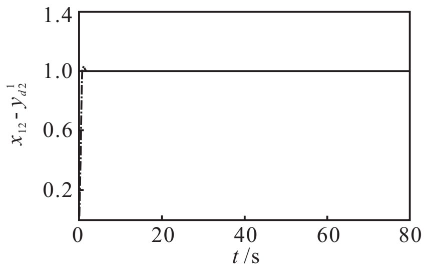

图 2 　x_{12} 跟踪期望轨迹 y_{d2}^1 的情况

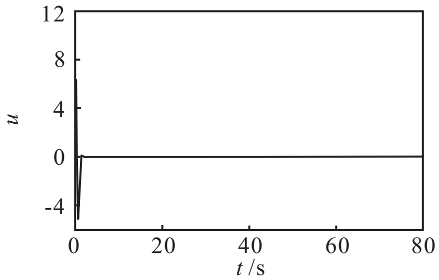

图 3　控制量 u 随时间变化的情况

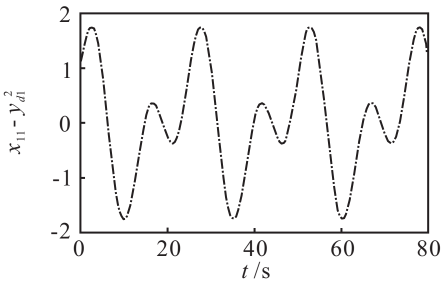

图 4　x_{11} 跟踪期望轨迹 y_{d1}^2 的情况

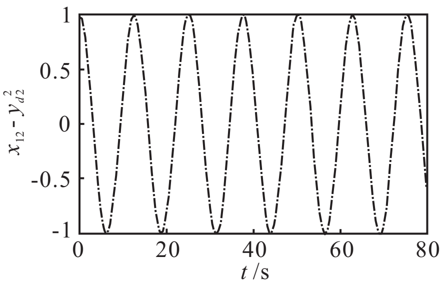

图 5　x_{12} 跟踪期望轨迹 y_{d2}^2 的情况

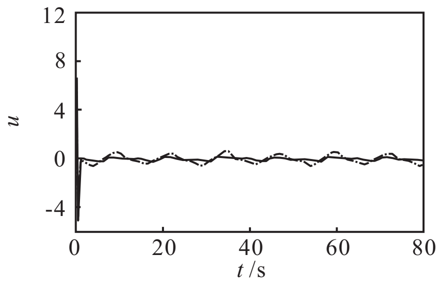

图 6　控制量 u 随时间变化的情况

从仿真结果可以看出, 由于缺乏非线性系统模型的先验知识, 在仿真的初始阶段存在一定的跟踪误差, 但经过一段时间的学习后, 跟踪效果很好, 充分表明了设计方法的有效性; 同时, 在跟踪不同的期望轨迹时, 不需要对控制器参数进行重新设计, 简化了设计过程, 充分表明了系统的鲁棒性.

5　结　论

本文对一类多输入多输出块控非线性系统的控制问题进行深入研究, 提出了一种基于反演自适应技术的控制器设计方案, 通过引入积分型Lyapunov函数, 在系统控制矩阵未知的情况下, 有效避免了估计控制参数时可能出现的奇异问题, 取得了全局稳定性. 该方法在利用系统已知信息方面有待进一步研究.

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