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Existence of positive periodic solution for a kind of predator-prey systems

LIU Xing-bo, HE Liu-rong

(*Department of Mathematics, East China Normal University, Shanghai* 200241, *China*)

Abstract: A non-autonomous predator–prey diffusive system of three species with delay was analyzed. By using Gaines and Mawhin's continuation theorem of coincidence degree theory, some sufficient conditions for the existence of positive periodic solution were established for the system.

Key words: predator-prey system; positive periodic solution; coincidence degree

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一类捕食-食饵系统正周期解的存在性

刘兴波, 何六荣

(华东师范大学 数学系, 上海 200241)

摘要: 讨论了具有时滞的非自治三种群捕食-食饵扩散系统. 利用重合度理论, 得到了正周期解存在的一些充分条件.

关键词: 捕食-食饵系统; 正周期解; 重合度

0 Introduction

The dynamic relationship between predators and their preys is an interesting mathematical problem and has attracted a great attention among mathematicians and biologists. Many good results have been obtained(see [1-4] and the references therein). Recently, the method of coincidence degree has been applied to study the existence of periodic solutions in predator-prey models (see [5-8]and the references therein).

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第一作者: 刘兴波, 男, 博士, 副教授, 研究方向为定性理论与分支理论.
E-mail: xbliu@math.ecnu.edu.cn.

第二作者: 何六荣, 女, 硕士, 研究方向为定性理论与分支理论.
E-mail: 51060601065@student.ecnu.edu.cn.

In this paper, we consider the following periodic predator-prey system with Michaelis-Menten type functional response

$$\begin{cases} \dot{x}_1 = x_1(t)\Big[a_1(t) - a_{11}(t)x_1(t) - \dfrac{k_1(t)x_1(t)x_3(t)}{n_1(t)x_3^2(t) + x_1^2(t)}\Big] + D_1(t)(x_2(t) - x_1(t)), \\ \dot{x}_2 = x_2(t)[a_2(t) - a_{22}(t)x_2(t)] + D_2(t)(x_1(t) - x_2(t)), \\ \dot{x}_3 = x_3(t)\Big[- a_3(t) + \dfrac{k_2(t)x_1^2(t-\tau_1)}{n_1(t)x_3^2(t-\tau_1) + x_1^2(t-\tau_1)} - \dfrac{k_3(t)x_4(t)x_3(t)}{n_2(t)x_4^2(t) + x_3^2(t)}\Big], \\ \dot{x}_4 = x_4(t)\Big[- a_4(t) + \dfrac{k_4(t)x_3^2(t-\tau_2)}{n_2(t)x_4^2(t-\tau_2) + x_3^2(t-\tau_2)}\Big], \end{cases} \tag{0.1}$$

where $x_i(t)$ $(i=1,2)$ represents the prey population in the ith patch; $x_i(t)$ $(i=3,4)$ represents the predator population; $\tau_i\ > 0$ $(i=1,2)$ is a constant delay due to gestation; $D_i(t)\ > 0$ $(i=1,2)$ is the dispersal rate of the prey in the ith path. The detailed biological meaning, we may refer to [8] and the references therein.

Suppose system (0.1) satisfies the following initial conditions:

$$x_i(s) = \varphi_i(s) \geqslant 0, \quad s \in [-\tau, 0], \quad \varphi_i(0) > 0, \quad i = 1,2,3,4, \quad \tau = \max\{\tau_1, \tau_2\}, \tag{0.2}$$

and $a_i(t),\ k_i(t)$ $(i=1,2,3,4)$, $n_i(t), D_i(t)$ $(i=1,2)$, $a_{11}(t),\ a_{22}(t)$ are positive continuous ω-periodic function. $\varphi_i, (i=1,2,3,4)$ are continuous functions. In what follows we will use the notations

$$\overline{f} = \frac{1}{\omega}\int_0^\omega f(t)\mathrm{d}t,\ \ f^M = \max_{t\in[0,\omega]} f(t),\ \ f^L = \min_{t\in[0,\omega]} f(t),$$

where $f(t)$ is a positive continuous ω-periodic function.

1 Existence of positive periodic solutions

Our purpose in this paper is, by using the continuation theorem of coincidence degree theory [9], to establish the existence conditions of at least one positive ω-periodic solution of model (0.1). Based on the coincidence degree theory, the case of existence of constant solution can not be excluded. But notice that in this case a very special relation must be satisfied for the periodic coefficient functions, then the coefficient functions can be taken to exclude this case. So we don't consider this case in our paper. For convenience, we introduce this theorem as follows

Let X and Y be two real Banach spaces, L: Dom $L \subset X \longrightarrow Y$ be a Fredholm mapping of index zero, and $P: X \longrightarrow X$, $Q: Y \longrightarrow Y$ be continuous projects such that Im P= Ker L, Ker Q = Im L, and X=Ker $L\oplus$ Ker P, Y=Im $L\oplus$Im Q. Denote by L_p the restriction of L to Dom $L\cap$ Ker P, by K_p: Im $L \longrightarrow$ Ker $P\cap$ Dom L the inverse of L_p, and by J: Im $L \longrightarrow$Ker P an isomorphism of Im Q onto Ker L.

Lemma 1.1[9] (Gaines and Mawhin's theorem) Let $\Omega \subset X$ be an open bounded set and let $N: X \longrightarrow Y$ a continuous operator which is L-*compact* on $\overline{\Omega}$ (*i.e.*, $QN: \overline{\Omega} \longrightarrow Y$ and $K_p(I-Q)N: \overline{\Omega} \longrightarrow X$ are compact). Assume

(a) for each $\lambda \in (0,1)$, $x \in \partial\Omega\cap$ Dom L, $Lx \neq \lambda Nx$;

(b) for each $x \in \partial\Omega \cap$ Ker L, $QNx \neq 0$;

(c) $\deg[JQN, \Omega \cap \operatorname{Ker} L, 0] \neq 0$.

Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap$ Dom L.

Next, we give the main result in this paper.

Theorem 1.1 Assume the following conditions are satisfied.

(N_1) $\left(\dfrac{a_1 - \frac{k_1}{2\sqrt{n_1}}}{a_{11}}\right)^L > 0$,

(N_2) $\overline{k}_4 - \overline{a}_4 > 0$,

(N_3) $\overline{k}_2 - \overline{a}_3 - \overline{\dfrac{k_3}{2\sqrt{n_2}}} > 0$,

then system (0.1) has at least one positive ω-periodic solution.

By making the change of variables; $x_i(t) = \mathrm{e}^{u_i(t)}$ $(i = 1, 2, 3, 4)$, the system (0.1) becomes

$$\begin{cases} \dot{u}_1 = a_1(t) - a_{11}(t)\mathrm{e}^{u_1(t)} - \dfrac{k_1(t)\mathrm{e}^{u_3(t)+u_1(t)}}{n_1(t)\mathrm{e}^{2u_3(t)} + \mathrm{e}^{2u_1(t)}} + D_1(t)(\mathrm{e}^{u_2(t)-u_1(t)} - 1), \\ \dot{u}_2 = a_2(t) - a_{22}(t)\mathrm{e}^{u_2(t)} + D_2(t)(\mathrm{e}^{u_1(t)-u_2(t)} - 1), \\ \dot{u}_3 = -a_3(t) + \dfrac{k_2(t)\mathrm{e}^{2u_1(t-\tau_1)}}{n_1(t)\mathrm{e}^{2u_3(t-\tau_1)} + \mathrm{e}^{2u_1(t-\tau_1)}} - \dfrac{k_3(t)\mathrm{e}^{u_3(t)+u_4(t)}}{n_2(t)\mathrm{e}^{2u_4(t)} + \mathrm{e}^{2u_3(t)}}, \\ \dot{u}_4 = -a_4(t) + \dfrac{k_4(t)\mathrm{e}^{2u_3(t-\tau_2)}}{n_2(t)\mathrm{e}^{2u_4(t-\tau_2)} + \mathrm{e}^{2u_3(t-\tau_2)}}. \end{cases} \tag{1.1}$$

It is easy to see that if system (1.1) has an ω-periodic solution $(u_1(t), u_2(t), u_3(t), u_4(t))^{\mathrm{T}}$, then $(x_1(t), x_2(t), x_3(t), x_4(t))^{\mathrm{T}}$ is a positive ω-periodic solution of system (0.1).

Lemma 1.2 Suppose $\lambda \in (0, 1)$ is a parameter, $(u_1(t), u_2(t), u_3(t), u_4(t))^{\mathrm{T}}$ is a ω-periodic solution of the system

$$\begin{cases} \dot{u}_1(t) = \lambda\left[a_1(t) - a_{11}(t)\mathrm{e}^{u_1(t)} - \dfrac{k_1(t)\mathrm{e}^{u_3(t)+u_1(t)}}{n_1(t)\mathrm{e}^{2u_3(t)} + \mathrm{e}^{2u_1(t)}} + D_1(t)(\mathrm{e}^{u_2(t)-u_1(t)} - 1)\right], \\ \dot{u}_2(t) = \lambda[a_2(t) - a_{22}(t)\mathrm{e}^{u_2(t)} + D_2(t)(\mathrm{e}^{u_1(t)-u_2(t)} - 1)], \\ \dot{u}_3(t) = \lambda\left[-a_3(t) + \dfrac{k_2(t)\mathrm{e}^{2u_1(t-\tau_1)}}{n_1(t)e^{2u_3(t-\tau_1)} + \mathrm{e}^{2u_1(t-\tau_1)}} - \dfrac{k_3(t)\mathrm{e}^{u_3(t)+u_4(t)}}{n_2(t)\mathrm{e}^{2u_4(t)} + \mathrm{e}^{2u_3(t)}}\right], \\ \dot{u}_4(t) = \lambda\left[-a_4(t) + \dfrac{k_4(t)\mathrm{e}^{2u_3(t-\tau_2)}}{n_2(t)\mathrm{e}^{2u_4(t-\tau_2)} + \mathrm{e}^{2u_3(t-\tau_2)}}\right], \end{cases} \tag{1.2}$$

then $|u_1(t)| + |u_2(t)| + |u_3(t)| + |u_4(t)| \leqslant R$, where $R = 2R_1 + R_2 + R_3$, and

$$R_1 = \max\left\{|\ln\left(\frac{a_1}{a_{11}}\right)^M|, |\ln\left(\frac{a_2}{a_{22}}\right)^M|, |\ln\left(\frac{a_2}{a_{22}}\right)^L|, |\ln\left(\frac{a_1 - \frac{k_1}{2\sqrt{n_1}}}{a_{11}}\right)^L|\right\},$$

$$R_2 = \frac{1}{2}\max\left|\ln\frac{k_2^M - \overline{a}_3}{\overline{a}_3 n_1^L}\right| + R_1 + \omega\overline{k}_2, \quad R_3 = \frac{1}{2}\max\left|\ln\frac{k_4^M - \overline{a}_4}{\overline{a}_4 n_2^L}\right| + R_2 + \omega\overline{k}_4.$$

Proof Since $u_i(t)$ $(i = 1, 2, 3, 4)$ are ω-periodic functions, we only need to prove the result in $[0, \omega]$. Choose $t_i \in [0, \omega] (i = 1, 2)$ such that $u_i(t_i) = \max\limits_{t \in [0, \omega]} u_i(t), (i = 1, 2)$, then it is clear that $\dot{u}_i(t_i) = 0$, $(i = 1, 2)$. In view of this and the first two equations of system (1.2), we have

$$\begin{cases} a_1(t_1) - a_{11}(t_1)\mathrm{e}^{u_1(t_1)} - \dfrac{k_1(t_1)\mathrm{e}^{u_3(t_1)+u_1(t_1)}}{n_1(t_1)\mathrm{e}^{2u_3(t_1)} + \mathrm{e}^{2u_1(t_1)}} + D_1(t_1)(\mathrm{e}^{u_2(t_1)-u_1(t_1)} - 1) = 0, \\ a_2(t_2) - a_{22}(t_2)\mathrm{e}^{u_2(t_2)} + D_2(t_2)(\mathrm{e}^{u_1(t_2)-u_2(t_2)} - 1) = 0. \end{cases} \tag{1.3}$$

If $u_1(t_1) > u_2(t_2)$, then $u_1(t_1) > u_2(t_1)$. It follows from (1.3) that $a_{11}(t_1)e^{u_1(t_1)} \leqslant a_1(t_1)$, which implies

$$u_2(t_2) < u_1(t_1) \leqslant \ln \frac{a_1(t_1)}{a_{11}(t_1)} \leqslant \ln \Big(\frac{a_1}{a_{11}}\Big)^M. \tag{1.4}$$

Similarly, if $u_1(t_1) < u_2(t_2)$, then $u_1(t_2) < u_2(t_2)$. By the second equation of (1.3), we have

$$u_1(t_1) < u_2(t_2) \leqslant \ln \frac{a_2(t_2)}{a_{22}(t_2)} \leqslant \ln \Big(\frac{a_2}{a_{22}}\Big)^M. \tag{1.5}$$

Now choose $s_i \in [0, \omega]$ $(i = 1, 2)$, such that $u_i(s_i) = \min\limits_{t\in[0,\omega]} u_i(t), (i = 1, 2)$, then $\dot{u}_i(s_i) = 0$. Similar to the discussion above, We can obtain

$$u_2(s_2) > u_1(s_1) \geqslant \ln \left(\frac{a_1 - \frac{k_1}{2\sqrt{n_1}}}{a_{11}}\right)^L, \quad \text{if} \quad u_1(s_1) < u_2(s_2); \tag{1.6}$$

$$u_1(s_1) > u_2(s_2) \geqslant \ln \frac{a_2(s_2)}{a_{22}(s_2)} \geqslant \ln \Big(\frac{a_2}{a_{22}}\Big)^L, \quad \text{if} \quad u_1(s_1) > u_2(s_2). \tag{1.7}$$

Take

$$R_1 = \max\left\{\Big|\ln \Big(\frac{a_1}{a_{11}}\Big)^M\Big|, \Big|\ln \Big(\frac{a_2}{a_{22}}\Big)^M\Big|, \Big|\ln \Big(\frac{a_2}{a_{22}}\Big)^L\Big|, \left|\ln \left(\frac{a_1 - \frac{k_1}{2\sqrt{n_1}}}{a_{11}}\right)^L\right|\right\}.$$

In view of (1.4)-(1.7), we have $|u_1(t)| < R_1$, $\quad |u_2(t)| < R_1$. On the other hand, by integrating the third and fourth equations of (1.2) over the interval $[0, \omega]$, we obtain

$$\begin{cases} \displaystyle\int_0^\omega \frac{k_2(t+\tau_1)\mathrm{e}^{2u_1(t)}}{n_1(t+\tau_1)\mathrm{e}^{2u_3(t)} + \mathrm{e}^{2u_1(t)}}\mathrm{d}t = \int_0^\omega \left[a_3(t) + \frac{k_3(t)\mathrm{e}^{u_3(t)+u_4(t)}}{n_2(t)\mathrm{e}^{2u_4(t)} + \mathrm{e}^{2u_3(t)}}\right]\mathrm{d}t, \\ \displaystyle\int_0^\omega \frac{k_4(t+\tau_2)\mathrm{e}^{2u_3(t)}}{n_2(t+\tau_2)\mathrm{e}^{2u_4(t)} + \mathrm{e}^{2u_3(t)}}\mathrm{d}t = \int_0^\omega a_4(t)\mathrm{d}t. \end{cases} \tag{1.8}$$

Using the mean value theorem for (1.8), it is clear that there exist two points $t_i^* \in [0, \omega], i = 1, 2$ such that

$$\frac{k_2(t_1^*+\tau_1)\mathrm{e}^{2u_1(t_1^*)}}{n_1(t_1^*+\tau_1)\mathrm{e}^{2u_3(t_1^*)} + \mathrm{e}^{2u_1(t_1^*)}} > \overline{a}_3, \qquad \frac{k_4(t_2^*+\tau_2)\mathrm{e}^{2u_3(t_2^*)}}{n_2(t_2^*+\tau_2)\mathrm{e}^{2u_4(t_2^*)} + \mathrm{e}^{2u_3(t_2^*)}}\mathrm{d}t = \overline{a}_4.$$

Hence, due to (N_2) and (N_3), we can get

$$|u_3(t_1^*)| < \frac{1}{2}\max\left|\ln \frac{k_2^M - \overline{a}_3}{\overline{a}_3 n_1^L}\right| + R_1, \ \ |u_4(t_2^*)| < \frac{1}{2}\max\left|\ln \frac{k_4^M - \overline{a}_4}{\overline{a}_4 n_2^L}| + |u_3(t_1^*)\right|. \tag{1.9}$$

Since for any $t \in [0,\omega]$,

$$|u_3(t)| \leqslant |u_3(t_1^*)| + \int_0^\omega |\dot{u}_3(s)|\mathrm{d}s, \quad |u_4(t)| \leqslant |u_4(t_2^*)| + \int_0^\omega |\dot{u}_4(s)|\mathrm{d}s.$$

Based on the third and fourth equation of (1.2), we obtain

$$|u_3(t)| \leqslant \frac{1}{2}\max_{t\in[0,\omega]}\left|\ln\frac{k_2^M - \overline{a}_3}{\overline{a}_3 n_1^L}\right| + R_1 + \omega\overline{k}_2 \triangleq R_2,$$

$$|u_4(t)| \leqslant \frac{1}{2}\max_{t\in[0,\omega]}\left|\ln\frac{k_4^M - \overline{a_4}}{\overline{a}_4 n_2^L}\right| + R_2 + \omega\overline{k}_4 \triangleq R_3.$$

Thus if we choose $R = 2R_1 + R_2 + R_3$, then $|u_1(t)| + |u_2(t)| + |u_3(t)| + |u_4(t)| \leqslant R$.

Lemma 1.3 Suppose $\mu \in [0,1]$ is a parameter and $(u_1,u_2,u_3,u_4)^{\mathrm{T}}$ is a constant solution of the system

$$\begin{cases} \overline{a}_1 - \overline{a}_{11}\mathrm{e}^{u_1} - \mu\left(\dfrac{1}{\omega}\displaystyle\int_0^\omega \frac{k_1(t)\mathrm{e}^{u_3+u_1}}{n_1(t)\mathrm{e}^{2u_3}+\mathrm{e}^{2u_1}}\mathrm{d}t + \overline{D}_1(\mathrm{e}^{u_2-u_1}-1)\right) = 0, \\ \overline{a}_2 - \overline{a}_{22}\mathrm{e}^{u_2} + \mu\overline{D}_2(\mathrm{e}^{u_1-u_2}-1) = 0, \\ -\overline{a}_3 + \dfrac{1}{\omega}\displaystyle\int_0^\omega \frac{k_2(t)\mathrm{e}^{2u_1}}{n_1(t)\mathrm{e}^{2u_3}+\mathrm{e}^{2u_1}}\mathrm{d}t - \mu\left(\dfrac{1}{\omega}\displaystyle\int_0^\omega \frac{k_3(t)\mathrm{e}^{u_3+u_4}}{n_2(t)\mathrm{e}^{2u_4}+\mathrm{e}^{2u_3}}\mathrm{d}t\right) = 0, \\ -\overline{a}_4 + \dfrac{1}{\omega}\displaystyle\int_0^\omega \frac{k_4(t)\mathrm{e}^{2u_3}}{n_2(t)\mathrm{e}^{2u_4}+\mathrm{e}^{2u_3}}\mathrm{d}t = 0, \end{cases} \tag{1.10}$$

then $|u_1| + |u_2| + |u_3| + |u_4| \leqslant R_0$, where

$$R_0 = 2R_4 + R_5 + R_6, \ \ R_4 = \max\left\{\left|\ln\frac{\overline{a}_1}{\overline{a}_{11}}\right|, \left|\ln\frac{\overline{a}_2}{\overline{a}_{22}}\right|, \left|\ln\frac{\overline{a_1 - \frac{k_1}{2\sqrt{n_1}}}}{\overline{a}_{11}}\right|\right\},$$

$$R_5 = \max\{|R_{50}|, |R_{51}|\}, \ \ R_{50} = \frac{1}{2}\ln\frac{\left(\overline{k}_2 - \overline{a}_3 - \overline{\frac{k_3}{2\sqrt{n_2}}}\right)B_1}{\left(\overline{\frac{k_3}{2\sqrt{n_2}}} + \overline{a}_3\right)n_1^M},$$

$$R_{51} = \frac{1}{2}\ln\frac{(\overline{k}_2 - \overline{a}_3)B_1}{\overline{a}_3 n_1^L}, \ \ B_1 = \mathrm{e}^{2u_1},$$

$$R_6 = \max\{\,|R_{60}|, |R_{61}|\,\}, \quad R_{60} = \frac{1}{2}\ln\frac{(\overline{k}_4 - \overline{a}_4)B_2}{\overline{a}_4 n_2^M},$$

$$R_{61} = \frac{1}{2}\ln\frac{(\overline{k}_4 - \overline{a}_4)B_2}{\overline{a}_4 n_2^L}, \quad B_2 = \mathrm{e}^{2u_3}.$$

Proof If $u_1 \geqslant u_2$, then from the first two equations of (1.10), we have

$$\begin{cases} \overline{a}_{11}\mathrm{e}^{u_1} = \overline{a}_1 - \mu\left(\dfrac{1}{\omega}\displaystyle\int_0^\omega \frac{k_1(t)\mathrm{e}^{u_3+u_1}}{n_1(t)\mathrm{e}^{2u_3}+\mathrm{e}^{2u_1}}\mathrm{d}t + \overline{D}_1(\mathrm{e}^{u_2-u_1}-1)\right) < \overline{a}_1, \\ \overline{a}_{22}\mathrm{e}^{u_2} = \overline{a}_2 + \mu\overline{D}_2(\mathrm{e}^{u_1-u_2}-1) > \overline{a}_2, \end{cases}$$

which implies that $\ln\frac{\overline{a}_2}{\overline{a}_{22}} \leqslant u_2 \leqslant u_1 \leqslant \ln\frac{\overline{a}_1}{\overline{a}_{11}}$. Similarly, if $u_1 \leqslant u_2$, it follows from (N_1) that

$$\ln\frac{\overline{a_1 - \dfrac{k_1}{2\sqrt{n_1}}}}{\overline{a}_{11}} \leqslant u_1 \leqslant u_2 \leqslant \ln\frac{\overline{a}_2}{\overline{a}_{22}}.$$

Thus, if we choose

$$R_4 = \max\left\{\left|\ln\frac{\overline{a}_1}{\overline{a}_{11}}\right|, \left|\ln\frac{\overline{a}_2}{\overline{a}_{22}}\right|, \left|\ln\frac{\overline{a_1 - \frac{k_1}{2\sqrt{n_1}}}}{\overline{a}_{11}}\right|\right\},$$

then, $|u_1| + |u_2| \leqslant 2R_4$. On the other hand, under the conditions (N_2) and (N_3), the third and fourth equation of (1.10) imply $R_{50} \leqslant u_3 \leqslant R_{51}$, $R_{60} \leqslant u_4 \leqslant R_{61}$, where

$$R_{50} = \frac{1}{2}\ln\frac{\left(\overline{k}_2 - \overline{a}_3 - \overline{\frac{k_3}{2\sqrt{n_2}}}\right)B_1}{\left(\overline{\frac{k_3}{2\sqrt{n_2}}} + \overline{a}_3\right)n_1^M}, \quad R_{51} = \frac{1}{2}\ln\frac{(\overline{k}_2 - \overline{a}_3)B_1}{\overline{a}_3 n_1^L},$$

$$R_{60} = \frac{1}{2}\ln\frac{(\overline{k}_4 - \overline{a}_4)B_2}{\overline{a}_4 n_2^M}, \quad R_{61} = \frac{1}{2}\ln\frac{(\overline{k}_4 - \overline{a}_4)B_2}{\overline{a}_4 n_2^L}, \quad B_1 = \mathrm{e}^{2u_1}, \quad B_2 = \mathrm{e}^{2u_3}.$$

It follows that $|u_3| \leqslant \max\{\,|R_{50}|, |R_{51}|\,\}$, $|u_4| \leqslant \max\{\,|R_{60}|, |R_{61}|\,\}$.

If we choose $R_0 = 2R_4 + R_5 + R_6$, where R_4, R_5, R_6, B_1, B_2 are defined as above, then $|u_1| + |u_2| + |u_3| + |u_4| \leqslant R_0$, This completes the proof of this lemma.

Proof of theorem 1.1 From previous discussion, we know that it suffices to show that the system (1.1) has at least one ω-periodic solution. To this end, we take

$$X = Y = \{(u_1(t), u_2(t), u_3(t), u_4(t))^{\mathrm{T}} \in C^1(\mathbf{R}, \mathbf{R}^4) | u_i(t+\omega) = u_i(t), i = 1, 2, 3, 4\},$$

and $\|\, (u_1(t), u_2(t), u_3(t), u_4(t))^{\mathrm{T}} \,\| = \sum_{i=1}^{4} \max\limits_{t\in[0,\omega]} |u_i(t)|$. With this norm, X and Y are Banach space. Let

$$L : \mathrm{Dom}\ L \cap X \longrightarrow Y, L(u_1(t), u_2(t), u_3(t), u_4(t))^{\mathrm{T}} = (\dot{u}_1(t), \dot{u}_2(t), \dot{u}_3(t), \dot{u}_4(t))^{\mathrm{T}},$$

where $\mathrm{Dom}\ L = \{(u_1(t), u_2(t), u_3(t), u_4(t))^{\mathrm{T}} \in C^1(\mathbf{R}, \mathbf{R}^4)\}$, and $N : X \longrightarrow X$,

$$N\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix} = \begin{bmatrix} a_1(t) - a_{11}(t)\mathrm{e}^{u_1(t)} - \dfrac{k_1(t)\mathrm{e}^{u_3(t)+u_1(t)}}{n_1(t)\mathrm{e}^{2u_3(t)} + \mathrm{e}^{2u_1(t)}} + D_1(t)(\mathrm{e}^{u_2(t)-u_1(t)} - 1) \\ a_2(t) - a_{22}(t)\mathrm{e}^{u_2(t)} + D_2(t)(\mathrm{e}^{u_1(t)-u_2(t)} - 1) \\ -a_3(t) + \dfrac{k_2(t)\mathrm{e}^{2u_1(t-\tau_1)}}{n_1(t)\mathrm{e}^{2u_3(t-\tau_1)} + \mathrm{e}^{2u_1(t-\tau_1)}} - \dfrac{k_3(t)\mathrm{e}^{u_3(t)+u_4(t)}}{n_2(t)\mathrm{e}^{2u_4(t)} + \mathrm{e}^{2u_3(t)}} \\ -a_4(t) + \dfrac{k_4(t)\mathrm{e}^{2u_3(t-\tau_2)}}{n_2(t)\mathrm{e}^{2u_4(t-\tau_2)} + \mathrm{e}^{2u_3(t-\tau_2)}} \end{bmatrix}.$$

Then system (1.1) can be written in the form $Lu = Nu$, $u \in \mathrm{Dom}\ L \cap X$. Clearly,

$$\mathrm{Im}\ L = \{(u_1(t), u_2(t), u_3(t), u_4(t))^{\mathrm{T}} \in X : \int_0^{\omega} u_i(t)\mathrm{d}t = 0, i = 1, 2, 3, 4\}$$

is closed in X, Ker $L=\mathbf{R}^4$, dim Ker L=codim Im $L=4$. Therefore, L is Fredholm mapping of index zero. Define two projects $P,Q:X\longrightarrow X$ as

$$P\begin{bmatrix}u_1(t)\\u_2(t)\\u_3(t)\\u_4(t)\end{bmatrix}=Q\begin{bmatrix}u_1(t)\\u_2(t)\\u_3(t)\\u_4(t)\end{bmatrix}=\begin{bmatrix}\overline{u}_1\\\overline{u}_2\\\overline{u}_3\\\overline{u}_4\end{bmatrix},$$

then Im P=Ker L, Ker Q=Im L, and X=Ker $L\oplus$ Ker P=Im $L\oplus$ Im Q. The isomorphism J from Im Q into Ker L can be the identity mapping since Im Q=Ker L. The inverse K_p of L_p is given by K_p: Im $L\longrightarrow$ Dom $L\cap$ Ker P,

$$K_p\begin{bmatrix}u_1(t)\\u_2(t)\\u_3(t)\\u_4(t)\end{bmatrix}=\begin{bmatrix}\int_0^t u_1(s)\mathrm{d}s-\frac{1}{\omega}\int_0^\omega\int_0^s u_1(s)\mathrm{d}t\mathrm{d}s\\\int_0^t u_2(s)\mathrm{d}s-\frac{1}{\omega}\int_0^\omega\int_0^s u_2(s)\mathrm{d}t\mathrm{d}s\\\int_0^t u_3(s)\mathrm{d}s-\frac{1}{\omega}\int_0^\omega\int_0^s u_3(s)\mathrm{d}t\mathrm{d}s\\\int_0^t u_4(s)\mathrm{d}s-\frac{1}{\omega}\int_0^\omega\int_0^s u_4(s)\mathrm{d}t\mathrm{d}s\end{bmatrix}.$$

By the Lebesgue convergence theorem, it is easy to see that QN and $K_p(I-Q)N$ are continuous and furthermore, by the Arzera-Ascoli theorem, $QN(\overline{\Omega})$ and $K_p(I-Q)N(\overline{\Omega})$are compact for any open bounded set $\Omega\subset X$. Hence, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega\subset X$. Particularly we take

$$\Omega=\{(u_1(t),u_2(t),u_3(t),u_4(t))^{\mathrm{T}}\in X:\|(u_1(t),u_2(t),u_3(t),u_4(t))^{\mathrm{T}}\|<R+R_0\},$$

where R and R_0 are defined by Lemma 1.2 and 1.3. Obviously, N is L-compact on $\overline{\Omega}$.

Next we show that the three conditions of Lemma 1.1 hold.

(1) Due to Lemma 1.2, we conclude that for each $\lambda\in(0,1)$, $u\in\partial\Omega\cap$Dom L, $Lu\neq\lambda Nu$.

(2) When $(u_1(t),u_2(t),u_3(t),u_4(t))^{\mathrm{T}}\in\partial\Omega\cap$ Ker L, $(u_1(t),u_2(t),u_3(t),u_4(t))^{\mathrm{T}}$ is a constant vector in $\mathbf{R}^4$ with the norm $R+R_0$. If $QN(u_1,u_2,u_3,u_4)=0$, then $(u_1,u_2,u_3,u_4)^{\mathrm{T}}$ is a constant solution of system (1.10) with $\mu=1$. From Lemma 1.3, we have $\|(u_1,u_2,u_3,u_4)^{\mathrm{T}}\|<R_0$, this contradiction implies that for each $x\in\partial\Omega\cap$ Ker L, $QN(x)\neq0$.

(3) By the definition of $QN(u)$, we know that there exists ξ_i $(i=1,2,3,4)\in[0,\omega]$, such that

$$QN(u)=\begin{bmatrix}\overline{a}_1-\overline{a}_{11}\mathrm{e}^{u_1}-\dfrac{\overline{k}_1\mathrm{e}^{u_3+u_1}}{n_1(\xi_1)\mathrm{e}^{2u_3}+\mathrm{e}^{2u_1}}+\overline{D}_1(\mathrm{e}^{u_2-u_1}-1)\\\overline{a}_2-\overline{a}_{22}\mathrm{e}^{u_2}+\overline{D}_2(\mathrm{e}^{u_1-u_2}-1)\\-\overline{a}_3+\dfrac{\overline{k}_2\mathrm{e}^{2u_1}}{n_1(\xi_2)\mathrm{e}^{2u_3}+\mathrm{e}^{2u_1}}-\dfrac{\overline{k}_3\mathrm{e}^{u_3+u_4}}{n_2(\xi_3)\mathrm{e}^{2u_4}+\mathrm{e}^{2u_3}}\\-\overline{a}_4+\dfrac{\overline{k}_4\mathrm{e}^{2u_3}}{n_2(\xi_4)\mathrm{e}^{2u_4}+\mathrm{e}^{2u_3}}\end{bmatrix}.$$

In order to verify the condition (c) in Lemma 1.1, we define $\phi : \mathrm{Dom}\, L \times [0,1] \longrightarrow X$.

$$\phi(u_1,u_2,u_3,u_4,\mu) = \begin{bmatrix} \overline{a}_1 - \overline{a}_{11}\mathrm{e}^{u_1} \\ \overline{a}_2 - \overline{a}_{22}\mathrm{e}^{u_2} \\ -\overline{a}_3 + \dfrac{\overline{k}_2\mathrm{e}^{2u_1}}{n_1(\xi_2)\mathrm{e}^{2u_3}+\mathrm{e}^{2u_1}} \\ -\overline{a}_4 + \dfrac{\overline{k}_4\mathrm{e}^{2u_3}}{n_2(\xi_4)\mathrm{e}^{2u_4}+\mathrm{e}^{2u_3}} \end{bmatrix} + \mu \begin{bmatrix} \dfrac{-\overline{k}_1\mathrm{e}^{u_3+u_1}}{n_1(\xi_1)\mathrm{e}^{2u_3}+\mathrm{e}^{2u_1}} + \overline{D}_1(\mathrm{e}^{u_2-u_1}-1) \\ \overline{D}_2(\mathrm{e}^{u_1-u_2}-1) \\ \dfrac{-\overline{k}_3\mathrm{e}^{u_3+u_4}}{n_2(\xi_3)\mathrm{e}^{2u_4}+\mathrm{e}^{2u_3}} \\ 0 \end{bmatrix},$$

where $\mu \in [0,1]$ is a parameter. When $(u_1,u_2,u_3,u_4)^{\mathrm{T}} \in \partial\Omega\cap$ Ker $L = \Omega \cap \mathbf{R}^4$, $(u_1,u_2,u_3,u_4)^{\mathrm{T}}$ is a constant vector with $\|(u_1,u_2,u_3,u_4)^{\mathrm{T}}\| = R + R_0$. From Lemma 1.3, we know $\phi(u_1,u_2,u_3,u_4,\mu) \neq (0,0,0,0)^{\mathrm{T}}$ on $\partial\Omega\cap$ Ker L. Because the algebra equations

$$\begin{cases} \overline{a}_1 - \overline{a}_{11}\mathrm{e}^{u_1} = 0, \\ \overline{a}_2 - \overline{a}_{22}\mathrm{e}^{u_2} = 0, \\ -\overline{a}_3 + \dfrac{\overline{k}_2\mathrm{e}^{2u_1}}{n_1(\xi_2)\mathrm{e}^{2u_3}+\mathrm{e}^{2u_1}} = 0, \\ -\overline{a}_4 + \dfrac{\overline{k}_4\mathrm{e}^{2u_3}}{n_2(\xi_4)\mathrm{e}^{2u_4}+\mathrm{e}^{2u_3}} = 0, \end{cases}$$

have a unique solution $(u_1^*,u_2^*,u_3^*,u_4^*) \in \Omega\cap$ Ker L. Hence, according to topological degree theory, we have

$$\begin{aligned} &\deg(JQN(u_1,u_2,u_3,u_4)^{\mathrm{T}}, \Omega\cap \mathrm{Ker}\, L, (0,0,0,0)^{\mathrm{T}}) \\ &= \deg(\phi(u_1,u_2,u_3,u_4,1), \Omega\cap \mathrm{Ker}\, L, (0,0,0,0)^{\mathrm{T}}) \\ &= \deg(\phi(u_1,u_2,u_3,u_4,0), \Omega\cap \mathrm{Ker}\, L, (0,0,0,0)^{\mathrm{T}}) \\ &= \mathrm{sign}\Big(\frac{4\overline{a}_{11}\overline{a}_{22}\overline{k}_2\overline{k}_4 n_1(\xi_2)n_2(\xi_4)\mathrm{e}^{3u_1^*+u_2^*+4u_3^*+2u_4^*}}{(n_1(\xi_2)\mathrm{e}^{2u_3^*}+\mathrm{e}^{2u_1^*})^2(n_2(\xi_4)\mathrm{e}^{2u_4^*}+\mathrm{e}^{2u_3^*})^2}\Big) \neq 0. \end{aligned}$$

Now we have verified all the conditions of Lemma 1.1, then system (1.1) has at least one ω-periodic solution. Therefore system (0.1) has at least one positive ω-periodic solution. This completes the proof of our main results.

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