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Left-connectedness of left cells in the Weyl Group of type E_6

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Abstract: We showed that all the left cells in the Weyl group E_6 were left-connected, verifying a conjecture of Lusztig in our case.

Key words: Weyl group; left cells; two-sided cells; left-connectedness
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E_6 型 Weyl 群中左胞腔的左连通性

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摘要: 首先通过计算机编程找出 E_6 型 Weyl 群左胞腔的所有极短元, 利用这些极短元证明了 E_6 型 Weyl 群的所有左胞腔都是左连通的, 从而证明了 Lusztig 关于左胞腔左连通性的一个猜想在 E_6 型 Weyl 群中是成立的.

关键词: Weyl 群; 左胞腔; 双边胞腔; 左连通性

0 Introduction

Let W be a Coxeter group with S its distinguished generator set. In [1], Kazhdan and Lusztig introduced the concept of left, right and two-sided cells of W, which provide certain representations of W and the associated Hecke algebra $\mathcal{H}$. Lusztig conjectured in [2] that any left cell of an affine Weyl group is left-connected. The left-connectedness should be a good structural property for a left cell. Though it has been verified in many special cases (see [3-6]), the conjecture still remains open up to now.

In the present paper, we consider the case where W is the Weyl group of type E_6 (we shall denote W simply by E_6). Tong described the left cells of E_6 by finding a representative set for those left cells and by drawing all the left cell graphs in [7]. Then Shi designed some

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algorithms and provided some criteria in his study of left-connectedness of left cells in [6,8]. Based on these results, we shall prove that all the left cells of the group E_6 are left-connected.

The contents of the paper are organized as follows. Section 1 is served as preliminaries, where we collect some concepts, terms and known results. Then we prove the left-connectedness for all the left cells in E_6 in Section 2.

1 Preliminaries

Let W be a Coxeter group with S its distinguished generator set. Let $\leqslant$ be the Bruhat-Chevalley order and ℓ the length function on W.

Let $A=\mathbb{Z}[q]$ be the polynomial ring in an indeterminate q with integer coefficients. To any $y,w\in W$, we associate some $P_{y,w}\in A$, called a *Kazhdan-Lusztig polynomial*, satisfying that $\deg P_{y,w}\leqslant\frac{1}{2}(\ell(w)-\ell(y)-1)$ if $y<w$, $P_{y,y}=1$ and $P_{y,w}=0$ if $y\nleqslant w$ (see [1]).

Write y—w for $y\neq w$ in W, if either $\deg P_{y,w}$ or $\deg P_{w,y}$ reaches its upper bound $\frac{1}{2}(|\ell(w)-\ell(y)|-1)$, where $|z|$ denotes the absolute value of $z\in\mathbb{Q}$. We have the following simple and useful fact:

(1.1.1) If $y<w$ and $\ell(w)-\ell(y)=1$, then y—w.

The preorders $\underset{L}{\leqslant}$, $\underset{R}{\leqslant}$, $\underset{LR}{\leqslant}$ and the associated equivalence relations $\underset{L}{\sim}$, $\underset{R}{\sim}$, $\underset{LR}{\sim}$ on W are defined as in [1]. The equivalence classes of W with respect to $\underset{L}{\sim}$ (respectively, $\underset{R}{\sim}$, $\underset{LR}{\sim}$) are called left cells (respectively, right cells, two-sided cells). The preorder relation $\underset{L}{\leqslant}$ (respectively, $\underset{R}{\leqslant}$, $\underset{LR}{\leqslant}$) on the elements of W induces a partial order relation on the left (respectively, right, two-sided) cells of W.

Let $\mathcal{L}(x)=\{s\in S\mid sx<x\}$ and $\mathcal{R}(x)=\{s\in S\mid xs<x\}$ for any $x\in W$. If $x,y\in W$ satisfy $x\underset{L}{\leqslant}y$ (respectively, $x\underset{R}{\leqslant}y$), then $\mathcal{R}(x)\supseteq\mathcal{R}(y)$ (respectively, $\mathcal{L}(x)\supseteq\mathcal{L}(y)$). In particular, if $x\underset{L}{\sim}y$ (respectively, $x\underset{R}{\sim}y$), then $\mathcal{R}(x)=\mathcal{R}(y)$ (respectively, $\mathcal{L}(x)=\mathcal{L}(y)$). So for any left (respectively, right) cell Γ of W, we may define $\mathcal{R}(\Gamma):=\mathcal{R}(x)$ (respectively, $\mathcal{L}(\Gamma):=\mathcal{L}(x)$) for any $x\in\Gamma$ (see [1, Proposition 2.4]).

In the remaining part of the section, we always assume W to be a Weyl group unless otherwise specified.

In [9], Lusztig defined a function $a:W\to\mathbb{N}$ and proved the following results involving the function a.

(1) For any $z\in W$, $a(z)\leqslant\frac{1}{2}|\Phi|$, where Φ is the root system of W.

(2) If $x,y\in W$ satisfy $x\underset{LR}{\leqslant}y$, then $a(x)\geqslant a(y)$. In particular, The condition $x\underset{LR}{\sim}y$ implies $a(x)=a(y)$. So we can define $a(\Gamma):=a(x)$ for any $x\in\Gamma$, where Γ is a left (respectively, right, two-sided) cell of W.

(3) If $x\underset{L}{\leqslant}y$ (respectively, $x\underset{R}{\leqslant}y$) and $a(x)=a(y)$, then $x\underset{L}{\sim}y$ (respectively, $x\underset{R}{\sim}y$).

(4) For any $I\subseteq S$, let w_I be the longest element in the subgroup W_I of W generated by I, then $a(w_I)=\ell(w_I)$.

(5) For any nonnegative integer i, let $W_{(i)}=\{w\in W|a(w)=i\}$, then $W_{(i)}$ is either empty or a union of some two-sided cells of W.

(6) If $W_{(i)}$ contains an element of the form w_I for some $I \subset S$, then $\{w \in W_{(i)} | \mathcal{R}(w) = I\}$ forms a single left cell of W.

(7) For any $x, y, z \in W$, we denote $z = x \cdot y$ if $z = xy$ and $\ell(z) = \ell(x) + \ell(y)$. In this case, we have $z \underset{R}{\leqslant} x$ and $z \underset{L}{\leqslant} y$, hence $a(z) \geqslant a(x), a(y)$. In particular, if $I = \mathcal{R}(z)$ (respectively, $I = \mathcal{L}(z)$), then $a(z) \geqslant \ell(w_I)$.

Let G be the connected reductive complex algebraic group with type dual to that of W. Then the following result is due to Barbasch, Vogan and Lusztig.

Lemma 1.1(see [10-13]) *There is a bijection* $\mathbf{u} \mapsto c(\mathbf{u})$ *from the set of special unipotent conjugacy classes in* G *to the set of two-sided cells in* W. *This bijection satisfies* $a(c(\mathbf{u})) = \dim \mathfrak{B}_{\mathbf{u}}$, *where* u *is any element in* $\mathbf{u}$ *and* $\dim \mathfrak{B}_u$ *is the dimension of the variety of all Borel subgroups of* G *containing* u.

Let K be a non-empty subset of W. Two elements $x, y \in K$ are called *left-connected in* K, written $x \underset{K}{—} y$, if there exists a sequence $x_0 = x, x_1, \cdots, x_r = y$ in K with some $r \geqslant 0$ such that $x_{i-1} x_i^{-1} \in S$ for $1 \leqslant i \leqslant r$. This defines an equivalence relation on K. Each equivalence class of K with respect to $\underset{K}{—}$ is called a *left-connected component* of K. The set K is called *left-connected*, if K consists of a single left-connected component.

$w \in W$ is said *fully commutative*, if w has no expression of the form $w = x \cdot w_{st} \cdot y$, where w_{st} is the longest element in the subgroup of W generated by s, t with $\ell(w_{st}) > 2$ (see [5]). We have the following result involving fully commutative elements.

Lemma 1.2([5, Theorem 2.1]) *Any left cell of* W *containing a fully commutative element is left-connected.*

Now assume the Weyl group (W, S) to be irreducible and of simply-laced type, where, by simply-laced type, we mean that the order $o(st)$ of the product st, for any $s \neq t$ in S, is not greater than 3, i.e., W is of type A, D or E. Let $s, t \in S$ satisfy $o(st) = 3$. By a *right* $\{s, t\}$*-string*, we mean a set $\{ys, yst\}$ with $y \in W$ satisfying $\mathcal{R}(y) \bigcap \{s, t\} = \emptyset$; by a *left* $\{s, t\}$*-string*, we mean a set $\{sy, tsy\}$ with $y \in W$ satisfying $\mathcal{L}(y) \bigcap \{s, t\} = \emptyset$.

We say that x is obtained from w by a *left* (respectively, *right*) $\{s, t\}$*-star operation*, if $\{x, w\}$ is a left (respectively, right) $\{s, t\}-$string. Note that the resulting element x for a left (respectively, right) $\{s, t\}$-star operation on w is always unique whenever it exists.

Sometimes we call a right $\{s, t\}-$string and a right $\{s, t\}$-star operation simply by a right string and a right star operation, respectively. Similarly for the left version of those terms.

We have the following result:

Lemma 1.3([14, Proposition 4.6]) *Let* $s, t \in S$ *be with* $o(st) = 3$. *Suppose that* $\{x_1, x_2\}$ *and* $\{y_1, y_2\}$ *be two right (respectively, left)* $\{s, t\}$*-strings. Then*

(a) $x_1 — y_1 \Leftrightarrow x_2 — y_2$;

(b) $x_1 \underset{L}{\sim} y_1 \Leftrightarrow x_2 \underset{L}{\sim} y_2$ (*respectively*, $x_1 \underset{R}{\sim} y_1 \Leftrightarrow x_2 \underset{R}{\sim} y_2$).

We say that $x, y \in W$ form a *right primitive pair*, if there exist two sequences $x_0 = x, x_1, \cdots, x_n$ and $y_0 = y, y_1, \cdots, y_n$ in W satisfying:

(a) For any $1 \leqslant i \leqslant n$, there exist some $s_i, t_i \in S$ with $o(s_i t_i) = 3$ such that both $\{x_{i-1}, x_i\}$ and $\{y_{i-1}, y_i\}$ are right $\{s_i, t_i\}-$strings.

(b) x_i——y_i for all $0 \leqslant i \leqslant n$.

(c) Either $\mathcal{R}(x) \nsubseteq \mathcal{R}(y)$ and $\mathcal{R}(y_n) \nsubseteq \mathcal{R}(x_n)$, or $\mathcal{R}(y) \nsubseteq \mathcal{R}(x)$ and $\mathcal{R}(x_n) \nsubseteq \mathcal{R}(y_n)$.

Note that any right string x, y of W form a right primitive pair with $n = 0$ in the above definition.

Similarly, we can define a left primitive pair in W.

Lemma 1.4([14, Proposition 4.1]) If x, y is a right (respectively, left) primitive pair, then $x \underset{R}{\sim} y$ (respectively, $x \underset{L}{\sim} y$).

To each $x \in W$, we define by $M(x)$ the set of all $y \in W$ satisfying the following condition: there is a sequence $x = x_0, x_1, \cdots, x_r = y$ in W with some $r \geqslant 0$ such that $x_{i-1}^{-1}x_i \in S$ and that both $\mathcal{R}(x_{i-1}) \nsubseteq \mathcal{R}(x_i)$ and $\mathcal{R}(x_{i-1}) \nsupseteq \mathcal{R}(x_i)$ hold for every $1 \leqslant i \leqslant r$.

A graph $\mathcal{M}(x)$ associated to each $x \in W$ is defined as follows. Its vertex set is $M(x)$, each $y \in M(x)$ is labeled by the set $\mathcal{R}(y)$; its edge set consists of all pairs $w, z \in M(x)$ with $\{w, z\}$ a right string.

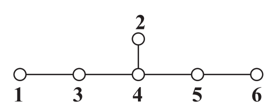
By a *path* in the graph $\mathcal{M}(x)$, we mean a sequence $z_0, z_1, \cdots, z_r$ in $M(x)$ such that $\{z_{i-1}, z_i\}$ is an edge of $\mathcal{M}(x)$ for any $1 \leqslant i \leqslant r$. We say that $x, x' \in W$ have the *same right generalized τ-invariants*, if for any path $z_0 = x, z_1, \cdots, z_r$ in $M(x)$, there is a path $z'_0 = x', z'_1, \cdots, z'_r$ in $M(x')$ with $\mathcal{R}(z'_i) = \mathcal{R}(z_i)$ for any $1 \leqslant i \leqslant r$, and if the same condition holds when the roles of x and x' are interchanged.

2 The left-connectedness of left cells in E_6

In this section, we concentrate ourselves to the Weyl group E_6. We shall prove the main result of the paper, which can be stated as follows.

Theorem 2.1 *Any left cell in E_6 is left-connected.*

Note that the left-connectedness of a left cell was conjectured by Lusztig for an affine Weyl group in [2]. Before showing our result, we need some preparation. The labels of the Coxeter generators s_i, $1 \leqslant i \leqslant 6$, of E_6 are coincident with the following Coxeter graph:



For simplifying the notation, we denote s_i by the boldfaced letter **i** for any $1 \leqslant i \leqslant 6$.

Following Shi in [6,8], we define, for any left cell L, any two-sided cell Ω of E_6 and any $i \in \mathbb{N}$, the following sets

$$E(L) := \{w \in L \mid a(sw) < a(w), \forall\, s \in \mathcal{L}(w)\},$$
$$E_{\min}(L) := \{w \in L \mid \ell(w) \leqslant \ell(x), \forall\, x \in L\},$$
$$E(\Omega) := \{w \in \Omega \mid a(sw) < a(w), \forall\, s \in \mathcal{L}(w)\},$$
$$F(\Omega) := \{w \in \Omega \mid a(sw), a(wt) < a(w), \forall\, s \in \mathcal{L}(w), t \in \mathcal{R}(w)\},$$

$$E(i) := \{w \in W_{(i)} \mid a(sw) < i, \forall\, s \in \mathcal{L}(w)\},$$
$$F(i) := \{w \in W_{(i)} \mid a(sw), a(wt) < i, \forall\, s \in \mathcal{L}(w), t \in \mathcal{R}(w)\}.$$

Recall the relation $\underset{K}{\text{---}}$ on a non-empty set K of W defined in the last section. The following result is crucial in proving the left-connectedness of a left cell of E_6.

Lemma 2.2 *Let L be a left cell of E_6. If $x \underset{L}{\text{---}} y$ for any $x \neq y$ in $E(L)$ then L is left-connected.*

Proof We need only to show that $x \neq y$ in L satisfy $x \underset{L}{\text{---}} y$. Take any $x \neq y$ in L. By the definition of the set $E(L)$, we can write $x = x' \cdot x''$ and $y = y' \cdot y''$ for some $x', y' \in E_6$ and some $x'', y'' \in E(L)$. We clearly have $x \underset{L}{\text{---}} x''$ and $y \underset{L}{\text{---}} y''$. Since $x'' \underset{L}{\text{---}} y''$ by our assumption, we get $x \underset{L}{\text{---}} y$, as required.

As a consequence of the results in [1,6,8], we have

Lemma 2.3([6, Condition C, Theorem A, Theorem B], [8, Section 4.6] and [1, Section 3.3]) (1) *Let w, L, Ω be an element, a left cell and a two-sided cell of E_6 respectively with $a(w), a(L), a(\Omega) \leqslant 6$. Then*

(1a) *w has an expression of the form $w = x \cdot w_J \cdot y$ for some $x, y \in W$ and some $J \subseteq S$ with $\ell(w_J) = a(w)$;*

(1b) *for any $w \in E(L)$, write $w = w_J \cdot y$ with $J = \mathcal{L}(w)$ for some $y \in E_6$. Then $\ell(w_J) = a(w)$;*

(1c) *if $E(L) = E_{\min}(L)$ then L is left-connected;*

(1d) *$F(\Omega) = \{w_J \in \Omega \mid J \subseteq S\}$;*

(2) *let w_0 be the longest element of E_6. Then the map $\psi: w \mapsto ww_0$ induces an involutive order-reversing permutation $\overline{\psi}: \Gamma \mapsto \Gamma w_0$ on the set of left (respectively, right, two-sided) cells of E_6 with respect to the partial order $\underset{L}{\leqslant}$ (respectively, $\underset{R}{\leqslant}$, $\underset{LR}{\leqslant}$). In particular, a left cell L of E_6 is left-connected if and only if so is $\overline{\psi}(L) = Lw_0$.*

Let Ω be a two-sided cell of E_6. In [8], Shi designed the following algorithm for finding the set $E(\Omega)$ from $F(\Omega)$.

Algorithm 2.4

(1) *Set $Y_0 = F(\Omega)$;*

Let $k \geqslant 0$. Suppose that the set Y_k has been found.

(2) *If $Y_k = \emptyset$, then the algorithm terminates;*

(3) *If $Y_k \neq \emptyset$, then find the set $Y_{k+1} = \{xs \mid x \in Y_k, s \in S \setminus \mathcal{R}(x); xs \in E(\Omega)\}$.*

The most technical part in applying Algorithm 2.5 is to determine whether or not an element xs is in the set $E(\Omega)$, that is, to determine if the inequality $a(tws) < a(ws)$ holds for any $t \in \mathcal{L}(ws)$. Consider the following result of Tong in [7].

Lemma 2.5([7, Theorem 7]) *Two elements $x, y \in E_6$ satisfy $x \underset{L}{\sim} y$ if and only if x and y have the same right generalized $\tau-$invariants.*

Since all the left cell graphs have been worked out by Tong in [7], it will be no difficulty to check if two elements of E_6 have the same generalized τ-invariants by using the computer programme MATLAB. Hence by Lemma 2.5, for any $t \in \mathcal{L}(ws)$, checking the validity of the inequality $a(tws) < a(ws)$ is amount to checking that tws and ws have different generalized τ-invariants.

Let $i \in \mathbb{N}$. From the knowledge of special unipotent conjugacy classes of the complex reductive algebraic group of type E_6 (see [15, Chapter 13]), we see by Lemma 1.1 that $W_{(i)} \neq \emptyset$ if and only if $i \in \{0,1,2,3,4,5,6,7,10,11,12,13,15,20,25,36\}$ and that $W_{(i)}$ is a single two-sided cell of E_6 unless $i=6$, and we also see that $W_{(6)} = \Omega_{6,1} \cup \Omega_{6,2}$, where $\Omega_{6,1}$ and $\Omega_{6,2}$ are two two-sided cells containing 81, 24 left cells respectively (see [7]).

Recall the involutive order-reversing permutation $\overline{\psi} : \Gamma \mapsto \Gamma w_0$ on the set $\Pi(E_6)$ of all two-sided cells of E_6 defined in Lemma 2.3(2). The followings are all the $\overline{\psi}$-orbits in $\Pi(E_6)$: $\{W_{(0)}, W_{(36)}\}$, $\{W_{(1)}, W_{(25)}\}$, $\{W_{(2)}, W_{(20)}\}$, $\{W_{(3)}, W_{(15)}\}$, $\{W_{(4)}, W_{(13)}\}$, $\{W_{(5)}, W_{(11)}\}$, $\{\Omega_{6,1}, W_{(10)}\}$, $\{\Omega_{6,2}, W_{(12)}\}$, $\{W_{(7)}\}$.

For $i \in \mathbb{N}$, let $\Sigma_{\leqslant i}$ (respectively, Σ_i) be the set of all left cells L of E_6 with $a(L) \leqslant i$ (respectively, $a(L) = i$). By Lemma 2.3 (2), to show Theorem 2.1, we need only to deal with all the left cells in $\Sigma_{\leqslant 7}$.

If $L \in \Sigma_{\leqslant 2}$, then L contains some fully commutative element of E_6. Hence L is left-connected by Lemma 1.2.

Now we prove the left-connectedness of left cells in $\Sigma_{\leqslant 7} \setminus \Sigma_{\leqslant 2}$ in the remaining part of the paper. By Lemma 2.3 (1d), we see that the set $F(i)$ for $3 \leqslant i \leqslant 6$ consists of all $w_J \in W_{(i)}$ with some $J \subseteq S$. So by the results of Tong in [7], we get

$$\begin{aligned}
F(W_{(3)}) &= \{w_{\mathbf{125}}, w_{\mathbf{126}}, w_{\mathbf{13}}, w_{\mathbf{146}}, w_{\mathbf{235}}, w_{\mathbf{236}}, w_{\mathbf{24}}, w_{\mathbf{34}}, w_{\mathbf{45}}, w_{\mathbf{56}}\},\\
F(W_{(4)}) &= \{w_{\mathbf{123}}, w_{\mathbf{124}}, w_{\mathbf{135}}, w_{\mathbf{136}}, w_{\mathbf{145}}, w_{\mathbf{156}}, w_{\mathbf{246}}, w_{\mathbf{256}}, w_{\mathbf{346}}, w_{\mathbf{356}}\},\\
F(W_{(5)}) &= \{w_{\mathbf{1235}}, w_{\mathbf{1236}}, w_{\mathbf{1246}}, w_{\mathbf{1256}}, w_{\mathbf{2356}}\},\\
F(\Omega_{6,1}) &= \{w_{\mathbf{134}}, w_{\mathbf{234}}, w_{\mathbf{245}}, w_{\mathbf{345}}, w_{\mathbf{456}}\},\\
F(\Omega_{6,2}) &= \{w_{\mathbf{1356}}\},
\end{aligned}$$

where we denote w_J by $w_{\mathbf{ijk}\cdots}$ for $J = \{s_i, s_j, s_k, \cdots\}$. We also have

$$\begin{aligned}
F(W_{(7)}) = &\{w_{\mathbf{1346}}, w_{\mathbf{4561}}, w_{\mathbf{2346}}, w_{\mathbf{2451}}, w_{\mathbf{13562}}, w_{\mathbf{243}} \cdot \mathbf{543}, w_{\mathbf{243}} \cdot \mathbf{542},\\
&w_{\mathbf{345}} \cdot \mathbf{243}, \mathbf{5631} \cdot w_{\mathbf{345}}, w_{\mathbf{245}} \cdot \mathbf{345}, w_{\mathbf{345}} \cdot \mathbf{245}, w_{\mathbf{245}} \cdot \mathbf{342}, w_{\mathbf{345}} \cdot \mathbf{1365}\}
\end{aligned}$$

by a result of Shi in [8, Example 4.10]. So we can perform Algorithm 2.4 to get $E(\Omega)$ for all two-sided cell Ω of E_6 with $3 \leqslant a(\Omega) \leqslant 7$ (see Tables 1, 2, 3, 4, 5, 6 for the results).

In Tables 1, 2, 3, 4, 5, 6, if $i \in \{3,4,5,7\}$, then we denote all the left cells in $W_{(i)}$ by $L_{i,j}$, $1 \leqslant j \leqslant n(i)$, where $n(i)$ is the number of left cells in $W_{(i)}$; if $i=6$, then we denote all the left cells in $\Omega_{6,k}$, $k=1,2$, by $L_{6k,j}$, $1 \leqslant j \leqslant n_k(6)$, where $n_1(6)=81$ and $n_2(6)=24$. For saving space in the tables, we denote $\{s_i, s_j, s_k, \cdots\}$ simply by $\mathbf{ijk}\cdots$ concerning the set $\mathcal{R}(L)$. For example, $\{s_1, s_2, s_3, s_5\}$ is denoted by $\mathbf{1235}$.

We observe from Tables 1, 2, 3, 4, 5, 6 that all the elements of $E(L)$ have the same length for any left cell L with either $a(L) \leqslant 5$ or $L \subset \Omega_{6,2}$. So for those left cells L, we have $E(L) = E_{\min}(L)$ and hence L is left-connected by Lemma 2.3 (1c). Let Λ be the set of all such left cells L of E_6 in $W_{(7)} \cup \Omega_{6,1}$ that the lengths of the elements in $E(L)$ are not all the same. Thus, to show Theorem 2.1, we need only to deal with all the left cells of E_6 in Λ. By Lemma 2.2, we shall prove the left-connectedness of those left cells L by showing that $x \underset{L}{—} y$ for any $x \neq y$ in $E(L)$ by a case-by-case argument.

We proceed our proof by constructing some connected graphs. Those graphs are named by F1, F2,$\cdots$, F54, respectively. One connected graph (say Fi) for each left cell L in Λ, each vertex of Fi represents an element (say z) of E_6 which is labeled by $\mathcal{L}(z)$, all the elements of $E(L)$ must occur as vertices in the graph Fi. There are two kinds of edges in the graph Fi: solid edges and dashed edges. Two vertices are joined by a solid edge if they form a left string and by a dashed edge if they form a left primitive pair but not a left string. The connectedness of the graph Fi implies that all the elements corresponding to the vertices of Fi belong to L by Lemma 1.4. Hence L is left-connected by Lemma 2.2.

Example 2.6 We take the graph F30 as an example to explain how we prove the left-connectedness for the left cell $L := L_{7,23}$. We have $E(L) = \{a, b, c\}$ with $a = \mathbf{124562423451}$, $b = \mathbf{145645242351}$ and $c = \mathbf{245234234561345}$ by Tab. 6. The elements a, b, c all occur as vertices of the graph F30 with labels $\boxed{\mathbf{1245}}$, $\boxed{\mathbf{1456}}$, $\boxed{\mathbf{245}}$, respectively.

As examples, we see that the vertices labeled by $\boxed{\mathbf{1456}}$ and $\boxed{\mathbf{1256}}$ in F30 are joined by a solid edge, the corresponding elements b and $b' = \mathbf{2}b$ form a left $\{\mathbf{2}, \mathbf{4}\}$-string, and that the vertices labeled by $\boxed{\mathbf{34}}$ and $\boxed{\mathbf{234}}$ in F30 are joined by a dashed edge, the corresponding elements $g = \mathbf{43}a$ and $h = \mathbf{2} \cdot g$ form a left primitive pair but not a left string. The fact of g, h forming a left primitive pair can be observed directly from the graph F30 as follows. There are two sequences: $g_0 = g, g_1 = 4g_0, g_2 = 3g_1 = a$ and $h_0 = h, h_1 = \mathbf{5}h_0, h_2 = \mathbf{3}h_1 = c$ satisfying $g_i — h_i$ for $0 \leqslant i \leqslant 2$ by (1.1.1) and Lemma 1.10; g_1, h_1 (respectively, g_2, h_2) can be obtained from g_0, h_0 (respectively, g_1, h_1) by a left $\{4, 5\}$- (respectively, $\{\mathbf{3}, \mathbf{4}\}$-) star operation respectively; $\mathcal{L}(h) = \{\mathbf{2}, \mathbf{3}, \mathbf{4}\} \nsubseteq \{\mathbf{3}, \mathbf{4}\} = \mathcal{L}(g)$ and $\mathcal{L}(g_2) = \{\mathbf{1}, \mathbf{2}, \mathbf{4}, \mathbf{5}\} \nsubseteq \{\mathbf{2}, \mathbf{4}, \mathbf{5}\} = \mathcal{L}(h_2)$ (see the last section).

We see from F30 that $b = \mathbf{256}a$ can be obtained from a by successively applying left $\{\mathbf{5}, \mathbf{6}\}$-, $\{\mathbf{4}, \mathbf{5}\}$-, $\{\mathbf{2}, \mathbf{4}\}$-star operations, that the elements g, h can be obtained from a, c by successively applying left $\{\mathbf{3}, \mathbf{4}\}$-, $\{\mathbf{4}, \mathbf{5}\}$-star operations respectively and that g, h form a left primitive pair. This implies by Lemma 1.4 that $b \underset{L}{—} a \underset{L}{—} g \underset{L}{—} h \underset{L}{—} c$. Hence $L = L_{7,23}$ is left-connected by Lemma 2.2.

Tab. 1 Description of left cells in $W_{(3)}$

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{3,1}$	**1463542**	**2**	$L_{3,5}$	**23465**	**5**	$L_{3,9}$	**242**	**24**	$L_{3,13}$	**454**	**45**	$L_{3,17}$	**126**	**126**
$L_{3,2}$	**12543**	**3**	$L_{3,6}$	**131**	**13**	$L_{3,10}$	**343**	**34**	$L_{3,14}$	**2346**	**46**	$L_{3,18}$	**146**	**146**
$L_{3,3}$	**2354**	**4**	$L_{3,7}$	**1254**	**14**	$L_{3,11}$	**14635**	**35**	$L_{3,15}$	**565**	**56**	$L_{3,19}$	**235**	**235**
$L_{3,4}$	**146354**	**4**	$L_{3,8}$	**1465**	**15**	$L_{3,12}$	**1463**	**36**	$L_{3,16}$	**125**	**125**	$L_{3,20}$	**236**	**236**

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{3,21}$	**13412,24231,34231**	**12**	$L_{3,24}$	**34563,45643,56543**	**36**	$L_{3,27}$	**1341,3431**	**14**	$L_{3,30}$	**134561,345631, 456431,565431**	**16**
$L_{3,22}$	**24562,45642,56542**	**26**	$L_{3,25}$	**2423,3423**	**23**	$L_{3,28}$	**3453,4543**	**35**			
$L_{3,23}$	**13451,34531,45431**	**15**	$L_{3,26}$	**2452,4542**	**25**	$L_{3,29}$	**4564,5654**	**46**			

Tab. 2　Description of left cells in $W_{(4)}$

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{4,1}$	135142	12	$L_{4,6}$	2565431	16	$L_{4,11}$	14543	35	$L_{4,16}$	1242	124	$L_{4,21}$	2462	246
$L_{4,2}$	12341	14	$L_{4,7}$	12423	23	$L_{4,12}$	256543	36	$L_{4,17}$	1351	135	$L_{4,22}$	2565	256
$L_{4,3}$	13514	14	$L_{4,8}$	24625	25	$L_{4,13}$	35654	46	$L_{4,18}$	1361	136	$L_{4,23}$	3463	346
$L_{4,4}$	123451	15	$L_{4,9}$	356542	26	$L_{4,14}$	25654	46	$L_{4,19}$	1454	145	$L_{4,24}$	3565	356
$L_{4,5}$	1234561	16	$L_{4,10}$	34635	35	$L_{4,15}$	1231	123	$L_{4,20}$	1565	156			

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{4,25}$	3462354, 2462354	4	$L_{4,32}$	135143, 145343	34	$L_{4,39}$	12452, 14542	125
$L_{4,26}$	1245234, 1454234	4	$L_{4,33}$	1234513, 1242345	35	$L_{4,40}$	13461, 34631	146
$L_{4,27}$	134615, 346351	15	$L_{4,34}$	2462543, 2564543	35	$L_{4,41}$	14564, 15654	146
$L_{4,28}$	4564, 5654	15	$L_{4,35}$	145643, 156543	36	$L_{4,42}$	346235, 246235	235
$L_{4,29}$	1351423, 1453423	23	$L_{4,36}$	12345613, 12423456	36	$L_{4,43}$	124523, 145423	235
$L_{4,30}$	3463542, 3564542	25	$L_{4,37}$	346354, 356454	45	$L_{4,44}$	34623, 24623	236
$L_{4,31}$	123413, 124234	34	$L_{4,38}$	246254, 256454	45			

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{4,45}$	134612543, 346235143, 246235143	3	$L_{4,52}$	12345134, 12452345, 14542345	45
$L_{4,46}$	124562345, 145642345, 156542345	5	$L_{4,53}$	12456234, 14564234, 15654234	46
$L_{4,47}$	13461254, 34623514, 24623514	14	$L_{4,54}$	123456134, 124523456,145423456	46
$L_{4,48}$	346235431, 246235431, 256453431	14	$L_{4,55}$	1346125, 3462351, 2462351	125
$L_{4,49}$	13514234, 12452342, 14534234	24	$L_{4,56}$	134612, 346231, 246231	126
$L_{4,50}$	34623542, 24623542, 35645242	24	$L_{4,57}$	124562, 145642, 156542	126
$L_{4,51}$	34623543, 24623543, 25645343	34	$L_{4,58}$	1245623, 1456423, 1565423	236

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{4,59}$	3462354231, 2462354231, 3564524231, 2564534231	12	$L_{4,62}$	123451342, 135142345, 124523452, 145342345	25
$L_{4,60}$	1346125431, 3462351431, 2462351431, 2564534131	13	$L_{4,63}$	1234561342, 1351423456, 1245234562, 1453423456	26
$L_{4,61}$	346235423, 246235423, 356452423, 256453423	23	$L_{4,64}$	1234561345, 1245623456, 1456423456, 1565423456	56

Tab. 3　Description of left cells in $W_{(5)}$

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{5,1}$	235645423413	3	$L_{5,13}$	1235143	34	$L_{5,25}$	123456142	126	$L_{5,37}$	124623	236
$L_{5,2}$	12462354	4	$L_{5,14}$	12345143	35	$L_{5,26}$	235654231	126	$L_{5,38}$	23565423	236
$L_{5,3}$	1234514234	4	$L_{5,15}$	23564543	35	$L_{5,27}$	12565431	136	$L_{5,39}$	1234561423	236
$L_{5,4}$	2356454234	4	$L_{5,16}$	1256543	36	$L_{5,28}$	1234514	145	$L_{5,40}$	2356542	246
$L_{5,5}$	123456142345	5	$L_{5,17}$	123456143	36	$L_{5,29}$	123461	146	$L_{5,41}$	2356543	346
$L_{5,6}$	123514	14	$L_{5,18}$	2356454	45	$L_{5,30}$	125654	146	$L_{5,42}$	12351	1235
$L_{5,7}$	23564542341	14	$L_{5,19}$	235654	46	$L_{5,31}$	12345614	146	$L_{5,43}$	12361	1236
$L_{5,8}$	1234615	15	$L_{5,20}$	12345614234	46	$L_{5,32}$	23565431	146	$L_{5,44}$	12462	1246
$L_{5,9}$	235645431	15	$L_{5,21}$	1235142	124	$L_{5,33}$	12345615	156	$L_{5,45}$	12565	1256
$L_{5,10}$	12351423	23	$L_{5,22}$	124625	125	$L_{5,34}$	1246235	235	$L_{5,46}$	23565	2356
$L_{5,11}$	124623542	24	$L_{5,23}$	12345142	125	$L_{5,35}$	123451423	235			
$L_{5,12}$	23564542	25	$L_{5,24}$	2356454231	125	$L_{5,36}$	235645423	235			

Continue of Tab. 3

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{5,47}$	**12462354231, 12564534231**	**12**	$L_{5,52}$	**124623543, 125645343**	**34**	$L_{5,57}$	**124625431, 125645431**	**135**
$L_{5,48}$	**1246235431, 1256453431**	**14**	$L_{5,53}$	**12346135, 12462345**	**35**	$L_{5,58}$	**1246254, 1256454**	**145**
$L_{5,49}$	**1246235423, 1256453423**	**23**	$L_{5,54}$	**12462543, 12564543**	**35**	$L_{5,59}$	**1234613, 1246234**	**346**
$L_{5,50}$	**1234613542, 1246234542**	**25**	$L_{5,55}$	**123461354, 124623454**	**45**	$L_{5,60}$	**123456135, 124623456**	**356**
$L_{5,51}$	**12345613542, 12462345642**	**26**	$L_{5,56}$	**1234561354, 1246234564**	**46**			

Tab. 4 Description of left cells in $\Omega_{6,1}$

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{61,1}$	**2452423413**	**13**	$L_{61,6}$	**234523456**	**46**	$L_{61,11}$	**2342345**	**235**	$L_{61,16}$	**345343**	**345**
$L_{61,2}$	**245242341**	**14**	$L_{61,7}$	**2345623456**	**56**	$L_{61,12}$	**2452423**	**235**	$L_{61,17}$	**456454**	**456**
$L_{61,3}$	**34534234**	**24**	$L_{61,8}$	**24524231**	**125**	$L_{61,13}$	**3453423**	**235**			
$L_{61,4}$	**24524234**	**34**	$L_{61,9}$	**134131**	**134**	$L_{61,14}$	**23423456**	**236**			
$L_{61,5}$	**23452345**	**45**	$L_{61,10}$	**234234**	**234**	$L_{61,15}$	**245242**	**245**			

L	$E(L)$	$\mathcal{R}(L)$	Figure
$L_{61,18}$	a=**3456342345134**, b=**13456123451234**, c=**45645342345134**, d=**234523456134524**, e=**245624523451234**	**4**	F1
$L_{61,19}$	a=**24562423451342**, b=**456452423451342**, c=**3456345234513412**, d=**13456123451234231**, e=**234562345613452431**	**12**	F2
$L_{61,20}$	a=**134512341**, b=**2342345134**, c=**3453412341**	**14**	F3
$L_{61,21}$	a=**2456242345134**, b=**45645242345134**, c=**345634523451341**, d=**1345612345123431**, e=**23452345613452431**	**14**	F2
$L_{61,22}$	a=**13456123451**, b=**234234561345**, c=**345634123451**, d=**4564534123451**	**15**	F4
$L_{61,23}$	a=**24562423451**, b=**456452423451**, c=**3456345234531**	**15**	F5
$L_{61,24}$	a=**3453423413**, b=**13451234123**, c=**234523451342**	**23**	F6
$L_{61,25}$	a=**45645423451342**, b=**245624523451342**, c=**345634523451342**, d=**1345612345123423**, e=**23456234561345243**	**23**	F7
$L_{61,26}$	a=**34563423451342**, b=**134561234512342**, c=**456453423451342**, d=**2345623456134524**, e=**2456245234512342**	**24**	F1
$L_{61,27}$	a=**3456342345**, b=**45645342345**, c=**245624523452**	**25**	F8
$L_{61,28}$	a=**13456123451342**, b=**234562345613452**, c=**345634123451342**, d=**4564534123451342**, e=**24562452345123452**	**25**	F9
$L_{61,29}$	a=**23456234561342**, b=**134561234561342**, c=**3456341234561342**, d=**45645341234561342**, e=**245624523451234562**	**26**	F10
$L_{61,30}$	a=**1345123413**, b=**23452345134**, c=**34534123413**	**34**	F3
$L_{61,31}$	a=**4564542345134**, b=**24562452345134**, c=**34563452345134**, d=**134561234512343**, e=**2345234561345243**	**34**	F7
$L_{61,32}$	a=**2345234513**, b=**13451234513**, c=**345341234513**	**35**	F11
$L_{61,33}$	a=**2456242345**, b=**45645242345**, c=**345634523453**	**35**	F5
$L_{61,34}$	a=**134561234513**, b=**2345234561345**, c=**3456341234513**, d=**45645341234513**, e=**245624523412345**	**35**	F9
$L_{61,35}$	a=**456454234513**, b=**2456245234513**, c=**3456345234513**, d=**13456123451243**, e=**234234561345243**	**35**	F7
$L_{61,36}$	a=**23452345613**, b=**134512345613**, c=**3453412345613**	**36**	F11
$L_{61,37}$	a=**45645423413**, b=**245624523413**, c=**345634523413**, d=**1345613451243**	**36**	F12
$L_{61,38}$	a=**4564542345**, b=**24562452345**, c=**34563452345**	**45**	F13
$L_{61,39}$	a=**1345612345134**, b=**23456234561345**, c=**34563412345134**, d=**456453412345134**, e=**2456245234512345**	**45**	F9

Continue of Tab. 4

L	$E(L)$	$\mathcal{R}(L)$	Figure
$L_{61,40}$	a=**456454234**, b=**2456245234**, c=**3456345234**	**46**	F13
$L_{61,41}$	a=**2345623456134**, b=**13456123456134**, c=**345634123456134**, d=**4564534123456134**, e=**24562452345123456**	**46**	F10
$L_{61,42}$	a=**1341231**, b=**23423413**	**123**	F14
$L_{61,43}$	a=**2342341**, b=**13412341**	**124**	F15
$L_{61,44}$	a=**345342341**, b=**1345123412**, c=**23423451342**	**124**	F6
$L_{61,45}$	a=**23423451**, b=**134123451**	**125**	F15
$L_{61,46}$	a=**34534231**, b=**134513412**	**125**	F16
$L_{61,47}$	a=**34563423451**, b=**134561234512**, c=**456453423451**, d=**2342345613452**, e=**2456245234512**	**125**	F1
$L_{61,48}$	a=**234234561**, b=**1341234561**	**126**	F15
$L_{61,49}$	a=**245624231**, b=**4564524231**	**126**	F17
$L_{61,50}$	a=**345634231**, b=**1345613412**, c=**4564534231**	**126**	F18
$L_{61,51}$	a=**1345131**, b=**34534131**	**135**	F19
$L_{61,52}$	a=**245624234513**, b=**4564524234513**, c=**34563452345131**, d=**134561234512431**, e=**2342345613452431**	**135**	F2
$L_{61,53}$	a=**13456131**, b=**345634131**, c=**4564534131**	**136**	F20
$L_{61,54}$	a=**24562423413**, b=**456452423413**, c=**3456345234131**, d=**13456134512431**	**136**	F12
$L_{61,55}$	a=**3453431**, b=**13451341**	**145**	F16
$L_{61,56}$	a=**234523451**, b=**1345123451**, c=**34534123451**	**145**	F11
$L_{61,57}$	a=**45645423451**, b=**245624523451**, c=**345634523451**, d=**1345612345124**, e=**23423456134524**	**145**	F21
$L_{61,58}$	a=**34563431**, b=**134561341**, c=**456453431**	**146**	F18
$L_{61,59}$	a=**1345612341**, b=**23423456134**, c=**34563412341**, d=**456453412341**	**146**	F4
$L_{61,60}$	a=**2345234561**, b=**13451234561**, c=**345341234561**	**146**	F11
$L_{61,61}$	a=**2456242341**, b=**45645242341**, c=**345634523431**	**146**	F5
$L_{61,62}$	a=**4564542341**, b=**24562452341**, c=**34563452341**, d=**134561345124**	**146**	F12
$L_{61,63}$	a=**45645431**, b=**345634531**, c=**1345613451**	**156**	F22
$L_{61,64}$	a=**23456234561**, b=**134561234561**, c=**3456341234561**, d=**45645341234561**	**156**	F23
$L_{61,65}$	a=**345634234513**, b=**1345612345123**, c=**4564534234513**, d=**23452345613452**	**235**	F24
$L_{61,66}$	a=**24562423**, b=**456452423**	**236**	F17
$L_{61,67}$	a=**34563423**, b=**456453423**	**236**	F25
$L_{61,68}$	a=**34563423413**, b=**134561234123**, c=**456453423413**, d=**2345234561342**, e=**2456245234123**	**236**	F1
$L_{61,69}$	a=**2456242**, b=**45645242**	**246**	F17
$L_{61,70}$	a=**345634234**, b=**4564534234**, c=**24562452342**	**246**	F8
$L_{61,71}$	a=**4564542**, b=**24562452**	**256**	F26
$L_{61,72}$	a=**3456343**, b=**45645343**	**346**	F25
$L_{61,73}$	a=**245624234**, b=**4564524234**, c=**34563452343**	**346**	F5
$L_{61,74}$	a=**13456123413**, b=**234523456134**, c=**345634123413**, d=**4564534123413**, e=**24562452341234**	**346**	F9
$L_{61,75}$	a=**4564543**, b=**34563453**	**356**	F27
$L_{61,76}$	a=**234562345613**, b=**1345612345613**, c=**34563412345613**, d=**456453412345613**, e=**2456245234123456**	**356**	F10
$L_{61,77}$	a=**13451231**, b=**234234513**, c=**345341231**	**1235**	F3
$L_{61,78}$	a=**134561231**, b=**2342345613**, c=**3456341231**, d=**45645341231**	**1236**	F4
$L_{61,79}$	a=**3456342341**, b=**13456123412**, c=**45645342341**, d=**234234561342**, e=**245624523412**	**1246**	F1
$L_{61,80}$	a=**456454231**, b=**2456245231**, c=**3456345231**, d=**13456134512**	**1256**	F12
$L_{61,81}$	a=**45645423**, b=**245624523**, c=**345634523**	**2356**	F13

Tab. 5 Description of left cells in $\Omega_{6,2}$

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{62,1}$	13561454234	4	$L_{62,9}$	13561452423456	35	$L_{62,17}$	13561454	145
$L_{62,2}$	1356145242341	14	$L_{62,10}$	135614542345	36	$L_{62,18}$	1356154	146
$L_{62,3}$	13561452423451	15	$L_{62,11}$	1356145423456	45	$L_{62,19}$	1356145423	235
$L_{62,4}$	135614524234561	16	$L_{62,12}$	1356145242345	46	$L_{62,20}$	135615423	236
$L_{62,5}$	13561452423	23	$L_{62,13}$	135614524231	123	$L_{62,21}$	1356154234	246
$L_{62,6}$	13561542345	25	$L_{62,14}$	1356145242	124	$L_{62,22}$	135615423456	256
$L_{62,7}$	135614524234	34	$L_{62,15}$	135614542	125	$L_{62,23}$	13561543	346
$L_{62,8}$	135614543	35	$L_{62,16}$	13561542	126	$L_{62,24}$	135615	1356

Tab. 6 Description of left cells in $W_{(7)}$

L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$	L	$E(L)$	$\mathcal{R}(L)$
$L_{7,1}$	123561454234	4	$L_{7,6}$	13461351	135	$L_{7,11}$	12452423	235	$L_{7,16}$	123561542	1246
$L_{7,2}$	124524234	34	$L_{7,7}$	123561454	145	$L_{7,12}$	1235615423	236	$L_{7,17}$	1346131	1346
$L_{7,3}$	1235614543	35	$L_{7,8}$	12356154	146	$L_{7,13}$	123561543	346	$L_{7,18}$	1456454	1456
$L_{7,4}$	234623454	45	$L_{7,9}$	23462345	235	$L_{7,14}$	14564543	356	$L_{7,19}$	2346234	2346
$L_{7,5}$	1235614542	125	$L_{7,10}$	12356145423	235	$L_{7,15}$	1245242	1245	$L_{7,20}$	1235615	12356

L	$E(L)$	$\mathcal{R}(L)$	Figure
$L_{7,21}$	a=1346123514, b=23462345134	14	F28
$L_{7,22}$	a=1245242341, b=2452342345134	14	F29
$L_{7,23}$	a=124562423451, b=1456452423451, c=245234234561345	15	F30
$L_{7,24}$	a=134612351423, b=2346234513423, c=14563453423413	23	F31
$L_{7,25}$	a=135614534234, b=134613514234, c=145634534234	24	F32
$L_{7,26}$	a=145645342345, b=1245624523452, c=13456135142345	25	F33
$L_{7,27}$	a=13461235143, b=234623451343	34	F28
$L_{7,28}$	a=23462345143, b=134612345143	35	F34
$L_{7,29}$	a=12456242345, b=145645242345	35	F35
$L_{7,30}$	a=234623456143, b=1346123456143, c=234562345623413	36	F36
$L_{7,31}$	a=14564542345, b=124562452345	45	F37
$L_{7,32}$	a=2346234564, b=2345623456234	46	F38
$L_{7,33}$	a=1456454234, b=12456245234	46	F37
$L_{7,34}$	a=34523423413, b=24523423413, c=2345234513412, d=2452423451342	123	F39
$L_{7,35}$	a=13461235142, b=234623451342, c=1456345342341	124	F40
$L_{7,36}$	a=3452342341, b=2452342341, c=24523423451342	124	F41
$L_{7,37}$	a=234623451, b=1346123451	125	F34
$L_{7,38}$	a=23452345231, b=34523423451, c=24523423451, d=245234123451342	125	F42
$L_{7,39}$	a=1346135142, b=145634534231	125	F43
$L_{7,40}$	a=13456135142, b=14564534231, c=345634513412	126	F44
$L_{7,41}$	a=234523456231, b=345234234561, c=245234234561, d=2345623456234231, e=2452341234561342	126	F45
$L_{7,42}$	a=12452423413, b=234523451341, c=345234123413, d=245242345134, e=245234123413	134	F46
$L_{7,43}$	a=23452345131, b=24524234513, c=3452341234513, d=2452341234513	135	F47
$L_{7,44}$	a=234523456131, b=245242345613, c=34523412345613, d=24523412345613, e=2345623456234131	136	F48
$L_{7,45}$	a=2345234531, b=2452423451, c=24523412345134	145	F49
$L_{7,46}$	a=134613514, b=14563453431	145	F43

Continue of Tab. 6

L	$E(L)$	$\mathcal{R}(L)$	Figure
$L_{7,47}$	a=**2346234514**, b=**13461234514**	**145**	F34
$L_{7,48}$	a=**1345613514**, b=**1456453431**, c=**34563451341**	**146**	F50
$L_{7,49}$	a=**23462345614**, b=**134612345614**, c=**23456234562341**	**146**	F51
$L_{7,50}$	a=**12456242341**, b=**145645242341**, c=**24523423456134**	**146**	F52
$L_{7,51}$	a=**23452345631**, b=**24524234561**, c=**23456234562341**, d=**245234123456134**	**146**	F53
$L_{7,52}$	a=**234562345631**, b=**245624234561**, c=**23456234562431**, d=**34563452345631**, e=**2452341234561345**	**156**	F54
$L_{7,53}$	a=**345234234**, b=**245234234**	**234**	F55
$L_{7,54}$	a=**2345234523**, b=**3452342345**, c=**2452342345**	**235**	F56
$L_{7,55}$	a=**13561453423**, b=**13461351423**, c=**14563453423**	**235**	F57
$L_{7,56}$	a=**124562423**, b=**1456452423**	**236**	F35
$L_{7,57}$	a=**23452345623**, b=**34523423456**, c=**24523423456**, d=**234562345623423**	**236**	F58
$L_{7,58}$	a=**1456453423**, b=**134561351423**	**236**	F59
$L_{7,59}$	a=**234523452**, b=**345342345**	**245**	F60
$L_{7,60}$	a=**2345234562**, b=**3453423456**, c=**23456234562342**	**246**	F61
$L_{7,61}$	a=**14564534234**, b=**124562452342**	**246**	F62
$L_{7,62}$	a=**23456234562**, b=**34563423456**, c=**2345623456342**, d=**2456245234562**	**256**	F63
$L_{7,63}$	a=**234523453**, b=**245242345**	**345**	F64
$L_{7,64}$	a=**1356145343**, b=**1346135143**, c=**1456345343**	**345**	F57
$L_{7,65}$	a=**2345234563**, b=**2452423456**, c=**23456234562343**	**346**	F65
$L_{7,66}$	a=**145645343**, b=**13456135143**	**346**	F59
$L_{7,67}$	a=**1245624234**, b=**14564524234**	**346**	F35
$L_{7,68}$	a=**23456234563**, b=**24562423456**, c=**2345623456243**, d=**3456345234563**	**356**	F66
$L_{7,69}$	a=**23456234564**, b=**234562345634**, c=**234562345624**, d=**245624523456**, e=**345634523456**	**456**	F67
$L_{7,70}$	a=**134612351**, b=**2346234513**	**1235**	F28
$L_{7,71}$	a=**124524231**, b=**245234234513**	**1235**	F68
$L_{7,72}$	a=**13461231**, b=**234623413**	**1236**	F28
$L_{7,73}$	a=**1245624231**, b=**14564524231**, c=**2452342345613**	**1236**	F52
$L_{7,74}$	a=**23462341**, b=**134612341**	**1246**	F34
$L_{7,75}$	a=**12456242**, b=**145645242**	**1246**	F35
$L_{7,76}$	a=**14564542**, b=**124562452**	**1256**	F37
$L_{7,77}$	a=**2346234561**, b=**13461234561**, c=**2345623456231**	**1256**	F51
$L_{7,78}$	a=**134561351**, b=**145645431**, c=**3456345131**	**1356**	F50
$L_{7,79}$	a=**234623456**, b=**234562345623**	**2356**	F69
$L_{7,80}$	a=**145645423**, b=**1245624523**	**2356**	F62

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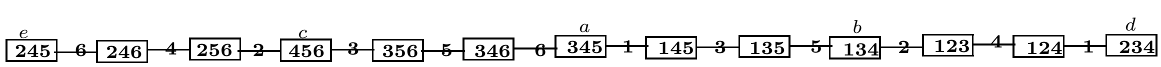

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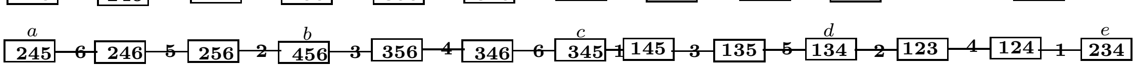

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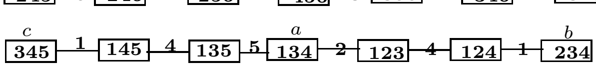

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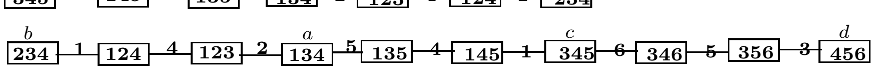

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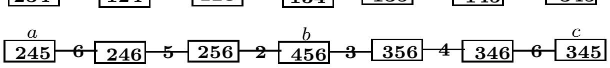

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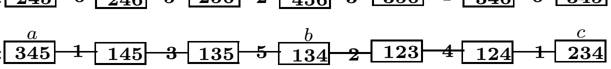

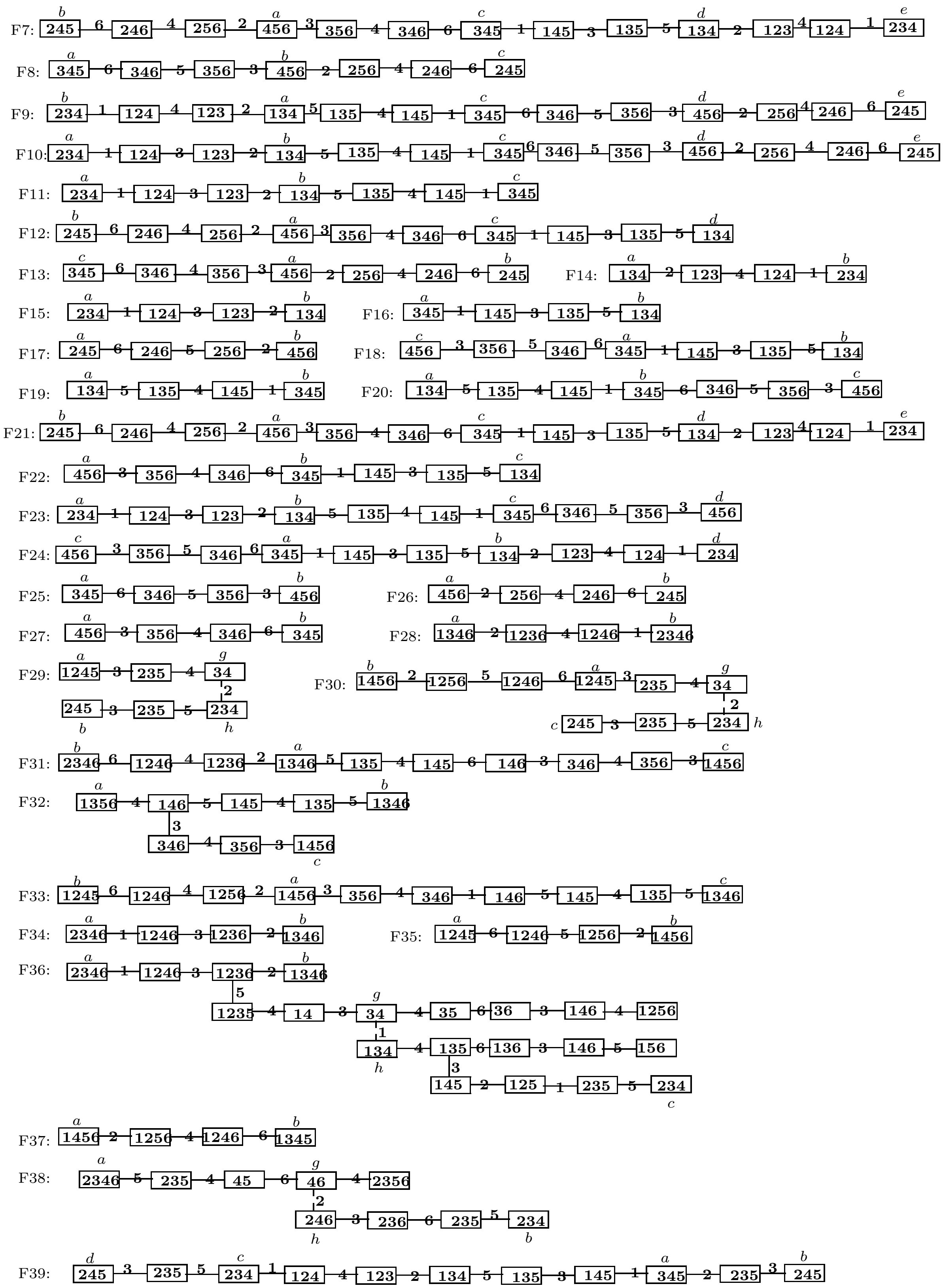
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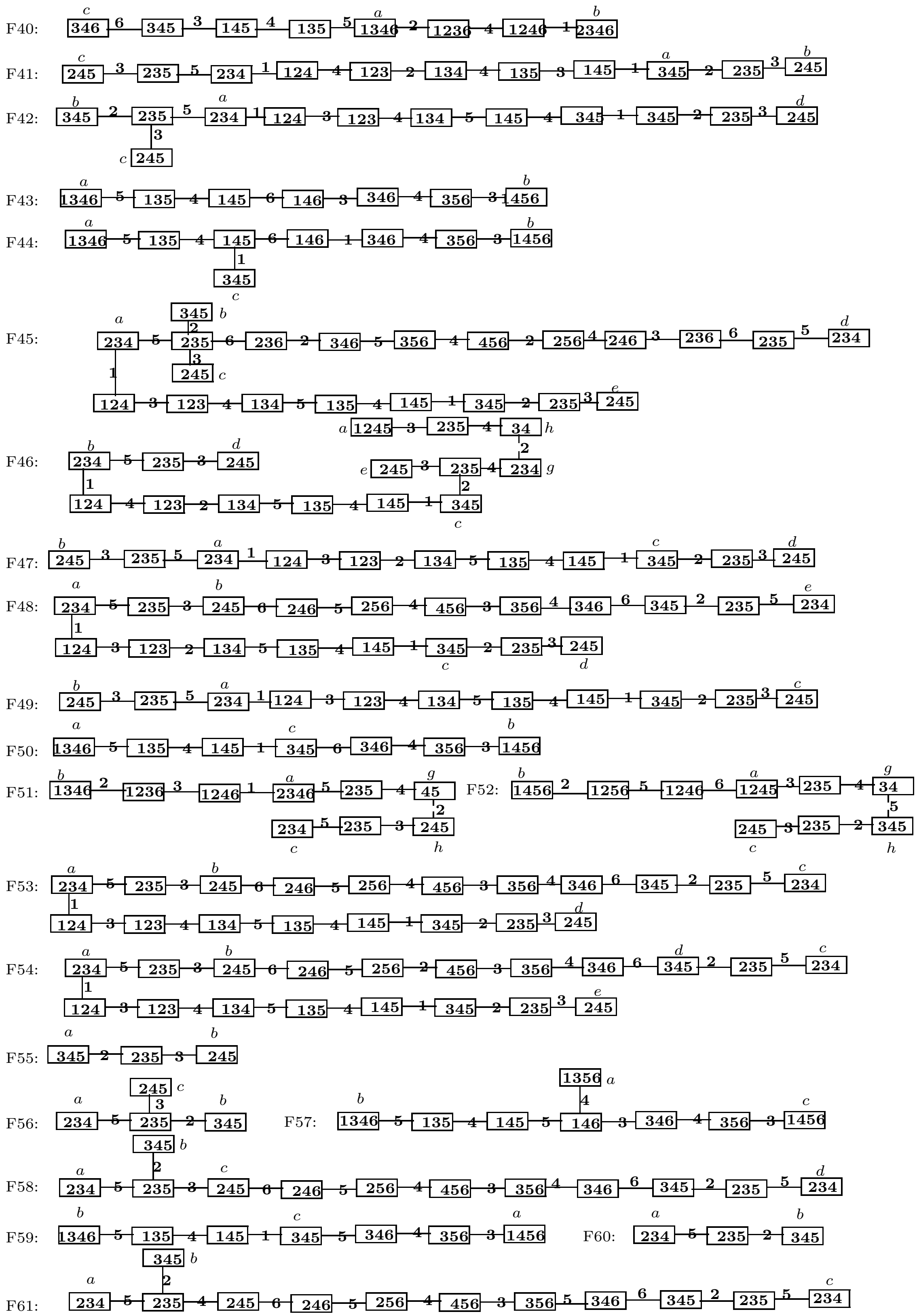
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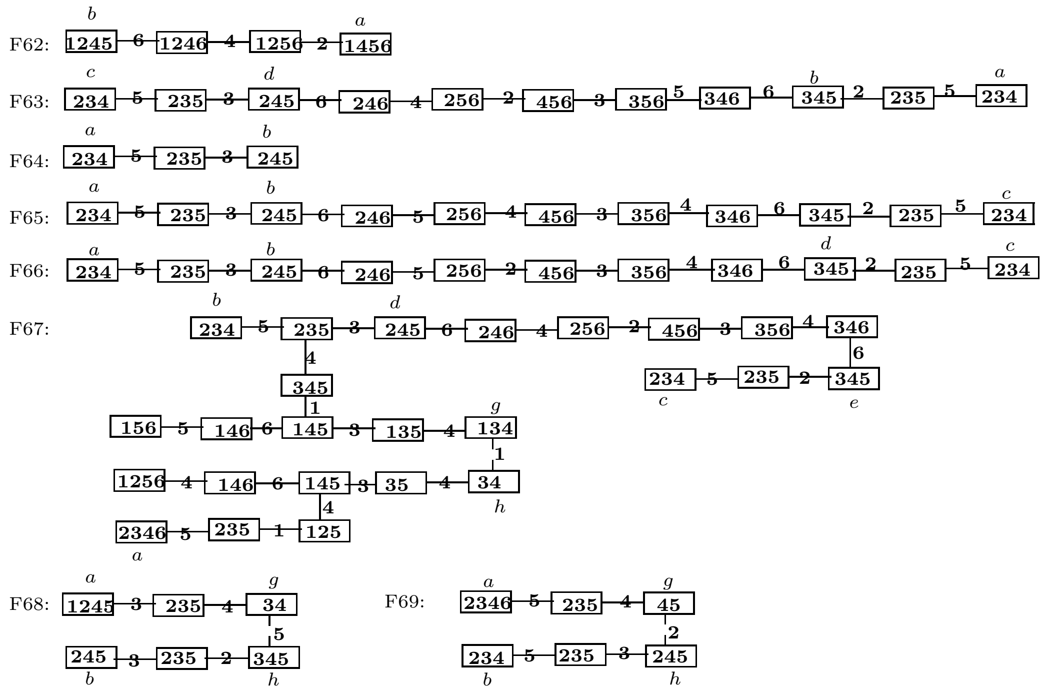

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