Ex Ante and *ex Post* Voting Power: A Method for Calculating Party Power in Party Government

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As political power tends to be wielded in the form of voting power in the national assembly, especially under the institutions of party government, one needs a method to calculate the voting power of political parties, both longitudinally and for a cross-section of European democracies. This paper suggests such a method, derived from the power index approach in cooperative game theory. The application of the method on the history of democracy in the two German nations results in party scores—mandates, *ex ante* and *ex post* voting power—that are much in congruence with the standard interpretations of the interwar period and the post-war politics in these two countries.

Keywords: Party Government; Government Coalitions and Coalitions in the Legislature; *Ex Ante* and *ex Post* Voting Power; Penrose-Banzhaf-Coleman Framework; Banzhaf Numbers or Scores; Double Banzhaf Games; Party Government in the Two German Speaking Nations

Introduction

Political power under a regime with party government in a multi-party system derives from the capacity of parties to enter into winning coalitions, either *ad hoc* in relation to the voting on decision issues in Parliament, or as a permanent member of a coalition in government. The Penrose-Banzhaf-Coleman model of coordination in N-person games—coalitions—offers a method to estimate this form of political power. In short: The cooperative game theory model measures the capacity of a party to contribute to the success of a coalition.

Here we wish to show how the power index method can be employed for the systematic and comparative analysis of the power of political parties in the typical regime of party government. We give as example a small study of party government in Germany—Weimar and FRG—as well as Austria—1st and 2nd Republic (1919-1933 and 1945-today). The research question can be posed as such: Have the various political parties been able to exercise voting power in proportion to their electoral support, as measured by the seats of the party in the nation's representative assembly?

Logic of Party Government: Double Banzhaf Games

In order to estimate the power of political parties under the regime of party government one needs to calculate Banzhaf numbers for the players involved in the games of party government, i.e. the political parties. In addition, one has to calculate the Banzhaf power for governments, whether majority or minority ones. All voting power derives from the capacity to form winning coalitions in the national assembly.

Banzhaf Voting Power

The Banzhaf voting power index (Banzhaf, 1965; Coleman, 1971; Felsenthal & Machover, 1988, 2002, 2004, 2005) models the marginal contribution of a player to the success of a coalition, given a social choice mechanism. Looking here at the so-called Penrose-Banzhaf-Coleman framework for solving one kind of N-person games, namely so-called *simple games*, one starts from the characteristic function of a game. Basically, one considers the set 2^N of all possible coalitions S employing the assumption of *equally likely* coalitions. From these 2^N coalitions, the Banzhaf score of player *i* is calculated as the number of coalitions in which *i* is *critical* according to:

$$(1) \quad \sum_{\substack{S \subseteq N \\ i \in S}} \left[\upsilon(S) - \upsilon(S \setminus \{i\}) \right]$$

that is, the number of coalitions that player *i* is able to *swing*. The Penrose-Banzhaf power index *β* of player *i* is defined by the ratio of swings *η* of *i* to the number of coalitions not containing the player *i*. Thus, we have:

$$(2) \quad \beta_i = \frac{\eta_i}{2^{n-1}}.$$

The employment of the Banzhaf formula results in voting power scores, either absolute or relative ones (normalised). Since we compare these scores with the percentage number of seats or mandates in Parliament, we use relative Banzhaf scores below. All data contained in the Tables below have been calculated with the help of a computer algorithm for voting power analysis by David Leech. Online: http://homepages.warwick.ac.uk/~ecaae/#Progam_List [last review: 29 January 2013].

Below political parties will be looked upon as organised players making coalitions involving a perfect degree of individual party member discipline.

Ex Ante and *ex Post* Voting Power

The representative assembly makes a huge number of decisions concerning for instance government formation and support, the budget and taxation as well as regulation in the form of legislation and policies. Additional functions of parliament include controlling government and articulating relevant problems as well as electing government. Under party government, the actors in collective decisions are the political parties that tend to vote with one voice, based on their differential number of mandates. Both the constitutional rules of decision-making and the strategy of coalition formation are decisive for the outcomes of parliamentary decision-making.

Most decisions in Parliament require simple majorities, which is why a minimum winning coalition is enough. Sometimes certain decisions of constitutional relevance are singled out requiring a heavier support or qualified majority. To prevail, a winning coalition would then have to be oversized. In the normal business of government, simple majority suffices in most parliamentary countries.

A coalition that is minimum winning does not need to be a minimum sized coalition. Actually, coalitions may be of many kinds, but the essential thing is that they win the voting sequences. Coalitions may be temporary, focusing on one voting sequence, or they may be permanent over an election period, such as for instance 4 years. Government coalitions are formed for the purpose of exercising power over a longer period, but government coalitions may break up prematurely, leading either to a new government or to new elections.

A political party maximizes its voting power by actively participating in the formation of coalitions. If it enters a government coalition, then it shares the voting power of the government with its governing partners. It may also exercise voting power by participating in temporary coalitions that achieve the minimum-winning format. Political power is mainly exercised through the capacity to influence voting in the national assembly. Typical for continental European democracies is the multi-party system, meaning that the parliamentary arena is comprised of three or more political parties, where no party has a majority position.

One may enquire after the differences in seats and voting power for the major political parties under the regime of party government by calculating two power index scores: *ex ante* power measures based upon the election outcomes before government formation, and *ex post* power measures based upon the pattern of government formation: minority, simple majority, oversized, etc., after government formation. The following distinctions can be made in this new method for the calculation the power of parties under party government:

a) The calculation of *ex ante* power scores is the straightforward solution of the Banzhaf game for parties in the legislature before the formation of a government or government coalition.

b) The calculation of the *ex post* power scores involves a *double Banzhaf game* for the parties after government formation, where the parties in a government coalition share whatever power government has in the legislature, on the basis of a unanimity game between the government coalition partners.

One may in principle distinguish between two parliamentary situations: a majority coalition and a minority coalition. The voting power score of the government coalition will here be assumed to be equally shared among the coalition parties on the basis of an imagined second Banzhaf game within the government—a unanimity game among equal coalition partners. Thus, we have:

1) Banzhaf number or score for a simple majority government = the party(ies) in government constitute the winning coalition in legislative voting (Banzhaf score = 1/number of parties in the government coalition).

2) Banzhaf number or score for a minority government = the party(ies) in government enter some of the winning coalitions in the legislature (Banzhaf score 0 < 1/number of coalition parties). When the coalition government does not control the voting in the legislature, it is forced to make *ad hoc* coalitions with the other parties—see Appendix for an example.

When there is a minority coalition government, other parliamentary actors like the opposition parties will receive Banzhaf scores as well, depending on whether they are decisive in coalitions.

c) One may further calculate the parties' Banzhaf power index numbers for both minority and majority coalitions for every legislative period and weighted for every single year, thus arriving at aggregated scores for legislative periods, and longer time spans.

d) By relating the *ex post* Banzhaf scores to the relative size of the parties' representation in the legislative assembly (seats or mandates), it can be shown how the parties' voting power differ from their sizes in Parliament, because some parties tend to dominate the political game, whereas others do not play a big role despite their relative strength.

Let us proceed to a couple of examples from the spectacular political history of democracy in Germany and Austria.

Example: Party Government in the Two German Speaking Nations

Germany

Proportional election formulas have been utilized in Germany to establish a legislative assembly. In compliance with parliamentarism, main power lies with the *Kanzler*. Two kinds of governments prevail: simple majority governments (FRG) and minority governments/coalitions (Weimar). There have only been a few consociational governments. In the Weimar Republic, the disproportion between the size of representation of the left-wing (seats, mandates) and its *ex post* Banzhaf voting power is striking. In the FRG, an effort was made to eliminate this effect, but has it been successful?

The Weimar Republic

In the Weimar Republic the smaller and right wing parties often augmented their voting power in comparison to their share of seats in the *Reichstag* by joining government. This discrepancy and disproportionality can be demonstrated by subtracting the parties' relative share of seats from its *ex post* Banzhaf points: Especially DDP and BVP enlarged their influence and voting power in comparison to their share of seats (DDP + 6.7 per cent, BVP + 5.3 per cent). But also larger conservative parties, that is *Zentrum* and DVP, increased their voting power by entering the mainly minority governments. Apparently, party coalitions, first minority governments, then

shifting majority coalitions, were the key to exercise power in the Weimar Republic, highly successfully accomplished by DVP's Gustav Stresemann between 1923-1929 (**Table 1**).

On the contrary, the Social Democratic party (SPD) and the

Table 1.
Banzhaf power distribution in the weimar republic 1919-1933.

Party		Modified aggregation	Deviations from proportionality (power – seats)
SPD	seats (rel.)	0.254736842	
	ex ante	0.291411	0.0366742
	ex post	0.179639259	−0.0750976
U SPD	seats (rel.)	0.154	
(1919-1924)	*ex ante*	0.139715	−0.014285
	ex post	0.1057972	−0.0482028
DNVP	seats (rel.)	0.138947368	
	ex ante	0.126354053	−0.0125933
	ex post	0.113363011	−0.0255843
Zentrum	seats (rel.)	0.139635857	
	ex ante	0.129780053	−0.0098558
	ex post	0.155244942	0.0156091
BVP	seats (rel.)	0.037010234	
	ex ante	0.034639444	−0.0023708
	ex post	0.090371939	0.0533617
DVP	seats (rel.)	0.084253906	
	ex ante	0.077607053	−0.0066469
	ex post	0.136149469	0.0518955
DDP	seats (rel.)	0.058270387	
	ex ante	0.052078947	−0.00619144
	ex post	0.124839478	0.066569091
NSDAP	seats (rel.)	0.132896772	
(1924-1933)	*ex ante*	0.148824857	0.015928085
	ex post	0.116784334	−0.016112438
Wirtschafts-	seats (rel.)	0.024752883	
partei	*ex ante*	0.023595211	−0.001157672
	ex post	0.029837938	0.005085055
KPD	seats (rel.)	0.096936858	
	ex ante	0.094663278	−0.00227358
	ex post	0.045465496	−0.051471362
Dt.-Hannov.	seats (rel.)	0.006994286	
Partei	*ex ante*	0.006280333	−0.000713953
	ex post	0.001587514	−0.005406772
Christl.-nat.	seats (rel.)	0.020770765	
Landvolk (1924-1933)	*ex ante*	0.018560071	−0.002210694
	ex post	0.012855946	−0.007914819
CSVD	seats (rel.)	0.015807965	
(1930-1933)	*ex ante*	0.014741333	−0.001066632
	ex post	0.013180917	−0.002627048
Konservative	seats (rel.)	0.001299827	
Volkspartei	*ex ante*	0.0034224	0.002122573
(1930-1932)	*ex post*	0.013798129	0.012498302
Deutsche	seats (rel.)	0.010404347	
Bauernpartei	*ex ante*	0.008974556	−0.001429791
(1928-33)	*ex post*	0.003109667	−0.00729468
Others	seats (rel.)	0.006491473	
	ex ante	0.00438	−0.002111473
	ex post	0.001472844	−0.005018629

Abbr.: SPD = Sozialdemokratische Partei Deutschlands (social democratic); U SPD = Unabhängige Sozialdemokratische Partei Deutschlands (independent, socialist); DNVP = Deutschnationale Volkspartei (nationalist, conservative); Zentrum = Center party (catholic); BVP = Bayerische Volkspartei (Bavarian, conservative); DVP = Deutsche Volkspartei (liberal); DDP = Deutsche Demokratische Partei (social liberal); NSDAP = Nationalsozialistische Deutsche Arbeiterpartei; Wirtschaftspartei = liberals; KPD = Kommunistische Partei Deutschlands (communists); Deutsch-Hannoversche Partei (local conservative party); Christl.-nat. Landvolk = Christlich-Nationale Bauern- und Landvolkpartei (agrarian, conservative); CSVD = Christlich sozialer Volksdienst (protestant conservative); Konservative Volkspartei = small conservative party; Deutsche Bauernpartei (agrarian).

Communist Party (KPD) respectively were often unable to turn their mandate strength in parliament effectively into *ex post* influence or voting power after the broad "Weimar coalition". Social democrats became frequently the largest party in parliament after elections. Additionally, their *ex ante* Banzhaf score was greater than their relative mandate strength. But even so, they seldom joined government. A loss of power can be stated from the *ex ante* to the *ex post* stage of government formation, which can be explained by a lack of coalition options from the mid-1920s onwards.

The nationalist DNVP was a large right-wing opposition party, which often supported one of the minority governments. But, in contrast to SPD or KPD, the DNVP shows only a small loss of power when comparing its relative proportion of seats and its voting power. Thus, only a small difference can be stated between the DNVP with its nearly equal *ex ante* and its post Banzhaf scores and the government parties.

The Federal Republic (FRG)

At first sight, the results for the FRG seem to differ from the findings for the Weimar republic: The Federal Republic developed a 2.5 party system (Poguntke, 1999), comprising two "catch all" parties (Kirchheimer, 1965), namely CDU respectively CSU and SPD, and the FDP as a 0.5 "pivotal" party (Keman, 1994).

But when applying the Banzhaf-Coleman approach to the distribution of power, we thus find a parallel between the Weimar republic and the FRG: Again, the political left shows a tendency of underrepresentation in governments on the federal level, also when the party system changed since the 1980s and 1990s.

As can be seen in **Table 2**, only the FDP has been able to increase its voting power in comparison to its relative mandate strength. The "Union", consisting of CDU and CSU, shows only small losses of power in the *ex ante* and *ex post* government estimates. The FDP has played a powerful role by means of its pivotal status. Taking the discrepancy between the relative share of seats and the *ex post* Banzhaf scores into account, it can be noted that only the liberal FDP (+28 per cent) and to a lesser degree the Greens (+4.6 per cent) are the parties whose Banzhaf *ex post* power scores increased between 1949 and 2012, compared with their share of the seats (mandates).

The Social Democrats have often been unable to transfer their profound mandate strength into power through an *ex post* government formation. Their voting power is small compared with their representation. Thus, the loss of power in *ex post* government formation is nearly twice as much (−18.1 per cent) as the Christian democratic loss (−9.5 per cent). Yet, the lack of power of the left-wing on the federal level is compensated by participation in several *Länder* governments and their power in the *Bundesrat*, although *Die Linke* is still regarded as "regierungsunfähig" (incapable of governing) by the other four parties at the federal level.

Austria

When looking at the First Republic in Austria, the right-wing parties were heavily overrepresented in comparison to their relative share of seats in the *Nationalrat*. On the other hand, in

Table 2.
Banzhaf power distribution in the FRG 1949-2012.

Party		Modified aggregation	Deviations from proportionality (power – seats)
CDU/CSU	seats (rel.)	0.52482205	
	ex ante	0.501040618	−0.023781432
	ex post	0.429696945	−0.095125105
SPD	seats (rel.)	0.435363492	
	ex ante	0.301324545	−0.134038947
	ex post	0.254545455	−0.180818037
FDP	seats (rel.)	0.111550297	
	ex ante	0.242562455	0.131012158
	ex post	0.395151491	0.283601194
Grüne (since 1983)	seats (rel.)	0.070312448	
	ex ante	0.153269967	0.082957519
	ex post	0.116666667	0.046354219
PDS (since 1991)**/**	seats (rel.)	0.055674471	
Linkspartei.PDS /	*ex ante*	0.072640727	0.016966256
Die Linke	*ex post*	0.000000000	−0.055674471
Others	seats (rel.)	0.01402697	
(1949-1961)	*ex ante*	0.01501596	0.00098899
	ex post	0.01301044	−0.00101653

Abbr.: CDU/CSU = Christdemokratische Partei Deutschlands/Christlichsoziale Partei Bayerns (Christian democratic); SPD = Sozialdemokratische Partei Deutschlands (Social democratic); FDP = Freie Demokratische Partei (liberal); Grüne (ecological); PDS = Partei des demokratisches Sozialismus, Linkspartei. PDS, Die Linke (left socialist).

the Second republic (1945-today) with its consociational governments, an opposing finding can be made: Both the Austrian social democrats (SPÖ) and the Christian democratic (ÖVP) are able to augment voting power from the *ex ante* to the *ex post* government formation. In contrast to the German system, the smaller parties (FPÖ, Greens, BZÖ) play a minor role in parliament and government.

The First Republic (1919-1933)

Between 1919 and 1933 especially three political blocks dominated the Austrian party system: a socialist (SDAP), a Christian-catholic (Christlichsoziale) and a German nationalist block, which included some smaller parties (Pelinka, 2004: p. 535).

Neglecting the two years, in which SDAP and Christlichsoziale formed a grand coalition before the constitution was passed in 1920, it was the Christian social party and the smaller German nationalist party group, which was participating in government on a regular basis. As can be seen in the Banzhaf power distribution in **Table 3**, the SDAP was heavily underrepresented in comparison to its relative share of seats (−25.9 per cent). The German nationalist party group, in contrast, is massively overrepresented with a plus of 31 per cent in relation to the mandate strength won in elections.

The Second Republic (1945-Today)

Germany is often described as the "Grand coalition state" (Schmidt, 1996). But, when comparing the number of grand coalitions in the FRG (2) and Austria (10), it has to be emphasized that this concept would fit Austria better. This is reflected in the Banzhaf numbers for the parties *ex post* government formation: in Austria, it is only the two "catch all" parties that augment their power by frequently forming government (SPÖ + 4.2 per cent, ÖVP + 2.7 per cent) while all other parties incur losses.

For a long time, the Austrian party system was a 2.5 party system like the German one, consisting of SPÖ, ÖVP and FPÖ. But in contrast to the FRG, the Austrian FPÖ never gained the "pivotal" status as the FDP could in Germany, because of the major parties exercising power in an oversized coalition (**Table 4**).

Conclusion

The Banzhaf-Coleman approach may be developed into a tool for the systematic and comparative analysis of the power of political parties under the institutions of party government. Voting power is wielded over legislation, budgets and taxation, policies and regulations. It derives from winning by simple majority in the legislative assembly, either as a single party or as a coalition partner in government, with majority or minority status.

Applying this new method to the political history of democracy in the two German nations, we observe quite a remarkable fit between the stylised images of the politics of these countries and the power scores for the political parties. Thus, the interwar period comprised highly skewed party government, with dis-

Table 3.
Banzhaf power distribution in Austria 1919-1933 (1st Republic).

Party		Modified aggregation	Deviations from proportionality (power – seats) modified aggregation
SDAP	seats (rel.)	0.326533333	
	ex ante	0.333333333	0.0068
	ex post	0.066666673	−0.259866666
Christlichsoziale	seats (rel.)	0.462751467	
	ex ante	0.333333333	−0.129418134
	ex post	0.466666666	0.003915199
Deutschnationale	seats (rel.)	0.13902	
	ex ante	0.3333333	0.1943133
	ex post	0.449999967	0.310979967
Landbund	seats (rel.)	0.14	
	ex ante	0	−0.14
	ex post	0	−0.14
Heimatbund/Heimwehr	seats (rel.)	0.04424	
	ex ante	0	−0.04424
	ex post	0.01777767	−0.02646233
Others	seats (rel.)	0.012	
	ex ante	0	−0.012
	ex post	0	−0.012

Abbr.: SDAP = Sozialdemokratische Arbeiterpartei (social democratic); Christlichsoziale = Christlichsoziale Partei (christian-conservative); Deutschnationle = Deutsch-Nationale Bewegung (nationalist); Landbund (agrarian); Heimatbund/Heimwehr (paramilitary, nationalist).

Table 4.
Banzhaf power distribution in Austria after 1945 (2nd Republic).

Party		Modified aggregation	Deviations from proportionality (power – seats)
KPÖ (1945-1959)	seats (rel.)	0.024675318	
	ex ante	0.035714293	0.011038975
	ex post	0	−0.024675318
Greens (since 1986)	seats (rel.)	0.074190836	
	ex ante	0.058608031	−0.015582805
	ex post	0	−0.074190836
SPÖ	seats (rel.)	0.433602479	
	ex ante	0.391257975	−0.042344504
	ex post	0.475124378	0.041521899
ÖVP	seats (rel.)	0.418770529	
	ex ante	0.366382331	−0.052388198
	ex post	0.445273631	0.026503102
VdU/FPÖ (since 1949)	seats (rel.)	0.104511154	
	ex ante	0.212018132	0.107506978
	ex post	0.084656084	−0.01985507
Liberales Forum (1994-1999)	seats (rel.)	0.055737705	
(1994-2002)	*ex ante*	0	−0.055737705
	ex post	0	−0.055737705
BZÖ (since 2006)	seats (rel.)	0.089253188	
	ex ante	0.142857	0.053603812
	ex post	0	−0.089253188

Abbr: KPÖ = Kommunistische Partei Österreichs (communist), Greens = Grünen – Die grüne Alternative (ecological); SPÖ = Sozialdemokratische Partei Österreichs (social democratic); ÖVP = Österreichische Volkspartei (Christian democratic); VdU/FPÖ = Verband der Unabhängigen (1945-1955)/since 1955: Freiheitliche Partei Österreichs (right-wing-demagogic, populist); Liberales Forum (liberal); BZÖ = Bündnis Zukunft Österreichs (nationalist-right-wing demagogic, economically liberal).

proportionate power of the centre (Germany) and the right-wing (Austria), indicating democratic instability. The post-war shows considerably more of balance between the right and the left, especially in Austria, where consociationalism provides the left with more power than the simple majority governments in the FRG.

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Appendix

Calculation of *ex ante* and *ex post* voting power in a 5 player legislature with parties of the following size: P1 = 35%, P2 = 25%, P3 = 18%, P4 = 12% and P5 = 10%.

	P1	P2	P3	P4	P5
Ex ante voting power:	0.38	0.23	0.23	0.077	0.077
Ex post voting power:					
Majority gov P1 + P2:	1/2	1/2	0	0	0
Minority gov with P1 + P5:	0.5/2	0.17	0.17	0.17	0.5/2