TR-3
1966

Texas Water Resources Institute
make every drop count

Techniques for Hydrograph Synthesis Based on Analysis of Data from Small Drainage Basins in Texas

M.D. Hudlow

Texas Water Resources Institute

Texas A&M University

TR#3

TECHNIQUES FOR HYDROGRAPH SYNTHESIS BASED ON ANALYSIS OF DATA FROM SMALL DRAINAGE BASINS IN TEXAS

A Thesis

By

MICHAEL DALE HUDLOW

Submitted to the Graduate College
of the Texas A&M University in partial
fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 1966

Major Subject: METEOROLOGY

TECHNIQUES FOR HYDROGRAPH SYNTHESIS BASED ON
ANALYSIS OF DATA FROM SMALL DRAINAGE
BASINS IN TEXAS

A Thesis

By

MICHAEL DALE HUDLOW

Approved as to style and content by:

(Chairman of Committee)

(Head of Department)

(Member)

(Member)

(Member)

May 1966

ACKNOWLEDGEMENTS

The author graciously acknowledges the guidance of Professor R. A. Clark. Appreciation is also extended to Professors J. F. Griffiths, V. E. Moyer, and W. B. Davis for their review and suggestions, and to Dr. J. J. O'Brien for his suggestions concerning the curve fitting procedure.

The author's graduate program was supported by a Texas A&M University Graduate College Fellowship. Further acknowledgement is extended to the Data Processing Center, Texas A&M University, for the use of their computer facilities and to the Water Resources Institute, Project No. 5002, Texas A&M University, for providing assistance in the reproduction. The streamflow and rainfall data were acquired from records maintained by the Blacklands Experimental Watersheds, Agricultural Research Service, U. S. Department of Agriculture, Riesel, Texas, Mr. R. W. Baird, Director.

The manuscript was typed by Mrs. D. Faver, Mr. D. Gramatges drafted the figures, and the duplication of the final manuscript was by Mr. F. T. Callahan.

To all my family, I extend my appreciation for encouragement throughout the years of my education. To my wife, Sarah, I especially am grateful; because without her patience and assistance this thesis would not have been possible.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

A	watershed area
A_A	area added to the unit hydrograph recession in order to ensure a volume of 1 in.
B	base width of unit hydrograph (hr, or for the TWC technique, min)
c, r	parameters in Pearson's equation
C_b	parameter in Sherman-Mayer infiltration equation
C_p, C_t	Snyder's coefficients
D	duration of rainfall excess (hr)
d_1, d_2	the deviations of r and c respectively, from the trial r and c values which were used for the origin of the multiple Taylor series expansion
f_{av}	average infiltration capacity (in./hr)
f_c	infiltration capacity (in./hr)
f_u	ultimate infiltration capacity (in./hr)
L	length of main stream from gaging station to divide (mi, or for the TWC technique, ft)
L_{ca}	length along main stream from the gaging station to a point opposite the centroid of the drainage area (mi)
P_r	hydrograph period of rise (hr, or for the TWC technique, min)
q_p	hydrograph peak discharge (cfs/mi^2)
Q	hydrograph discharge (cfs)
Q_a	discharge amounts to be added to the already existing discharge values in the unit hydrograph recession in order to insure a volume of 1 in. (cfs)

Q_p	hydrograph peak discharge (cfs)
Q_7	discharge for the seven points of the modified Snyder technique (cfs)
R	rainfall intensity (in./hr)
S	slope of main stream (ft/ft)
t_p	basin lag time (hr)
T	time (hr)
T_E	effective storm duration (hr)
T_u	time at which infiltration capacity reaches ultimate (hr)
T_{75}, T_0	times at 75 per cent peak discharge and assumed zero discharge, respectively (hr)
v	residual damping factor
W_{50}, W_{75}	hydrograph width at 50 and 75 per cent peak discharge, respectively (hr, or for the TWC technique, min)

CHAPTER I

INTRODUCTION

Need for the Study

The report by the Committee on Surface Water Hydrology of the American Society of Civil Engineers (ASCE) (2) has pointed out that one of the areas in hydrology lacking adequate research is the investigation of runoff from small areas. As indicated by Meier (22, p. 55) and Barnes (4, p. 55), more work is needed for establishing lag time[1] versus basin characteristics relationships. Meier also stated that a method should be perfected for fitting unit hydrograph[2] data to mathematical functions in order that computers can be used to generate synthetic unit hydrographs. This study has considered all three of these related areas.

Objective and Scope of Study

The objective of this study is hydrograph synthesis based on an investigation of runoff from small drainage

[1]Lag time as used in this study is equal to the period of rise of the hydrograph minus one-half the rainfall excess duration.

[2]The unit hydrograph may be defined as the discharge hydrograph resulting from 1 in. of direct runoff generated uniformly over the contributing area at a uniform rate during a specified period of time.

basins in Texas. In addition, a technique is developed for fitting unit hydrograph data to a mathematical function.

The analyses were made for small drainage basins, ranging in size from 0.5 to 75 mi^2, located in central Texas. However, it should be stressed that techniques used in this study are applicable also to larger basins.

Falling within the scope of this study is the selection of a satisfactory technique for obtaining the temporal distribution of infiltration for basins of the size under consideration. The period of rainfall excess can then be determined. Correlations were considered between the critical hydrograph parameter, lag, and basin characteristics. In order to broaden the scope of the study, information on lag obtained by two previous investigators (Meier, 22; Espey, Morgan, and Masch, 10) was compared with lag relationships developed in this investigation.

Related Studies

The unit hydrograph principle was presented in 1932 by Sherman (27). Since that time, the unit hydrograph theory has been accepted as one of the best methods available for relating a particular rainfall event to the hydrograph resulting from the event.

In 1939, Brater (5) studied the usefulness of the unit hydrograph principle when applied to small streams.

This work was carried out on basins that varied from 4.24 to 1875 acres. Brater concluded that the unit hydrograph technique may be applied successfully to small watersheds.

In many cases, due to the lack of runoff data, the designer must resort to some method of estimating hydrograph characteristics from physiographic features of the watershed. This is particularly true in southwestern United States. The first procedure developed for synthetically constructing a unit hydrograph was presented by Snyder (32) in 1938. From data for the Appalachian Mountain area, Snyder related basin lag time to watershed length parameters.

From 1938 to present, there have been numerous procedures proposed for synthetic unit hydrograph development. In 1962, Morgan and Johnson (25) made analyses utilizing four such synthetic unit-hydrograph methods. A comparison of Snyder's method (32), Common's method (8), Mitchell's method (24), and the SCS (Soil Conservation Service) method (29), was made to determine if one of these procedures would give a consistently better estimate of a unit hydrograph. Their study utilized data from 12 basins in Illinois that ranged in size from 10 to 100 mi^2. Morgan and Johnson concluded from their limited study that none of the four methods consistently gives better estimates.

One of the more recent contributions to hydrograph synthesis was made by Hickok, Keppel, and Rafferty (13). In a study of 14 watersheds in the arid Southwest that ranged in size from 11 to 790 acres, lag time was related to watershed area, average land slope, and drainage density. They found, as is implied by the procedure presented by Snyder, that the lag time is the major determinant of hydrograph shape. In fact, as pointed out by Linsley, Kohler, and Paulhus (21, p. 204), the key factor in most synthetic procedures has been the basin lag.

Minshall (23) has studied the effect of varying storm characteristics on the unit hydrograph for watersheds of 27, 50, and 290 acres. Plots are presented which relate lag time and unit hydrograph peak discharge to rainfall intensity.

Eagleson (9) made a study to determine a set of empirical equations to be used for sewered areas. His equations are analogous to those used in the Snyder technique for natural areas. Urban areas up to 7.5 mi^2 located in Louisville, Kentucky, were used.

Several attempts have been made to represent unit hydrographs by a mathematical model. One of the more recent attempts was made by Gray (11). In 1960, Gray studied 46 watersheds which ranged in size from 0.27 to 32.64 mi^2. He presented a method whereby the unit hydrograph for a small watershed can be synthesized from a representative dimensionless hydrograph. The dimensionless hydrograph was characterized by a two-parameter Gamma distribution; the parameters were determined from measurable topographic characteristics. Reich (26) performed a similar study for small watersheds. However, in contrast to Gray's approach, Reich did not utilize a representative dimensionless hydrograph. That is, the mathematical function used by Reich permits one to obtain directly a unit hydrograph if the period of rise, the peak discharge, and a shape parameter are known. Reich suggests that this mathematical function, known as the Pearson type III function, offers more flexibility because it is a three parameter rather than a two-parameter function. The Pearson type III function is discussed in more detail in Chapter II.

Recently, Meier (22, 1964) carried out hydrograph analyses for three basins in Texas of approximately 18, 70, and 75 mi^2. The objective of the Meier study was quite similar to the present study, viz., to examine the validity of techniques when applied to small watersheds in Texas and at the same time to observe the behavior of the parameters critical to hydrograph synthesis.

The most recent related study was carried out by Espey, Morgan, and Masch (10, 1965). In their study equations were developed which would permit the synthesis of unit hydrographs in a manner analogous to the approach used in the Snyder technique. Two sets of equations were derived, one set for small urban watersheds and the other for small natural watersheds. The equations for the natural watersheds were based on analyses from 11 areas ranging in size from 0.143 to 7.01 mi^2. Seven of these watersheds were located in Texas.

Although routing procedures will not be used in this study, it should be mentioned that routing is another way of obtaining a synthetic unit hydrograph for an ungaged basin. A detailed review of various routing procedures is presented by Laurenson (17).

CHAPTER II

DEVELOPMENT AND PROCEDURE

Source and Selection of Data

The basins used in this study fall into two categories:

1. Those for which complete analyses were performed. That is, for a selected number of storm events, infiltration and rainfall analyses were performed, and best estimates of the lag time for the basins were made. Also, an average unit hydrograph was determined for each basin.
2. Those for which analyses have been performed by other investigators, the results of these analyses being used in the present study.

The approximate location of each basin used in this study is shown in Figure 1. Characteristics of each basin are shown in Table 1, viz., area (A) in square miles, length of main stream (L) in miles along the main stream from the gaging station to the divide, the length in miles along the main stream from the gaging station to a point opposite the centroid of the drainage area (L_{ca}), and the slope of the main stream (S) in ft/ft, which is the

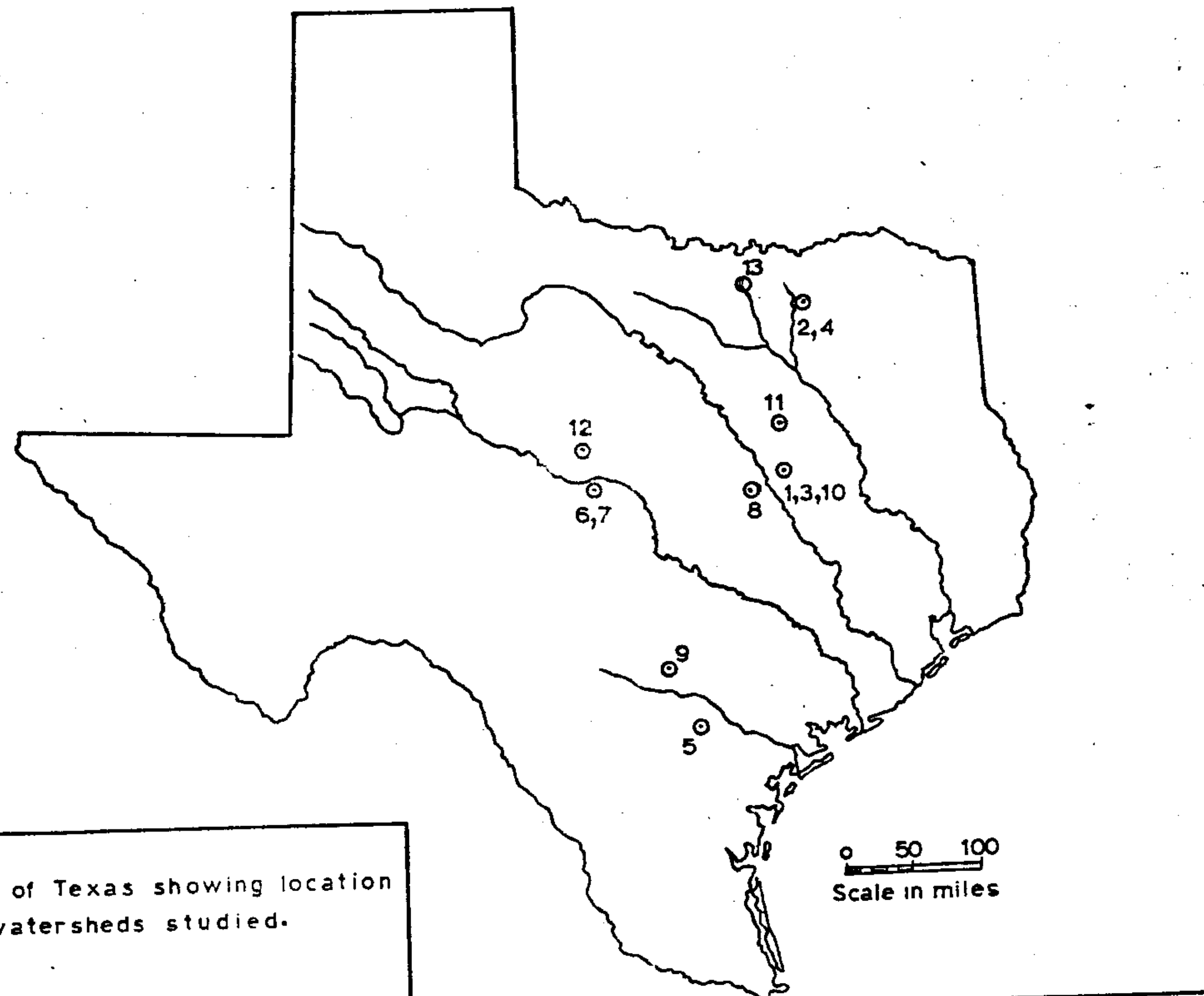

Fig. 1. Map of Texas showing location of watersheds studied.

Table 1. Watershed Characteristics

No.	Watershed Name	Major River Basins	Category	Reference	$A(mi^2)$	L(mi)	Lca(mi)	S(ft/ft)
1	Y	Brazos	1	Present Study	0.48	0.96	0.33	.0119
2	Honney Creek #12	Trinity	2	Espey, Morgan, & Masch (10)	1.26	1.53	0.91	.0150
3	D	Brazos	1	Present Study	1.73	2.23	0.87	.00612
4	Honney Creek #11	Trinity	2	Espey, Morgan, & Masch (10)	2.14	1.84	1.04	.0110
5	Escondido #1	San Antonio	2	"	3.29	2.22	1.33	.0133
6	Deep Creek #3	Colorado	2	"	3.4[illegible]	2.69	0.90	.0190
7	Deep Creek #8	Colorado	2	"	4.32	4.79	1.70	.0120
8	Cow Bayou #4	Brazos	2	"	5.25	4.11	1.70	.0167
9	Calaveras	San Antonio	2	"	7.01	3.50	1.70	.00793
10	J	Brazos	1	Present Study	9.16	6.78	3.93	.00377
11	Pin Oak Creek	Brazos	2	Meier (22)	17.60	7.92	3.75	.00173
12	Mukewater Creek	Colorado	2	"	70.00	19.20	8.50	.00228
13	Little Elm Creek	Trinity	2	"	75.50	25.00	14.00	.00122

difference in elevation between the divide and the gaging station divided by L. Also, the name of the watershed, its major river basin, the category of study, and the reference from which the information was derived are indicated.

The three basins that fall in Category 1 are located on the Brushy Creek watershed about 15 mi southeast of Waco, Texas, in the Brazos River Basin. This area, typical of the "Blacklands" of Texas, has a gently rolling relief of predominantly Houston black clay soil. A detailed description of the area can be found in USDA Bulletin No. 5 (31). Figure 2 is a map of the three basins illustrating the distribution of rain gages and location of stream gages. The rain gages are automatic recording gages of the Fergusson weighing type. Streamflow is measured with the use of the V-notch or Columbus weir and continuous stage recorders.

Data Analysis

Table 2 lists the storms analyzed for the three basins and the data source. Gray (11, p. 35) listed six criteria which should be followed in selecting hydrologic data suitable for unit hydrograph development:

1. The rain must have fallen within the selected time unit and must not have extended beyond the period of rise of the hydrograph.

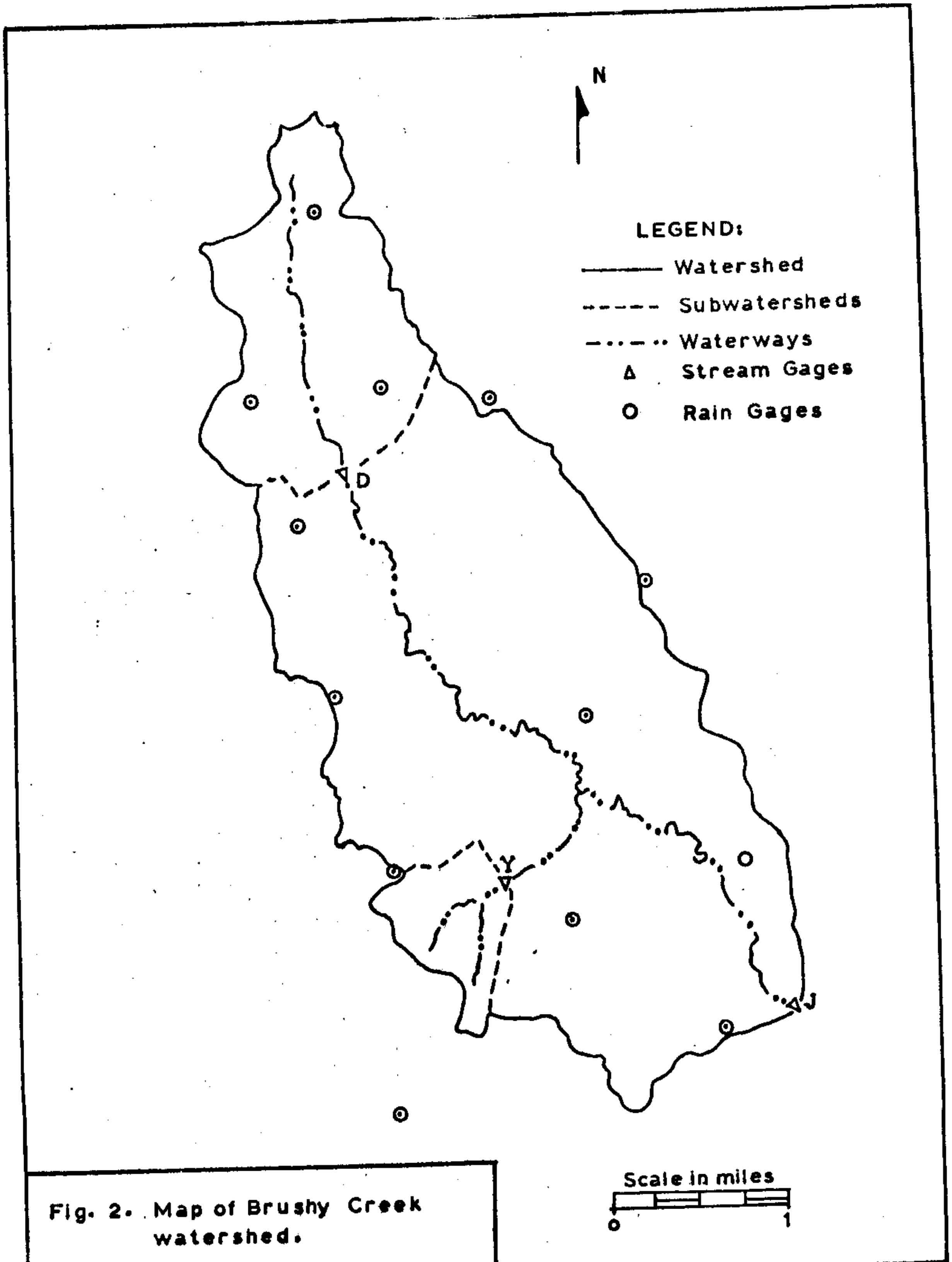

Fig. 2. Map of Brushy Creek watershed.

Table 2. Storms Analyzed and Data Sources

Basin	Date	Data Source
Y	March 28, 1938 May 17, 1939 May 18, 1939 May 20, 1939	Reference (30)
D	May 3, 1957 June 23, 1959 December 31, 1959	Reference (3)
J	January 23, 1938 February 16, 1938 March 28, 1938 May 18, 1939	Reference (30)

2. The storm must have been well distributed over the watershed, all stations showing an appreciable amount.
3. The storm period must have occupied a place of comparative isolation in the record.
4. The runoff following a storm must have been uninterrupted by the effects of low temperatures and unaccompanied by melting snow or ice.
5. The stage graphs or hydrographs must have a sharp, defined, rising limb culminating in a single peak and followed by an uninterrupted recession.
6. All stage graphs or hydrographs for the same

watershed must show approximately the same period of rise.

When all of these criteria could not be satisfied in the selection of the storms, it was necessary to examine closely the data from each storm to determine whether it was appropriate for use in this study.

Rainfall. The time distribution of basin average rainfall intensity was determined by Thiessen polygon weighting (21, p. 36). In addition, the areal distribution of each storm was studied by preparing isohyetal maps of total rainfall.

Infiltration. The problem of infiltration analysis is a complex one. If infiltrometer tests are not available, techniques of deriving infiltration rates from rainfall-runoff analyses must be used. Wisler and Brater (34, p. 113) discuss such a technique, which was suggested by Horner and Lloyd (14), for determination of the infiltration capacity curve for small drainage basins. They state that for small basins, in which the hydrograph quickly responds to the varying intensities of rainfall, the actual manner in which the infiltration varies throughout the storm often can be determined quite accurately. However, such response cannot be expected on basins larger than a few acres. Johnstone and Cross (16, p. 198) discuss a more elaborate technique for the computation of infiltration capacity curves for small natural areas based on rainfall-runoff data. Their technique appears suitable for areas less than about 1 mi^2.

In work with large areas an average infiltration capacity generally is sufficiently accurate. Wisler and Brater (34, p. 117) discuss a technique proposed by Horton (15) whereby the average infiltration capacity may be determined for large drainage basins in which the rain intensity is not uniform. Thus, for areas less than 1 mi^2 and for large basins one of the techniques discussed above, or one similar in nature, might be used. The primary problem is: what should be done for areas from 1 mi^2 to perhaps 25 mi^2?

As pointed out by Mitchell (24, p. 22), one of the most practical approaches to the determination of infiltration capacity curves was presented by Sherman and Mayer (28). Although the technique was based on infiltration analysis from basins of 10 to 3000 mi^2 in Oklahoma and Mississippi, it is felt that by virtue of the principles involved it can give good estimates for smaller areas and other localities. For this reason and because the technique can be readily adapted for computer analysis, the Sherman and Mayer technique has been used exclusively in this study. The methodology, as presented by Sherman and Mayer, makes use of a diagram whereby an infiltration capacity curve may be derived graphically from any given value of average infiltration capacity, f_{av}, and the ultimate infiltration capacity, f_u. Sherman and Mayer also report an equation derived by Escott (28, p. 667) which describes the shape of the curves for their diagram. By proper manipulation it is a simple matter to use the equation presented by Escott for predicting the infiltration capacity curve. This equation (see Appendix A) is simply an expression which defines the time distribution of infiltration capacity subject to the restraint of the two shape parameters, f_{av} and f_u. It is believed that this technique is particularly well suited to areas similar to those used in the present study. The accuracy of the technique hinges on the proper selection of f_u.

The soil for the central Texas area, as classified according to the ASCE Hydrology Handbook (1, p. 48), falls into the low infiltration capacity group. This indicates an f_u between 0.01 to 0.10 in./hr. The infiltration capacity curves presented by Linsley, Kohler, and Paulhus (20, p. 213) for Houston black loam give an f_u equal to about 0.06 in/hr. Since the Houston black clay should be even more impervious than the Houston black loam, a value of 0.04 in./hr was adopted. However, if f_u was actually as large as twice or as small as one half this value, the error involved in the determination of rainfall excess generally would be small. This is due to the relatively small magnitude of f_u for this class of soil.

A further description of the Sherman and Mayer technique is presented in Appendix A. Once the time distribution of rainfall intensity and infiltration capacity are determined, the duration of rainfall excess, D, is readily available. A typical distribution for basin D, with the rainfall excess duration indicated, is shown in Figure 3.

Synthetic procedures. As pointed out previously, streamflow data are not always available for a basin; therefore, synthetic procedures must be adopted. This is particularly true in the case of small watersheds. It was felt that it would be advantageous to examine certain synthetic procedures and apply them to the watersheds of the present study. Therefore, three procedures were examined. In addition, a method was perfected for fitting the unit hydrograph data given by two of these procedures to mathematical functions so that numerical methods could be used to generate synthetic unit hydrographs.

Butler (7, p. 309) points out that the only drainage basin characteristics that need to be considered for synthetic unit hydrograph determination are those which affect the rate of surface drainage of a given uniform rate of net rainfall. Climate and infiltration

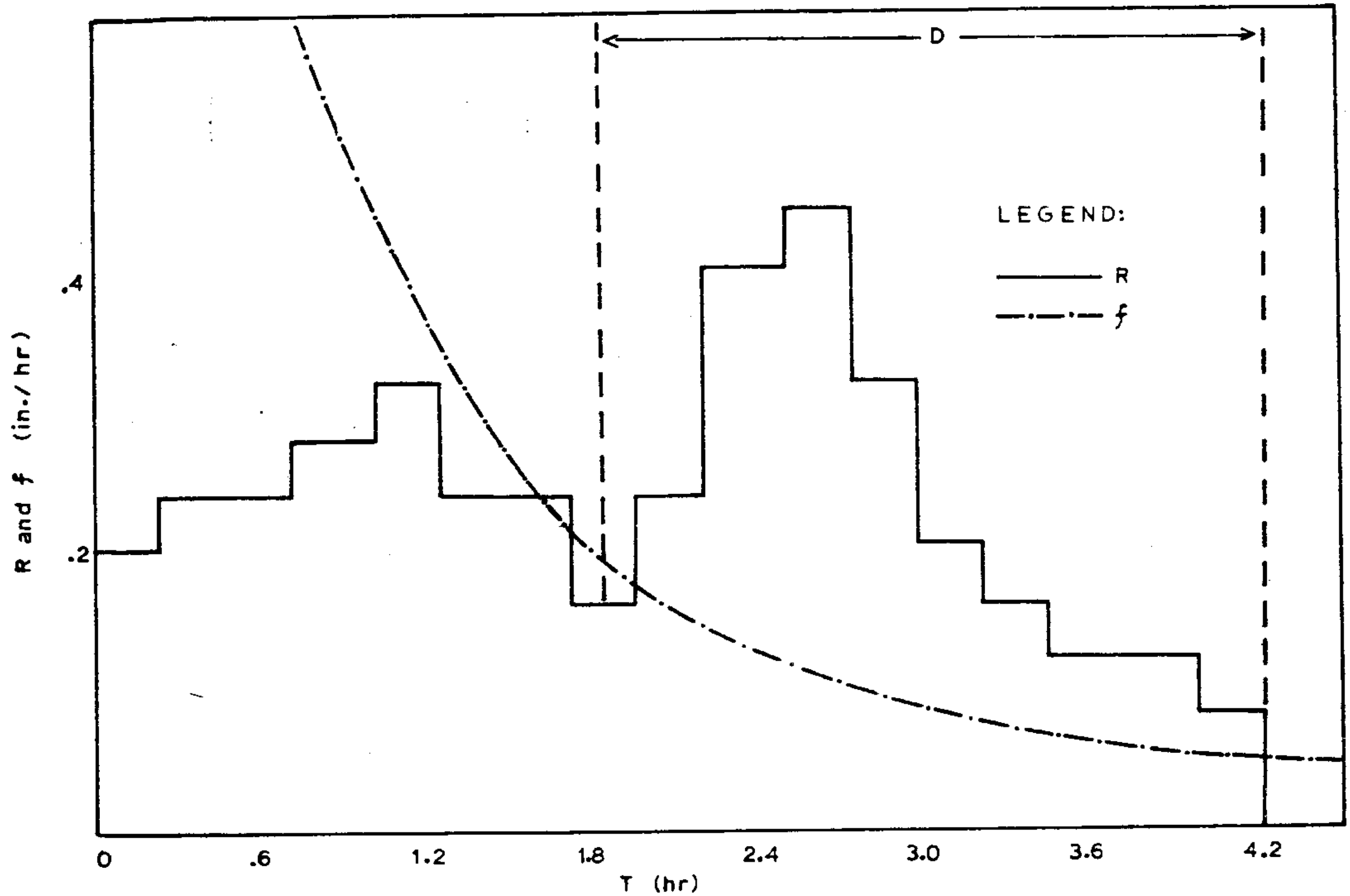

Fig. 3. Distribution of rainfall intensity and infiltration for basin D on December 31, 1959.

characteristics determine the amount of net rain but not its rate of drainage; therefore, they should not be involved in synthetic unit hydrograph determination. Rather, watershed characteristics such as area, basin shape, channel slope, land slope, size of channel, condition of channel, and stream density might be expected to have a pronounced effect on the unit hydrograph.

Snyder (32, 1938) presented the first set of equations for synthesizing a unit hydrograph from measurable topographic characteristics. Since that time there have been numerous other approaches. Some of these were mentioned in Chapter I. Snyder's procedure was selected as one of the techniques for examination in this study because:

1. It is one of the most widely utilized synthetic procedures in existence.
2. There have been many sets of equations quite similar to Snyder's original equations derived for different areas and conditions. For example, Linsley (19) has modified Snyder's equations to give better results for the Sierra Nevada and Coast Range. Eagleson (9) has obtained a set of equations analogous to Snyder's for sewered areas in Louisville, Kentucky. Another one of the techniques examined in this

study involves the use of a set of equations, proposed by Espey, Morgan, and Masch (10) for Texas, which are quite analogous to Snyder's equations.

3. Snyder's technique is easy to apply.
4. Although this is the oldest synthetic technique, it is still one of the most logical approaches available. Regardless of how elaborate the technique, the critical factor appears to be lag time. That is, if the basin lag time can be estimated accurately, a satisfactory unit hydrograph generally can be produced. Snyder used lag time as one of his defining equations.

For a duration of rainfall excess equal to $t_p/5.5$, which Snyder adopted for his study, Snyder's equations for the lag time and peak discharge are

$$t_p = C_t(L\ L_{ca})^{0.3} \tag{1}$$

and

$$Q_p = 640\ C_p A/t_p\ , \tag{2}$$

where t_p denotes the watershed lag time in hours; Q_p refers to the unit hydrograph peak discharge in cubic feet per second (cfs); C_t is a coefficient which represents differences in slope and channel storage between drainage basins; and C_p is a coefficient which represents the effects of such factors as channel storage on the flood wave. Snyder found C_t to vary from 1.8 to 2.2 and C_p to vary from 0.56 to 0.69[3] for the areas he studied.

From a study of unit hydrographs for a large number of drainage basins, the Corps of Engineers (33, p. 11) have obtained relationships for the determination of conservative estimates of the unit hydrograph widths at 50 and 75 per cent of peak flow. The curves for the unit graph widths at 50 and 75 per cent peak flow are presented in Linsley, Kohler, and Paulhus (21, p. 206). The widths at 50 and 75 per cent flow can be evaluated from the following equations:

$$W_{50} = 10^{(2.92 - 1.1 \log q_p)} \qquad (3)$$

and

$$W_{75} = 10^{(2.67 - 1.1 \log q_p)} , \qquad (4)$$

where W_{50} and W_{75} are the widths, in hours, at 50 and 75 per cent peak flow, respectively, and q_p is the unit hydrograph peak discharge in cfs/mi^2. The Corps of Engineers have suggested, as a guide for shaping the

[3]The Corps of Engineers prefers to use the expression 640 C_p rather than just C_p, where 640 is a conversion factor. The values then range from 360 to 440.

hydrograph, that the 50 and 75 per cent widths should be positioned so that one-third of the width is placed to the left and two-thirds of the width to the right of the peak. This procedure was adopted for this study.

One other relationship is needed, this being a relationship for the base width of the unit hydrograph. Following the suggestions of the SCS and the Corps of Engineers, a base width of five times the period of rise was adopted.

From the above relationships, seven points on the unit hydrograph may be determined. Considerable effort in the study was devoted to perfecting a technique for fitting a mathematical function to seven such points. The advantages of describing the complete hydrograph by such a function are:

1. The complete hydrograph can be described numerically with the aid of a computer. For most flood routing problems the complete hydrograph is needed. If iterative procedures are to be performed with the aid of a computer, a complete hydrograph frequently is needed.
2. Computer facilities are now available that will sketch desired curves. However, these plotters connect the consecutive points with straight

lines. Therefore, the more points that are available, the better the reproduction of the curve.

3. The shape of the hydrograph will be completely objective, once the basic equations are established.

The Pearson type III function was selected as the most appropriate mathematical function for fitting the seven points. This function can be expressed as

$$Q = Q_p(T/P_r)^r \exp\left[\frac{-(T - P_r)}{c}\right] , \qquad (5)$$

where Q is the discharge, in cfs, at any time T; T is the time in hours from beginning of rainfall excess; Q_p is the peak discharge in cfs; P_r is the period of rise of the hydrograph in hours; r is a dimensionless constant for a particular hydrograph; and c is a constant for a particular hydrograph expressed in units of hours. In Reich's study (26), c was taken to be the time from the occurrence of the peak discharge to the centroid of the hydrograph, and r was interpreted as being the ratio, P_r/c. However, in the present study these assumptions were not made. Instead, c and r are parameters, determined by trial and error, that give a least-squares fit of Equation (5) to the 7 points. Since the least-squares technique used for fitting the non-linear function, Equation (5), is not a conventional procedure, the procedure is illustrated in Appendix B. Also, the problem involved in obtaining the necessary area, 1 in. of runoff, under the unit hydrograph is discussed in Appendix B.

Another synthetic technique examined in this study is that presented by Espey, Morgan, and Masch (10) (their report was prepared under the sponsorship of the Texas Water Commission (TWC)). Since the procedure for using their method is identical to the procedure for the Snyder method, the equations will be presented without further discussion. They state that their equations are based on data from 11 rural watersheds in Texas, Oklahoma, and New Mexico and that the equations will predict hydrograph characteristics within ± 20 per cent two-thirds of the time. The equations for a 30-min unit hydrograph are:

$$P_r = 2.65L^{0.12}S^{-0.52} , \tag{6}$$

$$Q_p = 1700A^{0.88}P_r^{-0.3} , \tag{7}$$

$$B = 7410A^{0.64}Q_p^{-0.53} , \tag{8}$$

$$W_{50} = 73700A^{1.11}Q_p^{-1.13} , \tag{9}$$

and

$$W_{75} = 44600A^{1.06}Q_p^{-1.13} , \tag{10}$$

where P_r is the period of rise in minutes; L is the length of the main stream in feet; S is the slope of the main stream in ft/ft; Q_p is the peak discharge in cfs; A is the area of the basin in square miles; B is the base width in minutes; and W_{50} and W_{75} are the widths in minutes at 50 and 75 per cent peak flow, respectively.

The other synthetic technique that was examined was the SCS method (29). The SCS bases its procedure on an average dimensionless hydrograph for small watersheds developed from the analysis of a large number of actual unit hydrographs obtained from widely scattered geographical locations. Twenty-eight points defining the dimensionless hydrograph are presented by the SCS (29, p. 3.16-5). The ordinate is expressed as the ratio Q/Q_p and the abscissa as a ratio of T/P_r. After the period of rise is obtained, the following expression is given for determining the peak discharge:

$$Q_p = \frac{484A}{P_r} , \tag{11}$$

where Q_p is the peak discharge in cfs; A is the area in square miles; and P_r is the period of rise in hours.

The SCS reports (29, p. 3.15-1) that when runoff is uniform (or nearly so), it is usually sufficient to estimate the lag time from the empirical equation:

$$t_p = 0.6T_c \ , \qquad (12)$$

where t_p and T_c are the lag time and time of concentration,[4] in hours, respectively. Equation (12) and a nomogram by Kirpick (29, p. 3.15-7), which was based on the length and slope of the main stream, were used to obtain t_p.

From the foregoing discussion, it is apparent how the lag time and peak flow are estimated for the TWC and SCS techniques. However, the estimates for lag time and peak flow for the Snyder technique depend on the values of C_T and C_p. As reported by the Corps of Engineers (33, p. 11), values of $640C_p$ and C_T vary from 600 and 0.4, respectively, for southern California to 200 and 8.0, respectively, for states bordering on the eastern Gulf of Mexico. To provide a better estimate of these coefficients for the basins of this study, the correlations discussed in the following paragraphs were made.

Since Snyder's coefficient, C_T, in Equation (1) is affected to a large degree by the change in slope between drainage basins, a plot was made of C_T versus $\sqrt{S}$ for the watersheds in Category 2. The reason for expecting a good relationship is evident. Many equations for

[4]Time of concentration is the time it takes water to travel from the hydraulically most distant part of a watershed to the watershed outlet.

relating basin lag to the physical characteristics of the drainage basin have been proposed. Barnes (4, p. 55) has pointed out that the square root of the slope appears in most of these equations. Barnes states further that the slope factor is probably borrowed from the Chézy formula (21, p. 68). However, if Equation (1) is inspected, it can be seen that slope is not explicitly a factor. Therefore, the coefficient C_T must be a function of slope.

Other correlations also were examined in an attempt to obtain a reasonable estimate of lag. Meier reported that a linear relationship was found to exist between the period of rise and the dimensionless quantity $L^2/A\sqrt{S}$ for the three points of his study when the points were plotted on semilogarithmic paper. However, no usable correlation was found to exist when the other points of this study were added to Meier's. A plot also was made of lag time versus $L/\sqrt{S}$ on logarithmic paper. This particular relationship was tried because it can be seen from Equation (6) that the only basin characteristics involved are L and S. Finally, the most frequently used correlation was made, i.e., a logarithmic plot of lag time versus $LL_{ca}/\sqrt{S}$. These plots showed a very definite correlation.

It was decided that the plot of $LL_{ca}/\sqrt{S}$ would be used for estimating the lag time for the Snyder technique, since there was less scatter for this plot than for the plot of C_T versus $\sqrt{S}$. Although, in this modified Snyder method there remains only one of Snyder's original equations, as such, it still will be referred to as Snyder's method.

From examination of Equation (2), one expects C_p to be directly proportional to t_p and inversely proportional to A. When a plot was made of $640C_p$ versus $C_T(LL_{ca})^{0.3}/A$ a definite trend was found to exist. This relationship was used for estimating C_p for each basin.

Synthetic unit hydrographs were determined from each of three techniques using: (1) estimated lag time, determined as described in the previous discussions, and (2) observed lag time. The observed lag time was used for purposes of comparison. As pointed out previously, lag time is the key parameter in describing a unit hydrograph. Therefore, it was felt that the synthetic procedures should be examined with the actual lag time employed.

Dimensionless unit hydrographs. The dimensionless hydrograph technique developed by the Bureau of Reclamation (6) was used to convert all unit hydrographs to the desired duration. The unit hydrograph has been converted into dimensionless form by expressing the hydrograph abscissa in per cent of $(t_p + D/2)$ and the hydrograph ordinate as $Q(t_p + D/2)/\text{Volume}$, where Volume is the total runoff volume under the unit hydrograph. In addition, the dimensionless hydrographs were output in the computer program.

CHAPTER III

PRESENTATION AND DISCUSSION OF RESULTS

Correlation Studies

The data used in the correlations to determine the Snyder coefficients are presented in Table 3. The original source for this data is presented in Table 1.

A plot of C_T versus $\sqrt{S}$ is presented in Figure 4. There is considerable scatter about the "best-fit" straight line; however, a trend definitely is present. Intuitively, this is what one would expect. Since the lag time is simply a function of the time required for water to reach the outlet, it seems logical that the steeper the slope the faster the runoff will occur.

Figure 5 is a plot of $640C_p$ versus $C_T(LL_{ca})^{0.3}/A$. Examination of this figure reveals four points, 2, 4, 7, and 8 that are suspect. Although it is realized that hydrology is plagued with extremes, these points merit examination. Points 6 and 7 are for adjoining basins of similar physiographic characteristics. Point 6 falls within reasonable proximity of the line of "best-fit" and is not suspect; however, point 7 is extremely low. In the case of points 2, 4, and 8, values of $640C_p$ in excess of 700, to this writer's knowledge, rarely, if ever,

Table 3. Lag Times, Snyder's Coefficients, Shape Factor, and Dimensionless Hydrograph Peaks

No.	Watershed Name	t_p (Hours)	C_T	$640C_p$	L^2/A	$\frac{Q(t_p + D/2)}{Volume}$
1	Y	.33	.47	525	1.90	.86
2	Honney Creek #12	1.00	.91	735	1.85	1.25
3	D	1.40	1.15	426	2.87	.71
4	Honney Creek #11	1.42	1.17	772	1.58	1.30
5	Escondido #1	.75	.54	470	1.50	.79
6	Deep Creek #3	1.25	.96	445	2.10	.75
7	Deep Creek #8	.75	.40	200	5.30	.34
8	Cow Bayou #4	1.25	.70	785	3.21	1.33
9	Calaveras	2.25	1.32	523	1.75	.88
10	J	3.40	1.23	481	5.00	.81
11	Pin Oak Creek	5.10	1.85	520	3.56	.88
12	Mukewater Creek	8.50	1.85	570	5.26	.97
13	Little Elm Creek	13.10	2.26	480	8.27	.81

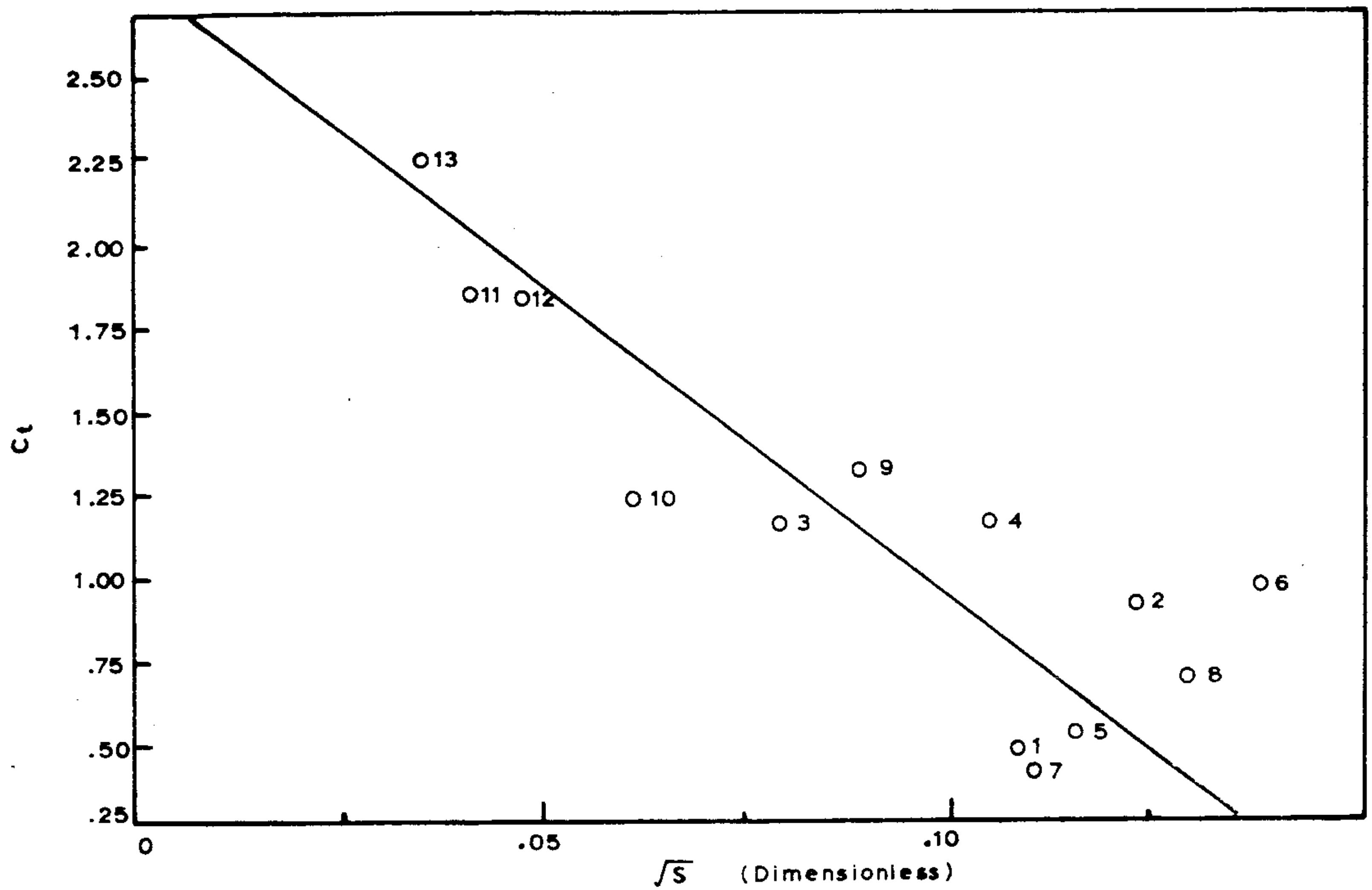

Fig. 4. Relationship between Snyder's coefficient, C_t, and the watershed slope.

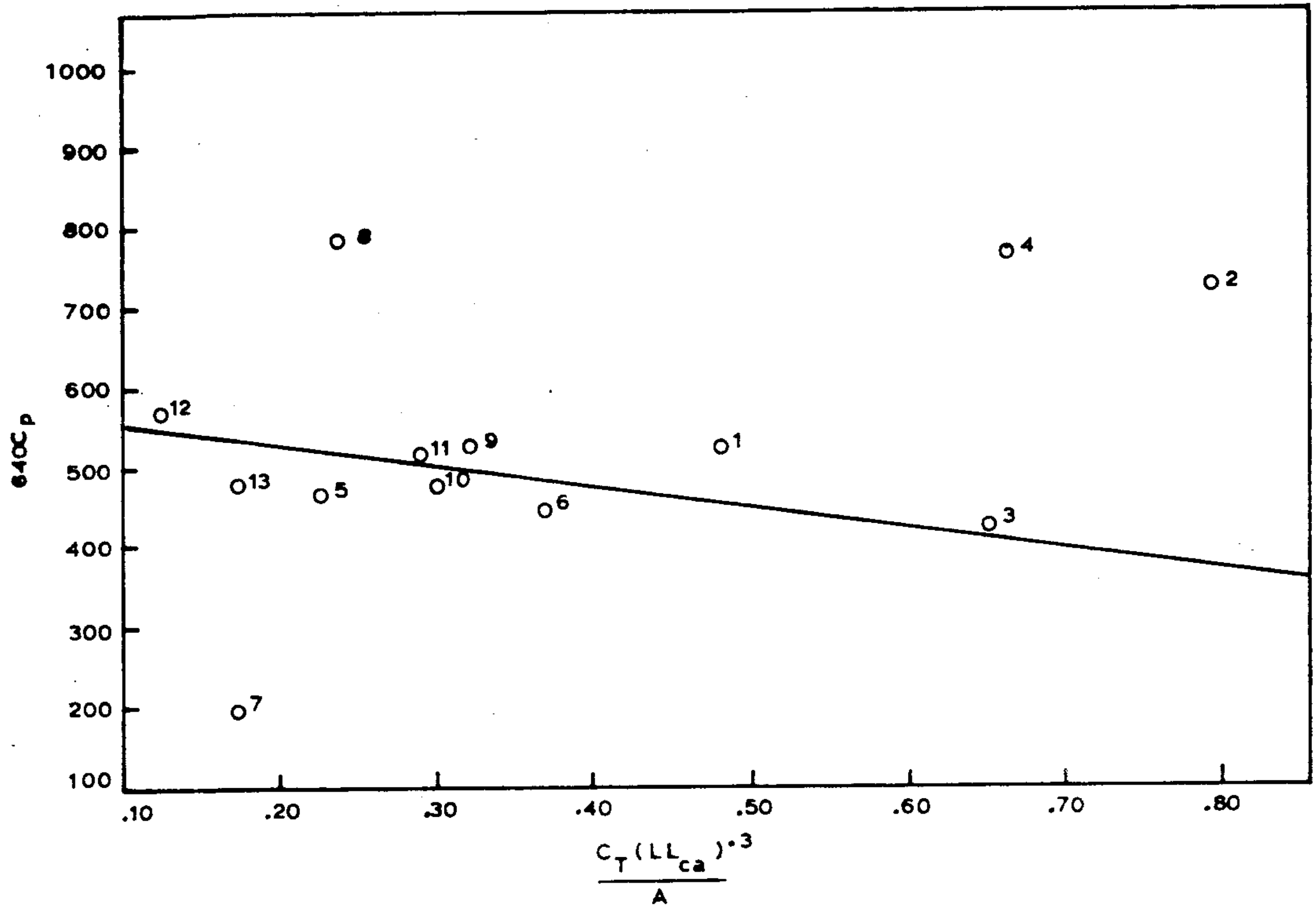

Fig. 5. Relationship between Snyder's coefficient, C_p, and basin characteristics.

exist. Since points 2, 4, 7, and 8 are suspect they were not used in obtaining the "best-fit" straight line of Figure 5.

A plot of t_p versus $L/\sqrt{S}$ on logarithmic paper is presented in Figure 6. This correlation was selected because the only physical parameters in Equation (6), for the TWC technique, are L and S. It was anticipated that the scatter would be significantly reduced by introducing L_{ca} into the numerator for the abscissa values. Examination of Figure 7 reveals this not to be the case. The reason for this is not apparent. At first, it was suggested that the reason might be because the basins all possess similar configurations. However, as can be seen from Table 3, the shape factors for the watersheds, L^2/A, vary considerably. Nevertheless, it appears that for the watersheds of this region the plot of t_p versus $L/\sqrt{S}$ gives approximately the same results as the plot of t_p versus $LL_{ca}/\sqrt{S}$. L and S are two of the most easily obtained physical characteristics for a watershed.

Comparison of Synthetic Procedures

Figures 8, 9, and 10 present a comparison of the unit hydrographs computed by the various synthetic procedures, utilizing estimated lag, and the average unit hydrographs obtained from observed data for basins Y, D,

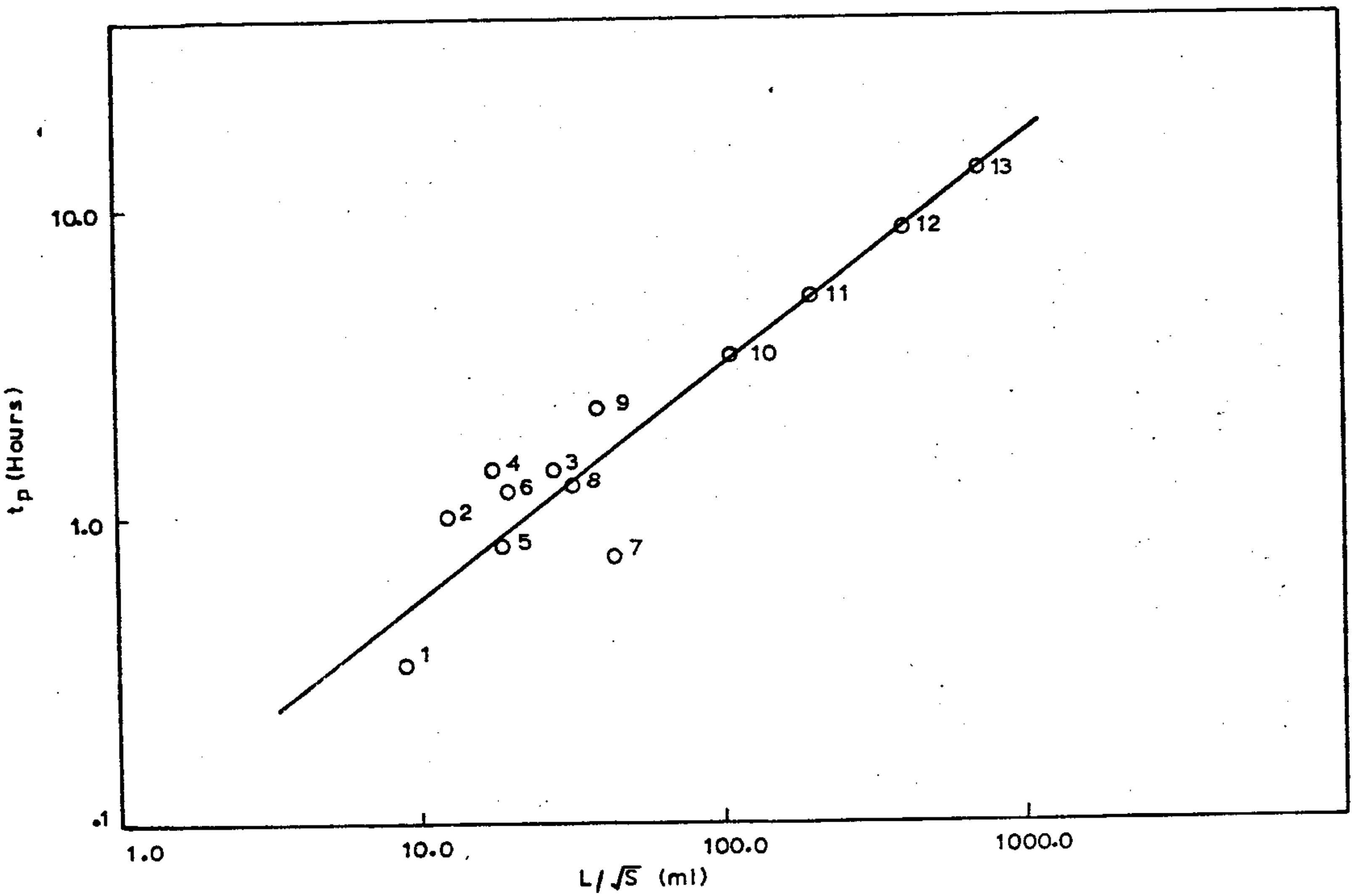

Fig. 6. Relationship between lag time and watershed characteristics.

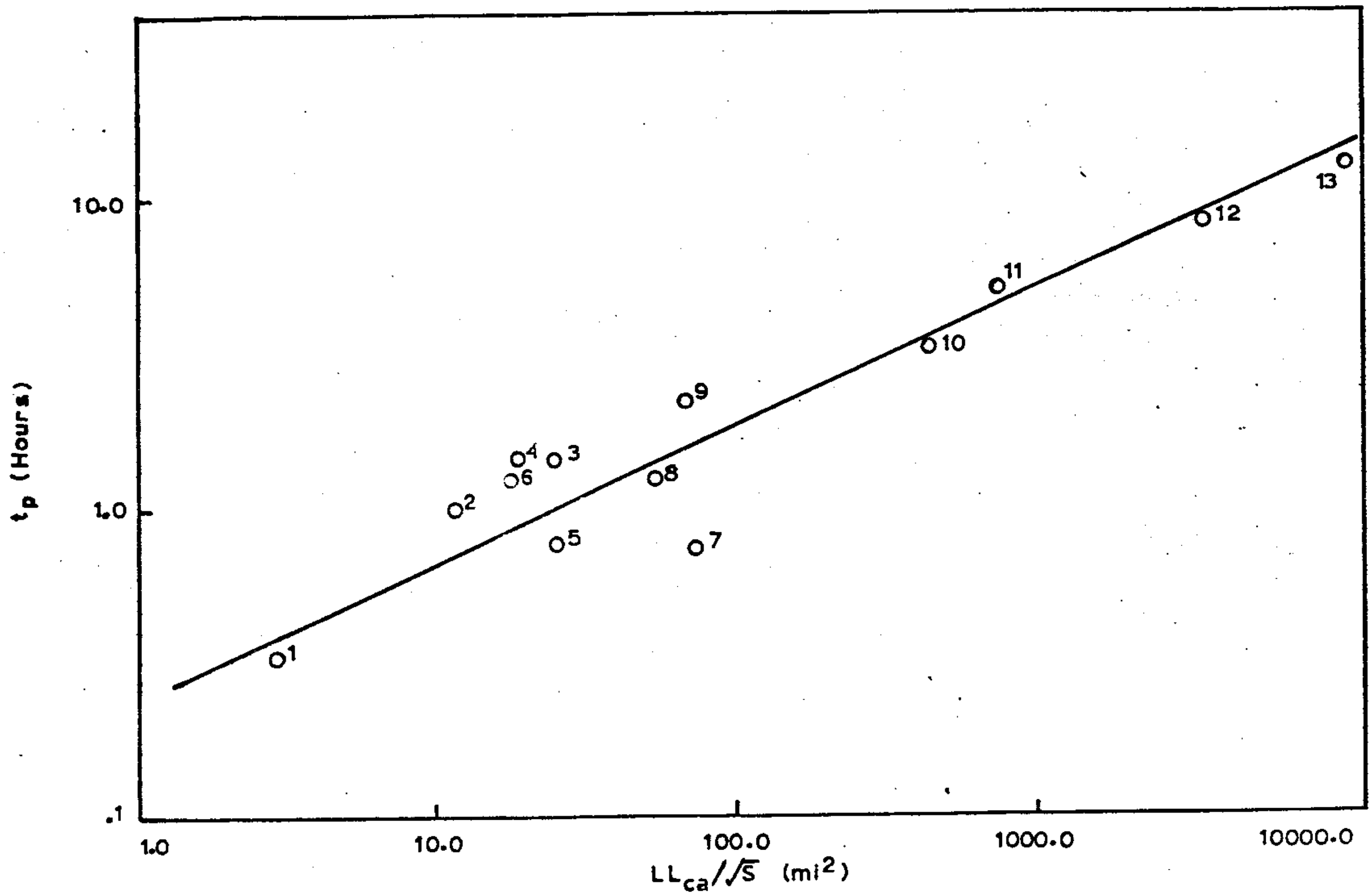

Fig. 7. Relationship between lag time and watershed characteristics.

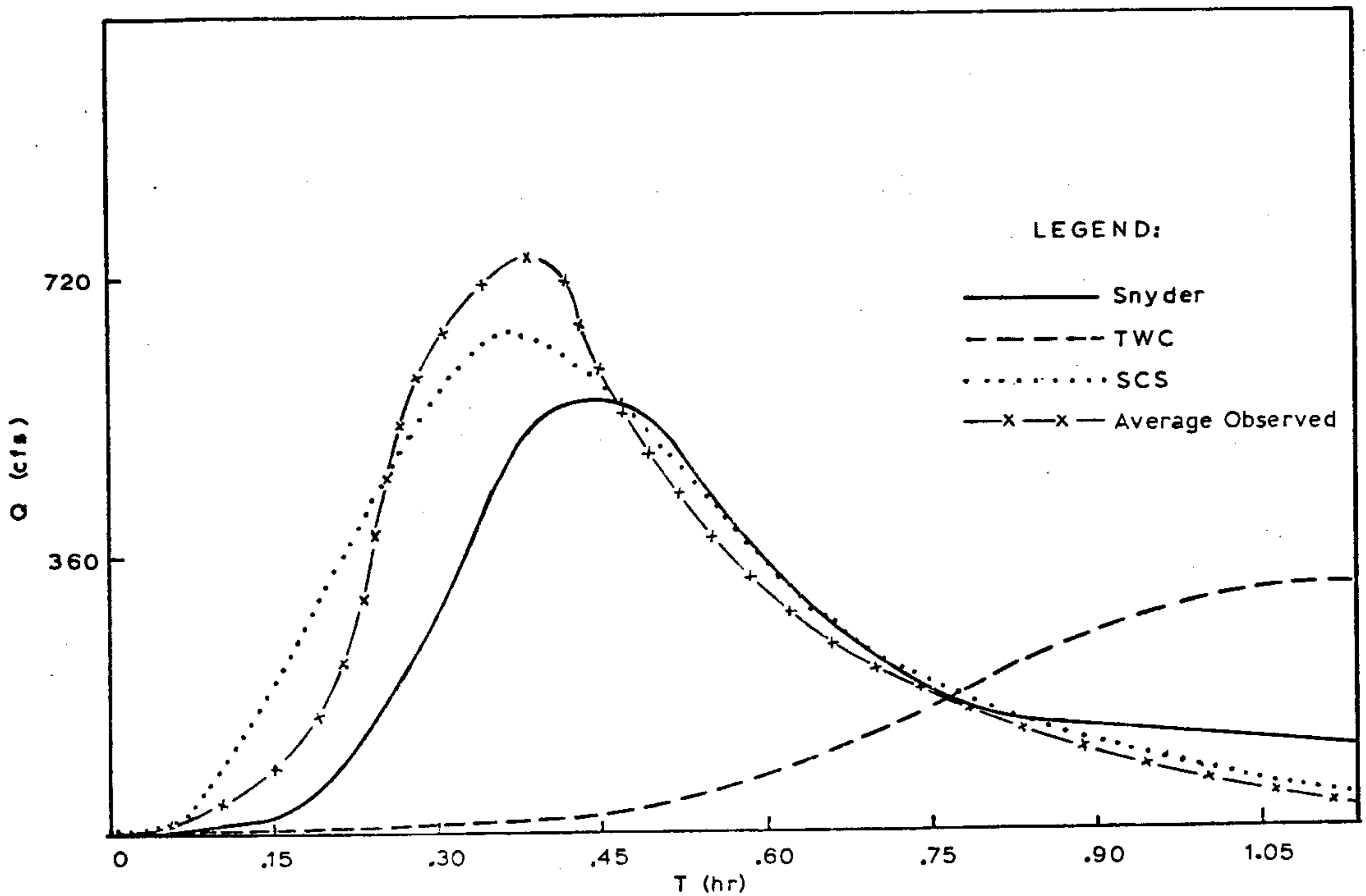

Fig. 8. Comparison of synthetic unit hydrograph techniques, computed using an estimated lag time, to an average unit hydrograph computed from actual data for basin Y.

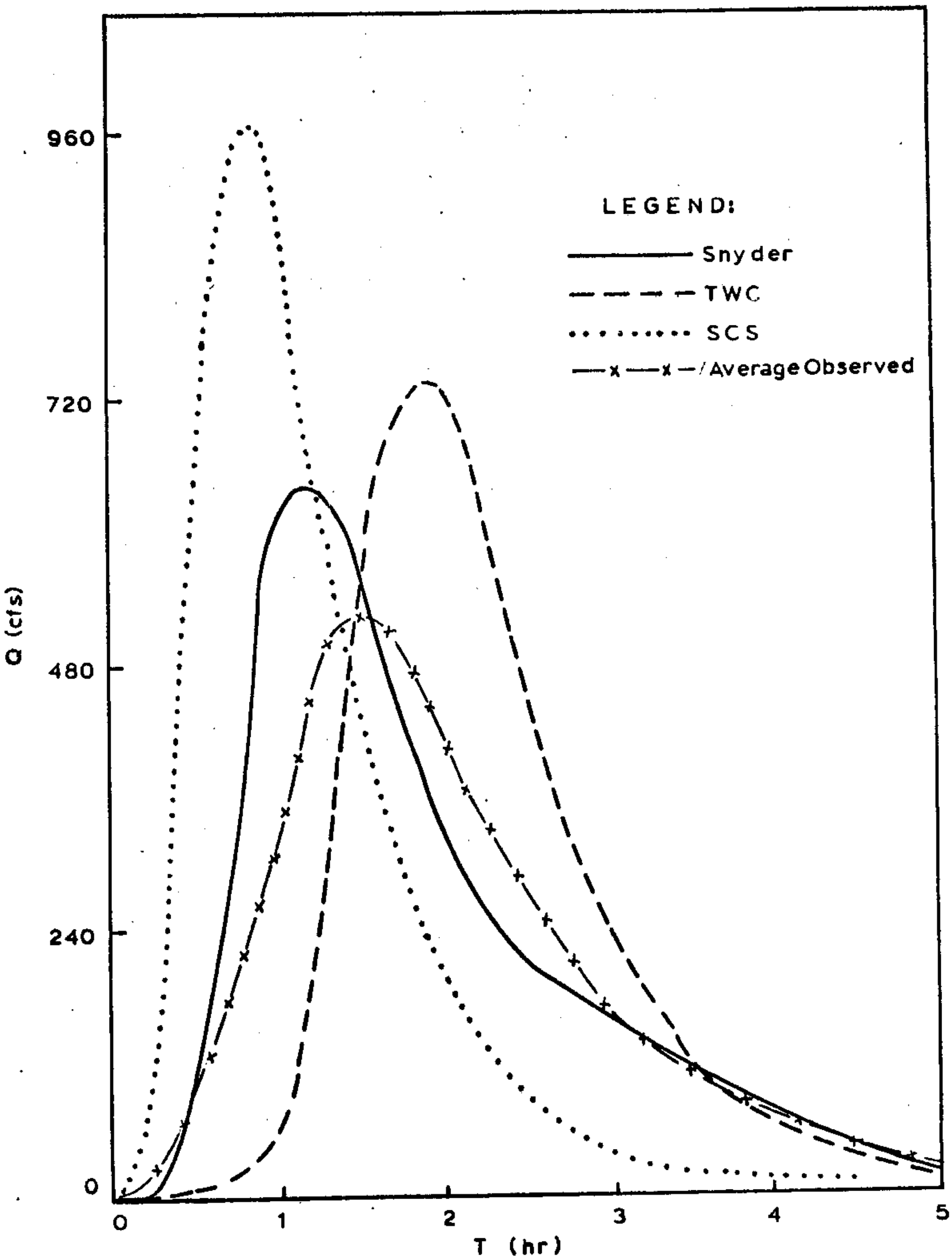

Fig. 9. Comparison of synthetic unit hydrograph techniques, computed using an estimated lag time, to an average unit hydrograph computed from actual data for basin D.

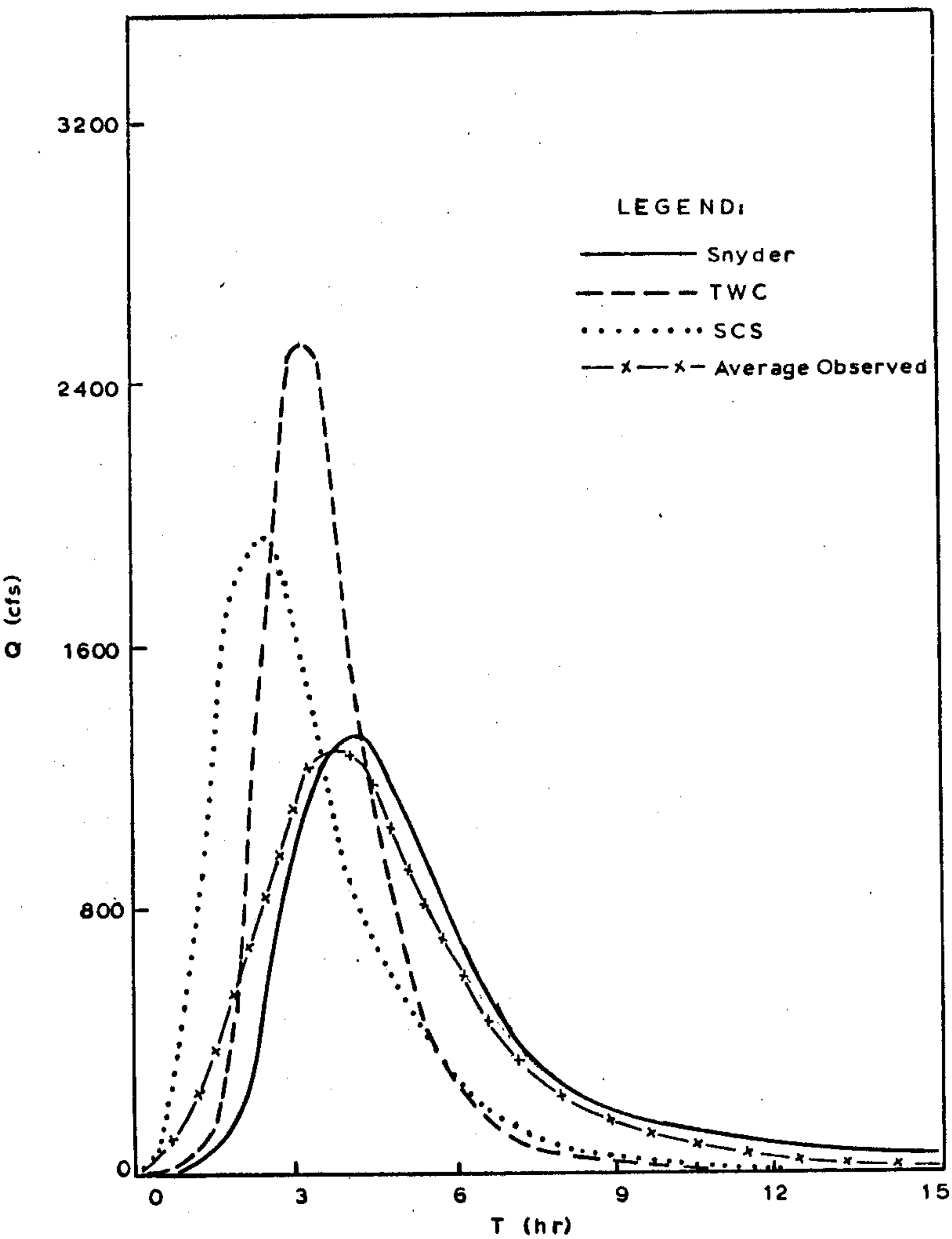

Fig. 10. Comparison of synthetic unit hydrograph techniques, computed using an estimated lag time, to an average unit hydrograph computed from actual data for basin J.

and J, respectively. Figures 11, 12, and 13 represent the same comparisons, except that a lag time based on observed data is used in the synthetic procedures. The unit hydrographs are for unit rainfall durations of 5, 15, and 45 min for basins Y, D, and J, respectively.

Upon examination of Figures 8 through 13, it appears that, on the average, the SCS method and the Snyder method used in conjunction with the Pearson type III function are compatible. However, a few peculiarities are noted, for example, the significant over estimate of the peak by the SCS method in Figure 9. A relationship, such as Kirpich's (29, p. 3.15-7), which was developed from data dissimilar in time and space from the watersheds in this study, should be used only if better means are not available. In practice, if sufficient information is available to construct relationships, such as the ones illustrated in Figures 4 through 7, this should be done regardless of the synthetic procedures employed.

Plots such as those presented in Figures 4 through 7 often prove to be the best means available, when streamflow data are lacking, for predicting lag time and peak discharge. It is apparent, however, that error also can be encountered in the use of these plots, for example, consider Figure 7. The correlation coefficient for this plot is 0.95. A correlation coefficient of one implies a

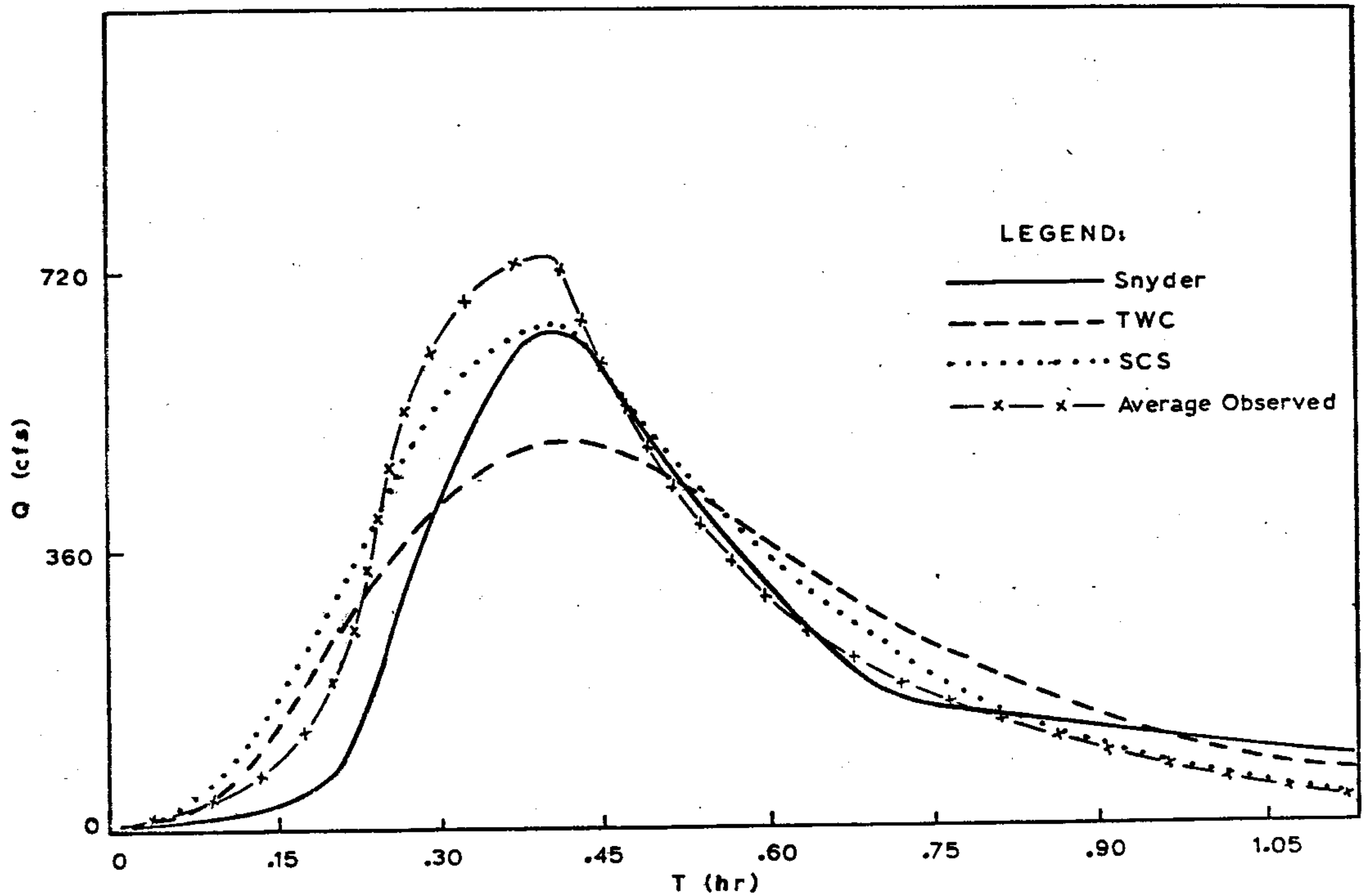

Fig. 11. Comparison of synthetic unit hydrograph techniques, computed using an observed lag time, to an average unit hydrograph computed from actual data for basin Y.

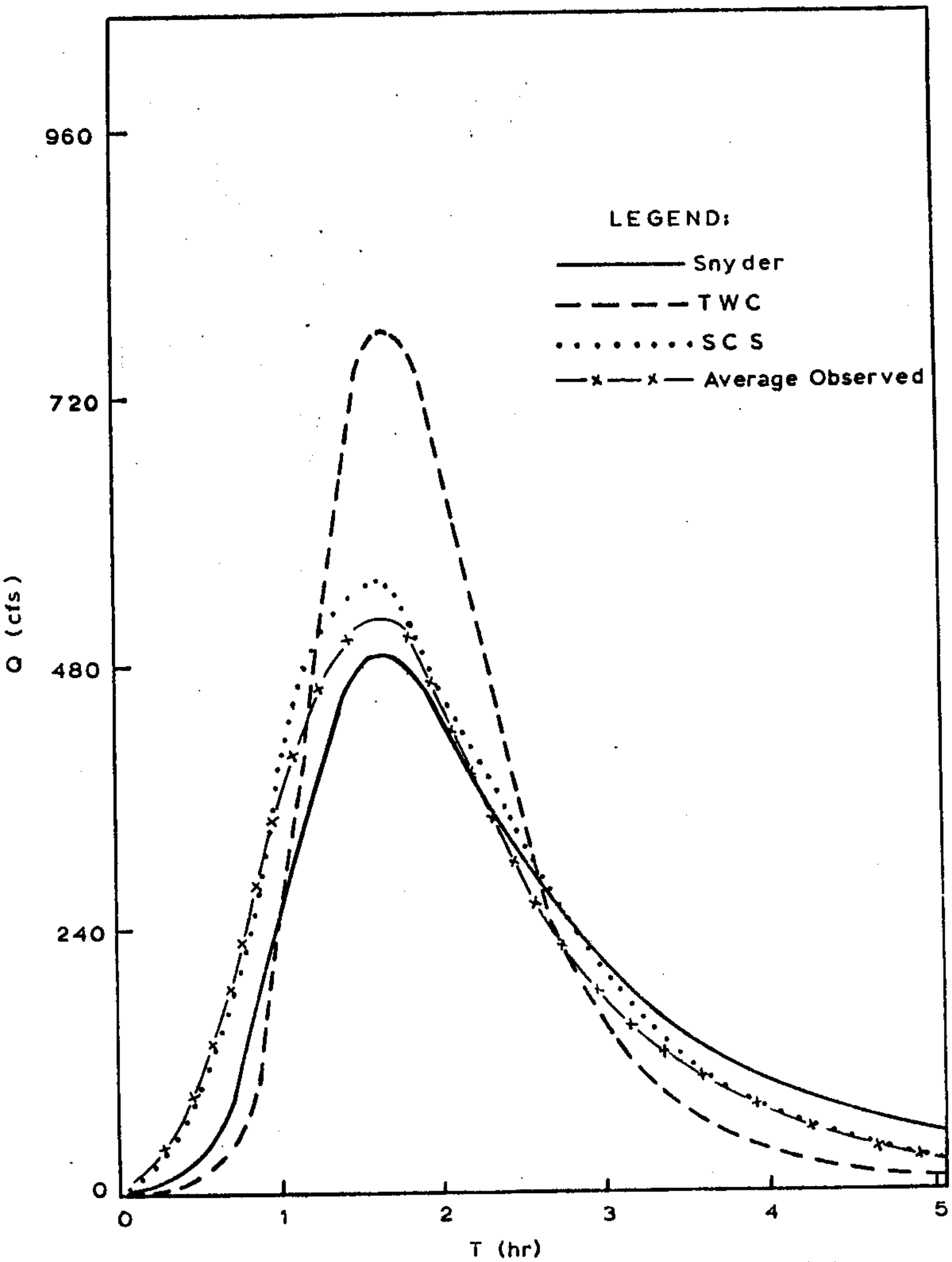

Fig. 12. Comparison of synthetic unit hydrograph techniques, computed using an observed lag time, to an average unit hydrograph computed from actual data for basin D.

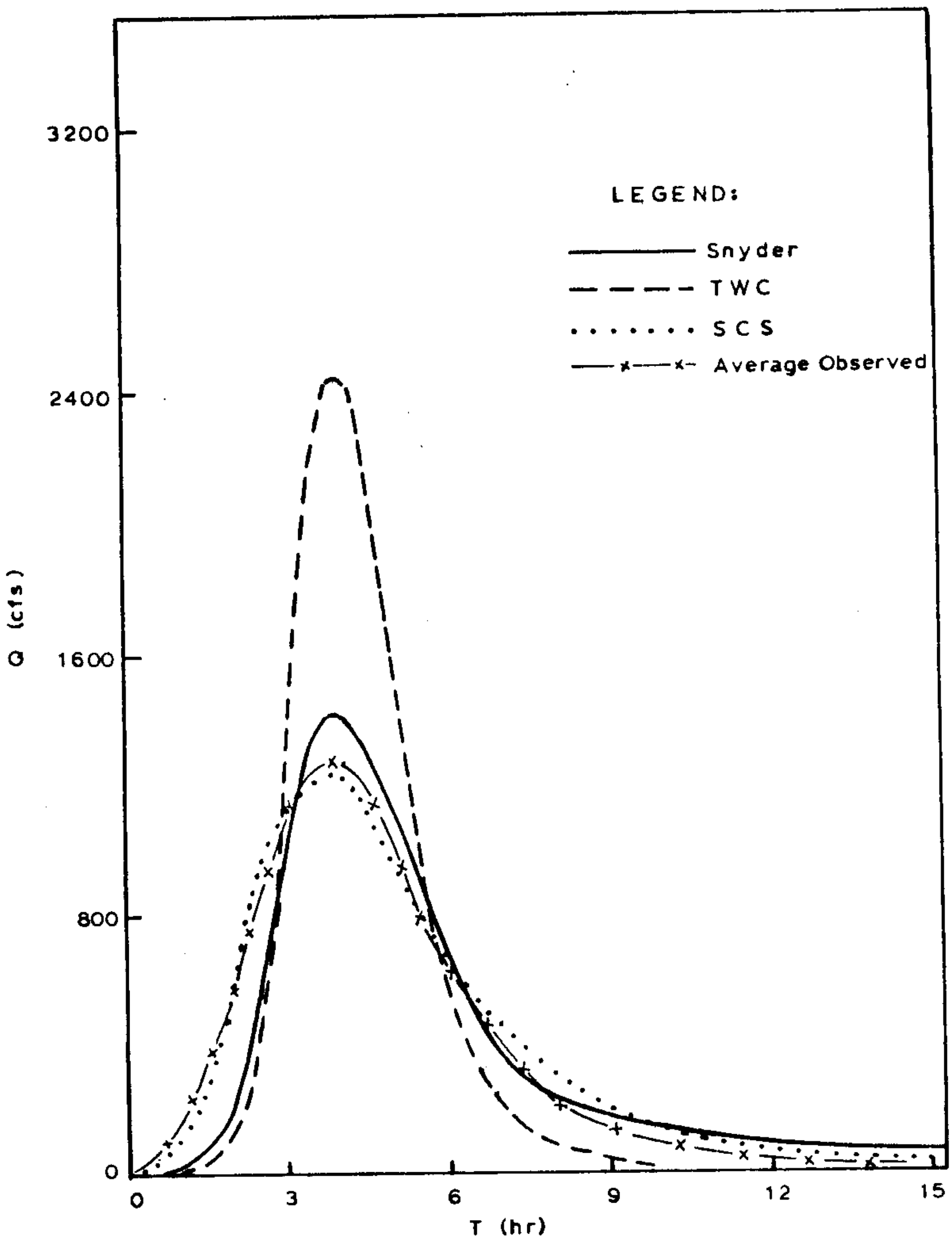

Fig. 13. Comparison of synthetic unit hydrograph techniques, computed using an observed lag time, to an average unit hydrograph computed from actual data for basin J.

perfect fit while zero indicates no correlation. Consequently, a correlation coefficient of 0.95 is a reasonably "good fit." However, this "best fit" line has a standard error of estimate of 0.66 hr; thus, errors may arise in its use. The standard error of estimate may be interpreted as follows. If the sample size is large enough, 68, 95, and 99.7 per cent of the sample points will fall within one, two, and three standard errors, respectively. For purposes of illustrating the error that might be encountered in the use of Figure 7 consider the mean lag time for all 13 basins (3.12 hr). If the physical characteristics of a basin are such that Figure 7 indicates a lag time of 3.12 hr while the actual lag time for the basin is one standard error away from this value, then a 21 per cent error is involved in the estimate of the lag time. Since the peak discharge is inversely proportional to the lag time, see Equation (2), this error also will be incorporated into the estimate of peak discharge.

It was stated, previously, that the SCS and Snyder techniques are, on the average, compatible. However, the TWC method does not give consistently good results. In fact, it consistently gave poor results for the cases studied. It was anticipated that the TWC method might prove the most successful of the techniques, since it was recently developed and the set of Equations (6) through

(10) were derived specifically for the region of this study. However, even when observed lag time was utilized, the peak values were still in considerable error. In Figure 11, the peak was under-estimated by 35 per cent while in Figures 12 and 13 the peak was over-estimated by 33 and 50 per cent, respectively.

Previous investigators, such as Morgan and Johnson (25, p. 17), Hickok, Keppel, and Rafferty (13, p. 610), and others, have indicated that the lag time is the major determinant of the shape of the unit hydrograph. This was also found to be true in the present study. Satisfactory unit hydrographs were reproduced by the SCS and Snyder methods when observed lag times were used. However, there is something basically lacking in the TWC set of equations. Three possibilities are:

1. The equations do not express the true physical relationship between the hydrograph parameters and the basin characteristics, even though they may reproduce the unit hydrographs satisfactorily for the basins from which they were derived.
2. The equations are based on an analysis of an insufficient number and type of watersheds.
3. Appreciable errors are made in the analyses of the unit hydrograph data used in the correlation.

Success of Curve Fitting

The "goodness-of-fit" of the Pearson type III function to the seven unit hydrograph points computed by Snyder and TWC methods was excellent in all cases studied. A total of 12 "fits" were made--six using the Snyder technique and six using the TWC technique. In all cases the correlation coefficient did not fall below 0.995 and the per cent standard error[5] was never greater than 3.5 per cent. The "goodness-of-fit" appeared to be slightly better for the shallow accession hydrographs, although the fit was excellent in all cases.

The "goodness-of-fit" of the seven points appears quite adequate; however, the question of how well the mathematical expression describes the actual unit hydrograph must be examined also. The best way to do this is to examine cases in which both the peak and lag determined from the synthetic procedure are approximately equal to the observed lag and peak. The hydrographs computed by Snyder's method in Figures 10, 12, and 13 approximate this situation. It appears that a slightly modified positioning of the widths at 50 and 75 per cent peak flow could improve the reproduction. Nevertheless, the variations

[5]Per cent standard error has been defined as the standard error of estimate divided by the unit hydrograph peak discharge.

from the actual unit hydrographs are not large. Additional cases should be studied to determine if the Pearson type III function will always give an adequate reproduction of the unit hydrograph, or if some other mathematical function might give better results.

Dimensionless Hydrographs

The average dimensionless unit hydrographs for basins Y, D, and J based on observed data are presented in Figure 14. In addition, peak values for the dimensionless hydrographs for all basins considered in this study are presented in Table 3. It is interesting to note that the peak values cluster around a mean peak of 0.82, if points 2, 4, 7, and 8 are ignored. The remaining nine peak values fall within ± 17 per cent of the mean peak (0.82).

The agreement of the dimensionless hydrographs for basins J and Y is remarkable. The lower peak value for the dimensionless hydrograph for basin D might be attributed to a change in watershed regime. The data used in this study for watershed D were taken in the late 1950's, while the data used for basins J and Y were taken in the late 1930's. The Brushy Creek watershed experienced a rather marked vegetation change from 1940 to the late 1950's. During the 1950's a large portion of the watershed was converted into grassland and placed under the

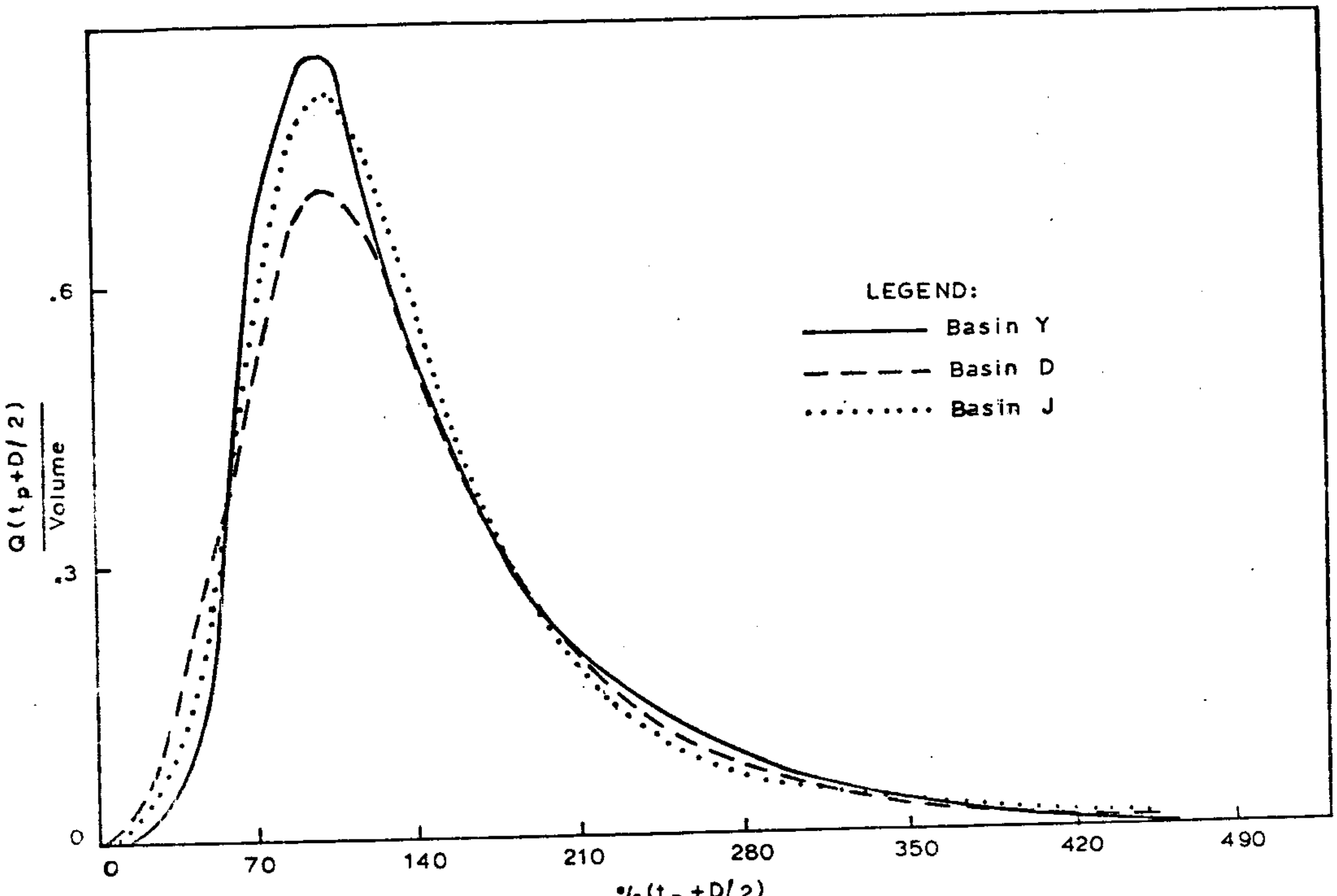

Fig. 14. Average dimensionless hydrographs for basins Y, D, and J.

Soil Bank Program. Also, a better erosion program has been maintained in the later years. These changes could account for the lower dimensionless unit hydrograph peak for watershed D.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The following conclusions are inferred from this study:

1. The Sherman and Mayer technique (28) appears to be the only practical method available for determining the temporal distribution of infiltration capacity from rainfall-runoff data for basins larger than 1 mi^2.
2. On the average, the SCS method and the Snyder method, when used in conjunction with the Pearson type III function, give comparable results.
3. The TWC technique consistently gave poor results.
4. If the lag time is estimated correctly, the Snyder and SCS methods will describe the unit hydrograph satisfactorily. Therefore, lag time appears to be the key hydrograph parameter.
5. The "goodness-of-fit" of the Pearson type III function to the seven unit-hydrograph points computed by the Snyder and TWC method is excellent.
6. The Pearson type III function, when fitted to

the seven unit-hydrograph points, satisfactorily describes the shape of the unit hydrograph for the cases of this study.

7. On the basis of the preceding two conclusions, it can be further concluded that the Pearson type III function might be used for fitting observed data when few data points are available.
8. The procedure presented in Appendix B works quite well for fitting a mathematical function, such as the Pearson type III function, to unit-hydrograph data.
9. There is considerable similarity between the dimensionless unit hydrographs of the region.

Recommendations

Some areas, suggested by this study, for further research are:

1. Work is needed on estimating the temporal distribution of infiltration capacity from rainfall-runoff data for basins from 1 to perhaps 25 mi^2.
2. Since lag time is a key determinant of hydrograph shape, more work is needed on the subject of lag relationships.
3. More cases should be studied to determine if the Pearson type III function will continue to

give an adequate reproduction of the unit hydrograph, or if some other mathematical function might give better results.

APPENDIX A

THE SHERMAN AND MAYER INFILTRATION ANALYSIS TECHNIQUE

Escott has derived an equation that can be used to describe the infiltration capacity curve obtained by the Sherman and Mayer (28, p. 667) technique. By proper manipulation, the equation presented by Escott can be written in the following form:

$$f_c = f_u + \frac{1}{106} \exp[(T_u - T)/C_b)] \ , \qquad (13)$$

where f_c is the infiltration capacity, in in./hr, at time T; f_u is the ultimate infiltration capacity in in./hr; T_u is the time, in hr, at which the infiltration capacity reaches ultimate; and C_b is given by

$$C_b = \frac{(f_{av} - f_u)}{f_{av}} \ , \qquad (14)$$

where f_{av} is the average infiltration capacity in in./hr. The first approximation for f_{av} is determined by subtracting the total runoff from the total rainfall and dividing this quantity by the effective duration of the storm. The effective duration of the storm in hours, T_E, is determined subjectively by subtracting light rainfall periods (or periods in which rainfall intensities are suspected to be less than the infiltration curve) at the beginning or ending of the storm from the total duration of the storm. The computer program, which is written in Fortran IV language and follows this Appendix, adjusts f_{av} to yield the observed runoff volume.

T_u may be expressed as

$$T_u = C_b \log\{\frac{106T_E(f_{av} - f_u)}{C_b[1 - \exp(-T_E/C_b)]}\} . \qquad (15)$$

An example of an infiltration capacity curve described by the Sherman and Mayer technique is presented in Figure 3, Chapter II.

```
$IBFTCXXX
C       THIS PROGRAM DETERMINES WEIGHTED RAINFALL INTENSITES BY THE
C       THIESSEN METHOD AND INFILTRATION CAPACITY CURVES BY THE SHERMAN-
C       MAYER TECHNIQUE.
        DIMENSION R(100),A(20),P(20,100),ARRAY(1000),T(100)
C       S = THE PORTION OF ONE HOUR USED FOR SPACING OF POINTS EXPRESSED
C       IN MINUTES. LTE = TIME FROM THE BEGINNING TO THE END OF RAINFALL
C       IN MINUTES, MULTIPLIED BY 1/S, WITH 1 ADDED (EQUALS NO. OF PTS.).
C       FU = ULTIMATE INFILTRATION CAPACITY. FAV = AVG. INFILTRATION CA-
C       PACITY. TE = EFFECTIVE LENGTH OF STORM IN HOURS, Q = TOTAL RUNOFF
C       IN INCHES. NS = NUMBER OF RAINGAGES. A(I) = AREA IN SQ. MI. FOR
C       EACH GAGE AS DETERMINED BY THE THIESSEN METHOD. P(I,J) = CUMULA-
C       TIVE RAINFALL UP TO THE END OF THAT PERIOD FOR EACH STATION, WHERE
C       I IS THE STATION NUMBER AND J IS CUMULATIVE RAINFALL IN INCHES.
     99 READ (5,105) LTE,FU,S,FAV,TE,Q,XMAX,YMAX,NS
    105 FORMAT (I3,2F4.2,5F6.2,I3)
        S=S/60.
        WRITE (6,106) LTE,FU,S,FAV,TE,Q,XMAX,YMAX,NS
    106 FORMAT (1H1,//60X,11HDATA OUTPUT,///40X,I3,2F4.2,5F6.2,I3)
        READ (5,107) (A(I),I=1,NS)
    107 FORMAT (13F6.2)
        WRITE (6,108) (A(I),I=1,NS)
    108 FORMAT (1H0,//,(28X,13F6.2))
        DO177I=1,NS
        READ (5,109)  (P(I,J),J=1,LTE)
    109 FORMAT (13F6.2)
        WRITE (6,110)  (P(I,J),J=1,LTE)
    110 FORMAT (1H0,//,(28X,13F6.2))
    177 CONTINUE
C       AT=TOTAL BASIN AREA.
        AT=0.0
        DO1I=1,NS
        AT=AT+A(I)
      1 CONTINUE
        LTEM1=LTE-1
```

```
      DO2J=1,LTEM1
      SUM=0.0
      DO2I=1,NS
      SUM= SUM+( ABS(P(I,J+1)-P(I,J))*(1.0/S)*A(I))/AT
C     R=RAINFALL INTENSITY.
      R(J)=SUM
    2 CONTINUE
      PT=0.0
      DO3I=1,NS
C     PT=AVERAGE TOTAL PRECIPITATION OVER BASIN.
      PT= PT+(P(I,LTE)*A(I))/AT
    3 CONTINUE
      WRITE (6,102) PT
  102 FORMAT (1H0,///6:X,3HPT=,F7.2)
C     CALLING INFILTRATION SUBROUTINE
      CALL INFIL (LTE,FU,S,FAV,TE,Q,XMAX,YMAX,R,ARRAY)
      SX=0.0
      DO4I=1,LTEM1
C     T=TIME.
      T(I)=SX
      SX=SX+S
    4 CONTINUE
      CALL PLOT 3 (1HX,T,R,LTEM1)
      SX=S
      DO5I=1,LTEM1
      T(I)=SX
      SX=SX+S
    5 CONTINUE
      CALL PLOT 3 (1HX,T,R,LTEM1)
      CALL PLOT 4 (18,18HR AND FC IN IN./HR)
      WRITE(6,104)
  104 FORMAT (1H0, //60X,12HTIME (HOURS))
      GO TO 99
      END
```

```
$IBFTCXXXX
C       SUBPROGRAM FOR DETERMINING INFILTRATION CAPACITY CURVE.
        SUBROUTINE INFIL (LTE,FU,S,FAV,TE,Q,XMAX,YMAX,R,ARRAY)
        DIMENSION FC(100),T(100),ARRAY(1000),FMR(100),R(100),FA(200),
       XRA(200)
C       FC = INFILTRATION CAPACITY.
        NF=1
     10 CB=(FAV-FU)/FAV
C       TU = TIME AT WHICH INFILTRATION CAPACITY REACHES THE ULTIMATE
C       VALUE (FU).
        TU=CB*ALOG((106.*TE*(FAV-FU))/(CB*(1.-EXP(-TE/CB))))
C       TS=TIME.
        TS=0.0
        DO5I=1,LTE
        FC(I)=FU+EXP((TU-TS)/CB)/106.
        TS=TS+S
      5 CONTINUE
C       CHECKING AREA ABOVE INFILTRATION CAPACITY CURVE TO SEE IF IT
C       EQUALS THE OBSERVED RUNOFF.  IF THIS VOLUME DOES NOT EQUAL THE
C       VOLUME OF RUNOFF, IT IS MADE EQUAL BY A TRIAL AND ERROR ALTERATION
C       OF FAV.
        LTEM1=LTE-1
        LTEMX2=2*LTEM1
        N=0
        DO11I=1,LTEM1
        N=N+2
        NM1=N-1
        DO11J=NM1,N
        RA(J)=R(I)
     11 CONTINUE
        N=1
        FA(1)=FC(1)
        DO12I=2,LTE
        N=N+2
        NM1=N-1
```

```
      DO12J=NM1,N
      FA(J)=FC(I)
   12 CONTINUE
      DO7I=1,LTEMX2
      FMR(I)=RA(I)-FA(I)
    7 CONTINUE
      AQ=0.0
      DO8I=2,LTEMX2,2
      IF(FMR(I).GT.0.0) GO TO 2
      GO TO 8
    2 CONTINUE
      IF (FMR(I-1).GE.0.0) AQ=AQ+(FMR(I)+FMR(I-1))*S/2.0
      IF(FMR(I-1).GE.0.0) GO TO 8
      AQ=AQ+(FMR(I)**2/(FMR(I)+ABS(FMR(I-1))))*S/2.0
    8 CONTINUE
      RQ=AQ-Q
      IF (ABS(RQ).LE..02) GO TO 99
      NF=NF+1
      IF(NF.GT.1000) GO TO 99
      IF (RQ.LT.0.0) GO TO 4
      FAV=FAV+.01
      GO TO 10
    4 FAV=FAV-.01
      GO TO 10
   99 CONTINUE
      WRITE (6,101) AQ,TU,NF,FAV,(FC(I),I=1,LTE)
  101 FORMAT (1H0,///45X,3HAQ=,F6.2,6H    TU=,F6.2,5H   NF=, I3,6H   FAV=,
     XF6.2,////47X,38HTHE VALUES FOR FC (FROM 0 TO LTE) ARE.,//,(/64X,F4
     X.2))
      SX=0.0
      DO6J=1,LTE
C     T=TIME.
      T(J)=SX
      SX=SX+S
    6 CONTINUE
```

```
      WRITE (6,102)
  102 FORMAT (1H1)
      CALL PLOT 2 (ARRAY,XMAX,0.0,YMAX,0.0)
      CALL PLOT 3 (1H*,T,FC,LTE)
      RETURN
      END
```

APPENDIX B

A LEAST-SQUARES TECHNIQUE FOR FITTING HYDROGRAPH DATA TO A MATHEMATICAL FUNCTION

The standard procedure for using the method of least squares, in which non-linear normal equations arise, depends upon a reduction of the residuals to a linear form by a first order Taylor series approximation, which is taken about an initial or trial solution for the parameters (Levenberg, 18).

To illustrate the procedure, a technique will be developed for fitting the Pearson type III function to the seven points furnished by the Snyder method, or a method analogous to Snyder's. However, the procedure is the same for any function which leads to non-linear normal equations and for varying quantities of data. It is assumed that the reader is familiar with the conventional least squares procedure.

The Pearson type III function was described in Chapter II. This function was defined mathematically as

$$Q = Q_p (T/P_r)^r \exp\left[\frac{-(T - P_r)}{c}\right] . \qquad (5)$$

The approximation to Equation (5) resulting from a first-order, multiple Taylor series expansion around trial values for the parameters, r and c, is given by

$$Q = (Q)_{r_o,c_o} + \left(\frac{\partial Q}{\partial r}\right)_{r_o,c_o}(r - r_o) + \left(\frac{\partial Q}{\partial c}\right)_{r_o,c_o}(c - c_o) \quad , \quad (16)$$

where $(\ )_{r_o,c_o}$ indicates that the quantity in parenthesis is evaluated with the parameters set equal to the trial parameters, r_o and c_o. Terms of a higher order have been ignored.

Let

$$d_1 = r - r_o \ ,$$

and

$$d_2 = c - c_o \ .$$

Denote

$$Q_r' = \frac{\partial Q}{\partial r} \ ,$$

and

$$Q_c' = \frac{\partial Q}{\partial c} \ .$$

Further simplification can be obtained by letting

$$(Q)_{r_o,c_o} = Q_o \ ,$$

$$(Q_r')_{r_o,c_o} = Q_{ro}' \ ,$$

and

$$(Q_c')_{r_o,c_o} = Q_{co}' \ .$$

Equation (16) now can be written in a simplified form as

$$Q = Q_o + Q'_{ro}d_1 + Q'_{co}d_2 \; . \tag{17}$$

Thus, Equation (5) can be approximated by a linear combination in Q_o, Q'_{ro}, and Q'_{co}. The problem is now one of minimizing

$$S = \sum_{i=1}^{7} (Q_{7i} - Q_i)^2$$

or

$$S = \sum_{i=1}^{7} (Q_{7i} - Q_{oi} - Q'_{roi}d_1 - Q'_{coi}d_2)^2 \; , \tag{18}$$

where the Q_7 points are obtained from the Snyder equations. The summation indices will not be carried beyond Equation (18). However, all indicated terms and subsequent products of these terms must be summed with i ranging from one to seven.

From the calculus, it can be shown that S will be a minimum when the partial derivatives of S with respect to d_1 and d_2 are zero. The resulting two equations are

$$S'_{d_1} = \Sigma(Q_7 - Q_o - Q'_{ro}d_1 - Q'_{co}d_2)(-Q'_{ro}) = 0 \tag{19}$$

and

$$S'_{d_2} = \Sigma(Q_7 - Q_o - Q'_{ro}d_1 - Q'_{co}d_2)(-Q'_{co}) = 0 \; . \tag{20}$$

Equations (19) and (20) can be rearranged to give

$$d_1\Sigma(Q'_{ro})^2 + d_2\Sigma(Q'_{co}Q'_{ro}) = \Sigma(Q_7 - Q_o)Q'_{ro} \qquad (21)$$

and

$$d_1\Sigma(Q'_{ro}Q'_{co}) + d_2\Sigma(Q'_{co})^2 = \Sigma(Q_7 - Q_o)Q'_{co} \; . \qquad (22)$$

Values for d_1 and d_2 can be obtained by simultaneous solution of Equations (21) and (22).

The procedure is as follows:

1. Select initial values for r and c. These values should be chosen judiciously, if the process is to converge. In all cases, it was found that initial values of 9.0 for r and 0.1 P_r for c were sufficient.
2. Solve for d_1 and d_2.
3. As pointed out by Hartley (12, p. 271), the magnitude of the corrections of the initial trial values of the parameters to obtain the next trial values is proportional to the d's. Also, the sign of the correction will be given by the sign of the d's. This can be expressed mathematically as

$$r^2 = r^1 + vd_1 \qquad (23)$$

and

$$c^2 = c^1 + vd_2 \quad , \qquad (24)$$

where the superscripts indicate the trial number and v takes on a value from zero to one. Levenberg (18) pointed out that when v is equal to one, the technique frequently will diverge because of over correction. Hartley (12) has proposed a sophisticated procedure for determining the v for each iteration so that Equation (18) is minimized. However, in all cases, a value for v of 0.4 was found to work sufficiently well.

4. The process is repeated until there is no significant change in the parameters (r and c).

<u>Obtaining 1 in. volume of runoff for the unit hydrograph</u>. The problem of obtaining exactly 1 in. volume of runoff under the curve of the unit hydrograph, which is required by definition, is not as simple as it may appear, particularly when the process is to be [illegible] by electronic computation. Two possible approaches to the problem will be discussed briefly in the following paragraphs.

First, the least-squares procedure can be modified to determine the regression constants, r and c, such that the area under the curve is restrained to be 1 in. However, it was found in this study that the "goodness-of-fit" was reduced by such an approach. The second approach is to distribute the required area over some portion of the hydrograph. The latter approach has been used in this study. This study utilized the generally accepted practice of adjusting the recession portion of the unit hydrograph to give the needed 1 in. Adjustment was made from the time when the discharge equals 75 per cent of peak discharge on the recession side to the end of the runoff period. The manner in which the adjustment was made is illustrated by Figure 15. T_{75} and T_0 are the times at 75 per cent peak discharge and assumed zero discharge, respectively. Q_a is the discharge amount that must be added to the already existing discharge values between T_{75} and T_0 to ensure the needed 1 in. A_A is the total area to be added. The amount of addition required was generally less than 0.1 in. In this discussion, no mention has been made of a possible subtraction. In all cases considered in this study, it was found that an addition of area was required. However, the same procedure would be used for a subtraction.

The proportional amounts for A_1 and A_2 are selected to give the smoothest curve. For this study, it

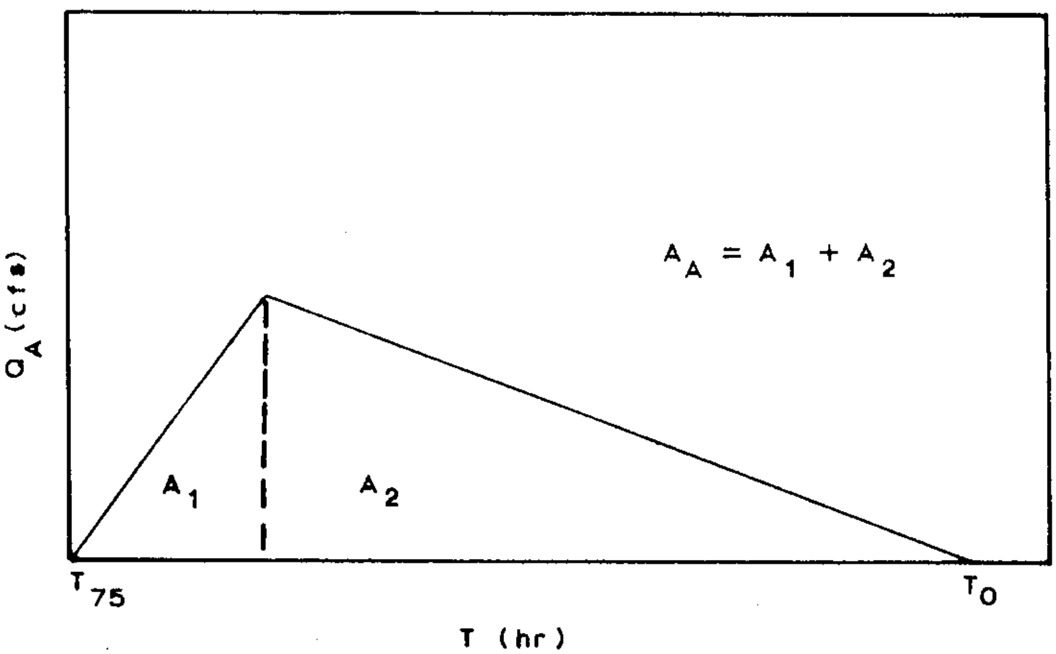

Fig. 15. Pictorial representation of the area added to the unit hydrograph recession in order to ensure a volume of one inch.

was found that A_1 and A_2 values of $0.3A_A$ and $0.7A_A$, respectively, gave satisfactory results.

The Fortran IV computer program, which utilizes the procedure outlined in this Appendix for fitting a mathematical function to hydrograph data is presented on the following pages.

```
$IBFTCXXX
C     THIS PROGRAM OUTPUTS A SYNTHETIC UNIT HYDROGRAPH, OF THE DESIRED
C     DURATION, BASED ON BASIN CHARACTERISTICS.  FIRST THE SEVEN POINTS,
C     FURNISHED BY A MODIFIED SNYDER APPROACH, ARE DETERMINED.  THEN, A
C     PEARSON TYPE THREE FUNCTION IS FITTED TO THESE 7 POINTS BY NON-
C     LINEAR LEAST SQUARES.  FINALLY, THIS UNIT HYDROGRAPH IS CONVERTED
C     TO THE DESIRED DURATION BY MEANS OF THE BUREAU OF RECLAMATION
C     DIMENSIONLESS HYDROGRAPH TECHNIQUE.
      DIMENSION Q(10),T(10),QF(300),TF(300)
  699 READ (5,105)   TL,CL,A,CT,CP,S,DSFUG,XMAX,XMAX1,YMAX2,YMAX1
  105 FORMAT (10F6.2,F9.2)
      S=S/60.
      DSFUG=DSFUG/60.
      WRITE (6,102)  TL,CL,A,CT,CP,S,DSFUG,XMAX,XMAX1,YMAX2,YMAX1
  102 FORMAT (1H1,//60X,11HDATA OUTPUT,///30X,10F6.2,F9.2)
C     TL = TOTAL LENGTH OF STREAM, IN MILES, MEASURED FROM THE DIVIDE TO
C     THE GAGING STATION. CL = LENGTH ALONG STREAM, IN MILES, FROM POINT
C     ON STREAM NEAREST CENTROID OF BASIN. A = BASIN AREA IN SQ. MI.
C     CT, CP = SNYDER COEFFICIENTS. S = THE PORTION OF ONE HOUR USED FOR
C     SPACING OF POINTS EXPRESSED IN MINUTES. DSFUG = DURATION  SELECTED
C     FOR UNIT HYDROGRAPH IN MINUTES.
C     DETERMINING 7 POINTS BY SNYDER TECHNIQUE.
C     TP=BASIN LAG IN HOURS.
      TP=CT*(TL*CL)**.3
C     CT WAS COMPUTED BY HAND TO GIVE LAGS DISCUSSED IN TEXT.
C     QP=PEAK DISCHARGE PER SQ. MI. IN CFS/SQ. MI.
      QP=640.*CP/TP
C     D= RAINFALL EXCESS DURATION.
      D=TP/5.5
C     PR=HYDROGRAPH PERIOD OF RISE.
      PR=D/2.0+TP
C     B= BASE LENGTH OF HYDROGRAPH EXPRESSED IN HOURS.
      B=5.0*PR
C     W50,W75= WIDTHS OF UNIT GRAPH AT .5QP AND .75QP RESPECTIVELY
C     EXPRESSED IN HOURS.
      W50=10.0**(2.92-1.1*ALOG10(QP))
```

```
      W75=10.0**(2.67-1.1*ALOG10(QP))
      QP=A*QP
C     Q = DISCHARGE IN CFS AND T = TIME IN HOURS.
      T(1)=0.0
      T(2)=PR-W50/3.0
      T(3)=PR-W75/3.0
      T(4)=PR
      T(5)=PR+W75*2.0/3.0
      T(6)=PR+W50*2.0/3.0
      Q(1)=0.0
      Q(2)=QP*.5
      Q(3)=QP*.75
      Q(4)=QP
      Q(5)=Q(3)
      Q(6)=Q(2)
      Q(7)=Q(1)
      T(7)=B
   14 WRITE (6,118) (Q(I), I=1,7)
  118 FORMAT (1H0,/30X,13HQ VALUES ARE.,(7F11.2))
      WRITE (6,119) (T(I),I=1,7)
  119 FORMAT (1H0, 30X,13HT VALUES ARE., (7F11.2))
      CALL NLLSF (R,C,V,SE,CC,PR,T,Q,QP,A, KT,B,S,QF,TF ,XMAX,YMAX1)
      CALL AREA (AH,QF,S,B,A)
      WRITE (6,117) TP,D,V,R,C,CC,SE,AH,KT
  117 FORMAT (1H0,12X,3HTP=,F6.2,2HD=,F5.2,2HV=,E11.3,2HR=,E11.3,2HC=,E
     X11.3,3HCC=,E11.3,3HSE=,E11.3,3HAH=,E11.3,3HKT=,I4)
   99 CONTINUE
      CALL DG (A,TP,D,QF,DSFUG,B,S,XMAX,YMAX1,AH,XMAX1,YMAX2)
      GO TO 699
      END
```

```
$IBFTCXX
C     FITTING PEARSON TYPE 3 FUNCTION TO THE 7 PTS. BY NON LINEAR LEAST
C     SQUARES.
      SUBROUTINE NLLSF (R,C,V,SE,CC,PR,T,Q,QP,A, KT,B,S,QF,TF ,XMAX,
     XYMAX1)
      DIMENSION Q(10),QF7(10),T(10),ARRAY(1000),QF(300),TF(300),QAA(300)
C     C IS THEORETICALLY THE TIME FROM THE PEAK TO THE CENTROID OF THE
C     UNIT HYDROGRAPH AND R IS THEORETICALLY THE RATIO, PR/C. HOWEVER,
C     IN THIS STUDY C AND R ARE PARAMETERS, DETERMINED BY TRIAL AND
C     ERROR, THAT ENSURE A LEAST-SQUARES FIT TO THE 7 POINTS.  R IS
C     DIMENSIONLESS WHILE C HAS THE SAME UNITS AS T AND PR (HR).
      KT=0
      R=9.0
      C=.1*PR
   10 CONTINUE
      AZ=0.0
      BZ=0.0
      CZ=0.0
      DZ=0.0
      EZ=0.0
      RZJ=0.0
      DO2I=1,7
C     QF= PEARSON TYPE 3 FUNCTION, THE 7 INDICATES FOR SPECIFIED SEVEN
C     POINTS, WHERE QF HAS THE UNITS OF QP (CFS).
      QF7(I)=QP*(T(I)/PR)**R/EXP((T(I)-PR)/C)
C     QR AND QC ARE THE FIRST DERIVITIVES OF Q WITH RESPECT TO R AND C
C     RESPECTIVELY.
      QR=QP*(T(I)/PR)**R*ALOG(T(I)/PR)/EXP((T(I)-PR)/C)
      QC=QP*(T(I)/PR)**R*((T(I)-PR)/C**2)/EXP((T(I)-PR)/C)
      AZ=AZ+(Q(I)-QF7(I))*QR
      BZ=BZ+QR*QR
      CZ=CZ+QR*QC
      DZ=DZ+(Q(I)-QF7(I))*QC
      EZ=EZ+QC*QC
      RZJ=RZJ+(Q(I)-QF7(I))**2
```

```
    2 CONTINUE
C       GENERATING POINTS
      P=B/S+1.0
      M=P
      MP1=M+1
C       QF IS DISCHARGE AT TIME TSH.
      TSH=0.0
      DO17K=1,M
      QF(K)=QP*(TSH/PR)**R/EXP((TSH-PR)/C)
      TSH=TSH+S
   17 CONTINUE
      QF(MP1)=QP*(B/PR)**R/EXP((B-PR)/C)
      CALL AREA (QFDT,QF,S,B,A)
C       D1,D2 = THE DEVIATIONS OF R AND C RESPECTIVELY FROM THE TRIAL R
C       AND C VALUES WHICH WERE USED FOR THE ORGIN OF THE MULTIPLE TAYLOR
C       SERIES EXPANSION.
      D1=(AZ*EZ-CZ*DZ)/(BZ*EZ-CZ*CZ)
      D2=(BZ*DZ-AZ*CZ)/(BZ*EZ-CZ*CZ)
C       V=DAMPING FACTOR.
      V=.4
      WRITE(6,100) D1,D2,V,QFDT
  100 FORMAT (1H0,35X,3HD1=,E11.3,3HD2=,E11.3,2HV=,E11.3,5HQFDT=,E11.3)
      RE=R
      CE=C
      R=R+(V*D1)
      C=C+(V*D2)
      KT=KT+1
      E1=ABS(R-RE)
      E2=ABS(C-CE)
      IF (E1.LE..01)GO TO 74
      IF (KT.GT.50) GO TO 12
      GO TO 10
   74 CONTINUE
      IF (E2.LE..01)GO TO 12
      IF (KT.GT.50) GO TO 12
```

```
      GO TO 10
   12 CONTINUE
      R=R- V*D1
      C=C- V*D2
      KT=KT-1
C     COMPUTING STANDARD ERROR AND CORRELATION COEFF.
C     SE=STANDARD ERROR OF ESTIMATE.
      SE=((RZJ)/5.0)**.5
      QBAR=0.0
      RZFB=0.0
      RZB=0.0
      DO77I=1,7
      QBAR=QBAR+Q(I)/7.0
   77 CONTINUE
      DO64I=1,7
      RZFB=RZFB+(QF7(I)-QBAR)**2.0
      RZB=RZB+(QF7(I)-QBAR)**2+(Q(I)-QF7(I))**2
   64 CONTINUE
C     CC=CORRELATION COEFFICIENT.
      CC=(RZFB/RZB)**.5
C     GENERATING HYDROGRAPH POINTS WITH SPACING S.
      TSH=0.0
      P=B/S+1.
      M=P
      MP1=M+1
      DO26K=1,M
C     QF IS DISCHARGE AT TIME TF.
      QF(K)=QP*(TSH /PR)**R/EXP((TSH-PR)/C)
      TF(K)=TSH
      TSH=TSH+S
   26 CONTINUE
      QF(MP1)=QP*(B/PR)**R/EXP((B-PR)/C)
      TF(MP1)=B
C     ADJUSTING THE TAIL OF THE UNIT HYDROGRAPH IN ODER TO OBTAIN ONE
C     INCH RUNOFF VOLUME UNDER THE UNIT HYDROGRAPH.
```

```
C     LX/10 AND (1-LX/10) ARE THE PROPORTIONS FOR THE DELTA SMOOTHING
C     FUNCTION (SEE FIGURE (15)).
      LX=3
      PA=(T(7)-T(5))/S
      N=PA
      MMN=M-N
      MMNM1=MMN-1
      N2=N*LX/10
      N2P1=N2+1
      SN=(N2P1+1)/2
      AH=QFDT
      ALX=LX
      AX=ALX/10.
      CN=(((1.-AH)*AX)*A*5280.**2)/(12.*S*3600.)
      DO115I=1,N2P1
      XN=N2P1
      XI=I
C     QAA=THE DISCHARGE VALUES TO BE ADDED TO THE QF VALUES FOR ENSURING
C     ONE INCH VOLUME.
      QAA(I)=(1./SN)*(XI/XN)*CN
  115 CONTINUE
      N2P2=N2P1+1
      NL=N-N2P2
      SN2=(NL+1)/2
      CN=CN*(1./AX-1.)
      DO113I=N2P2,N
      YN=NL
      YI=NL-(I-N2P2)
      QAA(I)=(1./SN2)*(YI/YN)*CN
  113 CONTINUE
      DO114I=MMN,M
      J=(I-MMNM1)
      QF(I)=QF(I)+QAA(J)
  114 CONTINUE
      QF(MP1)=QF(M)
```

```
      WRITE(6,101)
  101 FORMAT (1H1)
      CALL PLOT 2 (ARRAY,XMAX,0.0,YMAX1,0.0)
      CALL PLOT 3 (1HX,T,Q,7)
      CALL PLOT 3 (1H*,TF,QF,MP1)
      CALL PLOT 4 (36,36H                          DISCHARGE IN CFS)
      WRITE (6,120)
  120 FORMAT (1H0,   60X,12HTIME (HOURS))
      RETURN
      END
```

```
$IBFTCXXXX
C     DETERMINING VOLUME UNDERNEATH HYDROGRAPH BY TRAPEZOIDAL RULE.
      SUBROUTINE AREA (AH,QF,S,B,A)
      DIMENSION QF(300)
      P=B/S+1.0
      M=P
      MP1=M+1
      MM1=M-1
      PMM1=M-1
      VOL=0.0
      DO16L=2,MM1
   16 VOL=VOL+QF(L)
C     AH= AREA(VOLUME) UNDER THE HYDROGRAPH.
      AH=(S*(.5*QF(M)+VOL)+((QF(M)+QF(MP1))/2.)*(B-PMM1*S))*(3600.*12.)
     X/(A*(5280.)**2)
      RETURN
      END
```

```
$IBFTCXXXXX
C      THIS SUBROUTINE UTILIZIES THE BUREAU OF RECLAMATION DIMENSIONLESS
C      HYDROGRAPH PROCEDURE FOR CONVERTING TO A UNIT HYDROGRAPH OF THE
C      DESIRED DURATION.
       SUBROUTINE DG (A,TP,D,QF,DSFUG,B,S,XMAX,YMAX1,AH,XMAX1,YMAX2)
       DIMENSION TDHIP(300),TSU(300),QDH(300),QF(300),QSU(300),ARRAY(1000
      X)
       DHP= TP+D/2.0
C      PD=PROGRAM DURATION.
       PD=D
C      CF=CONVERSION FACTOR.
       CF=(A*(5280.)**2)/(12.*3600.)
       P=B/S+1.0
       M=P
       MP1=M+1
C      TDGS=DIMENSIONLESS HYDROGRAPH TIME IN HOURS.
       TDGS=0.0
       DO43I=1,M
C      TDHIP=DIMENSIONLESS HYDROGRAPH TIME IN PER CENT.
       TDHIP(I)=(TDGS*100.)/DHP
C      QDH=DIMENSIONLESS HYDROGRAPH DISCHARGE.
       QDH(I)=QF(I)*DHP/(CF*AH)
       TDGS=TDGS+S
    43 CONTINUE
       TDHIP(MP1)=(B*100.)/DHP
       QDH(MP1)=QF(MP1)*DHP/(CF*AH)
       WRITE(6,103)
   103 FORMAT (1H1)
       CALL PLOT 2 (ARRAY,XMAX1,0.0,YMAX2,0.0)
       CALL PLOT 3 (1H*,TDHIP,QDH,MP1)
       CALL PLOT 4 (7,7HQ*DHP/V)
       WRITE (6,104)
   104 FORMAT (1HO,60X,12HPER CENT DHP)
C      DSFUG= DURATION SELECTED FOR UNIT HYDROGRAPH.
       D= DSFUG
```

```
      DHP= TP+D/2.0
      DO49I=1,M
C     TSU=UNIT HYDROGRAPH TIME, IN HOURS, FOR SELECTED DURATION.
      TSU(I)= DHP*TDHIP(I)/100.
C     QSU=UNIT HYDROGRAPH DISCHARGE, IN CFS, FOR SELECTED DURATION.
      QSU(I)=QDH(I)*CF/DHP
   49 CONTINUE
      TSU(MP1)=DHP*TDHIP(MP1)/100.
      TSUM=TSU(MP1)
      QSU(MP1)=QDH(MP1)*CF/DHP
      D=PD
      PS=S
      S=TSU(2)-TSU(1)
      CALL AREA (AD,QSU,S,TSUM,A)
      S=PS
      WRITE (6,101) AD,TSUM,DSFUG
  101 FORMAT (1H1,40X,3HAD=,E11.3,5HTSUM=,E11.3,6HDSFUG=,E11.3)
      CALL PLOT 2 (ARRAY,XMAX,0.0,YMAX1,0.0)
      CALL PLOT 3 (1H*,TSU,QSU,MP1)
      CALL PLOT 4 (36,36H                         DISCHARGE IN CFS)
      WRITE (6,120)
  120 FORMAT (1H0,   60X,12HTIME (HOURS))
      RETURN
      END
```

LIST OF REFERENCES

1. American Society of Civil Engineers, Hydrology Handbook, Committee on Hydrology, Hydraulics Division, New York, 184 pp., 1949.

2. American Society of Civil Engineers, "Research Needs in Surface Water Hydrology," Committee on Surface Water Hydrology, J. Hydraulics Div., ASCE, 91(1): 75-83, January 1965.

3. Agricultural Research Service, Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1956-1959, U. S. Department of Agriculture, Washington D.C., 42 pp., 1963.

4. Barnes, Bertram S., "Consistency in Unitgraphs," J. Hydraulics Div., ASCE, 85(8):39-61, August 1959.

5. Brater, E. F., "The Unit Hydrograph Principle Applied to Small Water-Sheds," J. Hydraulics Div., ASCE, 105: 1154-1178, September 1939.

6. Bureau of Reclamation, Bureau of Reclamation Manual, U. S. Department of the Interior, Washington D.C., Vol. IV, Part 6, Chap. 6, 7, 1947.

7. Butler, Stanley S., Engineering Hydrology, Prentice-Hall, Inc., Englewood Cliffs, N. J., 356 pp. 1957.

8. Commons, G., "Flood Hydrographs," Civil Engineering, 12:571-572, 1942.

9. Eagleson, Peter S., "Unit Hydrograph Characteristics for Sewered Areas," J. Hydraulics Div., ASCE, 88(2): 1-25, March 1962.

10. Espey, William H., Morgan, Carl W., and Masch, Frank D., A Study of Some Effects of Urbanization of Storm Runoff from a Small Watershed, Texas Water Commission, Tech. Rep. HYD 07-6501, CRWR-2, Austin, 109 pp., 1965.

11. Gray, Don M., "Synthetic Unit Hydrographs for Small Watersheds," J. Hydraulics Div., ASCE, 87(4):33-53, July 1961.

12. Hartley, H. O., "The Modified Gauss-Newton Method for the Fitting of Non-Linear Regression Functions by Least Squares," Technometrics, 3(2):269-280, May 1961.

13. Hickok, R. B., Keppel, R. V., and Rafferty, B. R., "Hydrograph Synthesis for Small Arid-Land Watersheds," Ag. Eng., ASAE, 40:608-611, 615, 1959.

14. Horner, W. W., and Lloyd, C. Leonard, "Infiltration-Capacity Values as Determined from a Study of an Eighteen-Month Record at Edwardsville, Illinois," Trans. Am. Geophys. Union, 21(II):522-541, 1940.

15. Horton, R. E., "Determination of Infiltration Capacity for Large Drainage Basins," Trans. Am. Geophys. Union, 18(II):371-385, 1937.

16. Johnstone, Don, and Cross, Williams P., Elements of Applied Hydrology, The Ronald Press Company, New York, 276 pp., 1949.

17. Laurenson, E. M., "Hydrograph Synthesis by Runoff Routing," Water Research Laboratory, Report No. 66, The University of New South Wales, 248 pp., 1962.

18. Levenberg, Kenneth, "A Method for the Solution of Certain Non-Linear Problems in Least Squares," Quarterly of Applied Math., 2, 164-168, 1944.

19. Linsley, Ray K., "Application of the Synthetic Unit-Graph in the Western Mountain States," Trans. Am. Geophys. Union, 24:580-585, 1943.

20. Linsley, R. K., Kohler, M. A., and Paulhus, J. L. H., Applied Hydrology, McGraw-Hill, New York, 689 pp., 1949.

21. Linsley, R. K., Kohler, M. A., and Paulhus, J. L. H., Hydrology for Engineers, McGraw-Hill, New York, 340 pp., 1958.

22. Meier, W. L., Jr., Analysis of Unit Hydrographs for Small Watersheds in Texas, Texas Water Commission, Bulletin 6414, Austin, 58 pp., 1964.

23. Minshall, Neal E., "Predicting Storm Runoff on Small Experimental Watersheds," J. Hydraulics Div., ASCE, 86(8):17-38, August 1960.

24. Mitchell, William D., "Unit Hydrographs in Illinois," Illinois Division of Waterways, prepared in cooperation with U. S. Geological Survey, Springfield, 294 pp., 1948.

25. Morgan, Paul E., and Johnson, Stanley M., "Analysis of Synthetic Unit-Graph Methods," J. Hydraulics Div., ASCE, 88(5):199-219, September 1962.

26. Reich, B. M., "Design Hydrographs for Small Watersheds from Rainfall," Ph.D. Dissertation, Civil Engineering Section, Colorado State University, Fort Collins, Colorado, 57 pp., July 1962.

27. Sherman, L. K., "Stream Flow from Rainfall by the Unit-Graph Method," Engineering News-Record, 108:501-505, 1932.

28. Sherman, L. K., and Mayer, L. C., "Application of the Infiltration Theory to Engineering Practices," Trans. Am. Geophys. Union, 22(III):666-677, 1941., May 1941.

29. Soil Conservation Service, Engineering Handbook, Hydrology, Supplement A., Section 4: U. S. Department of Agriculture, Washington D.C., pp. 3.15-1 to 3.16-21, 1957.

30. Soil Conservation Service, Hydrologic Bulletin No. 2, U. S. Department of Agriculture, Washington D.C., 197 pp., 1947.

31. Soil Conservation Service, Hydrologic Bulletin No. 5, U. S. Department of Agriculture, Washington D.C., 38 pp., 1942.

32. Snyder, Franklin F., "Synthetic Unit-Graphs," Trans. Am. Geophys. Union, 19(I):447-454, 1938.

33. U. S. Army Corps of Engineers, Flood-Hydrograph Analyses and Computations, EM1110-2-1405, Washington D.C., 60 pp., 1959.

34. Wisler, Chester O., and Brater, Ernest F., Hydrology, John Wiley & Sons, Inc., New York, 408 pp., 1963.