doi:10.3788/gzxb20134207.0817

基底和膜层-基底系统的赝布儒斯特角计算

刘华松[1],姜玉刚[1,2],王利栓[1],姜承慧[1],季一勤[1,3]
(1 天津津航技术物理研究所 天津市薄膜光学重点实验室,天津 300192)
(2 同济大学 物理系 先进微结构材料教育部重点实验室,上海 200092)
(3 哈尔滨工业大学 光电子技术研究所 可调谐激光技术国家级重点实验室,哈尔滨 150080)

摘 要:对基底和膜层-基底系统的赝布儒斯特角进行了数值计算.结果显示:当基底的消光系数小于 0.01 时,基底的赝布儒斯特角主要是由折射率决定;当基底的消光系数大于 0.1 时,基底的赝布儒斯特角不仅与折射率有关,而且还与消光系数有关,随着消光系数发生后周期性变化.研究表明:单层膜-基底系统的赝布儒斯特角主要由膜层的物理厚度、折射率、基底的光学常量所决定;在 HfO_2-硅和 HfO_2-融石英基底系统中,赝布儒斯特角随着入射光波长和膜层厚度的变化呈现准周期性规律变化,可能是由入射光在膜层的干涉效应引起的.

关键词:光学常量;折射率;消光系数;膜层-基底系统;赝布儒斯特角
中图分类号:O484 **文献标识码**:A **文章编号**:1004-4213(2013)07-0817-6

Calculation of Pseudo-brewster Angle for Substrate and Thin Film-substrate System

LIU Hua-song[1], JIANG Yu-gang[1,2], WANG Li-shuan[1], JIANG Cheng-hui[1], JI Yi-qin[1,3]
(1 *Tianjin Key Laboratory of Optical Thin Film, Tianjin Jinhang Institute of Technical Physics, Tianjin* 300192, *China*)
(2 *Key Laboratory of Advanced Micro-Structure Materials (Ministry of Education), Department of Physics, Tongji University, Shanghai* 200092, *China*)
(3 *National Key Laboratory of Science and Technology on Tunable Laser, Institute of Optical-electronics, Harbin Institute of Technology, Harbin* 150080, *China*)

Abstract: Numerical calculation of pseudo-Brewster angles for substrate and thin-film substrate system is demonstrated. The results show: for only substrates, pseudo-Brewster angles are mainly affected by refractive index when the extinction coefficient is lower than 0.01; when the extinction coefficient is above 0.1, the pseudo-Brewster angle is also modulated by the extinction coefficient. The studies indicate: for thin film-substrate systems, pseudo-Brewster angle is influenced by both refractive index and physical thickness of thin film together with optical constants of substrate; in the HfO_2 film on silicon and fused silica substrate systems, pseudo-Brewster angles exhibit quasi-periodic characteristic with variation of incident wavelength or thin film thickness, as caused by interference effect.

Key words: Optical constant; Refractive index; Extinction coefficient; Thin-film substrate system; Pseudo-Brewster angle

0 Introduction

Brewster angle is an important parameter to describe reflection properties upon surface of optical medium. For instance, Wild et al.[1] found a large negative lateral shift near the Brewster

Foundation item: The National Natural Science Foundation of China (No. 61235011), Tianjin Municipal Government of China (Nos. 13JCYBJC17300, 12JCQNJC01200)
First author: LIU Hua-song (1980—), male, senior engineer, Ph. D. degree, mainly focuses on low loss laser coatings tchnology. Email: liuhuasong@gmail.com
Received: Jan. 21, 2013; **Accepted**: May 7, 2013

angle in a configuration with an absorbing dielectric reflecting medium, which has attracted some attention recently[2-5]. In fact, such a lateral shift occurs around the angle when the real part of the reflection coefficient vanishes. We would like to term this angle as pseudo-Brewster angle, which should be more pertinent than Brewster angle, since an absolute null reflection cannot exist for a dissipative reflecting medium generally[6]. Measurement of pseudo-Brewster angle is usually used as a method for determining the optical constants (refractive index n and extinction coefficient k)[7].

When p-polarized collimated light is incident upon the surface of an optical isotropic transparent medium, the reflected wave vanishes at a specific incident angle, which is known as Brewster angle. Brewster angle can be expressed as $\theta_B = \arctan^{-1}(n)$, where n is the refractive index of medium. For absorbing medium, refractive index is in complex form $N_s = n_s\text{-}ik_s$, where n_s and k_s are refractive index and extinction coefficient respectively. When p-polarized parallel light is incident upon the surface of such medium, the reflectance R_p is nonzero at all angles of incidence but can reach a minimum at one angle of incidence which is called as pseudo-Brewster angle[8]. In thin film-substrate system, if thin film and substrate are both absorbing, the pseudo-Brewster angle can be defined as the incident angle when R_p of thin film-substrate system reaches a minimum. This angle is useful for measurement of optical constants of thin film and substrate[9-12], and manufacturing of thin film polarization splitters[13].

1 Basic theory

For substrate, the amplitude reflection coefficient for p-polarized light can be given by

$$r_p = \frac{n_0 \cos\theta_s - N_s \cos\theta_0}{n_0 \cos\theta_s + N_s \cos\theta_0} \tag{1}$$

where n_0 is refractive index of incident medium, θ_0 is angle of incidence, θ_s is the refractive angle in the absorbing substrate and N_s is the complex refractive index of substrate.

For thin film-substrate system, the first step is to write the characteristic matrix of single thin film-substrate $\boldsymbol{M}$

$$\boldsymbol{M} = \begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} \cos\delta_f & i\sin\delta_f/\eta_f \\ i\eta_f \sin\delta_f & \cos\delta_f \end{bmatrix} \begin{bmatrix} 1 \\ \eta_s \end{bmatrix} \tag{2}$$

where the phase of retardation δ_f is

$$\delta_f = 2\pi N_f d_f \cos\theta_f / \lambda \tag{3}$$

where $\lambda, N_f, d_f, \theta_f$ are the wavelength in vacuum, complex refractive index of thin film, layer thickness, and complex angle of refraction in the thin film, respectively.

The parameter η_f and η_s is equivalent optical admittance at oblique incidence, as follows

$$\eta_f^p = N_f/\cos\theta_f,\ \eta_s = N_s/\cos\theta_s \tag{4}$$

where θ_f and θ_s are the complex angles of refraction in the thin film and substrate.

The next step is to calculate the equivalent optical admittance Y_p for the thin film-substrate system, as follows

$$Y_p = \frac{C}{B} = \frac{i\eta_f \sin\delta_f + \eta_s \cos\delta_f}{\cos\delta_f + i\eta_s \sin\delta_f/\eta_f} \tag{5}$$

So, the amplitude reflection coefficient and reflectance of the p-polarized parallel component can be given:

$$r_p = \frac{\eta_0 - Y_p}{\eta_0 + Y_p},\ R_p = \left(\frac{\eta_0 - Y_p}{\eta_0 + Y_p}\right)\left(\frac{\eta_0 - Y_p}{\eta_0 + Y_p}\right)^* \tag{6}$$

The pseudo-Brewster angle (θ_B) is defined by

$$\left.\frac{\partial R_p}{\partial \theta_0}\right|_{\theta_B} = 0 \tag{7}$$

Analytic solution of Eq. (7) is very complicate for absorbing film-substrate system, thus numerical calculation is adopted to obtain pseudo-Brewster angle. The process of calculation is shown in Fig. 1. The incident medium is considered as air, which means $n_0 = 1$.

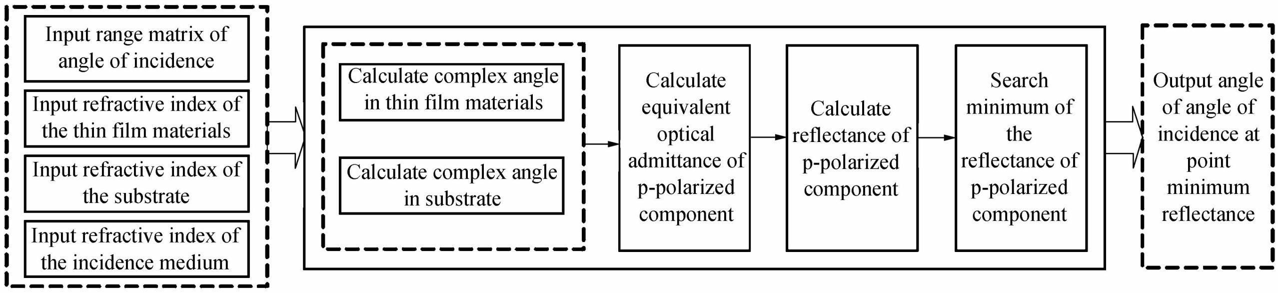
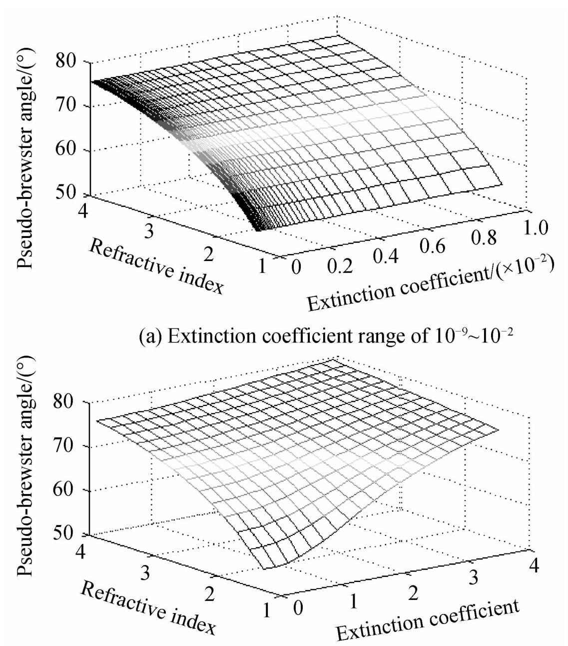


Fig. 1 Calculation process of pseudo-Brewster angle

2　Results and discussion

Influence of pseudo-Brewster angle by substrate refractive index and extinction coefficient is first analyzed. Fig. 2 (a) shows the 3D plot of pseudo-Brewster angle distribution with a substrate refractive index range of 1.38～4 and a substrate extinction coefficient range of 10^{-9} ～ 10^{-2} while Fig. 2 (b) shows that with another substrate extinction coefficient range of 0.1～4. The corresponding contour plot of pseudo-Brewster angle distribution with the same substrate refractive index and extinction coefficient ranges are shown in Fig. 3.

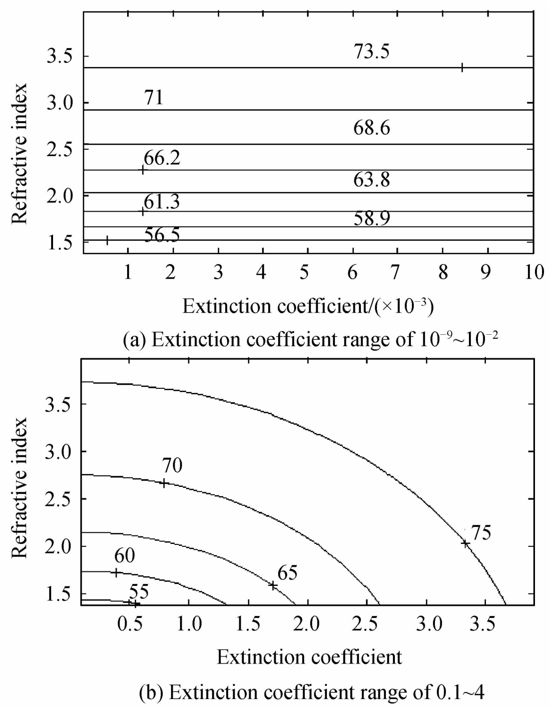


(a) Extinction coefficient range of 10^{-9}~10^{-2}

(b) Extinction coefficient range of 0.1~4

Fig. 2　3-D plot of pseudo-Brewster angle distribution with refractive index range of 1.38～4

It can learn from Fig. 2(a) and Fig. 3(a) that the values of pseudo-Brewster angels are mainly depended on the refractive index when the extinction coefficient is below 0.01, which stands for the weakly absorbing substrate. In such substrate, pseudo-Brewster angel increases with enlargement of the refractive index. For absorbing substrate with large extinction coefficient, pseudo-Brewster angel is modulated by not only the refractive index but also the extinction coefficient. They are demonstrated in Fig. 2(b) and Fig. 3(b) with the extinction coefficient being larger than 0.1. It can be seen that pseudo-Brewster angel becomes larger while both refractive index and extinction coefficient increase.

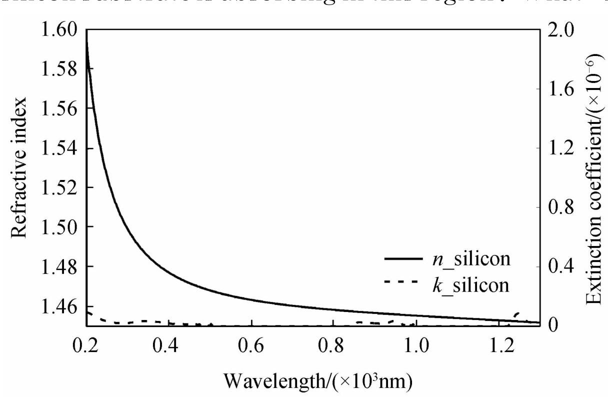


(a) Extinction coefficient range of 10^{-9}~10^{-2}

(b) Extinction coefficient range of 0.1~4

Fig. 3　Contour of 3-D of pseudo-Brewster angle distribution with refractive index range of 1.38～4

Typical examples are given with fused silica (quartz) and silicon, both of which are most common used substrates in thin film optics experiments and applications. The transparent region for silica substrate lies in near ultraviolet and visible range while silicon substrate is transparent in the wave range of 1 200～4 000 nm. Fig. 4(a) and (b) respectively shows the dispersion of optical constants of the silica and silicon substrates in the wavelength range of 200 ～ 1 300 nm. In the Fig. 4, solid line stands for the refractive index and the dashed line stands for the extinction coefficient. Silica substrate shows normal refractive index dispersion relationship with low extinction coefficient in its transparent region while silicon substrate is absorbing in this region. What′s



(a) Silica substrate

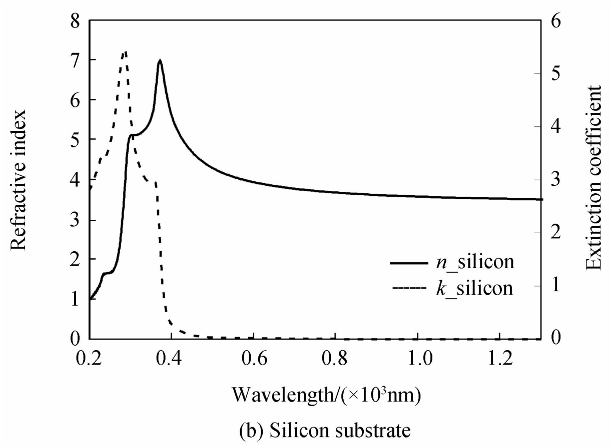


Fig. 4 Dispersion of optical constants of the substrates

more, silicon substrate shows abnormal dispersion relationship of optical constant in the wave range of 200 ～ 400 nm with very high extinction coefficient which is in the extinction coefficient range of Fig. 2(b) and Fig. 3(b), which makes silicon substrate as a typical absorbing medium in this wave range.

The dispersion of pseudo-Brewster angle with wavelength range from 200 nm to 2 000 nm for silica and silicon substrates are shown in Fig. 5(a) and (b), respectively. Pseudo-Brewster angle of silica decreases monotony with the decrease of refractive index at longer wavelength because of the low extinction coefficient in this region. Variation curve of pseudo-Brewster angle of silicon exhibits two peaks at the wave range 200～400 nm because of the abnormal dispersion relationship of

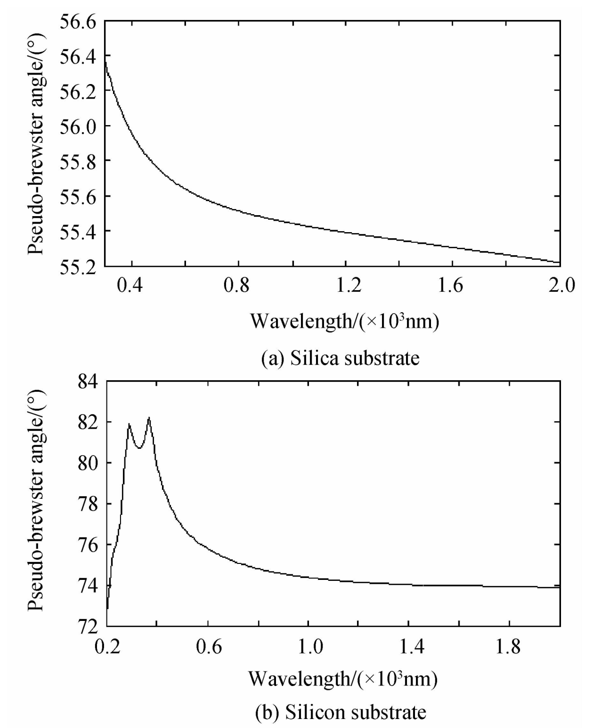


Fig. 5 Dispersion of pseudo-Brewster angle of the substrates

both extinction coefficient and refractive index. What′s more, the pseudo-Brewster angle of silicon substrate is significantly larger than that of silica substrate, which can be thought to be caused by the larger refractive index or extinction coefficient of silicon substrate than silica substrate.

In order to continue to research the characteristic of pseudo-Brewster angle, we choose a typical example of thin film-substrate system given by HfO_2 film and substrate, and the thicknesses of HfO_2 films were both about 500 nm. The variation of pseudo-Brewster angle of HfO_2-silica$_s$ystem and HfO_2-silicon$_s$ystem are shown in Fig. 6. From the Fig. 6(a), it can be seen that the pseudo-Brewster angle of HfO_2-silica$_s$ystem oscillates with wavelength of the incident light. The oscillation can be thought to be caused by interference effect, which has strong relation with the refractive index and thickness of the deposited films. For the HfO_2-silica$_s$ystem, the refractive index of HfO_2 is higher than the substrate for all wavelengths, thus pseudo-Brewster angle exists at non-zero degree. For HfO_2-silicon$_s$ystem, due to the refractive index of HfO_2 layer being lower than silicon substrate, so the optical thickness of thin film satisfies with antireflection conditions at some certain wavelengths. In such conditions, absolute minimum reflectance for p - polarized light is

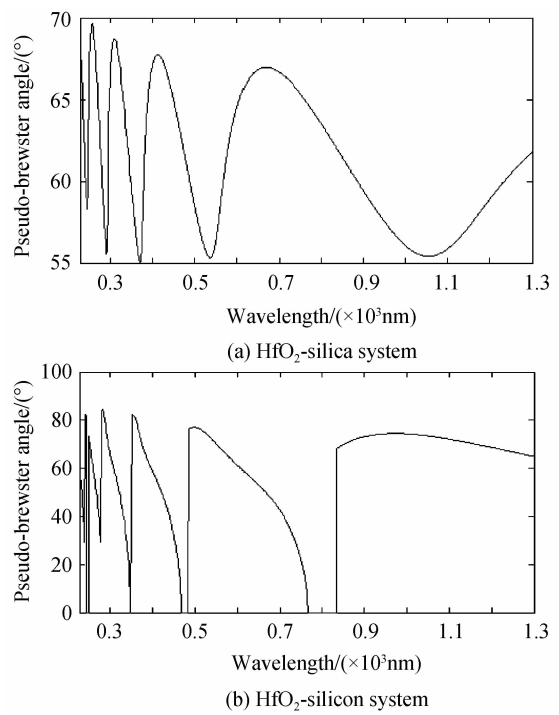


Fig. 6 Dispersion of pseudo-Brewster angle of thin film-substrate system

achieved at zero incident angles while only relative low reflection point was achieved at other incident angle.

It is also important to research the influence of thin film thickness on pseudo-Brewster angle. It is well known for thin film-substrate system that optical characteristics repeat when optical thickness of the film reaches integral multiples of $\lambda/4$, where λ stands for the reference wavelength. Although absolute transmittance degrades when the substrate and the films are absorbing mediums, the positions of extreme values do not change. It is constantly true that the film thickness is too large to yield interference effect. The quasi-periodic characteristics of pseudo-Brewster angle of HfO_2-silica and HfO_2-silicon system with the reference wavelength at 351 nm and 633 nm are shown in Fig. 7.

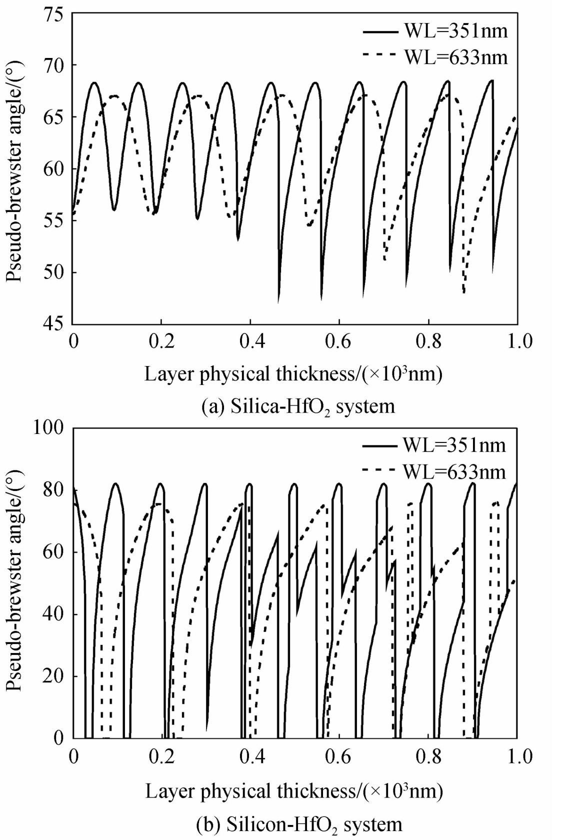


Fig. 7　Periodic characteristic of Pseudo-Brewster angle with reference wavelength at 351 nm and 633 nm

From Fig. 7(a), it can be seen that the pseudo-Brewster angle of HfO_2-silica system exhibits quasi-periodic characteristics as the film thickness increase, and the range of pseudo-Brewster angles are all almost between 50° and 70°. From Fig. 7(b), we can find that the pseudo-Brewster angle of HfO_2-silicon system also exhibits quasi-periodic characteristics as the film thickness increases, but there is the pseudo-Brewster angles of zero and the range of pseudo-Brewster angles cover from 0° to 80°. From both Fig. 7(a) and (b), it can be found that the period is short and the quasi-periodic characteristic will become clear when the reference wavelength is much shorter.

3　Conclusion

In conclusion, the pseudo-Brewster angle of substrate and thin film-substrate system are researched. The pseudo-Brewster angle of substrate is affected by the refractive index and extinction coefficient. For transparent substrate with extinction coefficient below 0.01, the pseudo-Brewster angle of substrate is mainly decided by refractive index. For absorbing substrate with extinction coefficient above 0.1, the pseudo-Brewster angle is both modulated by the refractive index and the extinction coefficient. For thin film-substrate system, the pseudo-Brewster angle is influenced both by the refractive index and thickness of thin film together with the refractive index of substrate. As caused by interference effect, pseudo-Brewster angle of thin film-substrate system exhibits quasi-periodic characteristic with the variation of the film thickness and wavelength of incident light. When the refractive index of thin film is larger than substrate, the minimum reflection of p-polarized light is achieved at non-zero incident angle. When the refractive index of thin film is smaller than the refractive index of substrate, minimum reflection of p-polarized light is achieved at zero incident angles when the anti-reflection condition is satisfied.

References

[1] WILD W J, GILES C L. Goos-Hanchen shifts from absorbing media[J]. *Physical Review A*, 1982, **25**(4): 2099-2101.
[2] LAI H M, CHAN S W. Large and negative Goos-Hanchen shift near the Brewster dip on reflection from weakly absorbing media[J]. *Optics Letters*, 2002, **27**(9): 680-682.
[3] WANG L G, ZHU S Y. Large positive and negative Goos-Hanchen shifts from a weakly absorbing left-handed slab[J]. *Journal of Applied Physics*, 2005, **98**(4): 043522(1-4).
[4] JORDAN P, TODD A G. Simple optical method to determine the porosity of porous silicon films[J]. *Thin Solid Films*, 2012, **520**(7): 2526-2531.
[5] CHIU M H, LAI C W, WANG S F, *et al*. Phase geographical map for determining the material type of a right-angle prism [J]. *Applied Optics*, 2006, **45**(26): 6781-6784.
[6] SHEN N H, CHEN J, WU Q Y, *et al*. Large Lateral shift near psoudo-Brewster angle on reflection from a weakly absorbing double negative medium[J]. *Optics Express*, 2006, **14**(22): 10574-10579.
[7] YOSHIOKA S, KINOSHITA S. Direct determination of the

refractive index of natural multilayer systems[J]. *Physical Review E*, 2011, **83**(5): 051917(1-7).

[8] ALI M A, AHMED S A. Second condition for the pseudo-Brewster angle of highly absorbing media[J]. *Journal of the Optical Society of America A*, 1991, **8**(9): 1419-1420.

[9] AZZAM R M A, ALSAMMAN A. Plurality of principal angles for a given pseudo-Brewster angle when polarized light is reflected at a dielectric-conductor interface[J]. *Journal of the Optical Society of America A*, 2008, **25**(11): 2858-2864.

[10] HONGYI C, SHIXIANG X, JINGZHEN L. Omnidirectional constant transmission and negative Brewster angle at planar interfaces associated with a uniaxial medium[J]. *Optics Express*, 2009, **17**(22): 19791-19797.

[11] BAHRIM C, HSU W T. Precise measurements of the refractive indices for dieletrics using an improved Brewster angle method[J]. *American Journal of Physics*, 2009, **77**(4): 337-343.

[12] OGUSU K, SUZUKI K, NISHIO H. Simple and accurate measurement of the absorption coefficient of an absorbing plate by use of the Brewster angle[J]. *Optics Letters*, 2006, **31**(7): 909-911.

[13] HU X H, HO K M. Brewster angle phenomenon in two-dimensional metallic photonic crystals and its application to polarization beam splitting[J]. *Appllied Physics Letters*, 2006, **89**(20): 201906-1-3.