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STOCHASTIC NATURE OF ENTRAINED SEDIMENT PARTICLES FROM THE BED

Keshavarzi ALIREZA
Assistant Professor, Irrigation Department, Shiraz University, Shiraz, Iran
Fax:009811-712-265-3322, E-mail:keshavrz@hafez.shirazu.ac.ir

Shirvani AMIN
Irrigation Dept., Shiraz University, Shiraz, Iran

Abstract: In this present study, the entrained sediment particles from the bed were recorded using image processing facilities and techniques. The instantaneous velocities of the flow were measured using electromagnetic velocity meter. The bed of the flume was covered by 6 mm sand particles. The entrainment of sediment particles from the bed was recorded using CCD camera. An image processing technique was used to derive the difference between images. The number of sediment particles from the bed was counted with time. The Poisson distribution function was applied to the number of entrained particles and entrainment intensity from the bed. It was found that the poission distribution well defined the entrainmnet process. For the experimental data and at a sequential time, the Markov process was applied to the data. The AIC and BIC criteria were tested to find the appropriate model. It was found that the first order Mrkove chain was the appropriate model for definition of the sequential occurrence of the bursting process.

Key words: Bursting events, Markov Chain, Poission Distribution

1. INTRODUCTION

The importance of the initiation of individual sediment particle motion arises from the stochastic nature of sediment entrainment from the bed. There is also a difficulty for the definition of incipient particle motion over a flat bed. The difficulty arises from the influence of turbulence on sediment motion which has been discussed by Ball and Keshavarzy (1995) and Keshavarzy and Ball (1995, 1999). The entrainment of particles from a mobile bed in an open channel flow has been investigated in several studies; for example by Einstein and Li (1958), where it was pointed out that this process is completely stochastic in nature due to the effect of turbulence One reason for this is the difficulty of observing sediment particles at the initiation of motion. Recently, attention has been focused on the use of image processing techniques for observing the motion of sediment particles. In several studies, for example those by Nelson et al. (1995), image processing techniques were used as a tool to investigate the intermittent nature of particle entrainment upstream and downstream of dunes. The application of image processing techniques has the potential to assist in understanding the processes influencing incipient of particle motion. This technique can show the entrainment of particles from the bed, settlement of particles, speed of particle movement, transport mode, and resting periods of a particle on the bed. With the capturing and collection of these data, it is possible to develop a statistical description of particle entrainment or particle initiation at the bed.

In this study attention is paid to define the initiation of sediment particle motion. An experimental test was undertaken over a mobile bed in a flume. In order to understand this process an image processing technique was used to observe the particle movement in an instant of time over a desired area. A probability distribution function was applied to the entrained sediment particles from the bed.

2. ANALYSIS OF IMAGES OF PARTICLE MOTION

To analyze the captured images, two different techniques were used in this study. These techniques were

- Probability analysis of the entrained particles determined by counting the number of particles in motion at an instant. This approach was useful to obtain an exceedance probability of particles in motion in time, respective to the exceedance probability of shear stresses of the bursting process at the bed.
- Application of some image processing technique to determine the displacement of particles between images.

The number of entrained and deposited particles was obtained by computing the difference between two sequential images. The subtraction technique was used for this purpose. Computing the difference between the light intensities at all pairs of corresponding pixels from image one and image two can compare two images. This technique was used and a sequence of images were compared to find the number of particles which entrained and deposited over specified area and in a given time increment. For analysis of the images, each image was digitized into an array of 384 by 288 pixels. Two different types of format were selected in digitizing process, a BMP format in color (24 b/p) and a PGM Grey scale (8 b/p) sub-format. The PGM Grey format was selected for its lower storage requirements and the ease of processing and file transfer between different computers.

The images were digitized using a Pentium II 950 MHz computer. Using a software to convert video film to a series of separate frames and hence to store them as a group of files in the computer. Initially this software captures a sequence of frames as a large file and then in another process converts this file to a series of frames with the desired format. It was these series of images that were analyzed to investigate the initiation of sediment motion. In this part of this study the purpose is to derive the difference between two sequential images in order to ascertain the number of particles entrained in an instant of time. To meet this aim and in order to analyze the images a specially written computer program was required to read the binary files and to process them for analysis. This program was developed in the C^{++} language due to its utility and capability for image processing.

The number of entrained particles over a specified area will vary with time. The entrained particles at any time depend on the instantaneous turbulent shear stress arising from the velocity fluctuations and the instantaneous shear stresses at the bed. Shown in Figure 1 are the velocity fluctuations of the flow with time and the corresponding instantaneous shear stress at the bed. This relation was investigated using a cross correlation analysis between instantaneous shear stress in sweep events and the number of particles entrained. The number of entrained particles were counted in a sequence of produced images derived from subtraction of sequential recorded images. The dimensionless shear stress in sweep events was computed also from a time series of the velocity fluctuations recorded at the same timescale of the images. Due to the temporal nature of turbulent boundary layer, definition of the initiation of sediment resting on the channel bed is very difficult and complicated. This arises from the need to consider the instantaneous turbulent shear stress, which is sometimes lower and sometimes higher than the critical shear stress for a particle. The concept of bursting process was introduced by Kline et al. (1967) as a process which consists of four categories of events; these categories are the sweep ($u'>0$, $v'<0$), ejection ($u'<0$, $v'>0$), outward interaction ($u'>0$, $v'>0$) and inward interaction ($u'<0$, $v'<0$) with each event having a different phase of action. The above concept was recognized as a means of momentum transfer between the turbulent and laminar regions near the boundary. Shown in Fig. 1 is a phase diagram with the zone of each event indicated. Bridge and Bennett (1992) noted that these four alternative types of bursting events have different effects on the mode and rate of sediment transport. Studies by for example; Thorne et al. (1989), Nelson et al. (1995), and Drake et al. (1988) indicated that close to the bed most of the sediment entrainment occurs during the bursting events. Also, the entrainment of particles from a mobile bed in an open channel flow has been investigated in several studies; for example by Einstein and Li (1958), where it was pointed out that this process is completely stochastic in nature due to the effect of turbulence.

Recently, the contributions of coherent structures, such as the sweep and ejection events, to momentum transfer have been extensively studied by quadrant analysis or probability analyses based on two-dimensional velocity information. Studies by Nakagawa and Nezu (1978) and Grass (1971) have indicated that just above the channel bed, the sweep event is more responsible than the ejection event for transfer of momentum into the bed layer. Nakagawa and Nezu (1978), Thorne et al. (1989) and Keshavarzy and Ball (1995, 1999) concluded that sweep and ejection event occurred more frequently than outward and inward interaction.

The four types of bursting events identified earlier have different influences on the rate, and mechanisms of sediment entrainment in a turbulent flow. Despite the importance of the bursting events in sediment transport, the statistical characteristics of bursting events have not been investigated in sufficient detail. The probability density function was adopted as an appropriate approach for the investigation of the contribution of bursting events in the entrainment process. This was pointed out by Nakagawa and Nezu (1978), Bridge and Bennett (1992), Grass (1971) who suggested that the contribution of bursting events must be treated in the form of probability function.

In this study, the first order Markov chain was applied to the time series of the events. Additionally, the organization of bursting events was investigated using conditional probability analysis. The movement probability between the events was also investigated.

3. EXPERIMENTAL APPARATUS AND SETUP

The experimental tests in this study were carried out in a non-recirculating tilting rectangular flume. To perform a series of the experimental tests with different bed roughness, the bed was covered by sand particles of D_{50}= 6mm. The longitudinal and vertical components of the instantaneous velocity were measured using a small electromagnetic velocity meter. The measurements were performed along centerline of the flume and at the location of ensured developed flow. The experiments were carried out with different flow condition. These recorded velocities were analyzed to calculate the time averaged velocity in the horizontal and vertical directions, the overall mean shear stress, turbulent velocity fluctuations, the mean shear stress for each event, and to count the number of bursting events during the sample period.

The captured data were analyzed to calculate the following characteristics of the flow:

Time-averaged velocity, velocity fluctuations, root mean square and the Reynolds shear stress.

- For the time averaged mean velocity of the longitudinal and vertical components

$$U = \frac{1}{N}\sum_{i=1}^{N} u_i \quad \text{and} \qquad V = \frac{1}{N}\sum_{i=1}^{N} v_i \tag{1}$$

where; u_i and v_i are instantaneous velocities, U and V are the local temporal mean velocities in the longitudinal and horizontal directions, respectively and N is the number of samples.

- For the velocity fluctuations in two components about the mean value were given by

$$u_i' = u_i - U \quad \text{and} \qquad v_i' = v_i - V \tag{2}$$

where u'_i and v'_i are the velocity fluctuations in the longitudinal and vertical directions, respectively. The experimental data were employed also to calculate the turbulence intensity of flow in the horizontal and vertical directions.

4. MARKOV CHAIN MODELS FOR BURSTING PROCESS

A discrete random variable was defined as $\{S_t\}$ in which S_t is a quadrant zone or event in time (t). Therefore, in an instant of the time the S_t can be in quadrant 1, 2, 3 and 4, (outward interaction, ejection, inward interaction and sweep), respectively. Here, the change in the situation or state of the events in time series is defined as movement situation and was investigated by Markov process. The nature of Markov process is governed by a set of probabilities which is called the transition probabilities and they are explained as;

- The zero order Markov chain $P(0)$ was also examined here for comparison. It means that the current situation does not depend on the previous situation and it depends only on the current situation. The probabilities for zero order Markov chain can be computed as;

$$\hat{P}_i = \frac{n_i}{n} \qquad i = 1,2,3,4 \tag{7}$$

where n_i is number of situation i and n equal total number of sampling data.

- The first order Markov chain is defined as;

$$pr\{s_{t+1} | s_t, s_{t-1}, \ldots, s_1\} = pr\{s_{t+1} | s_t\} \tag{3}$$

According to this concept, the probability of the next situation depends on the current situation, but it does not depend on the particular way that the model system arrived at the current situation. With the application of the maximum likelihood estimators, the transition probabilities of first order Markov chain can be computed as;

$$\hat{P}_{ij} = pr\{s_{t+1} = j | s_t = i\} = \frac{no.of\ \ j's\ \ following\ \ i's}{total\ \text{no.}of\ \ i's} = \frac{n_{ij}}{n_{i.}} \qquad i,j = 1,2,3,4 \tag{4}$$

Where; $n_{i,j}$ is the number of transition from situation i to situation j. The $n_{i.}$ is the number of i's in the series followed by another situations, so that $n_{i.} = n_{i1} + n_{i2} + n_{i3} + n_{i4}$. The $p_{i,j}$ is estimated as the fraction of points for which $S_t = i$ is followed by points with $S_{t+1} = j$.

- The second order Markov chain is also a stochastic process which defines the current situation based on the two previous situations. The present situation S_{t+1} can be found on the basis of the situations at S_t and S_{t-1}. It means that the situation at (t+1) depends on the situation at (t) and (t-1).

$$pr\{s_{t+1} | s_t, s_{t-1}, \ldots, s_1\} = pr\{s_{t+1} | s_t, s_{t-1}\} \tag{5}$$

The above probability shows that a situation at the time step (t) depends on the situation at time step (t) and (t-1). The transition probabilities of second order Markov chain were obtained from the conditional relative frequencies of transition counts. It can be computed as;

$$\hat{P}_{kij} = pr\{s_{t+1} = j | s_t = i, s_{t-1} = k\} = \frac{no.of\ \ j's\ following\ \ i's\ \ following\ \ k's}{total\ no.of\ \ i's\ following\ k's} = \frac{n_{kij}}{n_{ki.}} \qquad i,j,k = 1,2,3,4 \tag{6}$$

in which the value of the time series at time t-1 was $S_{t-1}=k$ and the value of time series at time t was $S_t=i$, the probability that future value of time series $S_{t+1}=j$ is P_{kij}.

5. RESULTS AND DISCUSSION

The analysis of the experimentally determined data resulted in the characteristics of the bursting events close to the bed. In order to investigate the probability of motion between the bursting events, the experimental data were analyzed to find the occurrence of the bursting process. As a result, a time series of the bursting events was obtained.

As previously mentioned, there are four quadrant zones as outward interaction, ejection, inward interaction and sweep which are occurring in zones 1,2,3 and 4 respectively. The probability of movement from a quadrant to the next is a focus of this study. This movement of the events was investigated with time and in four zones. Therefore, a spatiotemporally process can be applied to the bursting events using time series model. The spatial variation of the events means how the events are moving and allocating in quadrant zones. Thus a time series of the events were consisted a sequential of the events, which occur in time. The events may be classified as discrete or continuos. In this study the data was classified as discrete random variable. The common stochastic process which represents time series of discrete random variable is Markov chain process.

The zero order Markov chain model was applied to the movement of the events from a zone to the next. Table 1 shows the estimated transition probabilities of the occurrence of the bursting process for zero-order Markov chain.

Table 1 Estimation probabilities of zero-order Markov chain

Probability		
$\hat{p}_1 = 0.228$	$\hat{p}2 = 0.272$	$\hat{p}3 = 0.21$
	$\hat{p}_4 = 0.287$	

With the application of 1st order Markov chain to the time series of the events it was found that, the 16 elements of transition probabilities of the first order Markov chain P(I) are;

$$P(I) = \begin{bmatrix} 0.49 & 0.14 & 0.09 & 0.28 \\ 0.14 & 0.57 & 0.20 & 0.09 \\ 0.10 & 0.28 & 0.45 & 0.17 \\ 0.20 & 0.09 & 0.14 & 0.57 \end{bmatrix}$$

Using the above matrix, the next situations of the events can be found from the current situation. It means that at the present time *(t)*, the situation is at a known quadrant, at t_n the situation of the quadrant can be found using probability analysis.

With the application of 2nd order Markov chain to the data set, 64 elements of transition probabilities for second order Markov chain P(II) were computed and presented in the following matrix as;

$$P(II) = \begin{bmatrix} 0.45 & 0.17 & 0.11 & 0.27 \\ 0.03 & 0.59 & 0.26 & 0.12 \\ 0.05 & 0.29 & 0.56 & 0.10 \\ 0.14 & 0.09 & 0.13 & 0.64 \\ 0.44 & 0.17 & 0.05 & 0.34 \\ 0.12 & 0.60 & 0.20 & 0.08 \\ 0.11 & 0.28 & 0.41 & 0.20 \\ 0.20 & 0.14 & 0.14 & 0.52 \\ 0.54 & 0.12 & 0.08 & 0.26 \\ 0.17 & 0.62 & 0.16 & 0.05 \\ 0.10 & 0.30 & 0.40 & 0.20 \\ 0.29 & 0.05 & 0.17 & 0.49 \\ 0.59 & 0.06 & 0.06 & 0.29 \\ 0.22 & 0.44 & 0.27 & 0.07 \\ 0.17 & 0.27 & 0.44 & 0.12 \\ 0.21 & 0.10 & 0.14 & 0.55 \end{bmatrix}$$

To find the most appropriate model to define the stochastic process of the bursting events, two criteria must be applied to find the best order of the Markov chain. They are listed as below;

- The Akaike information criterion (AIC), (Akaike 1974; Tong, 1975)
- The Bayesian information criterion (BIC), (Schwartz, 1978; Katz, 1971)

Both are based on the log-likelihood functions for the transition probabilities of fitted Markov chains. These log-likelihood depends on the transition counts and the transition probabilities. The log-likelihood for the 0th , 1st and 2nd orders of Markov chains are;

$$L_0 = \sum_{i=1}^{4} n_i \ln(\hat{P}_i) \tag{8}$$

$$L_1 = \sum_{i=1}^{4} \sum_{j=1}^{4} n_{ij} \ln(\hat{P}_{ij}) \tag{9}$$

$$L_2 = \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{4} n_{kij} \ln(\hat{P}_{kij}) \tag{10}$$

Both criteria attempt to find the most appropriate order for Markov chains. The AIC and BIC statistics are then computed for each trial order m, using

$$AIC(m) = -2 * L_m + 2 * s^m * (s-1) \tag{11}$$

$$BIC(m) = -2 * L_m + s^m * (\ln(n)) \tag{12}$$

where s equal to 4 which are the four quadrant zones. Here, it was examined which order was best applied to the data. The order m which produces minimum value in either Equation (11) or (12) is the most appropriate order of the Markov chain. The AIC and BIC values for the 0th 1st and 2nd orders are computed and presented in the following table.

Order / Criteria	0th order Markov Chain	1st order Markov Chain	2nd order Markov Chain
AIC	3038	2524	2496
BIC	3041	2537	2548

As it is shown in the above table, the first order Markov chain has minimum value in both AIC and BIC. It was found that the first order Markov chain defined the stochastic process of the quadrant analysis of the bursting events. Therefore the first order Markov chain is an appropriate mathematical model for the definition of the bursting events. As the result it can be concluded that the situation of an event at an instant of time (t+1) depends only on the situation at time (t) and the situation at (t) depends on the situation at (t-1).

The n- step transition matrix ($P^n(I)$) was obtained by using a first-order Markov chain and the values are presented in appendix 1. The n-step probabilities are the elements of the type $P^n(I)$ where $P(I)$ is the one step transition matrix. The values of $P^n(I)$ were approximately constant after 17 steps and afterward. This constant value in each column of $P^n(I)$ after step 17 implies an independent probability of occurrence of a state from its initial state (outward interaction, ejection, inward interaction and sweep). Using the above steady state probabilities (0.25, 0.29, 0.22 and 0.31), the mean recurrence time period was calculated. It was found that mean recurrence time period for outward interaction, ejection, inward interaction and sweep are approximately 4, 3.4, 4.5 and 3.2 of the time steps, respectively. Therefore, it was found that the periodic occurrence of the quadrant 2 and 4 are less than quadrants 3 and 1. It means that quadrants 2 and 4 occur more frequently than quadrants 1 and 3.

POISSION PROCESS

In this study, also the poission distribution was applied to the particle entrainment from the bed for the experimental test. The $X(t)$ denotes the number of isolated entrained particle at the bed, which entrained with time in a given interval $[0,t]$. With the following assumption, it can be stated that:

1. The probability of entrained particles depends only on time interval($\Delta(t)$).
2. The entrained particles at each time interval is independent.
3. The entrained particles probability is aproximately to zero if $\Delta(t)\to 0$.

With above assumptions, the poission disturbution of $X(t)$ is defined as:

$$P_n(t) = \text{prob}[X(t)=n]$$

Where, $P_n(t)$=prob[non-entrained particles at interval$[0,t]$], $\lambda t=E[X(t)]=\mu$. The constant λ reflects the intensity of the piossion process, which is, called the parameter of poission distribution. Because λ is assumed to be constant over time and the increments are independent, therefore, no concern exists for the location of the interval, thus the model XPOI(λ) is applicable for any time interval of t such as $[s,s+t]$ with $\mu=\lambda t$.

Therefore, with the application of the poission disturbution to entrained particle from the bed, the following results were obtained. Table 2 shows the entrainment intensity of particles from the bed.

Table 2 Estimation of intensity particle for four experimental tests

Experimental Test	A4-1	A4-2	A4-3	A4-4
λ%	1.96	10.75	15.85	27.45

Fig. 1,2,3 and 4 show cumulative probability of entrained particles for exprimental testes. In this study, the range of $X(t)$ was between zero and sixty events , then the ange of poission disturbution is between zero and infinity.

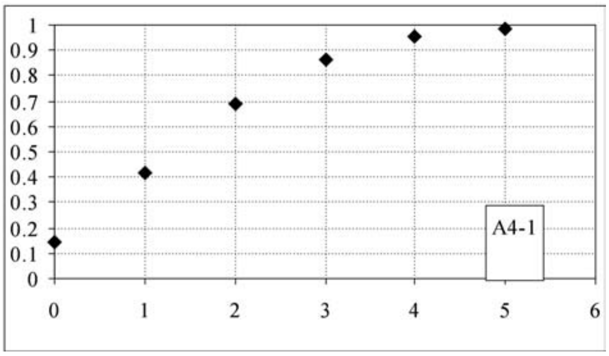

Fig. 1 Cumulative Probability of entrained Particles from the bed (Test A4-1)

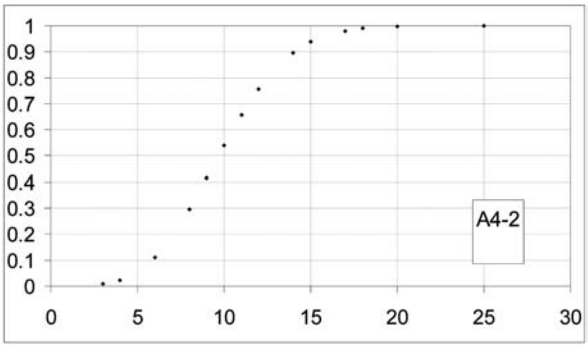

Fig. 2 Cumulative Probability of entrained Particles from the bed (Test A4-2)

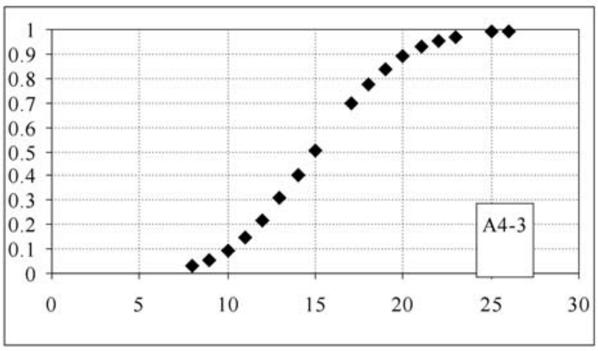

Fig. 3 Cumulative Probability of entrained

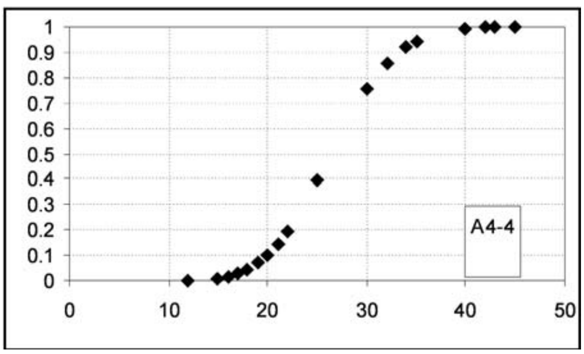

Fig. 4 Cumulative Probability of entrained

Particles from the bed (Test A4-3) Particles from the bed (Test A4-4)

6. CONCLUSION

In this study the poisson distribution was applied to the number of entrained particles from the bed and it was found that the poisson distribution well defined the process. The entrainment of sediment particles from the bed is stochastic in nature and it is strongly influenced by instantaneous shear stresses of the bursting process. Also the velocity fluctuations of bursting process were analyzed and a time series of the bursting events were produced. To find the frequency of the occurrence of the bursting events, the Markov chain stochastic model was applied to the data. It was examined which Markov chain best defines the occurrence of the bursting events. It was found that the first order Markov chain is the best model for the definition of the bursting process.

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