同济大学建校105周年学术报告会

Structural Response Uncertainty Propagation Using Evidence Theory

唐和生

结构工程与防灾研究所，土木工程学院
同济大学

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Outline

Motivations & objectives

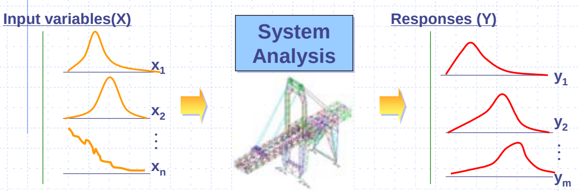
- The benefits of uncertainty quantification analysis
- Multiple types of uncertainty and propagation

Proposed methodology & Numerical example

- Engineering structural problem description in evidence theory
- Improvement of computational performance and accuracy
- Truss structure Example

Summary Remarks

Uncertainty Quantification (UQ) Analysis



Two Basic Assumptions in probabilistic analysis

- The law of large data
- Randomness of uncertain variables

Uncertainty Sources

Challenging problems are …

Various uncertainty sources

due to the restrictions of budget, resources, knowledge and time

System parameter

- Natural variability (dimensions, modulus, …)
- Imprecise statistical data from poor observations

Model form

- Faults in a conceptual and mathematical modeling
- Imperfect representation of boundary conditions

Scenario abstraction

- Unrealized & unexpected failure modes
- Undefined system behaviors

Uncertainties in a complex system

Challenging problems are …

Information

- Incompleteness Insufficiency
- Imprecise
- Inconsistent
- Interval information (w/o PDF)

Basic assumptions

in Probability Theory

- The law of large data
- Randomness

No longer valid

UQ Techniques

Parameter | Physical system Modeling | Scenario abstraction

Material properties, Loads, Geometries — Initial conditions, Model form — Failure modes

Sufficient data ? — Yes → Aleatory Uncertainty; No → Epistemic Uncertainty

Aleatory Uncertainty

Probabilistic Analysis

- MCS, ISM
- FORM, SORM
- Spectral stochastic FEM

Epistemic Uncertainty

Non-probabilistic Analysis

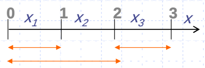
- Interval analysis
- Possibility Theory
- Evidence Theory

Evidence theory

Dempster-Shafer Theory (DST)

- **Frame of Discernment (X)**

  The set of mutually exclusive **elementary** propositions from given possible sets



$X = \{x_1, x_2, x_3\}$: Frame of discernment

$2^X = \{\varnothing, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_1\}, X$

- **Basic Belief Assignment (BBA)**

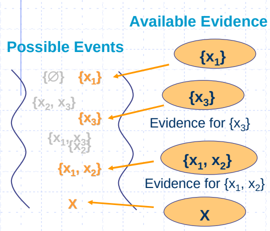
  The portion of total belief that is assigned exactly to a proposition through **basic belief assignment function** - *m*

$m : 2^X \rightarrow [0,1]$

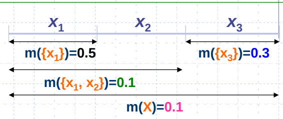
Axioms
I. $m(A) \geq 0$ for any $A \in 2^X$
II. $m(\varnothing) = 0$
III. $\sum_{A \in 2^X} m(A) = 1$

Evidence theory

Dempster-Shafer Theory (DST)



BBA structure



- **Additivity** does **not necessarily hold**
  $m(\{x_1\}) + m(\{x_2\}) \neq m(\{x_1, x_2\})$
- **Monotonicity** does **not necessarily hold**
  $m(\{x_1\}) \geq m(\{x_1, x_2\})$ even though $\{x_1\} \subset \{x_1, x_2\}$
- **It is not required** that $m(X)=1$, but $m(X) \leq 1$

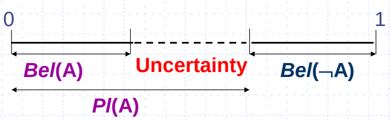
BBA ≠ Probability

Belief & Plausibility Functions

Likelihood for event *A* lies in the **interval [*Bel*(A), *Pl*(A)]**

$$Bel(A) = \sum_{C \subset A} m(C)$$

$$Pl(A) = \sum_{C \cap A \neq \varnothing} m(C)$$

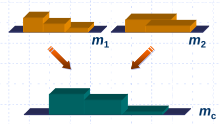


Dempster's Rule of Combining

$$m_c(A_i) = \frac{\sum_{B_j \cap C_k = A_i} m_1(B_j) m_2(C_k)}{1 - K}$$

$$K = \sum_{B_j \cap C_k = \varnothing} m_1(B_j) m_2(C_k)$$

BBA structures



Combined BBA structure

Dempster's rule disregards every contradiction

Algebraic properties : commutative and associative

Structural Problem Description in Evidence Theory

- **Epistemic parameter uncertainty**

Physical system :

Responses : $Y = [y_1, y_2, \ldots, y_n]$

Input data : $X = [x_1, x_2, \ldots, x_n]$

Structural Problem Description in Evidence Theory

- Epistemic parameter uncertainty with interval information
- Defining a **system failure set** with limit state value, Y_{fail}

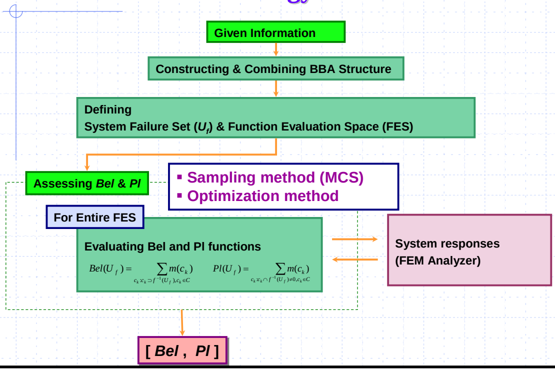
  $U_f = \{ Y : Y_{fail} - Y < 0 ,\ Y = f(c_k)$ and $c_k = [x_1, x_2, x_2, \ldots, x_n] \subset C\}$

  where, x is an uncertain parameter vector
- Resulting measurements, **Belief & Plausibility functions**

  Bounds [*Bel*, *Pl*]

$$Bel(U_f) = \sum_{c_k : c_k \supset f^{-1}(U_f), c_k \in C} m(c_k) \qquad Pl(U_f) = \sum_{c_k : c_k \cap f^{-1}(U_f) \neq 0, c_k \in C} m(c_k)$$

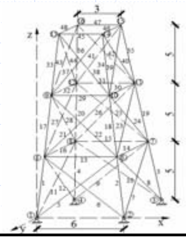
Numerical Methodology



Numerical Example

- **Epistemic parameter uncertainty**
  - Elastic modulus
  - Static load
- **Epistemic information Sources**
  - One expert
  - Interval information
- **Limit State**

$disp_{fail} = \{ disp_{tip} : | disp_{tip} | < 1m \}$



❑ Expert 1

Elastic modulus factor

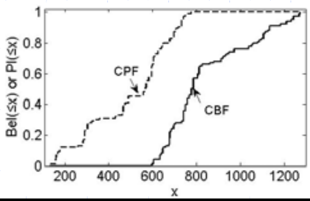
Static load factor PID BBA

	PID	BBA
E	[2.0,2.05][2.05,2.15][2.15,2.2]×10⁵MPa	0.25,0.5,0.25
F_x	[65,95][95,105][105,135]KN	0.25,0.5,0.25
F_y	[65,95][95,105][105,135]KN	0.25,0.5,0.25
F_z	[-135,-105][-105,-95][-95,-65]KN	0.25,0.5,0.25

- **Belief and Plausibility**

$Pl(disp_{fail})$	$Bel(disp_{fail})$
1.0	0.76

Complementary Cumulative Plausibility and Belief Function for the occurrence of limit tip displacement value y < *v*



Summary

- Evidence theory provides useful tool for partial and incomplete information situation.
- Evidence theory provides a Bound [*Bel, Pl*] for a uncertainty quantification problem, which has consistency with given incomplete information.

Thank you!