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量子理论新方法研究光的双缝衍射

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摘要:用量子理论新方法研究光的双缝衍射实验现象, 首先用光的量子理论计算光在缝中双缝衍射的波函数, 再由基尔霍夫定律计算光的衍射波函数, 由衍射强度正比于衍射波函数模方, 从而得到光双缝衍射强度的解析式, 把量子理论计算结果和经典电磁理论计算结果以及与实验数据三者进行比较, 发现量子理论结果与实验数据符合更好, 而经典电磁理论计算结果与实验有一定偏差. 从而说明量子理论更能精确解释光的衍射现象. 该方法还可进一步研究光的单缝、多缝以及光栅衍射的实验现象.

关键词:经典理论; 量子理论; 光双缝衍射; 基尔霍夫定律

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Photon Double-slit Diffraction with Quantum Theory Approach

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Abstract: In this paper, the light double-slit diffraction with quantum theory approach is studied. Firstly, the light wave function in slits is calculated, and the diffraction wave function is obtained with Kirchhoff's law. The diffraction intensity is proportional to the diffraction wave function da party. Futhermore, the intensity formula of light double-slit diffraction is obtained; by comparing the quantum theory calculation results and classical electromagnetic theory calculation results and the experimental date, it is found that the calculate result of quantum theory is in accordance with the experment data, but the classical electromagnetic theory calculation result with the experiment date has certain deviation. So, the quantum theory can explain the light diffraction more precisely. The proposed method can also further study the light of the single slit, many slits and grating diffraction experiment phenomena.

Key words: Classical theory; Quantum theory; Light diffraction; Kirchhoff's law

0 引言

在 19 世纪 70 年代, 麦克斯韦发展了电磁理论, 导致电磁波的发现, 并认为光也是电磁波, 即某一波段的电磁波. 光在传播过程中出现干涉、衍射现象. 如光的单缝、双缝、多缝、光栅衍射时, 以及 X 射线

在晶体中的衍射等大量光的衍射实验现象, 对这些实验的理论解释主要是经典电磁理论. 20 世纪初, 普朗克和爱因斯坦提出了光的量子理论. 从而揭示了微观客体具有波-粒二象性, 并得到大量实验的验证. 进而建立了微观客体的量子理论. 量子理论认为: 光、电子、中子、原子等都既具有波动性, 又具有

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粒子性,因而在一定条件下这些粒子可产生干涉、衍射现象.最近,我们用量子理论方法研究了电子、中子等物质波的衍射,理论计算结果与实验符合相当好.本文用量子理论新方法研究光在缝中的衍射,通过求解光的相对论波动方程,得到光在缝中波函数,通过基尔霍夫定律得到光的衍射波函数,该衍射波函数模方即几率正比于该点光衍射相对强度.经过理论推导我们也给出了光衍射强度与缝的长度、宽度、厚度、光的波长以及衍射角之间的解析关系,计算发现量子理论结果与实验数据符合较好,而经典电磁理论结果与实验有一定偏差.从而用量子理论方法能更好地解释了光衍射这一物理现象.

1 光在缝中的波函数 $\varphi(\mathbf{r}, t)$

假设缝的宽度均为 a ,长度均为为 b (见图 1).

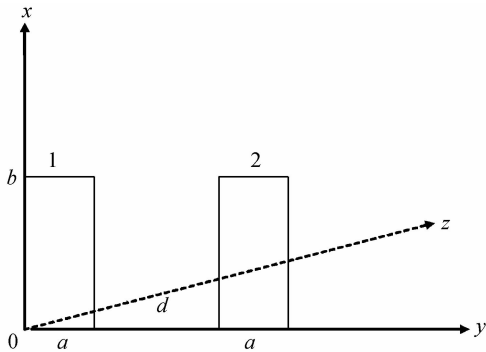


图 1 光子双缝衍射

Fig. 1 Light double-slit diffraction

取 x 轴沿缝宽方向, y 轴沿缝长方向.下面,我们就通过求解光的相对论波动方程而得到单缝中光的波函数.在 t 时刻,假设入射光沿 z 轴方向入射,波函数表示为

$$\varphi(\mathbf{r}, t) = \mathbf{A} e^{\frac{i}{\hbar}(p_z - E)t} = \sum_j \mathbf{A}_j e^{\frac{i}{\hbar}(p_z - E)t} \mathbf{e}_j = \sum_j \varphi_0 e^{-\frac{i}{\hbar} E t} \mathbf{e}_j \quad (1)$$

这里 $\varphi_0 = \mathbf{A}_j e^{\frac{i}{\hbar} p_z}$, $j = x, y, z$, \mathbf{A} 是常矢量,光的含时相对论量子波动方程为

$$i\hbar \frac{\partial}{\partial t} \varphi(\mathbf{r}, t) = c\hbar \nabla \times \varphi(\mathbf{r}, t) + V\varphi(\mathbf{r}, t) \quad (2)$$

其中, c 是光速,我们知道在单缝中,势能为

$$V(x, y, z) = \begin{cases} 0 & 0 \leq x \leq b, 0 \leq y \leq a, 0 \leq z \leq c' \\ \infty & \text{other} \end{cases} \quad (3)$$

在这里 c' 是缝的厚度,在缝中 $V=0$,光的相对论波动方程为

$$i\hbar \frac{\partial}{\partial t} \varphi(\mathbf{r}, t) = c\hbar \nabla \times \varphi(\mathbf{r}, t) \quad (4)$$

两边对时间求导得

$$\frac{\partial^2}{\partial t^2} \varphi(\mathbf{r}, t) = -c^2 [\nabla (\nabla \cdot \varphi(\mathbf{r}, t)) - \nabla^2 \varphi(\mathbf{r}, t)] \quad (5)$$

由于 $\varphi(\mathbf{r}, t) = \sqrt{\frac{\epsilon_0}{2}} (\mathbf{E}(\mathbf{r}, t) + ic\mathbf{B}(\mathbf{r}, t))$, 则有

$$\nabla \cdot \varphi(\mathbf{r}, t) = 0 \quad (6)$$

从式(5)和式(6)得到

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2\right) \varphi(\mathbf{r}, t) = 0 \quad (7)$$

式(7)与光的经典波动方程相同,但在这里,它是一个光的量子波动方程,它满足新的量子波动条件,当 $V(\mathbf{r}) \rightarrow \infty$ 时, $\varphi(\mathbf{r}, t) \rightarrow 0$ 时,有

$$\begin{cases} \varphi(0, y, z) = \varphi(b, y, z) = 0 \\ \varphi(x, 0, z) = \varphi(x, a, z) = 0 \end{cases} \quad (8)$$

而经典电磁波,以矩形波导中的电磁波为例,它的边界条件是

$$\begin{cases} E_x = E_z = 0, \frac{\partial E_x}{\partial x} = 0 & (x=0, a) \\ E_x = E_z = 0, \frac{\partial E_y}{\partial y} = 0 & (y=0, b) \end{cases} \quad (9)$$

显然式(8)和式(9)的边界条件是不同的,所以式(7)与经典电磁波动方程是不同的.

在 $\varphi(\mathbf{r}, t)$ 以一定频率变化情况下,则

$$\varphi(\mathbf{r}, t) = \varphi(\mathbf{r}) e^{-i\omega t} \quad (10)$$

把方程(10)代入方程(7),得

$$\frac{\partial^2 \varphi(\mathbf{r})}{\partial x^2} + \frac{\partial^2 \varphi(\mathbf{r})}{\partial y^2} + \frac{\partial^2 \varphi(\mathbf{r})}{\partial z^2} + \frac{4\pi^2}{\lambda^2} \varphi(\mathbf{r}) = 0 \quad (11)$$

如上所言,波函数 $\varphi(x, y, z)$ 满足边界条件

$$\varphi(0, y, z) = \varphi(b, y, z) = 0 \quad (12)$$

$$\varphi(x, 0, z) = \varphi(x, a, z) = 0 \quad (13)$$

光子波动函数 $\varphi(\mathbf{r})$ 可写为

$$\varphi(\mathbf{r}) = \varphi_x(\mathbf{r}) \mathbf{e}_x + \varphi_y(\mathbf{r}) \mathbf{e}_y + \varphi_z(\mathbf{r}) \mathbf{e}_z = \sum_{j=x, y, z} \varphi_j(\mathbf{r}) \mathbf{e}_j \quad (14)$$

把方程(14)代入方程(11)、(12)、(13),得

$$\frac{\partial^2 \varphi_j(\mathbf{r})}{\partial x^2} + \frac{\partial^2 \varphi_j(\mathbf{r})}{\partial y^2} + \frac{\partial^2 \varphi_j(\mathbf{r})}{\partial z^2} + \frac{4\pi^2}{\lambda^2} \varphi_j(\mathbf{r}) = 0 \quad (15)$$

$$\varphi_j(0, y, z) = \varphi_j(b, y, z) = 0 \quad (16)$$

$$\varphi_j(x, 0, z) = \varphi_j(x, a, z) = 0 \quad (17)$$

对式(15) $\varphi_j(\mathbf{r})$ 进行分离变量,则

$$\varphi_j(x, y, z) = X_j(x) Y_j(y) Z_j(z) \quad (18)$$

可得方程(15)的一般解为

$$\begin{aligned} \varphi_j(x, y, z) = & \sum_{mm} (D_{mmj} \sin \frac{n\pi x}{b} \sin \frac{m\pi y}{a} \cdot \\ & e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{n^2\pi^2}{b^2} - \frac{m^2\pi^2}{a^2}} z} + D'_{mmj} \sin \frac{n\pi x}{b} \sin \frac{m\pi y}{a} \cdot \\ & e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{n^2\pi^2}{b^2} - \frac{m^2\pi^2}{a^2}} z} \end{aligned} \quad (19)$$

其中第一项为透射波函数,第二项为反射波函数.由波函数在 $z=0$ 处连续

$$\varphi_0(x, y, z, t)|_{z=0} = \varphi(x, y, z, t)|_{z=0} \quad (20)$$

或

$$\varphi_{0j}(x, y, z)|_{z=0} = \varphi_j(x, y, z)|_{z=0} \quad j = x, y, z \quad (21)$$

得到

$$A_j = \sum_{nm} (D_{nmj} + D'_{nmj}) \sin \frac{n\pi x}{b} \sin \frac{m\pi y}{a} \quad (22)$$

再由波函数导数在 $z=0$ 处连续

$$\varphi'_0(x, y, z, t)|_{z=0} = \varphi'(x, y, z, t)|_{z=0} \quad (23)$$

或

$$\varphi'_{0j}(x, y, z)|_{z=0} = \varphi'_j(x, y, z)|_{z=0} \quad (24)$$

得到

$$A_j \frac{p}{\hbar} = \sum_{nm} (D_{nmj} - D'_{nmj}) \sqrt{\frac{4\pi^2}{\lambda^2} - \frac{n^2\pi^2}{b^2} - \frac{m^2\pi^2}{a^2}} \cdot \sin \frac{n\pi x}{b} \sin \frac{m\pi y}{a} \quad (25)$$

对方程(22)、(25)利用傅立叶变换,得到

$$D_{nmj} + D'_{nmj} = \frac{4}{ab} \int_0^a \int_0^b A_j \sin \frac{n\pi\xi}{b} \sin \frac{m\pi\eta}{a} d\xi d\eta = \begin{cases} \frac{16A_j}{mn\pi^2} & m, n = 1, 3, 5 \dots \\ 0 & m, n = 2, 4, 6 \dots \end{cases} \quad (26)$$

$$(D_{nmj} - D'_{nmj}) \sqrt{\frac{4\pi^2}{\lambda^2} - \frac{n^2\pi^2}{b^2} - \frac{m^2\pi^2}{a^2}} = \frac{4}{ab} \int_0^a \int_0^b A_j \frac{p}{\hbar} \sin \frac{n\pi\xi}{b} \sin \frac{m\pi\eta}{a} d\xi d\eta = \begin{cases} \frac{16A_j p}{mn\pi^2} & m, n = 1, 3, 5 \dots \\ 0 & m, n = 2, 4, 6 \dots \end{cases} \quad (27)$$

由方程(26)、(27)得到

$$D_{nmj} = \frac{8A_j}{(2m+1)(2n+1)\pi^2} (1+D) \quad (28)$$

$$D'_{nmj} = \frac{8A_j}{(2m+1)(2n+1)\pi^2} (1-D) \quad m, n = 0, 1, 2, 3 \dots \quad (29)$$

$$\text{其中 } D = \frac{p}{\hbar \sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}}}$$

把方程(28)和(29)代入方程(19)得

$$\varphi_j(x, y, z) = \sum_{nm} \frac{8A_j}{(2m+1)(2n+1)\pi^2} \sin \frac{(2n+1)\pi x}{b} \cdot \sin \frac{(2m+1)\pi y}{a} \left[(1+D) e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} + (1-D) e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} \right] \quad (30)$$

把方程(30)代入方程(10)得

$$\begin{aligned} \varphi_j(\mathbf{r}, t) = & -\frac{e^{ikR}}{4\pi R} e^{-i\omega t} \int_{s_0} e^{-ik_2 \cdot \mathbf{r}'} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{8A_j}{(2m+1)(2n+1)\pi^2} \left\{ iE e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} - \right. \\ & iF e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} + (i\mathbf{n} \cdot \mathbf{k}_2 - \frac{\mathbf{n} \cdot \mathbf{R}}{R^2}) (1+D) e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} + (1-D) \cdot \\ & \left. e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} \right\} \sin \frac{(2n+1)\pi}{b} x' \sin \frac{(2m+1)\pi}{a} y' dx' dy' \quad (36) \end{aligned}$$

$$\begin{aligned} \boldsymbol{\varphi}(x, y, z, t) = & \sum_{j=x, y, z} \boldsymbol{\varphi}_j(x, y, z, t) \mathbf{e}_j = \\ & \sum_{j=x, y, z} \sum_{nm} \sin \frac{(2n+1)\pi x}{b} \sin \frac{(2m+1)\pi y}{a} \cdot \\ & \frac{8A_j}{(2m+1)(2n+1)\pi^2} \left[(1+D) e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} + \right. \\ & \left. (1-D) e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} \right] e^{-i\omega t} \mathbf{e}_j \quad (31) \end{aligned}$$

方程(31)即为光在缝中波函数.

2 光的衍射波函数 $\Psi(\mathbf{r}, t)$

在上面,我们计算光子在缝中的波函数,下面将利用基尔霍夫公式求得光通过单缝后的衍射波函数.由基尔霍夫公式可得 $\psi_j(\mathbf{r}, t)$ 满足

$$\psi_j(\mathbf{r}, t) = -\frac{1}{4\pi s_0} \int \mathbf{n} \left[\nabla' \varphi_j + (ik - \frac{1}{r}) \frac{\mathbf{r}}{r} \varphi_j \right] ds \quad (32)$$

总的衍射波函数为

$$\boldsymbol{\Psi}(\mathbf{r}, t) = \psi_x(\mathbf{r}, t) \mathbf{e}_x + \psi_y(\mathbf{r}, t) \mathbf{e}_y + \psi_z(\mathbf{r}, t) \mathbf{e}_z = \sum_{j=x, y, z} \psi_j(\mathbf{r}, t) \mathbf{e}_j \quad (33)$$

设在单缝右边观察向 \mathbf{k}_2 方向传播的衍射波.如图2, \mathbf{r}' 是单缝中 $z=c'$ 平面上的一点(常量 c' 是单缝的厚度), P 是衍射空间的任意一点, \mathbf{n} 是垂直于单

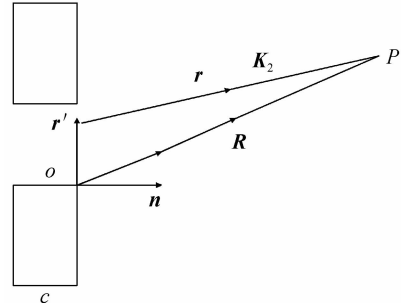


图2 光子单缝衍射

Fig. 2 Light single-slit diffraction

缝表面的单位矢量. \mathbf{k}_2 沿 \mathbf{r} 方向,且 $\mathbf{k}_2 = k \frac{\mathbf{r}}{r}$. 从图2可以得到

$$r = R - \frac{\mathbf{r}}{R} \cdot \mathbf{r}' \approx R - \frac{\mathbf{r}}{r} \cdot \mathbf{r}' = R - \frac{\mathbf{k}_2}{k} \cdot \mathbf{r}' \quad (34)$$

所以得

$$\frac{e^{ikr}}{r} = \frac{e^{ik(R - \frac{\mathbf{r}}{r} \cdot \mathbf{r}')}}{R - \frac{\mathbf{r}}{r} \cdot \mathbf{r}'} \approx \frac{e^{ikR} e^{-ik_2 \cdot \mathbf{r}'}}{R} \quad (|\mathbf{r}'| \ll R) \quad (35)$$

这里 $\mathbf{k}_2 = k \frac{\mathbf{r}}{r}$, 将方程(28)、(32)和(33)代入方程(30),得

$$\text{其中 } E = \sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} + \frac{p}{\hbar}$$

$$F = \sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} - \frac{p}{\hbar}.$$

设 \mathbf{k}_2 与 x 轴夹角为 $\frac{\pi}{2} - \alpha$, 与 y 轴夹角为 $\frac{\pi}{2} - \beta$, α 和 β 即为衍射波偏离 yz 面和 xz 面的角. 所以有

$$k_{2x} = k \sin \alpha, k_{2y} = k \sin \beta \quad (37)$$

$$\mathbf{n} \cdot \mathbf{k}_2 = k \cos \theta \quad (38)$$

这里 θ 是 \mathbf{k}_2 与 z 轴的夹角, 而且角 θ, α, β 满足

$$\cos^2 \theta + \cos^2 \left(\frac{\pi}{2} - \alpha \right) + \cos^2 \left(\frac{\pi}{2} - \beta \right) = 1 \quad (39)$$

将式(37)和式(38)代入式(36), 便得到

$$\begin{aligned} \psi_j(\mathbf{r}, t) = & -\frac{e^{ikR}}{4\pi R} e^{-i\omega t} \sum_{s_0} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{8A_j}{(2m+1)(2n+1)\pi^2} \int_0^b e^{-ik \sin \alpha x'} \sin \frac{(2n+1)\pi x'}{b} dx' \int_0^a e^{-ik \sin \beta y'} \cdot \\ & \sin \frac{(2m+1)\pi y'}{a} dy' \left\{ \left[iE + \left(ik - \frac{1}{R} \right) \sqrt{\cos^2 \alpha - \sin^2 \beta} (1+D) \right] e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} + \right. \\ & \left. \left[-iF + \left(ik - \frac{1}{R} \right) \sqrt{\cos^2 \alpha - \sin^2 \beta} (1-D) \right] e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} \right\} \end{aligned} \quad (40)$$

将式(40)代入式(33)可以得到

$$\begin{aligned} \Psi(x, y, z; t) = & -\frac{e^{ikR}}{4\pi R} e^{-i\omega t} \sum_{j=x, y, z} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{8A_j}{(2m+1)(2n+1)\pi^2} \int_0^b e^{-ik \sin \alpha x'} \sin \frac{(2n+1)\pi x'}{b} dx' \cdot \\ & \int_0^a e^{-ik \sin \beta y'} \sin \frac{(2m+1)\pi y'}{a} dy' e_j \left\{ \left[iE + \left(ik - \frac{1}{R} \right) \sqrt{\cos^2 \alpha - \sin^2 \beta} (1+D) \right] e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} + \right. \\ & \left. \left[-iF + \left(ik - \frac{1}{R} \right) \sqrt{\cos^2 \alpha - \sin^2 \beta} \cdot (1-D) \right] e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} \right\} \end{aligned} \quad (41)$$

方程(41)是在衍射区域的衍射波函数, 由波动方程我们便得到在显示屏上观察到的光的衍射强度 I 为

$$I \propto |\Psi(x, y, z; t)|^2 \quad (42)$$

3 光的双缝衍射波函数

由式(31), 我们可以知道在单缝中光的波函数

$\phi_1(x, y, z; t)$ 为

$$\begin{aligned} \phi_1(x, y, z; t) = & \sum_{j=x, y, z} \sum_{nm} \sin \frac{(2n+1)\pi x}{b} \cdot \\ & \sin \frac{(2m+1)\pi y}{a} \frac{8A_j}{(2m+1)(2n+1)\pi^2} \cdot \\ & \left[(1+D) e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} + \right. \\ & \left. (1-D) e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} \right] e^{-i\omega t} e_j \end{aligned} \quad (43)$$

通过坐标变换

$$\begin{cases} x' = x \\ y' = y - (a+d) \\ z' = z \end{cases} \quad (44)$$

得到第二个缝中光的波函数为

$$\begin{aligned} \phi_2(x, y, z; t) = & \sum_{j=x, y, z} \sum_{nm} \sin \frac{(2n+1)\pi x}{b} \cdot \\ & \sin \frac{(2m+1)\pi [y - (a+d)]}{a} \frac{8A_j}{(2m+1)(2n+1)\pi^2} \cdot \\ & \left[(1+D) e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} + \right. \\ & \left. (1-D) e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} z} \right] e^{-i\omega t} e_j \end{aligned} \quad (45)$$

由基尔霍夫定律, 和方程(41)相似, 我们能够得到在第一个缝和第二个缝中光的衍射波函数分别为

$$\begin{aligned} \psi_1(x, y, z; t) = & -\frac{e^{ikR}}{4\pi R} e^{-i\omega t} \sum_{j=x, y, z} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{8A_j}{(2m+1)(2n+1)\pi^2} \int_0^b e^{-ik \sin \alpha x'} \sin \frac{(2n+1)\pi x'}{b} dx' \cdot \\ & \int_0^a e^{-ik \sin \beta y'} \sin \frac{(2m+1)\pi y'}{a} dy' e_j \left\{ \left[iE + \left(ik - \frac{1}{R} \right) \sqrt{\cos^2 \alpha - \sin^2 \beta} \cdot (1+D) \right] e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} + \right. \\ & \left. \left[-iF + \left(ik - \frac{1}{R} \right) \sqrt{\cos^2 \alpha - \sin^2 \beta} \cdot (1-D) \right] e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} \right\} \end{aligned} \quad (46)$$

$$\begin{aligned} \psi_2(x, y, z; t) = & -\frac{e^{ikR}}{4\pi R} e^{-i\omega t} \sum_{j=x, y, z} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{8A_j}{(2m+1)(2n+1)\pi^2} \int_0^b e^{-ik \sin \alpha x'} \sin \frac{(2n+1)\pi x'}{b} dx' \cdot \\ & \int_{a+d}^{2a+d} e^{-ik \sin \beta y'} \sin \frac{(2m+1)\pi y'}{a} dy' e_j \left\{ \left[iE + \left(ik - \frac{1}{R} \right) \sqrt{\cos^2 \alpha - \sin^2 \beta} \cdot (1+D) \right] e^{i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} + \right. \\ & \left. \left[-iF + \left(ik - \frac{1}{R} \right) \sqrt{\cos^2 \alpha - \sin^2 \beta} \cdot (1-D) \right] e^{-i\sqrt{\frac{4\pi^2}{\lambda^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}} c} \right\} \end{aligned} \quad (47)$$

双缝衍射总的波函数为

$$\psi(x, y, z; t) = \psi_1(x, y, z; t) + \psi_2(x, y, z; t) \quad (48)$$

这样我们便得到在显示屏上能观察到光的衍射强度 I 为

$$I \propto |\psi(x, y, z, t)|^2 \quad (49)$$

4 结果与讨论

从方程(46)~(49)可知,光的相对衍射强度 I 是衍射角 α 和 β 的函数,文献[15]是光双缝衍射的实验,在文献[15]中,两缝的宽度均为 $a = 1.3 \times 10^{-4}$ m,两缝之间距离为 $d = 4 \times 10^{-4}$ m,缝到屏的距离 $r = 4$ m.实验用的光的波长为 $\lambda = 916 \times 10^{-9}$ m,在理论计算中,我们取与实验相同的以上参量,同时取缝长 $b = 2.0 \times 10^{-3}$ m,缝厚 $c = 8.8 \times 10^{-5}$ m,.经典电磁理论计算的双缝干涉强度公式为 $I = 4I_0 \frac{\sin^2(\pi a \sin \beta / \lambda)}{(\pi a \sin \beta / \lambda)^2} \cos^2(\pi d \sin \beta / \lambda)$.图3为理论与实验数据比较图.其中实曲线为量子理论新方法计算结果,虚线为经典电磁理论计算结果,点为实验数据.从图3可以看出,经典电磁理论计算结果对光的描述有一定偏差,主要表现在衍射强度最小值与最大值都小于实验值,因此经典电磁理论只是一种近似描述.由于光具有波粒二象性,因此,对光的干涉、衍射精确描述应该用量子理论方法,我们用量子理论方法计算的结果与实验符合更好.

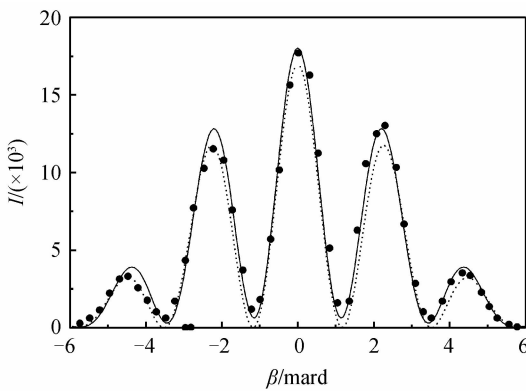


图3 理论与实验数据比较图,横坐标为衍射角,纵坐标为衍射强度

Fig.3 Theory and experiment data comparison chart, abscissa for diffraction angle, ordinate for diffraction intensity

5 结论

综上所述,我们用经典电磁理论和光的量子理论方法分别研究了光的双缝衍射.其中,量子理论方法给出了光的衍射强度与缝长、缝宽、缝厚、光的波长及衍射角之间的关系,并将理论计算结果与实验数据进行比较,我们发现量子理论方法的计算结果与实验数据符合更好,而经典电磁理论计算结果与

实验有一定偏差.因此,证明了光的干涉、衍射也是一种量子现象,更精确地描述也要用量子理论方法.同时,量子理论方法还可进一步研究光的多缝及光栅等光的衍射现象.

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