第 30 卷 第 6 期 量子电子学报 Vol.30 No.6
2013 年 11 月 CHINESE JOURNAL OF QUANTUM ELECTRONICS Nov. 2013

DOI: 10.3969/j.issn.1007-5461.2013.06.011

Characteristics of entanglement dynamics in linear and nonlinear cavity

DU Shao-jiang[1,2], *XIA Yun-jie*[2]

(1 Department of Physics and Information Engineering, Jining University, Qufu 273155, China ;
2 Shandong Provincial Key Laboratory of Laser Polarization and Information Technology,
School of Physics and Engineering, Qufu Normal University, Qufu 273165, China)

Abstract: It is significant that how to improve the property of quantum entanglement dynamics evolution by controlling the happening of entanglement sudden death. Initially entangled atoms interact with Jaynes-Cummings (J-C) model and nonlinear Jaynes-Cummings (N-J-C) model respectively. Using the method of concurrence the effect of nonlinearity and coupling of atom-field in N-J-C model and the effect of detuning were researched. It shows that entanglement sudden death appears in J-C model, however the meaningful result is that entanglement sudden death vanishes in N-J-C model by using the effect of nonlinearity and detuning in a certain condition. In addition, the atoms entanglement degree can reach to the original values.

Key words: quantum optics; controlling of entanglement sudden death; concurrence; Jaynes-Cummings model; nonlinear Jaynes-Cummings model

CLC number : O431.2 **Document code :** A **Article number:** 1007-5461(2013)06-0710-06

线性及非线性介质腔中量子纠缠动力学特征

杜少将 [1,2], 夏云杰 [2]

(1 济宁学院物理与信息工程系, 山东 曲阜 273155 ;
2 曲阜师范大学物理工程学院，山东省激光偏光及信息技术重点实验室, 山东 曲阜 273165)

摘 要： 如何抑制纠缠突然死亡现象的发生对提高量子纠缠动力学演化性能具有极大的意义，初始纠缠原子分别与非线性 N-J-C 模型及 J-C 模型进行相互作用，运用共生纠缠的度量方法分析非线性、耦合强度以及失谐量对纠缠原子动力学演化的影响，寻找避免纠缠突然死亡发生条件。在 J-C 模型中原子在纠缠演化中发生纠缠突然死亡现象，然而在 N-J-C 模型中利用介质的非线性和失谐量的影响可以避免纠缠突然死亡的发生，而且一定程度上几乎可以恢复到原子间纠缠的初始值。

关键词： 量子光学；纠缠突然死亡的控制；共生纠缠度； J-C 模型； N-J-C 模型

1 Introduction

Quantum entanglement has emerged as a key resource for all kinds of quantum progresses and has attracted more and more attention in recent years. The unavoidable coupling of a quantum system with its surrounding environment destroys the entanglement in the system. The effect of decoherence becomes critically detrimental in cases when the system's entanglement terminates abruptly, a phenomenon called finite-time disentanglement or entanglement sudden death. So far, many methods have been proposed to protect entanglement from decoherence. Several models are discussed in order to research the interaction between

Foundation item: Foundation of National Natural Science (61178012,11147019)
作者简介： 杜少将 (1975 -), 山东泗水人，硕士，讲师，主要从事量子光学与量子信息研究。 **E-mail:** dsjsd@126.com
Received date: 2013–03–18; **Revised date:** 2013–05–27

field and atoms. Interaction of a single mode of electromagnetic field with a two-level atom is the simplest problem introduced by Jaynes and Cummings[1]. Buck-Sukumar model is very interesting as it can be written as a combination of generators of SU(1,1) algebra[2~4]. Kerr-nonlinearity model has an effective Hamiltonian for a system in which the electromagnetic field mode is excited in a Kerr medium[5~7,17,21]. Dicke-Tavis-Cummings model researches the interaction between field and a group of two-level atoms[8,9,15,20]. Nonlinear Jaynes-Cummings model offers enormous possibilities to tailor the form of atom-field interaction[10,11].

In our work, the objective is to study the dynamics of entangled qubits, to prolong the time of entanglement of qubits, and to prevent the happening of entanglement sudden death. The interaction of a two-level atom with a single mode of electromagnetic field has already been discussed by Jaynes and Cummings, which called Jaynes-Cummings (J-C)model[1]. Subsequently another nonlinear Jaynes-Cummings (N-J-C)model was presented to discuss the interaction of qubits and fields. We'll investigate the evolution of two entangled atoms in the two different models and look for the difference between the two models.

The paper is organized as follows. In Sec. II, we investigate the dynamics of entangled atoms in J-C model and calculate the concurrence of entangled atoms in N-J-C model in Sec. III and compare the difference between them. A physical implementation of those models and discussion of the results are given in Sec.IV.

2 The dynamics of entangled atoms in J-C model

A model for the interaction of a single mode of electromagnetic field with a two-level atom is the simplest problem introduced by Jaynes-Cummings. In this section we analysis one of the entangled two atoms interacting with a single mode field of cavity. Initially state of the two-atom is produced in a Bell type state

$$|\Phi(0)\rangle_{\mathrm{AB}} = (\alpha|0,1\rangle + \beta|1,0\rangle)_{\mathrm{AB}}, \tag{1}$$

in which $|\alpha|^2+|\beta|^2=1$, the field in the cavity is in a number state $|n\rangle$, the state of total system can be written as

$$|\Phi(0)\rangle_{\mathrm{AB}} = (\alpha|0,1\rangle + \beta|1,0\rangle)_{\mathrm{AB}} \otimes |n\rangle, \tag{2}$$

and the Hamiltonian is given as

$$H = H_0 + H_{\mathrm{I}} = \frac{1}{2}\sum_{i=1}^{2}\omega_i\sigma_i^z + \omega a^+a + g(\sigma_{\mathrm{A}}^+a + \sigma_{\mathrm{A}}^-a^+), \tag{3}$$

ω_1, ω_2 are the essential transition frequencies of atoms and ω is the frequency of light field, g is coupling constant of atom and light field. $\sigma^z, \sigma^+, \sigma^-$ is spin operator of atom that $\sigma^z = |e\rangle\langle e| - |g\rangle\langle g|, \sigma^+ = |e\rangle\langle g|, \sigma^- = |g\rangle\langle e|$. $|e\rangle, |g\rangle$ is excited state and base state of atom.

At time t in the interaction picture, the state of system is depicted as

$$|\Phi(t)\rangle = C_1(t)|e_{\mathrm{A}}, e_{\mathrm{B}}, n-1\rangle + C_2(t)|e_{\mathrm{A}}, g_{\mathrm{B}}, n\rangle + C_3(t)|g_{\mathrm{A}}, e_{\mathrm{B}}, n\rangle + C_4(t)|g_{\mathrm{A}}, g_{\mathrm{B}}, n+1\rangle. \tag{4}$$

Using the Schrödinger equation of system $\mathrm{i}\frac{\partial|\Phi(t)\rangle}{\partial t} = H_{\mathrm{I}}|\Phi(t)\rangle$, we can get the concurrence of X-type state[12,13]

$$C(\rho_{\mathrm{AB}}) = 2\max\{0, |u| - \sqrt{xw}, |v| - \sqrt{yz}\}. \tag{5}$$

The letters x, y, z, w, u, v are the elements of density matrix. It represent as

$$\rho_{\mathrm{AB}} = \begin{pmatrix} x & 0 & 0 & v \\ 0 & y & u & 0 \\ 0 & u^* & z & 0 \\ v^* & 0 & 0 & w \end{pmatrix}.$$

3 The dynamics of entangled atoms in N-J-C model

The nonlinearity Jaynes-Cummings (N-J-C) model is depicted as follows. By including the motion of atom in the external field, the coupling is made position dependent. This offers enormous possibilities to tailor the form of atom-field interaction[6,14]. The general form for the Hamiltonian is

$$H_{\mathrm{NL}} = \omega a^+ a + \frac{1}{2}\omega_0\sigma_z + \chi a^{+2}a^2 + g(f(a^+a)a^m + adjoint), \tag{6}$$

here $f(a^+a)$ is an operator-valued function of the number operator a^+a and m is an integer. When $f(a^+a) = I$, the Hamiltonian models the interaction between a field in a Kerr medium and a two-level atom. The quadratic term $a^{+2}a^2$ accounts for the Kerr effect. Choosing $f(a^+a) = \sqrt{1+ka^+a}$, where $0 \le k \le 1$, then the Hamiltonian can be written as

$$H_{\mathrm{S}} = K^+K + \frac{\gamma}{2}\sigma_z + \lambda(K^+\sigma_- + K\sigma_+). \tag{7}$$

The suffix " S " indicates that the Hamiltonian describes the interaction of a single-mode of a cavity field with a two-level atom. The operator $K = \sqrt{1+ka^+aa}$ and $K^+ = a^+\sqrt{1+ka^+a}$ are deformed annihilation and creation operator respectively. $\chi = k\omega, g = \lambda\omega$ and $\gamma = \omega_0/\omega$. When $\gamma = 1$, the atom and cavity field are in resonance. Conveniently we set $\omega = 1$.

Our purpose is to study the dynamics of one of two entangled atoms interacting with a single mode of electromagnetic field. Let one of the entangled two atoms A and B interacting with the N-J-C mode. Initially state of the two-atom is expressed the same as equation (1). Subsequently we investigate the evolution of the entanglement of the two atoms.

Using the Schrödinger equation and combining equation (1),(2),(4),(5) and (7), we can get the concurrence of the atom and field

$$C = \frac{2\alpha\beta}{\Lambda\Xi}\Bigg\{\frac{\mu\sqrt{1+n}\sqrt{n(1+kn)}}{\sqrt{1+k+kn}}\sqrt{1-\cos(\Lambda t)}\sqrt{1-\cos\left[\frac{t}{2}(1+2\Xi)\right]}+ \\ \sqrt{1+4k^2n^2+\mu(n+1)-4kn(\gamma-1)-2\gamma+\gamma^2+\mu(n+1)\cos(\Lambda t)}+ \\ \sqrt{1+4k^2(n-1)^2+v+4k(n+\gamma-n\gamma-1)-2\gamma+\gamma^2+v\cos\left[\frac{t}{2}(1+2\Xi)\right]}\Bigg\}, \tag{8}$$

where $\mu = 2g^2\sqrt{1+kn}\sqrt{1+k+kn}, \quad v = 2g^2\sqrt{1+k(n+1)}\sqrt{n}\sqrt{n(1+kn)},$

$$\Lambda = \sqrt{4k^2n^2 + 4g^2(1+n)\sqrt{1+kn}\sqrt{1+k+kn} - 4kn(\gamma-1) + (\gamma-1)^2},$$

$$\Xi = \sqrt{4k^2(n-1)^2 + 4g^2\sqrt{1+k(n-1)}\sqrt{n}\sqrt{n(1+kn)} + (\gamma-1)^2 + 4k(n+\gamma-n\gamma-1)}.$$

4 Theoretical analysis

4.1 Dynamics of entanglement under the condition of resonance

We set $\gamma = 1$ and provide that the initial state of the atom and field are in maximum entanglement $\alpha = \sqrt{2}/2$.

1) Vacuum cavity field

The phenomenon of entanglement sudden death appears in J-C model (Fig.1 (a)) but it vanishes in N-J-C model (Fig.1 (b)) when the cavity field is in vacuum state under the condition of resonance between atom and field. It is to say that N-J-C model can make entanglement sudden birth in the region of sudden death of J-C model (Fig.1 (c)). Furthermore we investigate that the nonlinear coefficient has little effect to the concurrence under vacuum state.

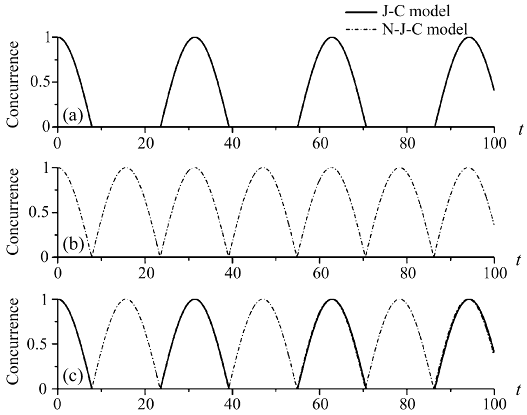

Fig.1 The evolution of entanglement in J-C model and N-J-C model under vacuum state (a) $n=0, g=0.2$, (b) $n=0, g=0.2, k=0.01$, (c) Overlapping of (a) and (b)

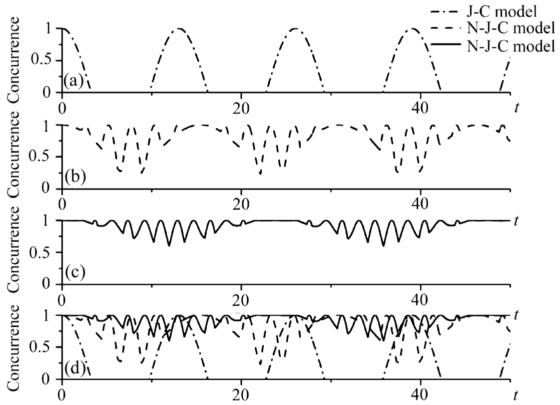

Fig.2 The evolution of entanglement in J-C model and N-J-C model under number state (a) $n=1, g=0.2$, (b) $n=5, g=0.2, k=0.01$, (c) $n=5, g=0.2, k=0.1$, (d) Overlapping of (a), (b), (c)

2) Number state of cavity field

When the cavity field is in number state, the phenomenon of entanglement sudden death also appears and the regions of it increase (Fig.2 (a)) comparing with the condition of vacuum state of J-C model (as it is shown in Fig.1(a)). Otherwise the phenomenon of entanglement sudden death disappears in the nonlinear Jaynes-Cummings model (Fig.2(b)). In addition, with the increasing value of n the concurrence is increasing. We also can see that with the increasing coefficient of the entanglement evolutes in high values (Fig.2 (c), (d)).

4.2 The evolution of entanglement under the condition of detuning

1) Vacuum state of cavity field

It is the same as shown in Fig.1(b) that when the atom evolutes in vacuum state of cavity with nonlinearity medium under the condition of resonance of atom and cavity field (Fig.3(a)). The atoms entanglement will increase when there is detuning between atom and cavity field (Fig.3(b)). With the increasing detuning parameter the concurrence between atom and field is increasing in N-J-C model (Fig.3(b),(c),(d)). Obviously the entanglement degree can also reach high value under the condition of high detuning coefficient.

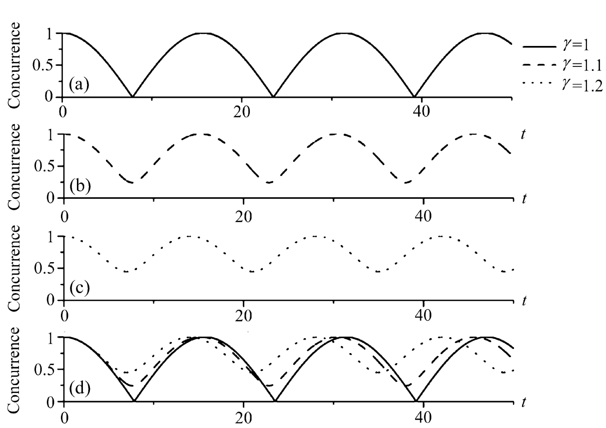

Fig.3 The evolution of entanglement in N-J-C model under the condition of detuning ($k=0.01, n=0, g=0.2$)

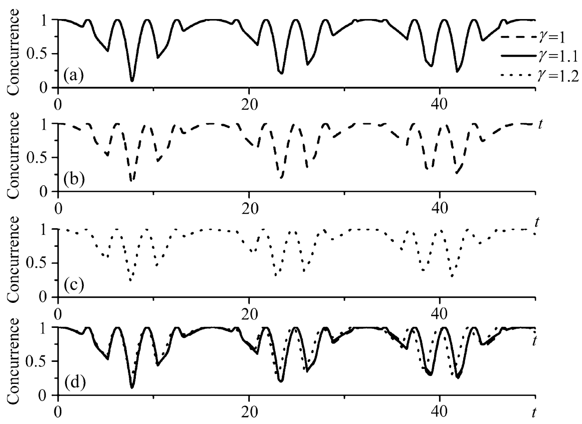

Fig.4 The evolution of entanglement in N-J-C model with number state of cavity under the condition of detuning ($k=0.01, n=3, g=0.2$)

2) Number state of cavity field

When the cavity field is in number state, it's found that the concurrence of the entanglement changes in a little region with the increasing detuning parameter, but it can also avoid happening of the entanglement sudden death under this condition. As it is shown in Fig.4.

4.3 The effect of nonlinearity to the entanglement

Entanglement sudden death can be prevented to happen with nonlinearity of cavity even the atom and cavity field are in resonance (Fig.5(a)). We can also find that the concurrence of the entanglement changes rapidly with the increasing nonlinearity parameter when the cavity field is in number state (Fig.5(b), (c)). With the increasing nonlinearity parameter the entanglement degree of atoms can almost reach the original value of concurrence (Fig.5(d)).

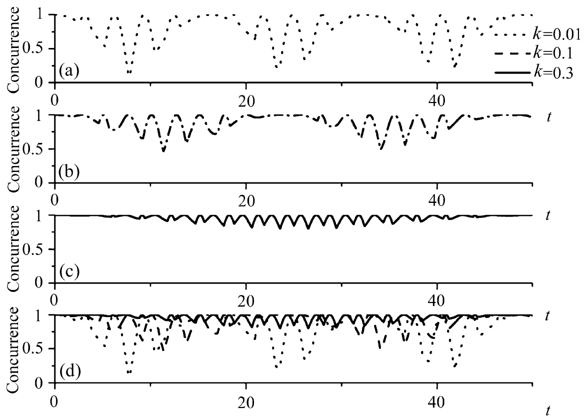

Fig.5 The effect of nonlinearity to the entanglement in N-J-C model ($n = 3, g = 0.2, \gamma = 1$)

5 Summary

Entanglement is an important resource in quantum communication and quantum computation, but the phenomenon of entanglement sudden death is harmful to them. We investigate the typical entanglement state of Bell-type state evolution in the usual J-C model and N-J-C model. Entanglement sudden death appears constantly in J-C model. However the phenomenon can vanish in nonlinear Jaynes-Cummings model. Furthermore we use a new Hamiltonian to research the dynamics of the entanglement in N-J-C model under different conditions and find that we can get high entanglement degree under certain conditions. The result shows that we can use N-J-C model to restore the quantum entanglement and in the following we will research the damped cavity interacting with the reservoir and study the quantum correlation transfer among the atoms, cavity and reservoir.

Reference:

[1] Jaynes E T, Cummings F W. Comparison of quantum and semiclassical radiation theories with application to the beam maser [J]. *Proc. IEEE,* 1963, 51(1): 89-109.

[2] Buck B, Sukumar C V. Exactly soluble model of atom-phonon coupling showing periodic decay and revival [J]. *Phys. Lett. A,* 1981, 81(2-3): 132-135.

[3] Buzek V. Jaynes-Cummings model with intensity-dependent coupling interacting with Holstein-Primakoff SU(1,1) coherent state [J]. *Phys. Rev. A,* 1989, 39(6): 3196-3199.

[4] Gerry C C. Two-photon Jaynes-Cummings model interacting with the squeezed vacuum [J]. *Phys. Rev. A,* 1988, 37(7): 2683-2686.

[5] Werner M J, Risken H. Quasiprobability distributions for the cavity-damped Jaynes-Cummings model with an additional Kerr medium [J]. *Phys. Rev. A,* 1991, 44(7): 4623-4632.

[6] Gora P, Jedrzejek C. Nonlinear Jaynes-Cummings model [J]. *Phys. Rev. A,* 1992, 45(9): 6816-6828.

[7] Joshi A, Puri R R. Dynamical evolution of the two-photon Jaynes-Cummings model in a Kerr-like medium [J]. *Phys. Rev. A,* 1992, 45(7): 5056-5060.

[8] Dicke R H. Coherence in spontaneous radiation processes [J]. *Phys. Rev.,* 1954, 93(1): 99-110.

[9] Tavis M, Cummings F W. Exact Solution for an N-molecule-radiation-field Hamiltonian [J]. *Phys. Rev.,* 1968, 170(2): 379-384.

[10] Vogel W, de Matos Filho. Nonlinear Jaynes-Cummings dynamics of a trapped ion [J]. *Phys. Rev. A*, 1995, 52(5): 4214-4217.

[11] Liu J, Wang Y. Motion-quantized Jaynes-Cummings models with an arbitrary intensity-dependent medium [J]. *Phys. Rev. A,* 1996, 54(3): 2326-2333.

[12] Wootters W K. Entanglement of formation of an arbitrary state of two qubits [J]. *Phys. Rev. Lett.,* 1998, 80(10): 2245-2248.

[13] Yu T, Eberly J H. Sudden death of entanglement [J]. *Science,* 2009, 323(5914): 598-601.

[14] Wang X, Sanders B C. Multipartite entangled coherent states [J]. *Phys. Rev. A,* 2001, 65 (1): 012303-012309.

[15] Li Xing, Cui Wenkai, Zhang Yingjie, Evolutional quantum entanglement character of a classical strongly driven cavity QED system [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2013, 30(2): 186-191 (in Chinese).

[16] Shi Peng, Li Jianjian, Chen Libo, *et al.* Dynamics of interaction between single photons and cavity-quantum dot system [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2012, 29(2): 165-170 (in Chinese).

[17] Wang Jianrong, Lai Yunzhong, Li Kun. Influence of detuning on entanglement evolution of cascade three-level atom in Kerr medium [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2011, 28(4): 434-438 (in Chinese).

[18] He Daiguo, Dong Yuli, Fang Jianxing, *et al.* Entangled coherent states: Generation [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2011, 28(4): 407-413 (in Chinese).

[19] Hu Ji, Fang Jianxin, Qian Li, *et al.* Thermal entanglement of Ising model with Dzyaloshinskii-Moriya interaction in an inhomogeneous magnetic field [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2011, 28(3): 329-334 (in Chinese).

[20] Bao Li, Sachuerfu, Wu Shumei. Quantum entanglement of the binomial field interacting with the moving atoms in the multiphoton Tavis-Cummings model [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2010, 27(5): 580-585 (in Chinese).

[21] Wang Qingshi, Guo Yunxiang, Lai Yunzhong. Quantum entanglement of two-mode SU(1,1) coherent states with three-level atoms in a Kerr-like-medium [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2009, 26(5): 577-584 (in Chinese).