NBER WORKING PAPER SERIES

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Working Paper 19667
http://www.nber.org/papers/w19667

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2013

We thank Pat Kehoe, Sam Kortum, Ben Moll, John Leahy, and Mike Waugh for their comments. The views expressed herein are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Chicago or the Federal Reserve System. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

At least one co-author has disclosed a financial relationship of potential relevance for this research. Further information is available online at http://www.nber.org/papers/w19667.ack

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NBER Working Paper No. 19667
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JEL No. F1,O11,O19,O33

ABSTRACT

We provide a theoretical description of a process that is capable of generating growth and income convergence among economies, and where freer trade has persistent, positive effects on productivity, beyond the standard efficiency gains due to reallocation effects. We add to a standard Ricardian model a theory of endogenous growth where the engine of growth is the flow of ideas. Ideas are assumed to diffuse by random meetings where people get new ideas by learning from the people they do business with or compete with. Trade then has a selection effect of putting domestic producers in contact with the most efficient foreign and domestic producers. We analyze the way that trade in goods, and impediments to it, affect this diffusion. We find that exclusion of a country from trade reduces productivity growth, with large long-term effects. Smaller trade costs have moderate effects on productivity.

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An online appendix is available at:
http://www.nber.org/data-appendix/w19667

The progress of a society is all the more rapid in proportion
as it is more completely subjected to external influences.
— Henri Pirenne

1 Introduction

In this paper we provide a theoretical description of a process of endogenous technology diffusion which we use to study the effects of trade barriers on productivity. Our theory complements the classical insights from international trade theories, in which gains from trade are due to re-allocation of resources using the same technology. Yet, we provide a model in which, as is widely and reasonably believed, trade serves as a vehicle for technology diffusion.[1]

Our starting point is a static trade model. For concreteness we follow the framework in Eaton and Kortum (2002) and Alvarez and Lucas (2007), EK-AL for short. In this model (as in many others) freer trade replaces inefficient domestic producers with more efficient foreign producers. We add to this familiar, static effect a theory of endogenous growth in which people get new, production-related ideas by learning from the people they do business with or compete with. Trade then has a selection effect of putting domestic producers in contact with the most efficient (subject to trade costs) foreign and domestic producers. The identification and analysis of these selection and learning effects is the new contribution of the paper.

Though constructed from familiar components, our model has a complicated, somewhat novel structure, and it will be helpful to introduce enough notation to describe this structure before outlining the rest of the paper. There are n countries, $i = 1, ..., n$, with given populations L_i and given iceberg trade costs, κ_{ij}. There are many goods produced in each country. The productivity of any good produced in i will be modeled as a draw from a country-specific probability distribution, defined by its cdf

$$F_i(z) = \Pr\{\text{productivity in } i \text{ of good drawn at random } \leq z\}.$$

We treat populations and trade costs as parameters and analyze the dynamics of the technology profiles $F = (F_1, ..., F_n)$ that serve as the state variables of the model. There are two steps in this analysis.

[1] It is certainly not the *only* vehicle: Think of the diffusion of nuclear weapons capabilities.

Given a profile F together with populations and trade costs we define a static competitive equilibrium for the world economy. We use the static model of international trade to determine the way a given technology profile F defines a pattern of world trade, including listings of which sellers in any country are domestic producers or exporters from abroad.

The second step in the analysis is based on a model of technology diffusion that involves stochastic meetings of individual people—we call them *product managers*—who exchange production-related ideas. We use a variation on the Kortum (1997) model, as developed in Alvarez et al. (2008). In this diffusion model, product managers in country i meet managers from some *source distribution* G_i at a given rate α_i and improve their own knowledge whenever such meetings put them in contact with someone who knows more than they do. In our application, this source distribution G_i is the technology profile of the set of sellers of any good who are active in country i, as determined by the trade theory applied in step 1. Under autarchy, then, the source distribution is simply the distribution F_i of domestic producers.[2] Trade improves on this source distribution by replacing some inefficient domestic sellers with more efficient foreigners, replacing F_i with a distribution G_i that stochastically dominates it, at least for high productivities. It is this selection effect that provides the link between trade volumes and productivity growth that we are seeking.

Technically, trade theory provides a map from a technology profile F to a profile $G = (G_1, ..., G_n)$ of source distributions. The diffusion model gives us a map from each pair (F_i, G_i) into a rate of change $\partial F_i(x,t)/\partial t$. Combining these two steps yields a law of motion for the technology profile F of all n countries together.

The organization of the rest of the paper is as follows. Section 2 introduces our model of technological change in the context of a closed economy, which will be later reinterpreted as a model of the entire world. For this case we present a complete characterization of the dynamics of a single economy that introduces many features that will be important in understanding the more general case. Section 3 characterizes the competitive static trade theory that maps an arbitrary technology profile F into a pattern of world trade. Section 4 then integrates the dynamics of technological change and static equilibrium implied by trade theory. We characterize the balanced growth path for a world economy and full dynamics of the right tail of the productivity distributions under constant trade costs and populations. This section includes a

[2]Kortum (1997) calls this distribution the technology frontier.

characterization of cases where trade costs have substantial effects on the stationary distribution, relative income, and the volume of trade, relative to the static trade theory of EK-AL. It also gives results for the case of costless trade. As we could predict on the basis of the static trade theory alone, a full analytical characterization of the dynamics in the general case is not a possibility, so we continue with numerical results.

In Section 5 we carry out some quantitative explorations to illustrate the effects of trade costs on income levels and growth rates. We calculate equilibrium paths for a symmetric world economy under different trade paths and compare the static and dynamic effects of tariff reductions in this context. We then consider catch-up growth when a small, poor, open economy is introduced into the otherwise symmetric world. These results are illustrated graphically. Section 6 provides a brief discussion of some substantive conclusions suggested by these exercises and of directions for future work that they suggest. Finally, in an Online Appendix we outlines a version of the model with Bertrand competition, and show numerical results for this alternative model.

Before continuing, we present a brief summary of the main results in the paper, and an overview of the related literature.

Preview of the Results This section review the main theoretical results, organizing them conceptually rather than in the order they are presented in the paper.

The initial conditions are given by the distribution of productivities $F_i(\cdot, 0)$. These govern all future behavior in our deterministic model. Their tail behavior can be characterized by level and curvature parameters, λ_i and θ_i respectively. In particular, θ_i is a measure of the concentration of firms in country i with very high productivities. Trade among countries leads to the immediate convergence across countries of the curvature of the tail to the common value θ, the maximum of the θ_i values in all n economies. Despite the continuous evolution of the distributions F_i through time this common tail parameter θ remains constant. [Propositions 1, 2, 7 and 8]

Under natural assumptions, the distributions of productivities, consumption and GDP converge to a unique balanced growth path with a common growth rate ν. The rate ν is given by the product of the sum of the countries' meeting rates, $\sum_i \alpha_i$ and the common tail parameter θ. Increases in the search efforts a_i or in the concentration θ of high productivities both stimulate faster growth. The model features a form of scale effect on growth. Adding more countries to the world in a way that enlarges the initial set of best practices increases the world growth rate, while arbitrarily partitioning a country into subregions leaves everything in the world unaltered. [Proposition 1, 2, 7 and discussion in Appendix C]

As with the case of the curvature parameter, the paths of the level parameters $\lambda_i(t)$ are independent of trade costs, as long as they are finite. Trade costs do not affect the diffusion of extremely productive technologies—which are the ones determining tail behavior—since for them productivity over-rides cost considerations. But in general trade costs do affect the diffusion of technologies, and higher trade costs imply that some more efficient foreign producers are replaced by inefficient domestic producers. This reduces per-capita income levels. [Propositions 8, 10 and 11, and figures 2 and 3.]

Both the effects of trade costs on output and the volume of trade depend on the elasticity of substitution across products, η. This parameter determines the weight that is given to goods from the left tail of the distribution of productivity. As η increases towards a critical value, expenditures become concentrated on goods with higher productivities. In this case, the behavior of trade volume and relative output on the model approaches the EK-AL Frechet case. [Proposition 9]

Our model has considerable flexibility because equilibrium wages are the only feature of the static trade theory that is relevant in determining the distribution of sellers in each country. No other feature of the equilibrium trade model is required to determine the diffusion of productivity. In particular, as we show in the Online Appendix, one can easily change the trade model from a competitive equilibrium to a Bertrand competition and retain tractability.

Related Literature We build on previous work on both trade and growth. We consider both perfect competitive setups from Eaton and Kortum (2002) and Alvarez and Lucas (2007), and the case with Bertrand competition as in (Bernard et al., 2003), Arkolakis et al. (2012), Holmes et al. (2012). We differ from these papers in that we allow for a more general distribution of productivities.

Our work is closely related to a number of papers on endogenous growth theory. Kortum (1997) considers a model of a closed economy in which innovators spend resources to draw ideas from an exogenous distribution of potential technologies. Unlike our model, innovators do not learn form other producers, but instead they learn form a set of exogenous ideas not embodied in goods, and thus there are no external effects. Yet the mathematical setup from which innovators learn best practices from disembodied exogenous ideas is similar to the one that producers in our set up use to learn from ideas currently embodied in goods. The derived stationary distribution of productivities is Frechet, but in their model there is no long-run growth unless there is population growth. We abstract from innovation, but allow for the diffusion of existing technologies across sectors and countries, and obtain a model with endogeous growth.

Luttmer (2012) extends his work on growth driven by innovators with heterogenous productivities and imitation by entrants. Luttmer (2007) considers imitation of incumbants, which is closely related to our mechanism for growth. His interest is to understand the contributions of both type of mechanisms to a balanced growth path. This is a broad topic that has been studied extensively. An important early contribution is Jovanovic and MacDonald (1994).

Weitzman (1998) gives a mathematical description of a process where the engine of growth is the repeated combination of pairs of original ideas from different areas of knowledge. Our modeling of diffusion, at that general level, is guided by the same mechanism: ideas embodied in different goods are pairwise combined to produce newer ideas. Nevertheless our specification of ideas is closer mathematically to the one in Kortum (1997), as extended by Alvarez et al. (2008).

Jovanovic and Rob (1989) have a model with a simpler demand side, where new ideas (productivities) are created by random meeting of existing ideas, but they allow for a richer set of possibilities after the meeting of two potential producers. In their model, potential producers can either implement their ideas or engage in costly search for a random meeting of a another holder of an unimplemented idea. Once ideas are implemented, they are no longer available to be combined with other ideas (implemented or not). Another difference in their outcome is that the ideas that are being recombined are the relatively bad ones, since the goods ones are implemented. Thus the initial set of ideas has an decreasing impact in the creating of new ideas as as time goes by. Furthermore their model, as in Kortum (1997) features an exogenous arrival of new ideas. Again, relative to our work, they abstract from trade.

A related problem is studied by Lucas and Moll (2011), who consider the problem of an individual which can produce or search for new ideas with variable intensity i.e in terms of the object of this model they select $\alpha(t) \in [0, 1]$, which is interpreted as fraction of time devoted to search for new ideas. The process of search is similar to the one in this paper. This allows the authors to study aggregate economic growth and cross sectional individual income differences. Perla and Tonetti (2012) study a similar problem, but there formulation is closer to a standard search model.

We relate to a broader literature that examines the connection between growth and trade, both theoretically and empirically.

Grossman and Helpman have several theoretical papers on growth and trade. The one that is closest to ours is Helpman and Grossman (1991). They consider a small open economy where reserchers develop new varieties of intermediate inputs. Technology is transferred from the rest of the world as an external effect. The pace of technology transfer is assumed to be proportional to the volume of trade. Their model abstracts from the selection effect which is at the core of our model, since the transmission depends on aggregate outcomes and affects all the entrants in the same way.

There is a larger empirical literature studying the relationship between trade, growth and development. On balance, they find relationships between trade flows, domestic and foreign innovation, and TFP (Coe and Helpman, 1995; Coe et al., 1997; Acharya and Keller, 2009). See Keller (2004) and Keller (2008) for reviews of this literature.

2 Technology Diffusion in a Closed Economy

We begin with a description of technology diffusion and growth in a closed economy. Consumers have identical preferences over a $[0,1]$ continuum of goods. We use $c(s)$ to denote the consumption of an agent of each of the $s \in [0,1]$ goods for each period t. There is no technology to transfer goods between periods. The period t utility function is given by

$$C = \left[\int_0^1 c(s)^{1-1/\eta} ds \right]^{\eta/(\eta-1)},$$

so goods enter in a symmetrical and exchangeable way. Each consumer is endowed with one unit of labor, which he supplies inelastically.

Each good s can be produced by many producers, each using the same labor-only, linear technology

$$y(s) = z(s) l(s) \tag{1}$$

where $l(s)$ is the labor input and $z(s)$ is the productivity associated with good s. Given the set-up, it is natural to assume that all the identical producers of good s behave competitively, an assumption we will maintain for the analysis of the paper.[3]

Using the symmetry of the utility function and the assumed competitive behavior we can group goods by their productivity z and write the time t utility as

$$C(t) = \left[\int_{\mathbb{R}_+} c(z)^{1-1/\eta} \; f(z,t) \; dz \right]^{\eta/(\eta-1)}, \tag{2}$$

where $c(z)$ is the consumption of any good s that has productivity z and $f(\cdot, t)$ is the productivity density. We assume that f is continuous. We use $F(z,t)$ for the cdf of productivity so that the productivity density is $f(z,t) = \partial F(z,t)/\partial z$.

In a competitive equilibrium the price of any good z will be $p(z) = w/z$ and the ideal price index for the economy at date t is

$$p(t) = \left[\int_{\mathbb{R}_+} p(z)^{1-1/\eta} f(z,t) dz \right]^{\eta/(\eta-1)}. \tag{3}$$

Real per capita GDP $y(t)$ equals the real wage $w/p(t)$ or

$$y(t) = \left[\int_{\mathbb{R}_+} z^{-1+1/\eta} f(z,t) dz \right]^{-\eta/(\eta-1)}, \tag{4}$$

provided the integral on the right converges.

In our model the analysis of the closed economy becomes a study of the evolution of the productivity distribution $F(z,t)$: a process of technological diffusion. We model diffusion as a process of search and matching involving the product managers of each of the $s \in [0,1]$ goods. We treat search as a costless activity, a by-product of production. We assume that managers interact with each other and exchange production-related ideas. When a manager of any good with productivity z meets a manager of any other good with productivity $z' > z$ adopts z' for the production of his own good.[4] After such a meeting the new technology z' is instantaneously diffused to all producers of the same good, thus keeping all the producers of the same good homogenous. We assume that entire set of managers of any single good has a total of

[3] In the Online Appendix we consider the case of Bertrand competition.

[4] Perhaps a more descriptive, yet less tractable model will distinguish between goods that are similar, in terms of how transferable is the technology.

α meetings per unit of time. While we refer to this process as technology *diffusion*, it might as well be called *innovation*, since the more advanced technology used for one good are adapted to a different good.[5]

Next we give a mathematical description of this process. To motivate a law of motion for the productivity distribution $F(z,t)$, we describe the discrete change between t and $t+\Delta$, and then derive its continuous-time limit. For a given level of the productivity z at date t, we assume that

$$\begin{aligned} F(z,t+\Delta) &= \Pr\{\text{productivity drawn at random } < z \text{ at } t+\Delta\} \\ &= \Pr\{\text{productivity} < z \text{ at } t\} \times \Pr\{\text{no greater draw in } (t,t+\Delta)\} \\ &= F(z,t)F(z,t)^{\alpha\Delta} \end{aligned}$$

The first term on the right reflects the option, which managers always have, to continue with their current productivity. The second is the probability that in $\alpha\Delta$ randomly drawn meetings an agent with productivity z does not meet anyone with a higher productivity. Given our assumption of independent draws, the fraction of managers with productivity below z at date $t+h$ is given by product of these two terms.

We have that
$$\frac{F(z,t+\Delta)-F(z,t)}{F(z,t)\Delta} = \frac{F(z,t)^{\alpha\Delta}-1}{\Delta}$$
and taking limits as $\Delta \to 0$ that:[6]

$$\frac{\partial \log F(z,t)}{\partial t} = \alpha \log\left(F(z,t)\right). \tag{5}$$

Then for any initial distribution (cdf) $F(z,0)$ the path of F is given by

$$\log(F(z,t)) = \log(F(z,0))e^{\alpha t}. \tag{6}$$

[5]The effect of indirect links, of the role of chance in our diffusion process, is familiar to us from the history of technology. Here is a nice example, taken from chapter 13 of Diamond (1998): "[N]ew technologies and materials make it possible to generate still other new technologies by recombination ... Gutenberg's press was derived from screw presses in use for making wine and olive oil, while his ink was an oil-based improvement on existing inks...."

[6]See Appendix A for an interpretation of the continuous time limit. A similar, but not identical, differential equation could be based on the more familiar assumption of Poisson arrivals, as opposed to the continuous arrivals postulated here. The formulation here has the convenient property of preserving distributions in the Frechet family. See Alvarez et al. (2008) and the Poisson extension in the Online Appendix.

It is evident from (6) that the law of motion (5) implies a non-decreasing level of real income $y(t)$. For empirical reasons, our interest is in sustained growth of economies that either grow at a fairly constant rate or will do so asymptotically. A central construct in our analysis will therefore be a *balanced growth path* (BGP), defined as a cdf Φ (with continuous density ϕ) and a growth rate $\nu > 0$ such that

$$F(e^{\nu t}z, t) = \Phi(z) \quad \text{for all } t$$

is a particular solution to (5). On a BGP

$$f(z,t) = \frac{\partial F(z,t)}{\partial z} = \phi(e^{-vt}z)e^{-vt}$$

also holds. Real GDP is

$$\begin{aligned} y(t) = & \left[\int_{\mathbb{R}_+} z^{-1+1/\eta}\phi(e^{-vt}z)e^{-vt}dz\right]^{-\eta/(\eta-1)} \\ = & e^{vt}\left[\int_{\mathbb{R}_+} x^{-1+1/\eta}\phi(x)dx\right]^{-\eta/(\eta-1)} \end{aligned} \tag{7}$$

provided the integral on the right converges. In the rest of this section we characterize (i) all pairs (Φ, ν) that are balanced growth paths and (ii) all initial distributions $F(z, 0)$ from which the solution $F(e^{vt}x, t)$ will converge asymptotically to $\Phi(x)$.

The possible balanced growth solutions to (5) are contained in the Frechet family of distributions, a two-parameter family defined by the cdfs:

$$F(z, 0) = \exp(-\lambda z^{-1/\theta}), \quad \theta, \lambda > 0. \tag{8}$$

Proposition 1. The cdf/growth rate pair (Φ, ν) is a balanced growth path of (5) if and only if Φ is a Frechet distribution with parameters $\lambda > 0$ and $\theta = \nu/\alpha$.

Proof: It is immediate from (6) that if $F(z, 0)$ is Frechet (λ, θ), $F(z, t)$ is Frechet $(e^{\alpha t}\lambda,\theta)$ for all t. Then if $\Phi(z) = F(e^{\alpha\theta t}z, t)$ is a BGP. Conversely, suppose $(\Phi, \alpha\theta)$ is a BGP so that $F(z, t) = \Phi(e^{-\alpha\theta t}z)$ solves (6). Then

$$\log\left(\Phi(e^{-\alpha\theta t}z)\right) = \log(\Phi(z))e^{\alpha t}.$$

Differentiating both sides with respect to t and setting $t = 0$ gives

$$\frac{d\log(\Phi(z))}{d\log(z)} = -\frac{1}{\theta}\log(\Phi(z))$$

which has the general solution

$$\log(\Phi(z)) = -\lambda z^{-1/\theta}. \quad \square \tag{9}$$

A Frechet distribution Φ has a "Pareto tail" which is to say that it has the property

$$\lim_{z\to\infty} \frac{1-\Phi(z)}{z^{-1/\theta}} = \lambda.$$

This is just a restatement of the solution (9) above, since $\Phi(z) \to 1$ as $z \to \infty$ and so the approximation $\log(\Phi(z)) \simeq -[1-\Phi(z)]$ becomes exact. The terminology stems from the fact that the numerator above is the right cdf of the Frechet distribution while the right cdf of the Pareto has the form $Az^{-1/\theta}$. The two tails are proportional. The parameter θ is often referred to as a "shape" or "tail" parameter.

This observation is central to a study of the stability of balanced growth paths, or to the question of what conditions on the initial distribution $F(z, 0)$ ensure that

$$\lim_{t\to\infty} \log F(e^{\alpha\theta t}z, t) = -\lambda z^{-1/\theta} \quad \text{for all} \quad z > 0 \tag{10}$$

for some $\lambda > 0$ and $\theta > 0$. The answer to this question is given by

Proposition 2. The solution (6) to (5) satisfies the stability condition (10) for some λ and θ if and only if the initial distribution $F(\cdot, 0)$ satisfies

$$\lim_{z\to\infty} \frac{1-F(z,0)}{z^{-1/\theta}} = \lim_{z\to\infty} \theta z^{1/\theta+1} f(z,0) = \lambda. \tag{11}$$

Proof. Using 6), (10) holds if and only if

$$\lim_{t\to\infty} \frac{\log\left[F(e^{\alpha\theta t}z, 0)\right] e^{\alpha t}}{z^{-1/\theta}} = -\lambda.$$

Use the change of variable $x = e^{\alpha t} z^{1/\theta}$ to get the equivalent statements

$$\lim_{x\to\infty} \frac{\log\left[F(x^{\theta}, 0)\right]}{x} = -\lambda$$

or

$$\lim_{z\to\infty} \frac{1 - F(z, 0)}{z^{-1/\theta}} = \lambda.\square$$

There are, of course, initial distributions that generate paths that do not converge in the sense of (10): any distribution with a bounded support, for example. At the opposite extreme, an example of an initial distribution that implies a growth rate that increases without bound can also be constructed, as we show in the Online Appendix.

3 Competitive Trade Model

In this section we move from the autarky model of Section 2 to a world economy of n countries. each with its own productivity distribution $F_i(\cdot, t)$ at any date t, all linked by trade in goods. The technology profile $F = (F_1, ..., F_n)$ will be the state variables of the world economy. We take iceberg trade costs and populations as given and construct a static trade equilibrium under the assumption of continuous trade balance. Among other things, this equilibrium will determine the productivity distribution of the product managers who are active in each country i. We label the cdfs of these distributions $G_i(\cdot, t)$. Product mangers in i, active and inactive, will meet managers from G_i at a given rate α and the outcomes of these meetings will provide a law of motion $\partial F_i(z, t)/\partial t$ for the technology profiles of all countries. This mapping is the topic of this section. Since the analysis is static, we temporarily suppress the time subscripts. The implied dynamics will be studied in Section 4.

The model is a variant of the trade theory of Eaton and Kortum (2000) and Alvarez and Lucas (2007). Each country under autarky is identical to the closed economy described in Section 2. We use the same notation here, adding the country subscript i to the variables $c_i(s)$, $z_i(s)$, $y_i(s)$, and ℓ_i. In this many-country case we group goods s which have the same profile $\mathbf{z} = (z_1, ..., z_n)$ of productivities across the n countries, where $z_i \ell_i$ is the production technology of the good $\mathbf{z}$ in country i. We assume that productivities are independently distributed across countries, and let $f(\mathbf{z}) = \prod_{i=1}^{n} f_i(z_i)$ denote the joint density of productivities. With this notation we can write the period utility as

$$C_i = \left[\int_{\mathbb{R}^n_+} c_i(\mathbf{z})^{1-1/\eta} \ f(\mathbf{z}) d\mathbf{z} \right]^{\eta/(\eta-1)},$$

where $c_i(\mathbf{z})$ is the consumption in country i of goods that have the cost profile $\mathbf{z}$.

We use w_i wage rates. We assume iceberg shipping costs: when a good is sent from country k a fraction κ_{ik} of the good arrives in i. The costs κ_{ik} are the same for all goods, and satisfy $0 < \kappa_{ij} \leq 1$ for all i, j and $\kappa_{ii} = 1$ for all i. Each good $\mathbf{z} = (z_1, ..., z_n)$ is available in i at the unit prices

$$\frac{w_1}{\kappa_{i1} z_1}, ..., \frac{w_n}{\kappa_{in} z_n},$$

which reflect both production and transportation costs.

We solve for equilibrium prices, given wages. Let $p_i(\mathbf{z})$ be the prices paid for good $\mathbf{z}$ in i :

$$p_i(\mathbf{z}) = \min_j \left[\frac{w_j}{\kappa_{ij} z_j} \right]$$

since agents in i buy the good at the lowest price. We let $\mathbf{B}_{ij} \subset \mathbf{R}^n_+$ be subset of the productivity (and goods) space

$$\mathbf{B}_{ij} = \{ z \in \mathbf{R}^n_+ : \frac{w_j}{\kappa_{ij} z_j} \leq \frac{w_k}{\kappa_{ik} z_k} \ \text{ for all } \ k \neq j \}.$$

for which j is the least cost vendor to i. Given prices $p_i(\mathbf{z})$, the ideal price index is the minimum cost of providing one unit of aggregate consumption C_i to buyers in i :

$$p_i^{1-\eta} = \int_{\mathbb{R}^n_+} p_i(\mathbf{z})^{1-\eta} \ f(\mathbf{z}) d\mathbf{z} = \sum_{j=1}^{n} \int_{\mathbf{B}_{ij}} \left(\frac{w_j}{\kappa_{ij} z} \right)^{1-\eta} f_j(z) dz$$

or

$$p_i^{1-\eta} = \sum_{j=1}^{n} \left(\frac{w_j}{\kappa_{ij}} \right)^{1-\eta} \int_0^\infty z^{\eta-1} f_j(z) \prod_{k \neq j} F_k \left(\frac{\kappa_{ij} w_k}{\kappa_{ik} w_j} z \right) dz. \tag{12}$$

With prices determined, given wages, we turn to the determination of equilibrium wages. Consumption of good $\mathbf{z}$ in country i equals

$$c_i(\mathbf{z}) = \left(\frac{p_i}{p_i(\mathbf{z})} \right)^\eta C_i = \left(\frac{p_i}{p_i(\mathbf{z})} \right)^\eta \frac{w_i L_i}{p_i}.$$

where the first equality follows from individual maximization and the second follows from the trade balance conditions $p_i C_i = w_i L_i$. The derived demand for labor in country i is thus

$$\begin{aligned}\sum_{j=1}^{n}\int_{\mathbf{B}_{ji}} c_j(\mathbf{z})\frac{1}{\kappa_{ji}z_i}f(\mathbf{z})d\mathbf{z} &= \sum_{j=1}^{n}\int_{\mathbf{B}_{ji}}\left(\frac{p_j}{p_j(\mathbf{z})}\right)^{\eta}\frac{w_jL_j}{p_j}\frac{1}{\kappa_{ji}z_i}f(\mathbf{z})d\mathbf{z} \\ &= \sum_{j=1}^{n}\left(\frac{\kappa_{ji}p_j}{w_i}\right)^{\eta}\frac{w_jL_j}{p_j}\frac{1}{\kappa_{ji}}\int_0^{\infty} z^{\eta-1}f_i(z)\prod_{k\neq i}F_k\left(\frac{\kappa_{ij}w_k}{\kappa_{ik}w_j}z\right)\ dz.\end{aligned}$$

Since labor is supplied inelastically, this implies

$$L_i = \sum_{j=1}^{n}\left(\frac{p_j}{w_i}\right)^{\eta}\frac{w_jL_j}{p_j}\kappa_{ji}^{\eta-1}\int_0^{\infty} z^{\eta-1}f_i(z)\prod_{k\neq i}F_k\left(\frac{\kappa_{ij}w_k}{\kappa_{ik}w_j}z\right)\ dz. \tag{13}$$

Given populations $L = (L_1, ..., L_n)$, trade costs $K = [\kappa_{ij}]$ and the distributions $F = (F_1, ..., F_n)$, equations (12) and (13) are $2n$ equations in wages $w = (w_1, ..., w_n)$ and prices $p = (p_1, ..., p_n)$ .as n equations in $w = (w_1, ..., w_n)$.

Definition. A *static equilibrium* is a wage $w = (w_1, ..., w_n) \in \mathbb{R}^n_+$ such that for some $p = (p_1, ..., p_n) \in \mathbb{R}^n_+$, (w, p) solves (12) and (13).

The next proposition states that a static equilibrium exists and that, provided $\eta \geq 1$, there is a unique static equilibrium.

Proposition 3: We take as given trade costs K, populations L, and distributions F. We assume that $L_i > 0$ and that $0 < \kappa_{ij} \leq 1$ and that the right cdfs $F = (F_1, ..., F_n)$ have continuous densities and satisfy

$$\lim_{z\to 0}\frac{f_i(z)z}{1 - F_i(z)} = \frac{1}{\theta_i} > \eta - 1 \tag{14}$$

for all $i = 1, ..., n$. Then there exists a static equilibrium wage w. Moreover, if $\eta > 1$, the excess demand system has the gross substitute property, and hence (i) the static equilibrium wage w is unique, and (ii) equilibrium relative wages are decreasing in population sizes:

$$\frac{\partial(w_j/w_i)}{\partial L_i} > 0$$

for all $j \neq i$.

Proof: See Appendix B.

4 Diffusion in a World Economy

The central idea of this paper is that trade in goods among countries stimulates the exchange of productivity-related ideas. In Section 2 we described a specific model of the exchange of ideas within a closed economy. In Section 3 we provided a model of trade in goods with many countries. Now we put these pieces together.

To do this, we replace the assumption of Section 2 that product managers in country i learn from the examples of other managers in their own country with the assumption that they learn from the managers of products that are *sold* in i, regardless of their origin. Instead of drawing from the distribution $F_i(\cdot, t)$ they draw from a distribution $G_i(\cdot, t)$ defined by the cdf

$$G_i(z,t) = \Pr\{\text{seller active in } i \text{ at } t \text{ has productivity } \leq z\}$$
$$= \sum_{j=1}^{n} \Pr\{\text{seller from } j \text{ is active in } i \text{ at } t \text{ and has productivity } \leq z\}$$
$$= \sum_{j=1}^{n} \int_0^z f_j(y,t) \prod_{k \neq j} F_k\left(\frac{w_k(t)\kappa_{ij}}{w_j(t)\kappa_{ik}} y, t\right) dy \qquad (15)$$

The probability that a producer in j with the productivity y exports to i is $f_j(y,t)$ times the probability that no producer elsewhere can offer a lower price. This will depend on all the productivity distributions plus trade costs plus equilibrium relative wages as determined in Section 3. The law of motion for productivity in each country then becomes

$$\frac{\partial \log(F_i(z,t))}{\partial t} = \alpha_i \log(G_i(z,t)) \qquad (16)$$

We are now in a position to define an equilibrium that describes the full dynamics of a world economy given parameters K, L and initial distributions $F(\cdot, 0) = (F_1(\cdot, 0), ..., F_n(\cdot, 0))$.

Definition. An *equilibrium* is a time path of wages $w(t) = (w_1(t), ..., w_n(t))$ and cdfs $F(\cdot, t)$ for all $t \geq 0$ such that

(i) $w(t)$ is a static equilibrium as defined in Section 3, and

(ii) given $w(t)$ the path $F(\cdot, t)$ satisfies (15) and (16).

As in Section 2, we are also interested in balanced growth equilibria. Here we define a BGP as a common growth rate ν and a profile of productivity distributions $\Phi = (\Phi_1 ..., \Phi_n)$ such that

$$\Phi_i(x) = F_i(e^{\nu t}x, t)$$

for all t. Along a BGP we can substitute $\Phi_i(e^{-\nu t}z)$ for $F_i(z,t)$ in (16) and $\phi_i(e^{-\nu t}z)e^{-\nu t}$ for $f_i(z,t)$, and using the homogeneity of the model we have that a BGP (Φ, ν) must satisfy[7]

$$\frac{\partial \log\left(\Phi_i(e^{-\nu t}z)\right)}{\partial t} = \alpha_i \log\left(\sum_{j=1}^{n}\int_0^{e^{-\nu t}z}\phi_j(y)\prod_{k\neq j}\Phi_k\left(\frac{w_k\kappa_{ij}}{w_j\kappa_{ik}}y\right)dy\right).$$

Letting $x = e^{-\nu t}z$, we have

$$\nu\frac{x\phi_i(x)}{\Phi_i(x)} = -\alpha_i \log\left(\Gamma_i(x)\right), \tag{17}$$

where

$$\Gamma_i(x) = \sum_{j=1}^{n}\int_0^{x}\phi_j(y)\prod_{k\neq j}\Phi_k\left(\frac{w_k\kappa_{ij}}{w_j\kappa_{ik}}y\right)dy. \tag{18}$$

Let $\gamma_i(x) = \partial\Gamma_i(x)/\partial x$ be the associated density.

The relations (15) and (18) of learning environments G_i to the profile F are complicated. A backward producer in a large, low wage economy could undercut a domestic producer with higher productivity or another foreign producer with higher trade costs. Little can be said in general, but the next result shows that all G_i are bounded from below by the joint distribution of sellers that would be active in i in a hypothetical world economy with no trade costs and a common labor market.

Proposition 4: $G_i(z; \mathbf{K}, \mathbf{w}) \geq \prod_{j=1}^{n} F_j(z)$, with equality if $\mathbf{K} = \mathbf{I}$ and $\mathbf{w} = \mathbf{1}$.

[7]Formally, let (w,p) be the equilibrium wages and prices for an economy with K, L, F. Let $\xi \in \mathbb{R}_{++}$ and define F_i^{ξ} as $F_i^{\xi}(z) = F_i(\xi z)$, for all i. Then (w,p) are also the equilibrium wages and prices for an economy with K, L, F^{ξ}.

Proof: Define the sets

$$M(z)=\left\{\mathbf{z}\in\mathbb{R}_+^n:\max\{z_1,...,z_n\}\leq z\right\}$$

and

$$B_i(z;\mathbf{w},\mathbf{K})=\left\{\mathbf{z}\in\mathbb{R}_+^n:z_{j^*}\leq z,\text{ where }j^*=\underset{j\in\{1,...,n\}}{\arg\min}\left\{\frac{w_j}{z_jk_{ij}}\right\}\right\}.$$

It is easy to see that $M(z)\subseteq B_i(z;\mathbf{w},\mathbf{K})$ since $\max\{z_1,...,z_n\}\leq z\Rightarrow z_j\leq z$, all $j=1,...,n$. Therefore,

$$G_i(z;\mathbf{K},\mathbf{w})=\int_{\mathbf{z}\in B_i(z;\mathbf{w},\mathbf{K})}f(\mathbf{z},t)d\mathbf{z}\geq\int_{\mathbf{z}\in M(z)}f(\mathbf{z},t)d\mathbf{z}=\prod_{j=1}^nF_j(z,t).\square$$

The next result is instructive, even though it is limited to the case of a symmetric world with costless trade.

Proposition 5: Assume that the n countries have the same size, $L_i=L$, and the same $\alpha=\alpha_i$, and that trade is costless, $\kappa_{ij}=1$, all i,j, and that the initial distributions are the same: $F_i(z,0)=F(z,0)$. Then wages will be identical and the equilibrium path of $F(z,t)$ is

$$\frac{\partial\log(F(z,t))}{\partial t}=\alpha n\log(F(z,t)).\tag{19}$$

Proof: The distribution of sellers varies across countries only through its dependence on country specific trade costs (see (15)). Therefore, in the case of costless trade all countries share the same distribution of sellers, $G_i(z)=G(z)$. In this case, the distribution of productivity for every country i solves

$$\frac{\partial\log(F_i(z,t))}{\partial t}=\alpha\log(G(z,t)).$$

In this symmetric case,

$$G(z,t)=\int_0^z nf(y,t)\left[F(y,t)\right]^{n-1}dy=F(z,t)^n.$$

We can drop the subscripts and (19) follows.$\square$

Along a BGP we can replace $F(z,t)$ with $\Phi(e^{-\nu t}z)$ and let $x = e^{-\nu t}z$ to obtain

$$\frac{\phi(x)x}{\Phi(x)} = -\frac{\alpha n}{\nu}\log(\Phi(x))$$

Then Proposition 1, with $n\alpha$ replacing α yields the

Corollary 1: The symmetric world economy described in Proposition 5 is on a balanced growth path if and only the cdfs are given by the Frechet cdf

$$\Phi_i(z) = \exp(-\lambda z^{-\frac{1}{\theta}})$$

with parameters λ and θ, and the growth rate of each of all economies is

$$\nu = n\alpha\theta.$$

The fact that the equilibrium growth rate is proportional to the number of economies n may require comment. We certainly do not believe that the division of Czechoslovakia into Slovakia and the Czech Republic led to an increase in world growth rates. In practice, ν would be identified with measured gdp growth and θ with a tail parameter (as discussed below) and the *product* $n\alpha$ with ν/θ. In everything that follows, we treat n as an unobservable constant.[8]

In Propositions 6-8 below, we develop some facts about much more general cases. The Frechet distribution does not obtain in these cases. But, perhaps surprisingly, the formula $\nu = n\alpha\theta$ (or more generally, $\nu = \theta\sum_{i=1}^{n}\alpha$) continues to describe the BGP of all economies.

The following condition on the tail behavior of these distributions will be used in deriving the next three results in this section.

Condition C: We say that a profile $F(z) = (F_1(z), ..., F_n(z))$ satisfies Condition C if

$$\lim_{z\to\infty}\frac{1 - F_i(z)}{z^{-1/\theta_i}} = \lambda_i < \infty$$

[8]In Appendix C we present an extension of the model with multiple locations per country to clarify the role of scale effects. There we show that, provided that the structure of transportation cost and labor markets across locations is kept constant, an equilibrium of the model is invariant to arbitrary division of locations into countries.

for all i,

$$\lim_{z\to\infty} \frac{1-F_i(z)}{z^{-1/\theta_i}} = \lambda_i > 0$$

for some i, and w_i and $\kappa_{ij} > 0$ for all i, j.

If Condition C holds no country is capable of ever-accelerating growth, at least one country is capable of sustained growth at a positive rate, and all countries are connected in the sense that it is possible for any country to trade with any other country.

The next result shows that the distributions G_i share a common right tail, with tail parameter $\theta = \max_i \theta_i$.

Proposition 6: Assume that the profile $F(z)$ satisfies Condition C. Then for all i the cdfs $G_1, ..., G_n$ satisfy

$$\lim_{z\to\infty} \frac{1-G_i(z,0)}{z^{-1/\theta}} = \lambda > 0, \tag{20}$$

where $\theta = \max_i \theta_i$ and $\lambda = \sum_j \lambda_j$.

Proof: We show that

$$\lim_{z\to\infty} \frac{g_i(z,0)}{(1/\theta)\, z^{-1/\theta-1}} = \lambda \tag{21}$$

for all i which will obviously imply (20). Differentiating both sides of (15) with respect to z and dividing through by $(1/\theta)\, z^{-1/\theta-1}$ where $\theta = \max_i \theta_i$ we obtain

$$\lim_{z\to\infty} \frac{g_i(z,0)}{(1/\theta)\, z^{-1/\theta-1}} = \sum_{j=1}^{n} \lim_{z\to\infty} \frac{f_j(z,0)}{(1/\theta)\, z^{-1/\theta-1}}$$

since the cdfs $F_k \to 1$ as $z \to \infty$ under the assumption that w_i and $\kappa_{ij} > 0$. Condition C requires that

$$\lim_{z\to\infty} \frac{g_i(z,0)}{(1/\theta)\, z^{-1/\theta-1}} = \sum_{j=1}^{n} \lambda_j \lim_{z\to\infty} \frac{(1/\theta_j)\, z^{-1/\theta_j-1}}{(1/\theta)\, z^{-1/\theta-1}}.$$

The terms in the sum on the right are zero if $\lambda_j = 0$ or if $\theta_j < \theta$ and equal to $\lambda_j > 0$ otherwise. This verifies (21) and completes the proof. $\square$

Proposition 6 describes the instantaneous effects of anything that alters the static trade equilibrium. Suppose, for example, that the initial distributions $F_1(\cdot, 0), ...F_n(\cdot, 0)$ represent a situation of autarky that is ended at $t = 0$ by an opening to trade. Then *immediately* all countries will have access to the source distributions $G_i(\cdot, 0)$ characterized in Proposition 6, all which have the same "Pareto tail." For the analysis of trade dynamics, then, any initial differences in the initial tail parameters θ_i cease to matter, and we use θ to denote *only* $\theta = \max_i \theta_i$.

Proposition 7. Assume that $\alpha_i > 0$, all i, and that the profile of stationary distributions $\Phi = (\Phi_1(z), ..., \Phi_n(z))$ satisfies Condition C. Then the growth rate on the balanced growth path equals

$$\nu = \theta \sum_{i=1}^{n} \alpha_i \tag{22}$$

and

$$\lim_{x\to\infty} \frac{1 - \Phi_i(x)}{x^{-1/\theta}} = \frac{\alpha_i}{\sum_j \alpha_j} \sum_j \lambda_j. \tag{23}$$

Proof: For large x, (17) and (18) imply

$$\begin{aligned} \nu \frac{\phi_i(x)}{x^{-1/\theta-1}} &\simeq - \alpha_i \frac{\log\ (\Gamma_i(x))}{x^{-1/\theta}} \\ &= \alpha_i \sum_j \lambda_j \end{aligned} \tag{24}$$

where the second line follows from Proposition 6 and Condition C. Condition C also implies that for large x, $\phi_i(x) \simeq (\lambda_i/\theta)\, x^{-1/\theta-1}$ so

$$\nu \lambda_i = \theta \alpha_i \sum_{j=1}^{n} \lambda_j.$$

Summing both sides over i yields (22) and then (23) follows from (24). □

As in Section 2, we are interested in conditions on the initial knowledge distributions $F_i(z, 0)$ that will imply convergence to a balanced growth path, in the sense that

$$\lim_{t\to\infty} \frac{\log\left[F_i\left(\left(e^{(\nu/\theta)t}x\right)^{\theta}, t\right)\right]}{x^{-1}} \tag{25}$$

is *constant* for all $x > 0$, i.e., that the normalized productivity $x(t) \equiv e^{-\nu t} z(t)$ has a Frechet distribution as in equation (10). Here we assume profiles for initial distributions $F_i(z, 0)$ that satisfy Condition C. Proposition 7 then implies that equations (22)-(23) hold for the common value $\theta > 0$ and positive values $\lambda_1, ..., \lambda_n$. In this n country case, $\nu = \theta \sum_{i=1}^n \alpha_i$. It is certainly not the case that these conditions will imply (25) for all values of x but the next result shows that (25) holds in the limit as $x \to \infty$ and provides a characterization of the dynamics of the right-tail of the productivity distributions $F_i(z, t)$.

Proposition 8: Assume that $\sum_{i=1}^n \alpha_i > 0$, that $F(z, 0)$ satisfies Condition C, and that there exist a solution to (16) that is twice continuously differentiable with respect to z and t. Let

$$\lambda_i(t) = -\lim_{x \to \infty} \frac{\log \left[F_i((e^{-(\nu/\theta)t} x)^\theta, t) \right]}{x^{-1}}$$

and let

$$\lambda_i^* = \frac{\alpha_i}{\sum_{j=1}^n \alpha_j} \sum_{j=1}^n \lambda_j(0).$$

Then, for all t

$$\lambda_i(t) - \lambda_i^* = \left[\lambda_i(0) - \lambda_i^* \right] e^{-(\nu/\theta)t}. \tag{26}$$

Proof: See Appendix B.

Note that neither wages w_j nor trade costs κ_{ij} affect the right tail of productivity in the balanced growth path. The relative level of the right tail across countries depends only on the relative value of α_i, the rate at which technology diffusion opportunities arrive. If all the α_i are the same, then the right tail (of normalized) productivity converges to the same value for all countries. Otherwise, countries with more opportunities for technology diffusion converge to a permanently higher productivity, which translates into higher income levels. As in the closed economy case, the level of the initial right tail of productivity has a permanent effect on the long run distribution, except that in the multi-country case it is the sum of the (normalized) right tails which matters in the long run. In addition, note that if $\alpha_i = 0$ for some country i then $\lambda_i^* = 0$, as this country's technology is constant, and hence gets farther and farther behind the rest.

To better understand the role of cross country differences on the arrival of technology diffusion opportunities consider the case where trade is costless, so all $\kappa_{ij} = 1$, but where the value of α_i differs across countries. With costless trade $G_i(z) = G_j(z)$ for all z, and hence from (17) and (18) we obtain that for all $z > 0$:

$$\Phi_i(z) = \Phi_j(z)^{\frac{\alpha_i}{\alpha_j}}$$

or equivalently

$$\lim_{t\to\infty} \frac{\log\left[F_i\left(\left(e^{(\nu/\theta)t}z\right)^{\theta}, t\right)\right]}{\log\left[F_j\left(\left(e^{(\nu/\theta)t}z\right)^{\theta}, t\right)\right]} = \frac{\alpha_i}{\alpha_j} \tag{27}$$

Thus if $\alpha_i > \alpha_j$ country i has a distribution of productivity that is stochastically better than j. Hence with costless trade the ratio of (25) equals the ratio of the α's not only as $z \to \infty$ but for *all* values of $z < \infty$.[9]

The next proposition analyzes the effect of the elasticity of substitution η in imports elasticities, equilibrium wages, and relative GDP's. As a preliminary step we define the total value I_{ij} of purchases of country i from j as function of trade cost $\mathbf{K}$ and wages $\mathbf{w}$ by

$$\begin{aligned} I_{ij} &= \int_{\mathbf{B}_{ij}} p_i(\mathbf{z}) c_i(\mathbf{z}) f(\mathbf{z}) d\mathbf{z} \\ &= \int_0^{\infty} \left(\frac{w_j}{z_j\,\kappa_{ij}\,p_i}\right)^{1-\eta} w_i L_i\, f_j(z_j) \prod_{k\neq j} F_k\left(\frac{w_k\kappa_{ij}}{w_j\kappa_{ik}}\ z_j\right) dz_j\ , \end{aligned}$$

Remark 1: For the case when all F_i are given by Frechet distributions with scale parameters λ_i and the same shape parameter θ, i) the Armigton's elasticity, i.e. the price elasticity of imports demand, which equals $1/\theta$, (iii) equilibrium wages, and iv) relative real GDP's, and thus gains from trade, are all independent of η, and relative GDP's are given by the ratio of λ's (Eaton and Kortum, 2002; Alvarez and Lucas, 2007).

[9] In the case of different α's the value of (25) for country i is *not* constant across z, i.e. the distributions Φ_i is not Frechet.

We use the notation $\tilde{I}_{ij}$, $\tilde{w}_i$, $\tilde{p}_i$ and $\tilde{y}_i$ to refer to imports of i from j, country's i equilibrium wages, price level, and real GDP for the Frechet case with tail parameter θ. The same three objects without a tilde can be interpreted as the long run distribution of a world economy that starts with Frechet distributions in all countries with the same tail parameter θ.

Proposition 9: Let $\theta_i = \theta$ for all countries, and $0 < \lambda_i < \infty$. Then

$$\begin{aligned}
\lim_{\eta \to 1+1/\theta} I_{i,j}(\mathbf{K}, \mathbf{w}; \eta) &= \tilde{I}_{i,j}(\mathbf{K}, \mathbf{w}) = \frac{\lambda_j \left(\frac{w_j}{\kappa_{ij}}\right)^{-1/\theta}}{\sum_{s=1}^{n} \lambda_s \left(\frac{w_s}{\kappa_{is}}\right)^{-1/\theta}} w_i L_i \,, \\
\lim_{\eta \to 1+1/\theta} w_i(\mathbf{K}; \eta) &= \tilde{w}_i(\mathbf{K}) \,, \\
\lim_{\eta \to 1+1/\theta} \frac{p_i(\mathbf{K}; \eta)}{p_j(\mathbf{K}; \eta)} &= \frac{\tilde{p}_i(\mathbf{K})}{\tilde{p}_j(\mathbf{K})} \,, \\
\lim_{\eta \to 1+1/\theta} \frac{y_i(\mathbf{K}; \eta)}{y_i(I_{n \times n}; \eta)} &= \frac{\tilde{y}_i(\mathbf{K})}{\tilde{y}_i(I_{n \times n})} \,.
\end{aligned}$$

Proof: See Appendix B.

This proposition is useful because the effects for the Frechet case are simple and well understood. The logic behind the result is clear: as $\eta \to 1 + 1/\theta$ the goods became such good substitutes that demand is concentrated on the best products. The result follows because the long run distributions of productivities have the same behavior in the tail as a Frechet distribution. The proposition is stated in terms of ratios for two reasons: first in the Frechet case the effect of η is multiplicative, and hence it cancels in this form. Second, the levels of GDP and imports as η tends to this limit diverge to infinity.

Corollary 2: For the case of costless trade, $\mathbf{K} = I_{n \times n}$, in the long-run

$$\lim_{\eta \to 1+1/\theta} \frac{y_i(I_{n \times n}; \eta)}{y_j(I_{n \times n}; \eta)} = \left(\frac{\lambda_i^*}{\lambda_j^*}\right)^{\theta/(1+\theta)} = \left(\frac{\alpha_i}{\alpha_j}\right)^{\theta/(1+\theta)} .$$

This corollary is a direct consequence of the last two propositions and equation (27).

Summarizing, as explained in Remark 1, Eaton and Kortum (2002) takes as given that productivity distribution given by Frechet with common tail θ and country specific λ_i which determines the Armington elasticities, and per-capita income differences. Our technology diffusion model gives a simple theory of θ and λ_i (propositions 7 and 8). In this theory, heterogeneity in α's imply differences in the level of income per-capita, while the sum of α's affects the common growth rate of the countries in the world.

We finish the theoretical exploration of the model by studying the effects of trade cost and wages in the neighborhood of costless trade, which would be helpful to interpret some of the numerical results that follow. If we start from a situation with costless trade and equal wages, a marginal increase in trade cost or wages has a negligible effect in the distribution of sellers.

Proposition 10: Take an arbitrary profile of productivity distribution $F(z)$ and consider the distribution of seller to country i given a profile of equal wages and costless trade. Then, the distribution of sellers to country i is invariant to small changes in trade cost or wages, i.e.,

$$\left.\frac{\partial G_i(z;\mathbf{K},\mathbf{w})}{\partial \kappa_{ij}}\right|_{\mathbf{K}=\mathbf{I},\mathbf{w}=\mathbf{1}} = \left.\frac{\partial G_i(z;\mathbf{K},\mathbf{w})}{\partial w_j}\right|_{\mathbf{K}=\mathbf{I},\mathbf{w}=\mathbf{1}} = 0.$$

Proof: Differentiating (18) with respect to κ_{ij}

$$\frac{\partial G_i(z;\mathbf{K},\mathbf{w})}{\partial \kappa_{ij}} = \int_0^z \left[f_j(y) \sum_{k\neq j} \frac{w_k}{w_j \kappa_{ik}} y f_k\left(\frac{w_k \kappa_{ij}}{w_j \kappa_{ik}} y\right) \prod_{l\neq j,k} F_l\left(\frac{w_l \kappa_{ij}}{w_j \kappa_{il}} y\right) \right.$$
$$\left. + \sum_{k\neq j} f_k(y) \left(-\frac{w_j \kappa_{ik}}{w_k} \frac{1}{\kappa_{ij}^2} y\right) f_j\left(\frac{w_j \kappa_{ik}}{w_k \kappa_{ij}} y\right) \prod_{l\neq j,k} F_l\left(\frac{w_k \kappa_{il}}{w_l \kappa_{ik}} y\right)\right] dy.$$

Evaluating at $\mathbf{K} = \mathbf{I}$ and $\mathbf{w} = \mathbf{1}$ and rearranging terms

$$\begin{aligned} \left.\frac{\partial G_i(z;\mathbf{K},\mathbf{w})}{\partial \kappa_{ij}}\right|_{\mathbf{K}=\mathbf{I},\mathbf{w}=\mathbf{1}} &= \int_0^z \left[-y f_j(y) \sum_{k\neq j} f_k(y) \prod_{l\neq j,k} F_l(y) \right. \\ &\quad \left. + y f_j(y) \sum_{k\neq j} f_k(y) \prod_{l\neq j,k} F_l(y) \right] dy \\ &= 0. \end{aligned}$$

A similar analysis follows by differentiating (18) with respect to w_j □

Proposition 10 holds independently of the profile of cdfs $F(z)$, but it takes as given the profile of wages $\mathbf{w}$ which we know is determined by the profile $F(z)$. We complement this result in Proposition 11, which studies the comparative static for the stationary distribution. That proposition establishes that when starting from a world with symmetric countries and costless trade, so that equilibrium wages are equal, changes in trade cost or in the size of an individual country have a negligible effect on the profile of stationary distributions of productivity of each country.

Let the parameters of a world economy be given by n, α, $\mathbf{K}$ and $\mathbf{L}$. We are interested in the comparative statics of the profile of stationary distributions $\phi(x;\mathbf{K},\mathbf{L})$ with respect to $\mathbf{K}$ and $\mathbf{L}$, evaluated at the case of a world of n equal size economies with costless trade.

Proposition 11: Consider a world economy with matrix of trade cost $\mathbf{K}$ and vector of population $\mathbf{L}$. Let $\phi(z;\mathbf{K},\mathbf{L})$ be the stationary distribution of such an economy where the corresponding equilibrium wages ensure balance trade for each country. Assume that for each z the density $\phi(z;\mathbf{K},\mathbf{L})$ is differentiable. Then, for all z

$$\left.\frac{\partial \phi_i(z;\mathbf{K},\mathbf{L})}{\partial \kappa_{ij}}\right|_{\mathbf{K}=\mathbf{I},\mathbf{L}=\mathbf{1}} = \left.\frac{\partial \phi_i(z;\mathbf{K},\mathbf{L})}{\partial L_j}\right|_{\mathbf{K}=\mathbf{I},\mathbf{L}=\mathbf{1}} = 0.$$

Proof: Notice first that $\phi(z;\mathbf{K},\mathbf{L})$ is the solution to the system of non-linear differential equations given by: equation (17) defining a balance growth path, equation (18) defining the distribution of sellers, equation (13) giving the solution to the static trade equilibrium wages, $\nu = n\alpha\theta$ defining the growth rate of a balance growth path, and $\lim_{x\to 0} \frac{x\phi_i(x)}{\Phi_i(x)} = \theta$ and $\lim_{x\to 0} \theta x^{\theta-1}\phi_i(x^\theta) = \lambda$ giving the boundary conditions for the densities. The result follows from totally differentiating the system of non-linear differential equations and Proposition 10. □

Propositions 10 and 11 taken together give the precise sense in which in a homogeneous world small trade costs have no effect on the diffusion of productivity. In the quantitative exploration that follows, we find that the lack of first order effects is clearly visible in a large range of parameters.

5 Quantitative Exploration

In this section we present numerical examples to illustrate the effect of trade costs. We consider two cases: a world consisting of n symmetric countries facing trade costs $\kappa_{ij} = \kappa$, and, a world consisting of one asymmetric and $n-1$ symmetric locations facing trade costs $\kappa_1 = \kappa_{1j} \le \kappa_{ji} = 1$, $j = 2, ..., n$, $i \neq j$. We also illustrate the effect on the diffusion of technology of heterogeneous arrival rates α_i and wages rates w_i, driven by differences in the size of countries L_i.

Calibration and Interpretation of Parameters

We can gain some understanding of the order of magnitude of the parameters α and θ by using information of the long-run growth rate of the economy ν, and information on θ which instead can be obtained either from the dispersion of productivities, or the tail of the size distribution of firms/plants, or the magnitude of trade elasticities. We turn to the description of each of these approaches.[10]

First, since we show above that asymptotically z is Frechet distributed, then $\log(z)$, the log of productivity, has standard deviation equal to $\theta\pi/\sqrt{6}$, see chapter 3.3.4 of Rinne (2008). Hence we can take measures of dispersion of (log) productivity to calibrate θ. The dispersion of (log) productivity range from $0.6 - 0.75$ when measured as the value-added per worker – see Bernard et al. (2003) Table II – and

[10]Our calibration strategy follows that in Lucas (2009), although our focus is on the distribution of productivity across "product managers" instead of individual workers. To operationalize the notion of a "product manager" we interpret a plant or firm as one manager, but we acknowledge the limits of this analogy.

around 0.8 when measures of physical total factor productivity are obtained using data on value-added, capital and labor inputs, and assumptions about the demand elasticities – see Hsieh and Klenow (2009) Table I, dispersion of TFPQ.[11] These numbers suggest a value for θ in the range $[0.5, 0.6]$.

Second, using that productivity is asymptotically distributed Frechet, and that the tail of the Frechet behaves as that of a Pareto distribution with tail coefficient $1/\theta$, we can use data on the tail of the distribution of productivity to directly infer θ. Lacking direct information on physical productivities, we can use information on the tail of the distribution of employment, together with a value for the elasticity of demand.[12] The tail of the size (employment) distribution of firms is approximately equal to 1.06 – see Figure 1 in Luttmer (2007). Therefore for the values of demand elasticities typically considered in the literature, say $\eta \in [3, 10]$, (see Broda and Weinstein (2006), Imbs and Mejean (2010), or Hendel and Nevo (2006)) it would imply a value for θ in the range $[0.1, 0.5]$.

Third, as showed before, in the case of a model with several symmetrical locations, θ is approximately the Armington trade elasticity, which will also give us another way to measure it. This method would suggest a value for θ in $[0.1, 0.25]$ (Alvarez and Lucas, 2007).

Once we have an estimate of θ, whatever its source, together with an estimate of long term growth of output ν, we can estimate the value of α, using that $\nu = n\alpha\theta$. For instance, if we take the long-run growth to be 0.02, $n\alpha$ would be in the range $[0.03, 0.2]$. Note that with a value of $n\alpha = 0.1$, which is the object that governs the speed of convergence of the λ's in general as shown in Equation (26), and of GDP in the setup of Corollary 2, the half-life to convergence will be approximately 5 years.

Based on this discussion, we set $\theta = 0.2$ to be consistent with the available evidence on the right tail of the distribution of productivity, and set $\alpha = 0.02/(\theta\, n)$, to match a growth rate 0.02. We consider a world consisting of $n = 50$ economies symmetric in all dimensions, with the possible exception of their trade cost. Given our choice of n, in a world with symmetric trade cost each economy has a relative GDP similar to

[11] Using data for eleven products for which direct measures of physical output are available Haltiwanger et al. (2008) calculate true measures of physical total factor productivity. They find that the dispersion of (log) true physical productivity is 20% higher than that measured using just value-added information.

[12] The CES structure implies that employment at industry/firm with cost x satisfies $l(z) \propto (1/z)^{\eta-1}$.

that of Canada or South Korea.

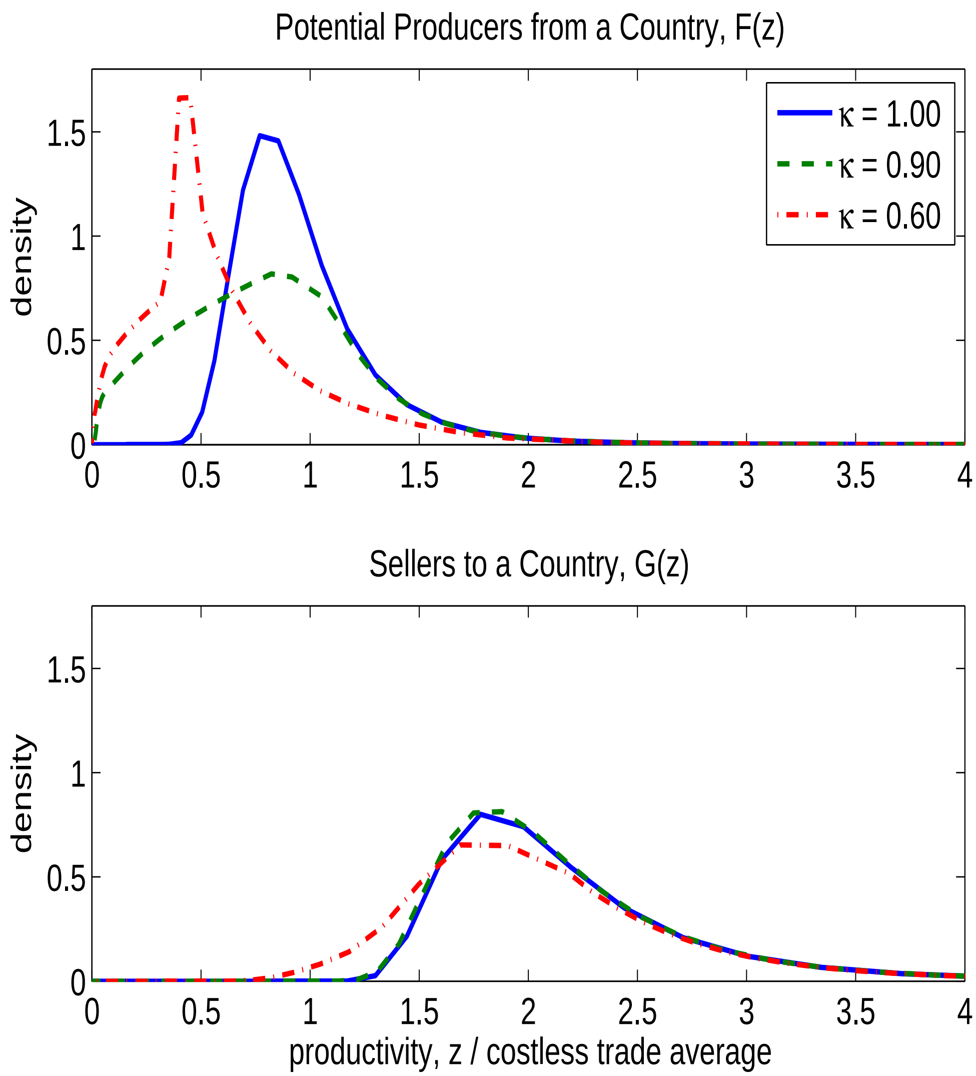
Symmetric World

Figure 1 illustrates the long run effect on the distribution of productivities z of introducing trade costs in a symmetric word of n countries. The thought experiment is to go from costless trade to a case where κ_{ij} takes a common value $\kappa < 1$ for all $j \neq i$. On the x-axis we measure the value of productivity, expressed as a ratio to the average productivity in the economy with costless trade ($\kappa = 1$). On the y-axis we display the density of relative productivities for different values of κ. The top panel shows the densities of productivities of the potential producers, the density ϕ (or f). The bottom panel shows the density of productivities of the sellers active in each country, the common density γ (or g). We have chosen the value of λ for the initial distribution so that with costless trade the average value of productivity z is equal to one.

Note first that, due to the selection effect, the density of sellers is stochastically larger than that of potential producers for each κ. The difference between the two densities increases for larger trade cost (for lower values of κ). Second, note that for $\kappa = 1$ the densities are Frechet, as we showed in Corollary 1. Third, for larger trade cost (lower κ) both densities have a thicker left tail, especially so for potential producers. Fourth, the change in the distribution of potential producers as κ varies illustrates the effect of trade costs on the diffusion of technologies, the main feature of the model in this paper. Finally, we note that these distributions are independent of the value of η, as equations (15) and (18) are independent of η, and $w_i = 1$ in a symmetric world.

Figure 2 illustrates the effect of introducing symmetric trade costs on real gdp in the top panel and in the ratio of imports to gdp in the bottom panel in a symmetric word of n countries. On the x-axis we measure trade cost. On the y-axis we measure real gdp, relative to gdp under costless trade (top panel) or the trade share, relative to the costless trade benchmark (bottom panel). In each panel the solid line displays the impact effect of introducing the trade costs, calculated by holding the distribution of productivity fixed at its distribution under of costless trade. As shown above, this benchmark has a Frechet distribution. The other lines in each panel show the effect of introducing trade cost on the balanced growth path. Each line is for a different value

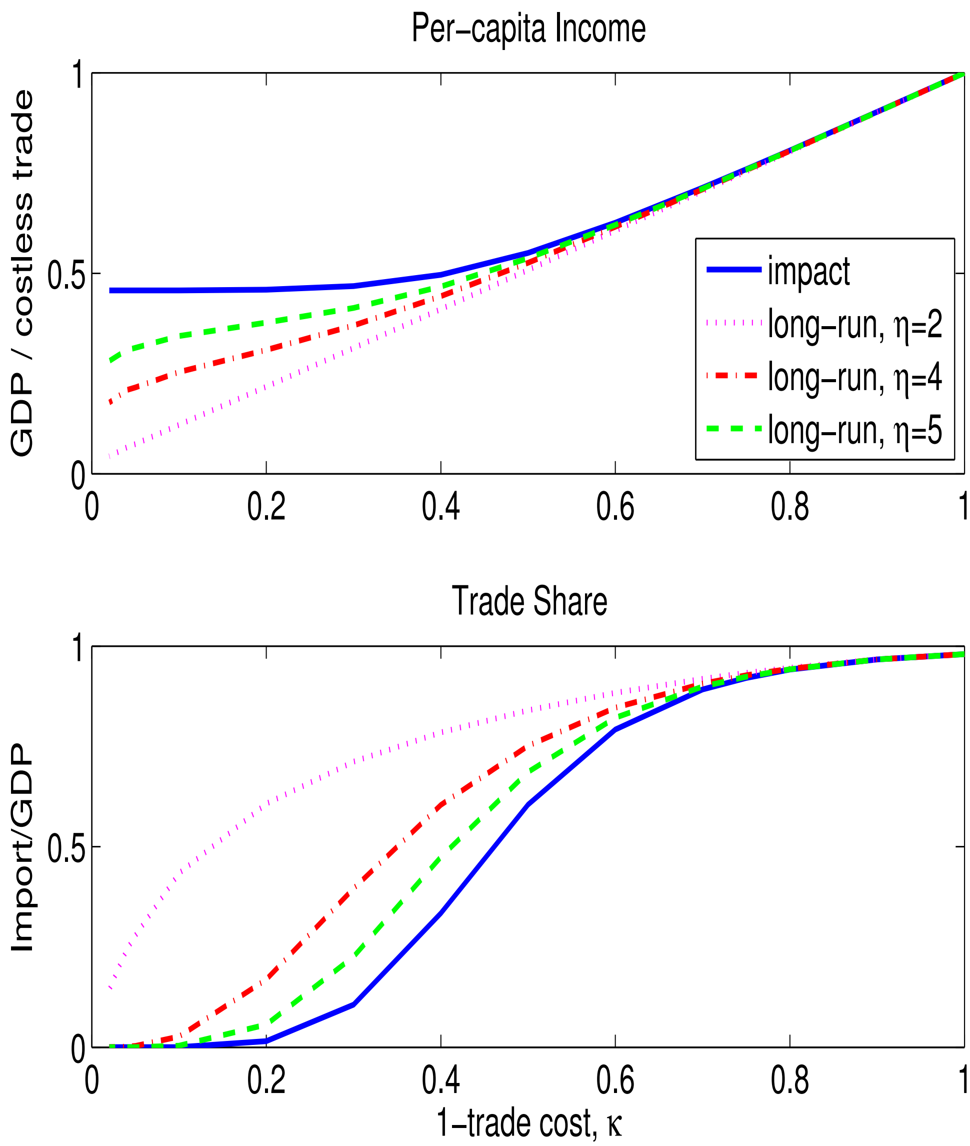
Figure 1: Long Run Effect of Trade Cost on the Distribution of Productivity



The top panel displays the density of the stationary distribution of normalized productivity z of potential producers in a country. for different value of trade cost, $\kappa = 1, 0.9, 0.6$. The bottom panel displays the stationary density of normalized productivities for the in each country. The productivities in the x-axis are measured relative to the expected value of the stationary distribution of potential producers in the case of costless trade. We consider a world economy with $n = 50$ symmetric locations with parameter values $\theta = 0.20$, $\alpha = 0.002$.

of the elasticity of substitution η. Recall that the growth rate of the world economy is unaffected by the introduction of finite trade cost, as long as $\kappa > 0$, so the ratios on these panels should be interpreted as level effects around the common balanced growth path.

Figure 2: Impact and Long run effect of introducing trade cost



The top panel shows the effect of trade cost on per-capita income on impact (solid), and the long-run effect for various values of $\eta = 2, 4, 5$. The comparison, and the initial condition, is given by the model with costless trade, i.e., $\kappa = 1$. The bottom panel shows the effect of trade cost on the volume of trade, measured as import to GDP. The effect on impact is the same regardless of the value of η. We consider a world economy with $n = 50$ symmetric locations with parameter values $\theta = 0.20$, and $\alpha = 0.002$.

The solid lines showing the impact effects correspond to the familiar effects of trade cost in the Ricardian trade theory of Eaton and Kortum (2002) and Alvarez and Lucas (2007). In this case there is an analytical expression for the GDP of an economy relative to case of costless trade:[13]

$$\frac{C(\kappa)}{C(1)} = \frac{\left[1 + (n-1)\kappa^{\frac{1}{\theta}}\right]^{\theta}}{n^{\theta}}.$$

The output effects of trade cost depend only on θ, the country size, $1/n$, and the value of the trade cost κ. As it has been noted, this expression does not depend on the value of the substitution elasticity η. In contrast, in the long-run, once the distributions of productivity adjust, the value of η does matter, as showed by the dashed lines. These effects of trade costs on gdp are larger the more difficult it is to substitute domestic goods for imports. The long-run calculations include the effects of the changes in the distribution of productivity due to the diffusion of technology, which are the contribution of this paper. As trade cost increases, individuals in each country meet relatively more unproductive sellers, and therefore the good technologies diffuse more slowly.

This panel also shows that the difference between the effect on impact (solid line) and the long-run effects (any of the other lines) are extremely small in the neighborhood of costless trade. This is to be expected, since Proposition 11 shows that around the symmetric costless trade, trade cost have only second order effects on productivity. Indeed, Figure 1 shows that the lesson drawn from Proposition 11 applies for a large range of trade costs, say even trade cost as large as $\kappa \geq 0.5$.

The effect of trade cost on the volume of trade is shown in the bottom panel of Figure 2.[14] The impact effect and long-runs effects are defined as in the top panel. Note that the impact effect of trade is the same as in Alvarez and Lucas (2007), since the distribution of productivities is Frechet, and it is given by[15]

$$v = \frac{(n-1)\kappa^{1/\theta}}{1 + (n-1)\kappa^{1/\theta}}$$

The long run effect of trade cost on the volume of trade is smaller than its effect

[13]This formula follows from specializing equation (6.10) in Alvarez and Lucas (2007) to a world without intermediate goods, non-tradable goods, and tariffs.

[14]Total imports in country j are $I_j = \sum_{i=1, i\neq j}^{n} I_{ji}$ and volume of trade, defined as imports relative to GDP, is given by $v_j = 1/(1 + I_{jj}/I_j)$.

[15]See Alvarez and Lucas (2007) expression (6.11) for the case of $\beta = \omega = 1$ and $\alpha = 0$.

on impact. This is due to the fact that a higher trade cost leads to a distribution of productivity for potential producers with a thicker left tail and the same right tail (see Figure 1), i.e. they lead to larger dispersion of productivities. A larger dispersion of productivity is associated with larger gains from trade. In addition, the difference between the long-run and the impact effect is larger the lower the elasticity of substitution η. As discussed before, the distribution of productivities are independent of the value of η, but the gains from trade are not independent of η, and with a lower elasticity of substitution for any given κ there is more trade.

For the interpretation of the magnitude of trade cost, the reader should remember that, for simplicity, in our model *all* goods are tradable. Compare our model with Alvarez and Lucas (2007) which includes a non-tradable sector, one with infinite trade cost, and with a fixed (i.e. Cobb-Douglas) share of expenditure. Thus our model's closest counterpart in the data will be the volume of trade for the tradable sector. Alternatively one could introduce a non-tradable sector in our model, and match it with the volume of trade of the whole economy, as in Alvarez and Lucas (2007).[16]

In the Online Appendix we also explore the robustness of the welfare results presented above to the case with Bertrand competition. We conclude there that for small and medium size trade cost, i.e. $\kappa \geq 0.5$, the welfare difference between perfect and Bertrand competition is a pure level effect, i.e. independent of κ, both on impact and on the long run. We reach this conclusion by analyzing the case of 25 symmetric countries, each with two locations, so that the total number of locations are the same as in the previous examples. For each elasticity η, we compare the ratio of the consumption using Bertrand competition to perfect competition for a given common trade cost $\kappa < 1$, with the same ratio for a zero trade cost ($\kappa = 1$). These ratios were between .98 and .95. For large trade cost, $\kappa < 0.5$, the effect of changes in trade cost on this ratio is more significantly, although the pattern depends on the particular value of the elasticity of substitution, taking values between 0.87 and 0.99.

[16]For instance, suppose non-tradables have a share ξ of expenditures, with labor freely mobile across sectors. Furthermore, suppose a form of Balassa-Samuelson hypothesis where there is no diffusion in non-tradables. Then, the model applies literally to a fraction $1 - \xi$ of the economy, and the trade share for the whole economy will be $(1 - \xi)$ times the trade share of our. Yet a better model, which has a similar effect but that requires more analysis, is one where diffusion occurs across both tradables and non-tradeables. The effect on this model on trade volume will be similar, but the analysis of the dynamics of diffusion is more complicated. We skip the inclusion of non-tradables for simplicity.

Asymmetric Trade Barriers

In the previous exercises we illustrated the effect of symmetric trade barriers. We now explore the impact of unilateral trade barriers by considering a world economy consisting of n countries, $n-1$ of which face symmetric trade cost κ_1 when trading among themselves, and a single country that faces a relatively larger cost to trade from and to this country, $\kappa_n \leq \kappa_1$. We refer to the first group as the $n-1$ symmetric countries, and to the later as the single deviant economy. Given our choice of $n = 50$, the single deviant economy is a country of the size of Canada or South Korea.[17] We interpret the $n-1$ relatively open countries as developed economies. Following Alvarez and Lucas (2007), we calibrate their trade cost to $\kappa_1 = 0.75$.[18]

In Figure 3 we illustrate, for different levels on initial trade cost, how the balanced growth path of these $n-1$ economies is affected by changes in the cost of trade with the single deviant economy, κ_n. In the top panel we show the effect on the per-capita income of the $n-1$ symmetric countries (solid line) and the single deviant economy (dashed line). Similarly, in the bottom panel we show the effect on the volume of trade. Most of the impact occurs in the single deviant economy which has higher trade cost. For the $n-1$ symmetric economies the goods produced by the single deviant economy are a small fraction of their consumption. As before there are two effects on real consumption from reductions in trade costs. The first is the effect captured in the traditional trade model: i.e., individuals consume goods that are less costly. The second effect is that the distribution of productivity get better as domestic producers interact with more productive sellers.

Figure 4 displays the dynamic path following a trade liberalization of the single deviant economy whose initial balanced growth path is described in Figure 3, for three values of the pre-liberalization trade cost. In particular, Figure 4 displays the dynamic effects of a once-and-for-all trade liberalization in the single deviant economy, taking the form of a reduction of its trade costs to the level of the advanced economies. These dynamics are shown for three different initial conditions (pre-liberalization),

[17] For our benchmark parameter values this size of the single deviant economy is far from the theoretical *small open economy* limit discussed in Appendix C. In that limit case there should be no effect on GDP or volume of trade after impact. Instead for the size of this single deviant economy there is a non negligible dynamic effect.

[18] This value is a compromise between low values of κ obtained from indirect estimates using gravity equations and higher ones using direct evidence of transportation costs, e.g., freight charges, imputed time costs on cargo in transit.

corresponding to the balance growth path with three alternative values of trade cost $\kappa_n(0) = 0.05, 0.30, 0.50$. In the top panel we show the value of GDP per-capita relative to the case with costless trade for the single deviant economy in the pre-liberalization and in the first 20 years following the trade liberalization. The bottom panel shows the initial and post liberalization dynamics of the volume of trade. In the x-axis we show the years that elapse since the trade liberalization.

The main message from Figure 4 is that a large part of the output gains from a reduction in trade costs occur immediately. The distribution of productivity of the single deviant economy is not affected on impact, but this economy is no longer forced to rely on its own producers for most of the goods it consumed, and can therefore discontinue its most unproductive technologies. In the model, this effect happens immediately. This is the effect captured by the standard trade model.[19] Thereafter, the distribution of productivity continues to improve due to the diffusion. This affects the whole distribution with the exception of the right tail (see Proposition 7 and 8, and Figure 1). These effects are persistent. The half lives are 10 years and longer. The magnitude of the effect on the distribution on per-capita income will depend on the value of η. For instance, if η is close to $1/\theta+1$, per-capita income is only a function of the tail of the distribution, which is not affected by trade cost (see Proposition 9).

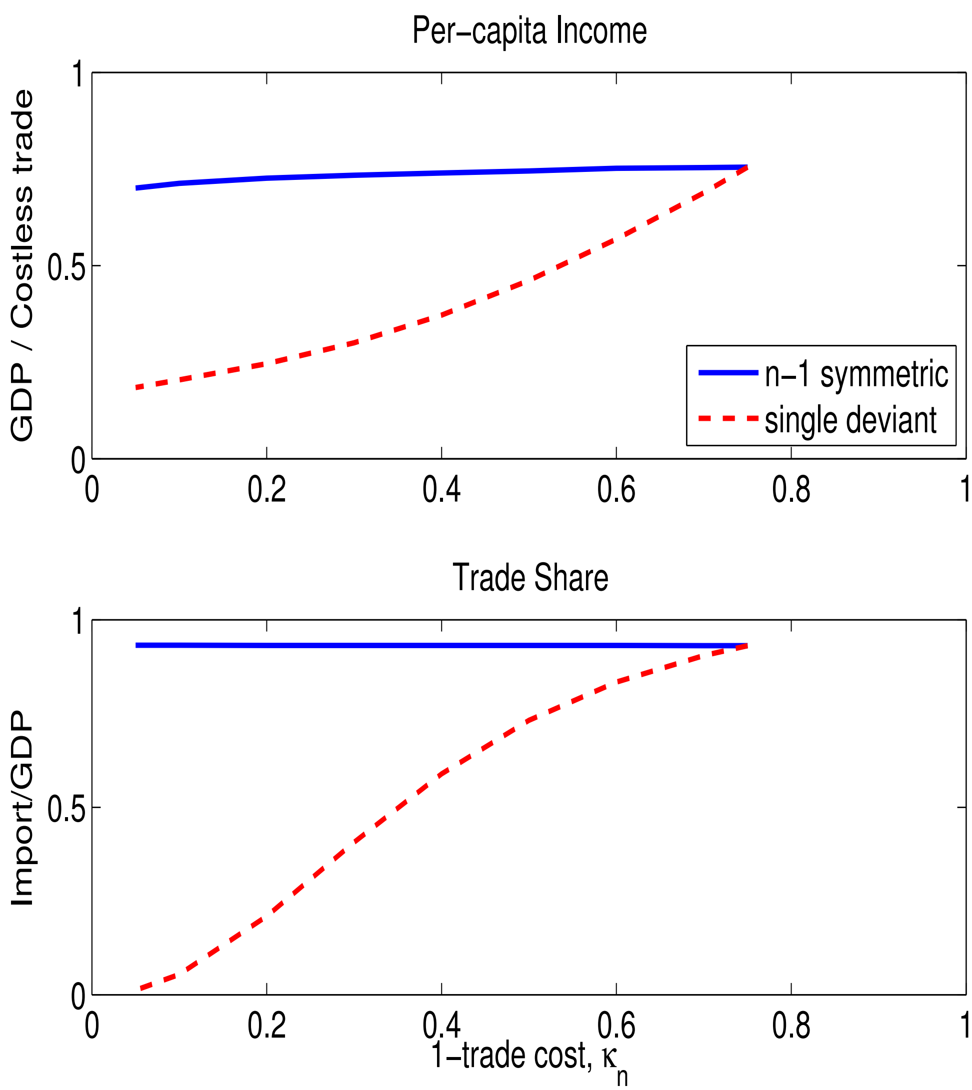
Heterogeneity in the Diffusion Rates α

In this section we explore the effect of cross country heterogeneity on the diffusion rate α of the long run relative real income levels. This analysis complements the cases of either large elasticity of substitution, $\eta \to 1/\theta + 1$, or a small open economy of Corollary 2 and Remark 2 in Appendix C, where we obtain the following analytical result: The ratio of long term real GDP equals $y_i/y_j = (\alpha_i/\alpha_j)^{\theta/(1+\theta)}$. We focus on a case where there are two values of the diffusion rate $\alpha^L < \alpha^H$, where half of the countries have one value and the other half the other, and where countries are identical in all other respects. We focus on costless trade so that the only departure from a Frechet distribution of productivities is due to the heterogeneity on the meeting rate. We conclude that the differences with the analytical cases discussed above are small.

Figure 5 plots the ratio of the long term real gdp levels for the countries with α^L

[19]These effects are larger than those predicted by a model with Frechet distribution (Eaton and Kortum, 2002), as the initial distribution of the single deviant economy has substantially more mass in the left tail than a Frechet distribution, similarly to the examples described in Figure 1.

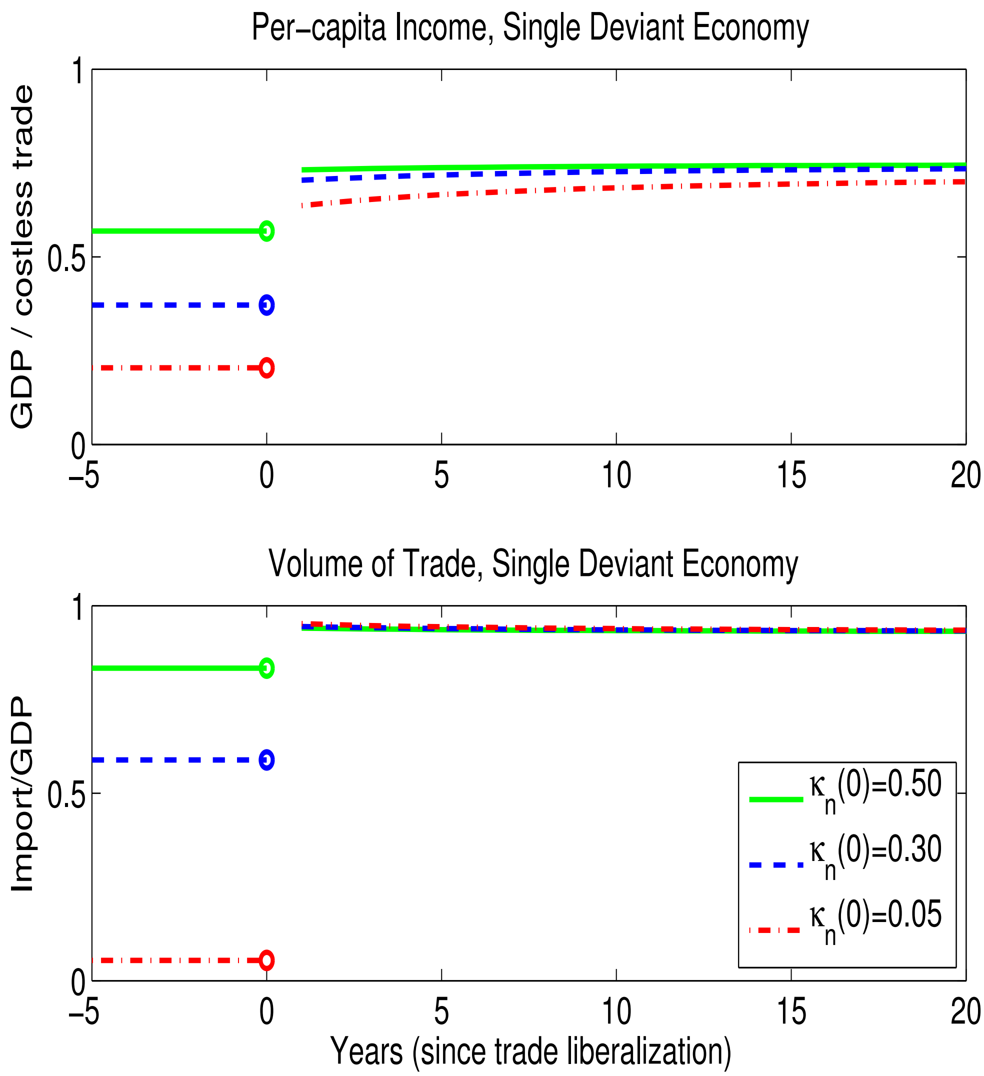
Figure 3: Long run effect of increasing trade costs in a single deviant economy



The top panel shows the effect of increasing the trade cost of a single deviant economy, κ_n, on per-capita income. The trade cost of the $n-1$ remaining countries is fixed at $\kappa_1 = 0.75$. The solid line shows the effect on the remaining $n-1$ symmetric countries. The dashed line shows the effect on the n^{th} single deviant economy. The per capita income are compared with the value they would have had if there trade cost will be zero in all countries, i.e. $\kappa_1 = \kappa_n = 1$. The bottom panel shows the effect of increasing the trade cost of the single deviant economy on the volume of trade, measured as imports to GDP. We consider a world economy with $n = 50$ countries. We use $\theta = 0.20$, $\alpha = 0.002$, $\eta = 3$.

relative to those with α^H. The rest of the parameters are $n = 50$, the growth rate is $\nu = 0.02$ and the value of $\theta = 0.2$. For each ratio of the α's we chose the sum of

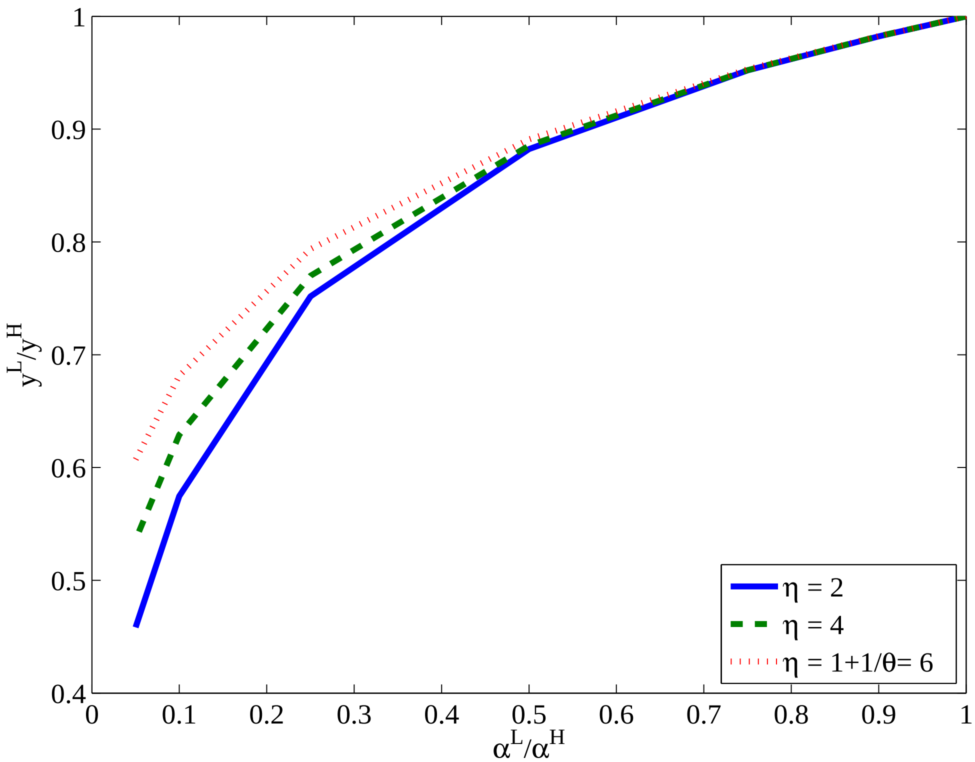
Figure 4: Transitional Dynamics Following a Reduction in Trade Cost of a Single Deviant Economy



The top panel shows the dynamics of per-capita GDP of the n^{th} originally deviant country following a reduction in trade cost, $\kappa_n(0) \rightarrow \kappa_n = 0.75$, for three initial levels of trade cost, $\kappa_n(0) = 0.05, 0.30, 0.50$. The trade cost of the $n-1$ symmetric countries is fixed at $\kappa_1 = 0.75$. Per-capita GDP is measured relative to the value in a world with costless trade, i.e., $\kappa_1 = \kappa_n = 1$. The bottom panel shows the corresponding dynamics of the volume of trade, measured as imports to GDP. We consider a world economy with $n = 50$ countries. We use $\theta = 0.20$, $\alpha = 0.002$, $\eta = 3$.

them so that the balanced growth rate is $\nu = 0.02$. We display the ratio of the α's in the horizontal axis. Each curve corresponds to a different value of the elasticity of

Figure 5: Effects of Heterogeneous α's on Per-Capita Income for Alternative Values of η.



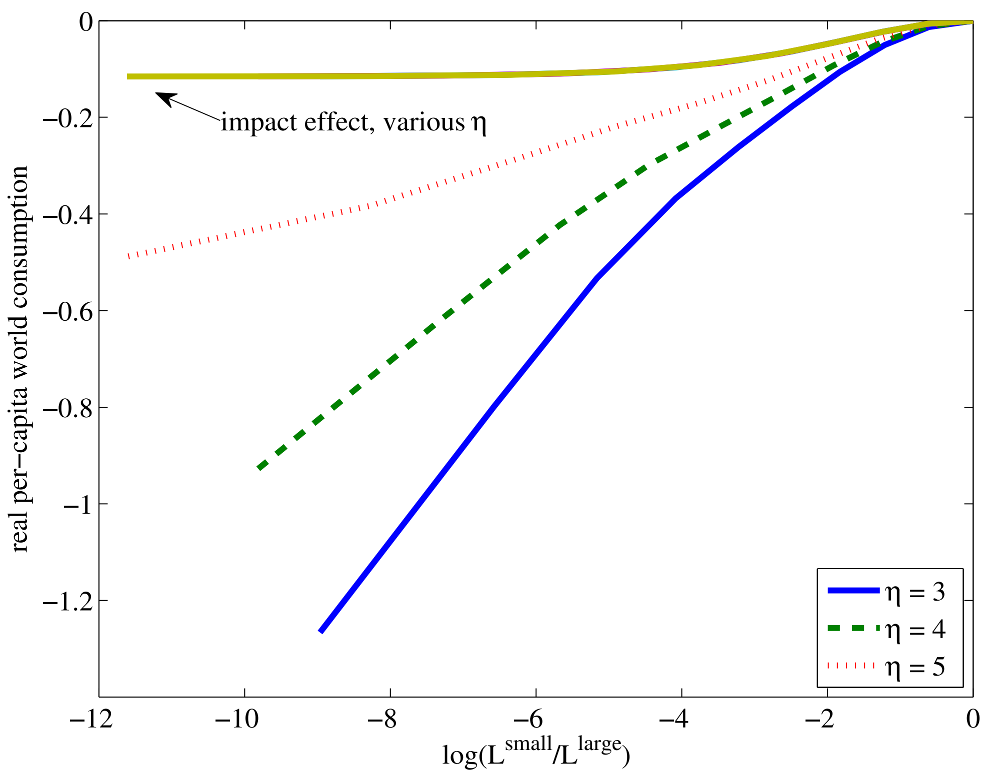
In the y-axis we plot the ratio of the long term real gdp levels for the countries with α^L relative to those with α^H. We display the ratio of the α's in the horizontal axis. Each curve corresponds to a different value of the elasticity of substitution η. The rest of the parameters are $n = 50$, the growth rate is $\nu = 0.02$ and the value of $\theta = 0.2$. For each ratio of the α's we chose the sum of them so that the balanced growth rate is $\nu = 0.02$.

substitution η.

We make two remarks on Figure 5 . First, the line for $\eta = 1/\theta + 1$ corresponds to the case of Corollary 2 and Remark 2 in Appendix C. Note that, for instance, if half of the countries have a diffusion rate ten times smaller, then the ratio of real incomes is about 60% of the one with the large diffusion rate. Instead if this diffusion rate is half, their real gpd is about 85% of the leaders. This is a numerical illustration of the comments right after Corollary 2. Second, the effect of a smaller η in relative gpd is small. In the case where the elasticity of substitution across goods is lower than the critical value, i.e., $\eta < 1/\theta + 1$, the effect beyond the right tail have to be taken into account. Yet, the quantitative effects, i.e., the vertical distance between the lines, are small.

Technology Diffusion and Wages

Figure 6: Effects of Heterogeneous Population Size's on Per-Capita World Income for Alternative Values of η.



The horizontal axis of Figure 6 has the ratio of the size of the countries, in log scale. The vertical axis plots the real aggregate per capital world consumption, as defined in (28). We normalize c for the static case to 1, and plots the log of this quantity. The solid line shows the impact effect, i.e., the effect when we take as given the initial Frechet distribution. The growth rate is $\nu = 0.02$ and the value of $\theta = 0.2$, and therefore, we set $\alpha = 0.05$. We set $L^{small} + L^{large} = 1$.

This section assesses the effect of different wages on the diffusion of technology. Recall that our assumption is that technology is diffused to the potential producers in a location from cost efficient producers located everywhere. Since sellers are determined by a comparison of relative cost of a particular good across locations, which depends on *wages*, thus differences in wages translate into difference in diffusion of technologies. In particular we explore the difference in diffusion due to difference in wages, itself caused by difference in country size, i.e. L_i. To isolate from other features we consider the case of no trade cost, i.e. $\kappa_{j,i} = 1$ and of only two large countries (or equivalently two group of countries, each group made of countries of the same size).

We fixed L, a vector of sizes normalized so that $\sum_i^n L_i = 1$, solve for the equilibrium wages $w(L)$, and consider the per-capita world real consumption as a measure of productivity, i.e.:

$$c(L) = \sum_i w(L)L_i/p\left(w(L)\right) \tag{28}$$

where $p(w(L))$ is the equilibrium common price level, which depends only on wages. We compare the effect of variations on relative size (and hence wages) on the real per capita world consumption both on impact and in the long run. For the impact effect we use that the distribution of productivities is Frechet. For the long run effect, which takes into account the effect on diffusion, we use the invariant distribution that solves equations (17) and (18). Thus the interpretation of the experiment is that we start from a world with two countries of the same size and compare the effects of "moving" people from one country to the other.[20]

The horizontal axis of Figure 6 has the ratio of the size of the countries, in log scale. The vertical axis plots the real aggregate per capital world consumption, as defined in (28). We normalize c for the static case to 1, and plots the log of this quantity. The solid line shows the impact effect, i.e., the effect when we take as given the initial Frechet distribution. As shown in Alvarez and Lucas (2007), in this case the per-capita world consumption equals:

$$c(L) = \lambda^{\theta/(1+\theta)} \sum_i L_i^{1/(1+\theta)}.$$

Clearly this expression is maximized when $L_i/\sum_j L_j = 1/n$, for all i. This is because we assume that costs are independently distributed across locations, thus if one region has a small share of the population it can only produce a small share of the products,

[20]The computations of these examples are facilitated greatly by using the converse of Proposition 3 stated and proven in the Online Appendix, so that we can actually compute the invariant distribution of productivity for a given w and then find the vector L that supports the equilibrium with balanced trade.

and hence requiring the more populous region to do it – which incidentally will be reflected in the equilibrium value of relative wages.

Our interest is on the additional difference between the impact effect and the long run effect. As explained in Proposition 11 there the effects are of second order for small change in wages. For change of wages of 50 %, i.e. those changes that correspond to different of size of about 92%, we found that the extra effect of diffusion is of 15 percentage points.

6 Conclusions

We have proposed and studied a new theory of cross-country technology diffusion, constructed by integrating two existing models: a static model of international trade based on the Ricardian framework introduced by Eaton and Kortum (2002) and a stochastic-process model of knowledge growth introduced in Kortum (1997) in which individuals get new ideas through their interactions with others. The new feature that connects these two models is a *selection effect* of international trade: Trade directly affects productivity levels by replacing inefficient domestic producers with more efficient foreigners and so increasing every country's contacts with best-practice technologies around the world.

The theory implies a long-run equilibrium in which all economies share a common, constant, endogenously determined growth rate, provided they are all connected in some degree through trade. Differences in trade cost will induce differences in income *levels* but not, in the long run, in rates of growth. This feature is shared with the von Neumann (1927) model and with the Parente and Prescott (1994) model of "barriers to riches." The transition dynamics following changes in trade costs, both world wide and by an individual country, are illustrated through stylized numerical examples. These dynamics are a mixture of static gains from trade that occur instantaneously under the trade model we use and gradual change that results from to changes in the intellectual environment that trade brings to individual countries. Improvements in technology arise from interactions among people who are brought together by the prospects of gains from trade and who get new ideas by adapting better technologies currently used in other locations and/or in the production of other goods.

The model of this paper is general enough to support a fairly realistic calibration to the world economy (as in Alvarez and Lucas (2007)) but our numerical illustrations here should not be viewed as an attempt to do this. The trade shares in the figures are much larger than those we observe. Adding a non-tradeables sector would remedy this, and would also reduce the size of the jump in production that follows a trade liberalization, but we have not done this. The model of technological change that we have adopted from Kortum (1997) is one of many possibilities—see, for example, the ones explored in Lucas and Moll (2011)—and we have not yet sought a parameterization that matches up to observations on actual catch-up growth. These are but two of the many directions that would be interesting to pursue further.

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A Interpretation of the Continuous Time Limit

For some readers the continuous time law of motion of $F(z,t)$ may seem odd, since for small Δ there are a "fractional" number of meetings. Here we show that our limit as Δ goes to zero can be regarded as simply an "extrapolation" of the law of motion to all values of t, with no change on the substance –provided the value of α is adjusted accordingly–, but with a simpler mathematical formalization. To see this consider the following discrete time law of motion for the right CDF of a closed economy:

$$F(z,j+1) = F(z,j)F(z,j) = F(z,j)^2, \text{ for all } j = 0,1,2,3,...$$

where we are measuring time in units so that there is *exactly* one meeting per period. In this case j is also the number of meetings since time zero. Continuing this way, and taking logs

$$\log F(z,j+1) = 2\log F(z,j) = 2^j \log F(z,0)$$

If we now measure time t in natural units (say years) and we assume that there are α' meetings per unit of time, we can write that j periods correspond to $t = j/\alpha'$ (years) and replacing in the previous expression

$$\log F(z,j) = 2^{\alpha' t} \log F(z,0)$$

Compare this with the continuous time limit we obtain in Section 2:[21]

$$\log F(z,t) = e^{\alpha t} \log F(z,0)$$

Thus both expression for the law of motion give identical expression (on integers values of t/α) if

$$\log(2) = \alpha/\alpha'.$$

In other words, the continuous time value of α has to be smaller than the discrete time value to take into account the "compounding" effect of the meetings, but otherwise they give the same answer.

[21]To be more precise, $\log \tilde{F}(z,t) = \log F(z,t\alpha') = e^{\alpha t} \log F(z,0) = e^{\alpha t} \log \tilde{F}(z,0)$.

B Additional Proofs

Proof of Proposition 3: To establish existence we show that the the excess demand system satisfies i) Walras' law, i.e. $\sum_{i=1}^{n} w_i Z_i(w) = 0$ for all w, ii) that the functions Z are continuous and homogenous of degree zero in w, iii) that $Z(w)$ are bounded from below, and iv) that $\max_j Z(w) \to \infty$ as $w \to w^0$ where w^0 is on the boundary of the n dimensional simplex.

Part (i) follows from replacing p_i in the expression for Z_i, (ii) continuity is immediate since the functions F_i are differentiable, and homogeneity is immediate by inspection of (12) and (13). For (iii), we can take $-\max_j L_j$ to be the lower bound. For (iv) we assume, without loss of generality, that $0 = w_1^0 \leq w_2^0 \leq \cdots \leq w_n^0 = 1$, and show that $Z_1(w) \to +\infty$. For any w we have

$$\begin{aligned} & Z_1(w) - L_1 \\ \geq \; & \left(\frac{w_n}{w_1}\right)^{\eta} \left(\frac{w_n}{p_n(w)}\right)^{1-\eta} L_n \, \kappa_{n1}^{\eta-1} \int_0^{\infty} z_1^{1-\eta} f_1(z_1) \prod_{k \neq 1} F_k \left(\frac{w_1 \kappa_{nk}}{w_k \kappa_{n1}} \; z_1\right) dz_1 \end{aligned}$$

Note that for all i we have

$$p_i \leq (w_n/\kappa_{in}) \left[\int_0^{\infty} z^{1-\eta} f_n(z) \; dz\right]^{1/(1-\eta)} ,$$

where the left hand side is the price that would be obtained by consumers in country i if they restrict themselves to buy only from country n. Considering $w = w^r$ we have that $w_n/p_n(w)$ is uniformly bounded from above by the previous expression, setting $i = n$ for all r large enough since $w_n^0 = 1$. Finally, for any $\epsilon > 0$, $w_1/w_k \leq 1 - \epsilon$ for all r large enough, and hence $F_k\left(\frac{w_1 \kappa_{nk}}{w_k \kappa_{n1}} \; z_1\right) > 0$ for all finite z_1. Using that $\eta > 1$ and taking limits we obtain the desired result. Given (i)-(iv), existence of an static trade equilibrium wage follows from Proposition 17.C.1 in Mas-Colell et al. (1995).

To establish the gross substitute property, since the excess demand system satisfies Walras' law, it suffices to show that $\partial Z_i(w)/\partial w_r > 0$ for all $i, r = 1, ..., n$ and $i \neq r$. First notice that $p_j(w)$ is increasing in each of the components of w and homogenous of degree one in w for all j. This implies that $w_r/p_r(w)$ is increasing in w_r. We have:

$$\begin{aligned}
\frac{\partial Z_i(w)}{\partial w_r} &= \sum_{j=1, j\neq r}^{n} \frac{\partial}{\partial w_r}\left[\left(\frac{w_i}{p_j(w)}\right)^{-\eta} \frac{w_j}{p_j(w)} L_j\right] \int_0^\infty \left(\frac{z_i}{\kappa_{ji}}\right)^{1-\eta} f_i(z_i) \prod_{k\neq i} F_k\left(\frac{w_i}{w_k}\frac{\kappa_{ik}}{\kappa_{ij}} z_i\right)\, dz_i \\
&+ \sum_{j=1, j\neq r}^{n} \left(\frac{w_i}{p_j(w)}\right)^{-\eta} \frac{w_j}{p_j(w)} L_j \int_0^\infty \left(\frac{z_i}{\kappa_{ji}}\right)^{1-\eta} f_i(z_i) \prod_{k\neq i} \frac{\partial}{\partial w_r}\left[F_k\left(\frac{w_i}{w_k}\frac{\kappa_{ik}}{\kappa_{ij}} z_i\right)\right] dz_i \\
&+ \frac{\partial}{\partial w_r}\left[\left(\frac{w_i}{p_r(w)}\right)^{-\eta} \frac{w_r}{p_r(w)} L_r\right] \int_0^\infty \left(\frac{z_i}{\kappa_{ri}}\right)^{1-\eta} f_i(z_i) \prod_{k\neq i} F_k\left(\frac{w_i}{w_k}\frac{\kappa_{ik}}{\kappa_{ir}} z_i\right)\, dz_i \\
&+ \left(\frac{w_i}{p_r(w)}\right)^{-\eta} \frac{w_r}{p_r(w)} L_r \int_0^\infty \left(\frac{z_i}{\kappa_{ri}}\right)^{1-\eta} f_i(z_i) \prod_{k\neq i} \frac{\partial}{\partial w_r}\left[F_k\left(\frac{w_i}{w_k}\frac{\kappa_{ik}}{\kappa_{ir}} z_i\right)\right] dz_i
\end{aligned}$$

For $j \neq r$, using that $\eta > 1$ and $p_j(w)$ in increasing, we get $\frac{\partial}{\partial w_r}\left[\left(\frac{w_i}{p_j(w)}\right)^{-\eta} \frac{w_j}{p_j(w)} L_j\right] > 0$. For $j = r$, using that $\eta > 0$, that $w_r/p(w)$ is decreasing in w_r we get that $\frac{\partial}{\partial w_r}\left[\left(\frac{w_i}{p_r(w)}\right)^{-\eta} \frac{w_r}{p_r(w)} L_r\right] > 0$. For $k = r \neq i$ we have that $\frac{\partial}{\partial w_r}\left[F_k\left(\frac{w_i}{w_k} z_i\right)\right] > 0$ since F_k is decreasing.

That w_i/w_j, relative wages of country i respect to any country j, are decreasing in L_i, follows from the strong gross substitute property. In particular, form an an application of the Hick's law of demand, since the excess demand of country i decreases with L_i, while the excess demand for any other country increases with L_i, –see, for example, first corollary of Theorem 3 in Quirk (1968). □

Proof of Proposition 8: Decomposing the time derivative of $F_i((e^{\alpha n t} z)^\theta, t)$ in the usual way, we have

$$\begin{aligned}
\frac{dF_i\left(\left(e^{(\nu/\theta)t} z\right)^\theta, t\right)}{dt} &= \frac{\partial F_i\left(\left(e^{(\nu/\theta)t} z\right)^\theta, t\right)}{\partial z} \frac{d\left(e^{(\nu/\theta)t} z\right)^\theta}{dt} + \frac{\partial F_i\left(\left(e^{(\nu/\theta)t} z\right)^\theta, t\right)}{\partial t} \\
&= (\nu/\theta)\, f_i\left(\left(e^{(\nu/\theta)t} z\right)^\theta, t\right) \theta \left(e^{(\nu/\theta)t} z\right)^{\theta-1} e^{(\nu/\theta)t} z \\
&\quad + \frac{\partial F_i\left(\left(e^{(\nu/\theta)t} z\right)^\theta, t\right)}{\partial t}
\end{aligned}$$

and so, dividing by $F_i(\left(e^{(\nu/\theta)t}z\right)^{\theta}, t)$ and applying (16) to the last term on the right,

$$\begin{aligned}\frac{d\log F_i\left(\left(e^{(\nu/\theta)t}z\right)^{\theta},t\right)}{dt} &= \frac{(\nu/\theta)\, f_i\left(\left(e^{(\nu/\theta)t}z\right)^{\theta},t\right)\theta\left(e^{(\nu/\theta)t}z\right)^{\theta-1}e^{(\nu/\theta)t}z}{F_i\left(\left(e^{(\nu/\theta)t}z\right)^{\theta},t\right)} \\ &\quad +\alpha_i\log G_i\left(\left(e^{(\nu/\theta)t}z\right)^{\theta},t\right).\end{aligned}$$

Let $x = e^{(\nu/\theta)t}z$ and multiplying through by x to obtain

$$\frac{d}{dt}\frac{\log F_i(x^{\theta},t)}{x^{-1}} = (\nu/\theta)\frac{f_i(x^{\theta},t)\theta x^{\theta+1}}{F_i(x^{\theta},t)} + \alpha_i\frac{\log G_i(x^{\theta},t)}{x^{-1}}.$$

Now let $x \to \infty$

$$-\frac{d}{dt}\lim_{x\to\infty} f_i(x^{\theta},t)\theta x^{\theta+1} = (\nu/\theta)\lim_{x\to\infty} f_i(x^{\theta},t)\theta x^{\theta+1} - \alpha_i \lim_{x\to\infty} g_i(x^{\theta},t)\theta x^{\theta+1},$$

where $\lim_{x\to\infty}\frac{d}{dt}\frac{\log F_i(x^{\theta},t)}{x^{-1}} = \frac{d}{dt}\lim_{x\to\infty}\frac{\log F_i(x^{\theta},t)}{x^{-1}}$ since $F_i(.,.)$ is assumed to be twice continuously differentiable. Reversing signs to conform to the definition of $\lambda_i(t)$,

$$\frac{d}{dt}\lambda_i(t) = -(\nu/\theta)\lambda_i(t) + \alpha_i\sum_{j=1}^{n}\lambda_j(t). \tag{B.1}$$

Summing both sides over i we have

$$\frac{d}{dt}\sum_{i=1}^{n}\lambda_i(t) = -(\nu/\theta)\sum_{i=1}^{n}\lambda_i(t) + \sum_{i=1}^{n}\alpha_i\sum_{j=1}^{n}\lambda_j(t) = 0\ ,$$

where the last equality uses the fact that $\nu = \theta\sum_{i=1}^{n}\alpha_i$. Then $\sum_i \lambda_i(t)$ stays constant and using the definition of λ_i^*, (B.1) implies that

$$\begin{aligned}\frac{d\lambda_i(t)}{dt} &= \frac{d\left(\lambda_i(t)-\lambda_i^*\right)}{dt} \\ &= -(\nu/\theta)\left(\lambda_i(t)-\lambda_i^*\right) - \frac{\nu}{\theta}\lambda_i^* + \alpha_i\sum_{j=1}^{n}\lambda_j(t) \\ &= -(\nu/\theta)\left(\lambda_i(t)-\lambda_i^*\right).\end{aligned}$$

Integrating gives equation (26). □

Proof of Proposition 9: We first remind the reader that for the case of a Frechet distribution

$$\tilde{I}_{ij}(\mathbf{K},\mathbf{w}) = \left(\frac{w_j}{\kappa_{ij}\,\tilde{p}_i}\right)^{1-\eta} w_i L_i \int_0^\infty \frac{\lambda_j}{\theta} y^{-1+\eta-1/\theta-1} \exp\left[-\sum_{k=1}^n \lambda_k \left(\frac{w_k \kappa_{ij}}{w_j \kappa_{ik}}\right)^{-1/\theta} y^{-1/\theta}\right] dy$$

doing a change of variables

$$\tilde{I}_{ij}(\mathbf{K},\mathbf{w}) = \left(\frac{w_j}{\kappa_{ij}}\right)^{-1/\theta} \left(\frac{1}{\tilde{p}_i}\right)^{1-\eta} w_i L_i \frac{\lambda_j}{\left[\sum_k \lambda_k \left(\frac{\kappa_{ik}}{w_k}\right)^{1/\theta}\right]^{1+\theta(1-\eta)}} \int_0^\infty t^{\theta(1-\eta)} \exp\left[-t\right]\, dt.$$

Similarly for the price index,

$$\tilde{p}_i^{1-\eta} = \frac{\int_0^\infty t^{\theta(1-\eta)} \exp\left[-t\right]\, dt}{\left[\sum_k \lambda_k \left(\frac{\kappa_{ik}}{w_k}\right)^{1/\theta}\right]^{1+\theta(1-\eta)}} \sum_{s=1}^n \lambda_s \left(\frac{w_s}{\kappa_{is}}\right)^{-1/\theta}.$$

Which implies

$$\tilde{I}_{ij}(\mathbf{K},\mathbf{w}) = \frac{\lambda_j \left(\frac{w_j}{\kappa_{ij}}\right)^{-1/\theta}}{\sum_{s=1}^n \lambda_s \left(\frac{w_s}{\kappa_{is}}\right)^{-1/\theta}} w_i L_i.$$

In the general case

$$\begin{aligned} I_{ij}(\mathbf{K},\mathbf{w};\eta) &= \left(\frac{w_j}{\kappa_{ij}\,p_i}\right)^{1-\eta} w_i L_i \int_0^\infty y^{-1+\eta} f_j(y) \prod_{k\neq j} F_k\left(\frac{w_k\kappa_{ij}}{w_j\kappa_{ik}}\, y\right) dy\ , \\ &= \left(\frac{w_j}{\kappa_{ij}}\right)^{1-\eta} w_i L_i \frac{\int_0^\infty y^{-1+\eta} f_j(y) \prod_{k\neq j} F_k\left(\frac{w_k\kappa_{ij}}{w_j\kappa_{ik}}\, y\right) dy}{\sum_{s=1}^s \left(\frac{w_s}{\kappa_{is}}\right)^{1-\eta} \int_0^\infty y^{-1+\eta} f_s(y) \prod_{k\neq j} F_k\left(\frac{w_k\kappa_{is}}{w_s\kappa_{ik}}\, y\right) dy} \end{aligned}$$

where the second equality follows by substituting the expression for the price level.

Defining

$$\underline{I}_{ij}(\underline{z}) = \lambda_j \left(\frac{w_j}{\kappa_{ij}}\right)^{-1/\theta} w_i L_i \times \frac{\int_0^{\underline{z}} \frac{1}{\theta} y \exp\left[-\sum_{k=1}^n \lambda_k \left(\frac{w_j \kappa_{ik}}{w_k \kappa_{ij}}\right)^{1/\theta} y^{-1/\theta}\right] M_{ij}(y)dy}{\sum_{s=1}^n \lambda_s \left(\frac{w_s}{\kappa_{is}}\right)^{-1/\theta} \int_0^{\underline{z}} \frac{1}{\theta} y \exp\left[-\sum_{k=1}^n \lambda_k \left(\frac{w_s \kappa_{ik}}{w_k \kappa_{is}}\right)^{1/\theta} y^{-1/\theta}\right] M_{is}(y)dy}$$

where

$$M_{ij}(z) = \frac{f_j(z) \prod_{k \neq j} F_k \left(\frac{w_k \kappa_{ij}}{w_j \kappa_{ik}} z\right)}{\frac{\lambda_j}{\theta} z^{-1/\theta+1} e^{-\lambda_j z^{-1/\theta}} \prod_{k \neq j} e^{-\lambda_k \left(\frac{w_j \kappa_{ik}}{w_k \kappa_{ij}}\right)^{1/\theta} z^{-1/\theta}}}$$

a function that converges to 1 as $z \to \infty$ given our assumption on the behavior of f_j. Furthermore, notice that

$$\lim_{\underline{z} \to \infty} \frac{\int_0^{\underline{z}} \frac{1}{\theta} y \exp\left[-\sum_{k=1}^n \lambda_k \left(\frac{w_j \kappa_{ik}}{w_k \kappa_{ij}}\right)^{1/\theta} y^{-1/\theta}\right] M_{ij}(y)dy}{\int_0^{\underline{z}} \frac{1}{\theta} y \exp\left[-\sum_{k=1}^n \lambda_k \left(\frac{w_s \kappa_{ik}}{w_k \kappa_{is}}\right)^{1/\theta} y^{-1/\theta}\right] M_{is}(y)dy} = 1.$$

which implies

$$\lim_{\eta \to 1/\theta+1} I_{ij}(\mathbf{K}, \mathbf{w}, \eta) \equiv \lim_{\underline{z} \to \infty} \underline{I}_{ij}(\underline{z}) = \frac{\lambda_j \left(\frac{w_j}{\kappa_{ij}}\right)^{-1/\theta}}{\sum_{s=1}^n \lambda_s \left(\frac{w_s}{\kappa_{is}}\right)^{-1/\theta}} w_i L_i = \tilde{I}_{i,j}(\mathbf{K}, \mathbf{w}).$$

From this follows that the equilibrium wages are the same. The result for the ratio of price levels follows a similar argument. Finally, the claim for the ratio of the real GDP follows from the previous two results. □

C Multiple Locations per Country

In order to clarify the role of scale effects and the interpretation of countries of different sizes we introduce the notion of a location within a country. We consider a world economy consisting of n countries, where each country i contains m_i locations. To simplify the analysis, we assume that a country is defined by a set of locations satisfying the following conditions:[22]

1. within each country there is a common labor market across the m_i locations;
2. there are no trading cost between locations within a country;
3. locations within a country face the same trading cost when trading with locations in other countries.

Denoting by $L_{i,l}$ the labor force in location l of country i, we can write the labor force of country i as

$$L_i = \sum_{l=1}^{m_i} L_{i,l}. \tag{C.1}$$

Since we assume that all locations within a country share the same labor market, there is a unique wage w_i for all location within a country. Likewise, given that there are no transportation cost within a country and all locations within a country face the same trading cost when trading with locations in other countries, all locations within a country face the same prices for all goods.

Denoting by $F_{i,l}(z,t)$ the distribution of cost in location l of country i, we can write the distribution of best practices in country i as

$$F_i(z,t) = \prod_{l=1}^{m_i} F_{i,l}(z,t). \tag{C.2}$$

Notice that the right hand size of (C.2) is the distribution of the minimum labor requirement over all locations within a country. This follows from the fact that all locations within a country share the same wages, there is no transportation cost between location within a country, and that all locations within a country share the same transportation cost vis-a-vis all other countries. Thus, the distribution $F_i(z,t)$ of cost of a country is all we need to know to calculate a static trade equilibrium. In particular, given the distribution of cost in each country and the size of the labor force of each country, L_i, we can calculate a static trade equilibrium as described in

[22]It is straightforward to extend the analysis to the case where labor is not mobile across location and there are arbitrary transportation costs across locations. In this case an equilibrium is given by a wage vector of dimension $\sum_{i=1}^{n} m_i$.

Section 3. In particular, the equilibrium values of $(w_i(t), p_i(t), C_i(t))_{i=1,...,n}$ are only function of the $(F_i(\cdot, t), L_i)_{i=1,...,n}$ and independent of the decomposition of countries into locations as long as equations (C.1) and (C.2) hold.

Assuming that the arrival rate of ideas in location l of country i equals $\alpha_{i,l}$, we can aggregate the evolution of best practices of all locations in a country to obtain the law of motion of best practices in country i:

$$\sum_{l=1}^{m_i} \frac{\partial \log (F_{i,l}(z,t))}{\partial t} = \sum_{l=1}^{m_i} \alpha_{i,l} \log [G_i(z,t)]$$

or

$$\frac{\partial \log (F_i(z,t))}{\partial t} = \alpha_i \log [G_i(z,t)]$$

where $\alpha_i = \sum_{l=1}^{m_i} \alpha_{i,l}$.

Furthermore, assuming that countries are aggregates of different different number of symmetric locations in terms of their population and number of technology managers, $L_{i,l} = L$ and $\alpha_{i,l} = \alpha$, we have that countries of different size are obtained by scaling their population $L_i = m_i L$ and the arrival rate of ideas $\alpha_i = m_i \alpha$.

It should be also clear that, provided that the structure of transportation cost and labor markets across locations is kept constant, an equilibrium of the model is invariant to arbitrary division of locations into countries. For instance, a country with m_i locations can be divided into m_i individual countries, each of them with a population of size L, receiving α ideas per period, and having a distribution of best practices $F_{i,l}(z,t) = (F_i(z,t))^{\frac{1}{m_i}}$.

We use the notion of locations to study the effect of changes in the arrival rate of product diffusions of a single small open into this country's output and welfare. To do so we first consider a limit set-up where all countries are identical and very small. Then we let one of these small economies to differ in its own meeting rate. Consider a sequence of worlds, each one indexed by n. We split the world economy into n identical locations, so that each country corresponds to one location, but where otherwise every other aspect of the world is kept the same. In particular fix $\bar{\alpha}$, $\bar{L}$ and a right cdf $\bar{F}(z,t)$. Using our previous result, for each world made of n countries let $L_i = \bar{L}/n$, $\alpha_i = \bar{\alpha}/n$ and $F_i(z,t) = \left(\bar{F}(z,t)\right)^{\frac{1}{n}}$ for each country $i = 1, ..., n$. Furthermore assume that $\kappa_{ij} = 1$ for all i, j regardless of the number of countries n. Clearly equilibrium wages are $w_i = 1$ for every country in every world with n countries. Moreover, for any n the stationary distribution of sellers, Γ_i in equation (18), is independent of i and is given by the same Frechet distribution with shape parameter θ and scale parameter λ derived from $\bar{F}(\cdot, 0)$. The common growth rate of each country is $\nu = \bar{\alpha}\theta$ for every n. The case of n small open economies is obtained when we let n be very large, so that each country i has negligible effect on common distribution Γ faced by each country. This is the same concept used in section 8 of Alvarez and Lucas (2007) to study optimal tariff rates.

Remark 2. Consider n small open economies, for large n. Assume that country 1 has $\alpha_1 < \bar{\alpha}/n \equiv \alpha_2 = \alpha_3 = \cdots = \alpha_n$. Since each economy is very small, the stationary distribution of sellers Γ in every country i is not affected by α_1. Equations (17)-(18) implies that the stationary distribution of country 1 is Frechet with the same shape parameter θ but with scale parameter $\lambda_1^* = (\alpha_1/\alpha_j)\,\lambda < \lambda = \lambda_j^*$, for $j \neq 1$. Relative GDP's equals the relative $\lambda's$ to the power $\theta/(1+\theta)$, and the relative $\lambda's$ equal the relative $\alpha's$. Moreover, we can study the dynamics of the effect of a permanent change in α_1 using Equation (26) from Proposition 8. Hence country's 1 real income level converges to $(\alpha_1/\alpha_j)^{\theta/(1+\theta)}$ at rate $\nu/\theta = \bar{\alpha}$ for $j \neq 1$.

Remark 2 and Corollary 2 give two different setups where relative meeting rates for diffusion opportunities, i.e. relative α's, determine exactly relative income levels.