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评价油井重复压裂前剩余可采储量的GM(1,1)模型

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摘　要　应用灰色系统理论，由油井重复压裂前的递减产量，生成相应的一阶累加序列，由累加序列建立了灰色预测模型GM(1,1)。据此模型研究油井由重复压裂前至废弃为止的生产动态。显然重复压裂前油井的日产油量在短期内可能波动较大，但经适当处理并从较长一段时间看，它有递减的趋势。经理论分析认为，生成的一阶累加序列有最大值，由此通过相关计算就可确定油井的剩余可采储量。此外，由于采用日产油量的累加序列建立模型，弱化了原始数据的随机波动性，强化了其规则有序性，而生成的一阶累加序列能更好地满足灰指数规律，适用于GM(1,1)模型计算。矿场实践表明，该模型所需原始数据少，但预测的精度高，可作为重复压裂选井选层的重要评判依据；也可与其它预测剩余可采储量的方法相互比较和确认，以便对重复压裂进行更加充分的论证。

主题词　灰色系统　模型　重复压裂　递减产量　可采储量

1　前　言

目前，国内绝大多数油气田已进入开发的中后期，作为增产挖潜、稳油控水的主要措施，重复压裂业已提到重要的议事日程上。要搞好重复压裂，首要的问题就是选井选层，而选井选层的关键便是对重复压裂井的剩余可采储量进行评价，它是压裂能够长期增产的物质基础。迄今为止，国内外评价剩余可采储量的常规方法大多采用物质平衡法，即先由不同有效厚度类别的单储系数及相应的有效厚度值及单井控制面积，算出单井控制地质储量后，与油田在目前开发水平下的采收率值相乘，再减去该井的累积产油量，便得出油井的剩余可采储量。该方法简单适用，故一直沿用至今。但这种方法也有一些缺点，因采收率值的大小对剩余可采储量的评价影响较大，而某一油田或区块的采收率值，大多用室内岩心驱替的办法作出的，由于所取岩心数量毕竟有限，能否代表整个油田或区块的整体情况尚未可知，此外，岩心取到地面，呈应力松驰状态，难以反映地下真实的应力状态，加上实验设备性能和人为因素的制约，因而由实验室得出的采收率值难免存在一定程度的偏差。鉴此，有必要对油井的剩余可采储量进行深入的研究和评估，尤其是新理论新方法的探讨，一方面能和传统方法相互比较和确认，另一方面能对重复压裂的选井选层进行更充分的论证，以保证取得更好的复压效果。本文正是基于这一思想，尝试用复压前短短几个月或几年的生产数据，预测剩余可采储量。

众所周知，油井重复压裂前的生产动态数据，尤其是日产油量，是一项综合的指标参数，它包含有效渗透率、有效厚度、地下流体性质参数、油藏非均质性及注水开发水平等众多因素的影响。可以认为，其随开发时间的变化是有一定规律的，有时即使因数据少而且波动，似乎无一定规律可循，但本文通过一阶累加序列建立日产油量的灰色模型，可达到弱化其随机性，强化其规律性的目的。这正是灰色建模理论的优越之处。相信它对目前国内油田重复压裂的选井选层工作，具有重要的指导作用和现实意义。

2　基本假设

(1)油井已进行过首次压裂，并且在重复压裂之前，未进行过任何增产措施(如洗井、解堵等)。

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(2)重复压裂前生产数据的选取,宜少不宜多,一般选取半年或一年内的生产数据。即使上下波动也不要紧,但总的趋势应是随生产时间的延长,产油量有降低的趋势。

(3)在选取计算的时间段内,对应注水井上未进行过重大举措,如小规模压裂、加强或限制日注水量等。

(4)重复压裂层位最好是全井段合压,如只是重复压裂一口油井中的部分层段,则重复压裂层段的日产油数据,一是根据该井的产液剖面资料,如无此项资料,则根据重复压裂层段的地层系数占全井总地层系数的百分比来确定。

3　GM(1,1)模型的特点

(1)模型具有微分、差分、指数兼容的性质,即性质是灰的(非唯一);

(2)模型参数是可调的,非唯一的,即参数是灰的;

(3)模型结构随时间而变,即结构是灰的;

(4)模型形似微分方程却不是一般微分方程;形似指数函数却不完全是指数函数,表明模型机理的非唯一性,即模型机理是灰的;

(5)模型是常系数性质的,但却排斥某些系数,因此,模型与参数的包涵关系不是唯一的,即模型参数分布是灰的。

4　灰色模型GM(1,1)的建立[1]

(1)先由下述时间序列

$\{Q_0^{(0)}(t), t=1,2,\cdots\cdots N\}$形成相应的一阶累加序列$\{Q_0^{(1)}(t), t=1,2,\cdots\cdots N\}$

其中

$$Q_0^{(1)}(t) = \sum_{K=1}^{t} Q_0^{(0)}(K)$$

(2)构造矩阵B和Y_N

$$B = \begin{bmatrix} -\frac{1}{2}[Q_0^{(1)}(1) + Q_0^{(1)}(2)] & 1 \\ -\frac{1}{2}[Q_0^{(1)}(2) + Q_0^{(1)}(3)] & 1 \\ \vdots & \\ -\frac{1}{2}[Q_0^{(1)}(N-1) + Q_0^{(1)}(N)] & 1 \end{bmatrix}$$

$$Y_N = [Q_0^{(0)}(2), Q_0^{(0)}(3)\cdots\cdots Q_0^{(0)}(N)]^T$$

(3)计算B^TB,并求B^TB的逆矩阵$(B^TB)^{-1}$

(4)计算B^TY_N

(5)计算参数列$\hat{a}$　　$\hat{a}=\begin{pmatrix} a \\ u \end{pmatrix}=(B^TB)^{-1}\cdot B^T\cdot Y_N$

(6)列出灰微分方程　　$\frac{\mathrm{d}Q_0^{(1)}}{\mathrm{d}t}+aQ_0^{(1)}=u$(7)求时间响应函数　　求解(6)中的灰微分方程,得

$$\hat{Q}_0^{(1)}(t+1) = \left(Q_0^{(1)}(0) - \frac{u}{a}\right)e^{-at} + \frac{u}{a}$$

式中可令$Q_0^{(1)}(0)=Q_0^{(1)}(1)$

只要原始数列$\{Q_0^{(0)}(t), t=1,2,\cdots\cdots N\}$总体趋势是随时间$t$的增加而递减的,那么解出的$a$值便是正的,

此时，$\left\{Q_0^{(1)}(0)-\frac{u}{a}\right\}$的值一般是负值，当$t\to\infty$时$e^{-at}\to 0$，即累加序列$\{\hat{Q}_0^{(1)}(t+1), t=0,1,2,\cdots\cdots\infty\}$的最大值就是$\frac{u}{a}$。如果日产油量是按月为单位统计的，那么剩余可采储量就是$30\left(\frac{u}{a}-Q_0^{(1)}(N)\right)$；如果日产油量是按年为单位统计的，那么剩余可采储量就是$365\left(\frac{u}{a}-Q_0^{(1)}(N)\right)$，依此类推。

5 应用举例及结果讨论

为了增强现场可操作性，列出阿南油田阿3-34井详细计算步骤，然后依此类推，给出了其余7口井的最后计算结果及其与常规物质平衡方法的计算结果对比。

（1）阿3-34井基本情况及生产数据

该井于1989年4月18日压裂全井段（13～26号层）投产，生产两年多后，于1991年8月6日重复压裂13～26层。该井满足上述一切假设，适于用GM(1，1)模型计算。本文选取重复压裂前7个月日产油量数据（表1）。

表1　阿3-34井重复压裂前7个月日产油量

Table 1　The daily oil production rate of A3-34 well seven months before refracturing

时间(mon)	1	2	3	4	5	6	7
日产油量(t/d)	11.6	11.2	10.5	10.0	6.9	8.3	10.8

（2）对原始序列$Q_0^{(0)}$作$1-AG_0$（一阶累加生成），即

$$
\begin{aligned}
Q_0^{(1)}(1) &= Q_0^{(0)}(1) = 11.6\\
Q_0^{(1)}(2) &= Q_0^{(1)}(1) + Q_0^{(0)}(2) = 11.6 + 11.2 = 22.8\\
Q_0^{(1)}(3) &= Q_0^{(1)}(2) + Q_0^{(0)}(3) = 22.8 + 10.5 = 33.3\\
Q_0^{(1)}(4) &= Q_0^{(1)}(3) + Q_0^{(0)}(4) = 33.3 + 10.0 = 43.3\\
Q_0^{(1)}(5) &= Q_0^{(1)}(4) + Q_0^{(0)}(5) = 43.3 + 6.9 = 50.2\\
Q_0^{(1)}(6) &= Q_0^{(1)}(5) + Q_0^{(0)}(6) = 50.2 + 8.3 = 58.5\\
Q_0^{(1)}(7) &= Q_0^{(1)}(6) + Q_0^{(0)}(7) = 58.5 + 10.8 = 69.3
\end{aligned}
$$

（3）构造数据矩阵B及数据向量Y_N

$$
B = \begin{bmatrix}
-\frac{1}{2}(Q_0^{(1)}(1)+Q_0^{(1)}(2)) & 1\\
-\frac{1}{2}(Q_0^{(1)}(2)+Q_0^{(1)}(3)) & 1\\
-\frac{1}{2}(Q_0^{(1)}(3)+Q_0^{(1)}(4)) & 1\\
-\frac{1}{2}(Q_0^{(1)}(4)+Q_0^{(1)}(5)) & 1\\
-\frac{1}{2}(Q_0^{(1)}(5)+Q_0^{(1)}(6)) & 1\\
-\frac{1}{2}(Q_0^{(1)}(6)+Q_0^{(1)}(7)) & 1
\end{bmatrix}
= \begin{bmatrix}
-17.2 & 1\\
-28.05 & 1\\
-38.3 & 1\\
-46.75 & 1\\
-54.35 & 1\\
-63.9 & 1
\end{bmatrix}
$$

$$
Y_N = (Q_0^{(0)}(2), Q_0^{(0)}(3), Q_0^{(0)}(4), Q_0^{(0)}(5), Q_0^{(0)}(6), Q_0^{(0)}(7))^T = (11.2, 10.5, 10, 6.9, 8.3, 10.8)^T
$$

（4）计算B^TB

$$
B^TB = \begin{pmatrix} 11772.2 & -248.6\\ -248.6 & 6 \end{pmatrix}
$$

(5) 求$(B^TB)^{-1}$

$$(B^TB)^{-1}=\begin{pmatrix}11772.2 & -248.6\\ -248.6 & 6\end{pmatrix}^{-1}=\begin{pmatrix}0.00068 & 0.0282\\ 0.0282 & 1.333\end{pmatrix}.$$

(6) 求 B^TY_N

$$B^TY_N=\begin{pmatrix}-2334.0\\ 57.7\end{pmatrix}$$

(7) 计算参数 $\hat{a}$　$\hat{a}=\begin{pmatrix}a\\ u\end{pmatrix}=(B^TB)^{-1}\cdot(B^TY_N)=\begin{pmatrix}0.04002\\ 11.095\end{pmatrix}$　即 $a=0.04002$　$u=11.095$

(8) 列出灰微分方程

$$\frac{\mathrm{d}Q_0^{(1)}}{\mathrm{d}t}+a\cdot Q_0^{(1)}=u \text{ 得 } \frac{\mathrm{d}Q_0^{(1)}}{\mathrm{d}t}+0.04002\cdot Q_0^{(1)}=11.095$$

(9) 求时间响应函数　解灰微分方程时间响应函数

$$\hat{Q}_0^{(1)}(t+1)=\left(Q_0^{(1)}(0)-\frac{u}{a}\right)e^{-at}+\frac{u}{a}$$

令 $Q_0^{(1)}(0)=Q_0^{(0)}(1)=11.6$ 代入上式，得 $\hat{Q}_0^{(1)}(t+1)=-265.775e^{-0.04002t}+277.236$

(10) 求剩余可采储量 N_{PN} 本文剩余可采储量的定义是重复压裂前至日产油量为0时累积产油量。由(8)中可知当 $t\to\infty$ 时，$-265.775e^{-0.04002t}\to0$。此时，$N_{PN}=(277.236-69.3)\times30=6238$t。

按上述示例方法，本文计算了华北二连阿南油田阿 3、阿 11 两个断块共计 8 口油井在重复压裂前的剩余可采储量，并相应给出了按物质平衡方法计算出的结果(表 2)。两种方法计算结果基本吻合。

表 2　比较两种方法得出的剩余可采储量

Table 2　The comparision of remaining recoverable reserves estimated by two ways

井　号	剩余可采储量(t)		误差 (%)
	GM(1,1)模拟法	物质平衡法	
阿 11-5	6238	6980	−10.6
阿 11-1	14641	14771	−0.9
阿 3-34	7817	8217	−4.9
阿 3-58	20258	19533	+3.7
阿 3	29080	34941	−16.8
阿 3-35	9199	10085	−8.8
阿 3-32	4037	4296	−6.0
阿 3-46	7166	8563	−16.3
平　均	12304.5	12973.3	−5.16

6　结　论

1. 本文应用GM(1,1)灰色模型，初步探讨了评价油井在重复压裂前剩余可采储量的新方法。该方法建模所需原始数据少，并采取了将原始数据累加生成的方法，由生成数据来建立模型，这就弱化了数据的随机性，强化了规律性。与物质平衡法对比，结果基本吻合。可作为重复压裂选井选层的重要评判依据。

2. 采用GM(1,1)模型来预测油井的剩余可采储量时，如原始数据忽大忽小，甚至随时间呈上升趋势，此时应采取相应的处理办法，如可用三个月、半年或一年的平均值作为一个原始数据点，这样更增强了原始数据的规则和有序性，能极大地提高预测精度。

3. 建议今后在重复压裂选井选层时，尝试采用GM(1,1)模型，并力争用各种方法对油井重复压裂前的剩余可采储量进行对比和确认，以便取得更好的复压效果。

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NUMERICAL SIMULATION OF WATERFLOODING FOR HETEROGENEOUS OIL RESERVOIR IN LAMINA SCALE **ACTA** 1998,19(1)

Yu Qitai et al. (*Scientific Research Institute of Petroleum Exploration and Development, Beijing*)

At present, the studies of waterflooding and distribution of remaining oil that from "Mega" of meter-hundred meters scale turns to "Macro" of centimeter-decimeter scale is the developing tendency for studying reservoirs and the devdeloping performance are in the high water stage. The numerical simulation water-flooding research for heterogeneous oil reservoir in lamina scale was conducted. An expert knowledge general models of heterogeneous oil reservoir in lamina scale were established. The depositional characteristics of actual reservoirs of these models were analysed, it is considerd that the reservoir of different rhythm types such as positive and opposite rhythm all consist of some small units of positive rhythm. Distribution water-flooding remaining oil in lamina scale was studied, 36 programs were calculated altogether. The effects of rhythmicity (positive and opposite rhythm), beddings (horizontal, oblique and cross bedding), oil viscosity, ratio of vertical and horizontal permeability, permeability, permeability contrast and capillary pressure on water displacement performance and distribution of waterflooding remaining oil were studied. The obtained results showed that the oil viscosity is a very important factor to effect waterflooding characteristic of reservoir, and waterflooding effect for oblique and cross beddings are better than horizontal bedding. Lamina controls subtle distribution of remaining oil which have guiding sense for developing remaining oil in centimeter scale.

Key words lamina rhythy heterogeneity water displacement performance remaining oil numerical simulation

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THE GM(1,1) MODEL OF EVALUATING OIL WELL REMAINING RECOVERABLE RESERVES BEFORE REFRACTURING **ACTA** 1998,19(1)

Jiang Tingxue et al. (*Wanzhuang Branch of Research Institute of Petroleum E.&D.*)

A grey predicting model of GM(1,1) can be established with the accumulated sequence generated by the oil well decline rate before refracturing according to the theory of grey system. The model can be applied to the study of the oil wells production performance to its abandonment rate. Although the oil well's production rate may fluctuate in short term before refracturing, it has a decline tendency during a longer time if the source data are processed properly. Theoretically, if the data are declined, the accumulated sequence must have a maximum value from which the remaining recoverable reserves may be deduced. On the other hand, the data used in the model are accumulated sequence of daily oil production rate, so that the generated one become more regular, and it satisfied grey exponential law, as is suitable to be calculated by grey model of GM(1,1). It has been testified by field cases that the model needs less source data, but its precision is relatively high. It is possible to be used as a basis in selecting refracturing wells and payzones. Comparision and confirmation can be made with other estimation of remaining recoverable reserves in order to demonstrate refracturing more completely.

Key words grey system model refracturing decline rate recoverable reserve

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THE FULLY COUPLED MATHEMATICAL MODEL OF THE FLUID-SOLID IN AN OIL RESERVOIR AND ITS FINITE ELEMENT EQUATIONS **ACTA** 1998,19(1)

Dong Pingchuan et al. (*Northeast University, Shenyang*)

A numerical model, based on the finite element method, is established to describe a fluid flow in a deforming saturated oil reservoir. The mathematical formulation describes a fully coupled governing equation system which consists of the equilibrium of rock skeleton and continuity for a fluid flow in a porous medium. An elasto-plastic model is utilized on a basis of Mohr-Coulomb yield surface. The Galerkin finite element method is applied to obtain simultaneous solutions in the space domain to the governing equations where the displacements and the fluid pressures are the primary unknowns. The final discretized equations are sovled in the time domain by using a fully implicit numerical scheme and the fully coupled finite element equations are obtained. A linear analysis is used to study the stability conditions of the present model. This model has a wide range of applications in the field of