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基于区间二元语义的动态灰色关联群决策方法及应用

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摘 要: 在动态群决策中属性值往往为区间语言信息,而数值决策模型难以处理,鉴于此,构建了一种动态灰色关联群决策模型。首先利用区间二元语义信息的运算法则与性质,集结信息评价矩阵;针对时间权重和专家权重已知而属性权重信息完全未知的情形,设计各时间段的正、负理想方案,以与正理想方案灰色关联度偏差最小化为目标构建多目标规划模型,确定属性权重;基于各时间段各方案对正、负理想方案的区间二元语义灰色关联度,构建方案优属度的优化模型,进而获得方案优属度的表达式。最后以案例验证了所提方法的有效性和可行性。结果表明,所提方法能够较为精确地处理语言信息,在一定程度上克服基于以往语言信息处理方法造成的信息扭曲和损失。

关键词: 灰色关联分析; 区间二元语义; 最小偏差; 优属度

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Dynamic grey incidence group decision making methodology based on interval two-tuple linguistic information processing and application

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Abstract: With respect to the problem that index value is ofter given in the form of interval linguistic assessment information in the dynamic multiple attribute group decision making, a novel dynamic grey incidence group decision making methodology based on interval two-tuple linguistic information is proposed. Firstly, the algorithms and properties of the interval two-tuple linguistic information are used to aggregate the information evaluation matrix. Secondly, the positive and negative ideal schemes for each time period are designed in order to deal with the problem that the expert weights and time weights are known, while index weights are unknown, so that the multi-objective programming model with the minimum deviation of the grey incidence degree between each scheme and the positive ideal scheme for each stage is built to determine the index weights. Thirdly, the grey incidence each stage degree with interval two-tuple linguistic information between each scheme and the positive and negative ideal scheme for each stage is calculated to establish the optimization model of the optimal membership degree for scheme, so that the expressions form of the optimal membership degree for scheme is determined to solve the optimal membership degree for scheme. Finally, an example validates the feasibility and effectiveness of the novel model. The results show that the proposed method can more precisely deal with the language information and avoid information distortion and loss based on the previous method of processing the language.

Keywords: grey incidence analysis; interval two-tuple linguistic information; minimum deviation; optimal membership degree

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0 引 言

在现实决策中，由于决策问题自身的复杂性和决策信息的模糊性、不确定性，决策者面对不确定性需要采取科学合理的决策方法进行决策，而采用过于主观的定性决策方法和非常客观的定量化决策方法去描述决策信息，往往造成决策信息丢失和扭曲，使得决策结果与事实不符。为解决这一决策问题，许多学者采用定性的语言，构建基于语言评价信息的群决策方法，处理现实的决策问题。然而对于语言评价信息的处理，早期主要是基于扩展原理的分析方法和基于符号转移的方法，这往往会造成信息丢失和扭曲，为了更有效地处理决策信息、减少信息的流失和误导，文献[1]针对提出的不同语言评价集给出各自的语言评价信息，并给出二元语义信息的融合方法[2-3]，该方法很快应用到决策分析、工程评价、信息检索、绩效评价、数据挖掘、风险评估、推荐系统等领域。

目前，二元语义在决策领域的运用主要体现在两个方面，一个方面是基于二元语义的信息表达，进行决策和决策分析；另一方面是利用二元语义，构建模糊系统，进行模糊建模，进而进行方案排序和选择最优方案。基于二元语义进行决策和决策分析的相关研究主要是从群决策[4-20]、多标准决策[11-13,15,20-26]、信息集结[2-3,12,14,18,21,27-35]、构建评价模型[36-37]等几个方面展开，对于基于二元语义的模糊建模，由于篇幅所限，不再赘述。纵观基于二元语义决策问题的相关文献，其大多是针对属性权重已知的静态问题的研究，对于具有区间语言评价信息的动态多属性群决策问题鲜有研究，尤其是针对时间权重和专家权重已知而属性权重信息完全未知的情形更是少之又少。鉴于此，本文提出了一种基于区间二元语义的动态灰色关联群决策模型。利用灰色系统理论的思想和方法，分别构建与正理想解灰色关联度偏差最小的规划模型和方案优属度优化模型，用以确定指标权重和方案优属度，并通过案例验证了所提模型的有效性和可行性。

1 预备知识

作为一种有效处理语言信息的方法，二元语义以短语和实数值的二元形式来反映语言信息集结信息，其能有效避免在语言评价信息集结和运算过程中造成的信息损失和扭曲，使得运算结果更客观准确[2-3]。

定义 1[2] 设 $s_k \in S$ 为语言短语，则有

$$\theta: S \to S \times [-0.5, 0.5) \tag{1}$$

$$\theta(s_i) = (s_i, 0), s_i \in S \tag{2}$$

定义 2[2] 设 $\beta \in [0, T]$ 为语言评价集 S 集结后的数值，T 为语言评价集 S 中的元素个数，则有

$$\Delta: [0, T] \to S \times [-0.5, 0.5) \tag{3}$$

$$\Delta = \begin{cases} s_k, \ k = round(\beta) \\ a_k = \beta - k, \ a_k \in [-0.5, 0.5) \end{cases} \tag{4}$$

式中，$round$ 表示取整算子，遵循四舍五入原则。

定义 3[2] 设 (s_k, a_k) 为二元语义，则有

$$\Delta^{-1}: S \times [-0.5, 0.5) \to [0, T] \tag{5}$$

$$\Delta^{-1}(s_k, a_k) = k + a_k = \beta \tag{6}$$

式中，s_k 为语言评价集 S 中的第 k 个元素；$a_k \in [-0.5, 0.5)$；Δ^{-1} 为逆函数。

对两个二元语义 (s_k, a_k) 和 (s_l, a_l) 进行比较，有如下规定[9]：

(1) 若 $k < l$，则 $(s_k, a_k) < (s_l, a_l)$。

(2) 若 $k = l$

① $a_k = a_l$，则 $(s_k, a_k) = (s_l, a_l)$；

② $a_k < a_l$，则 $(s_k, a_k) < (s_l, a_l)$；

③ $a_k > a_l$，则 $(s_k, a_k) > (s_l, a_l)$。

下面给出区间二元语义的概念及运算法则。

定义 4[2-3] 设 $(\underline{s}_k, \underline{a}_k)$，$(\overline{s}_k, \overline{a}_k)$ 为 2 个二元语义信息，$\underline{s}_k, \overline{s}_k \in S^{\mathrm{T}}$，$\underline{a}_k, \overline{a}_k \in [-0.5, 0.5)$，且 $(\underline{s}_k, \underline{a}_k) \leqslant (\overline{s}_k, \overline{a}_k)$，则称 $(s_k, a_k) = [(\underline{s}_k, \underline{a}_k), (\overline{s}_k, \overline{a}_k)]$ 为一个区间二元语义。

定义 5[2-3] 设 (s_k, a_k) 和 (s_l, a_l) 为任意 2 个区间二元语义信息，则规定运算法则如下：

(1) 当且仅当 $(\underline{s}_k, \underline{a}_k) = (\underline{s}_l, \underline{a}_l)$ 和 $(\overline{s}_k, \overline{a}_k) = (\overline{s}_l, \overline{a}_l)$ 成立时，$(s_k, a_k) = (s_l, a_l)$；

(2) $(s_k, a_k) + (s_l, a_l) = \{\Delta[\Delta^{-1}(\underline{s}_k, \underline{a}_k) + \Delta^{-1}(\underline{s}_l, \underline{a}_l)], \Delta[\Delta^{-1}(\overline{s}_k, \overline{a}_k) + \Delta^{-1}(\overline{s}_l, \overline{a}_l)]\}$；

(3) $\lambda(s_k, a_k) = \{\Delta[\lambda\Delta^{-1}(\underline{s}_k, \underline{a}_k)], \Delta[\lambda\Delta^{-1}(\overline{s}_k, \overline{a}_k)]\}$。

定义 6[2-3] 设 (s_k, a_k) 和 (s_l, a_l) 为任意 2 个区间二元语义信息，则称

$$p[(s_k, a_k) \geqslant (s_l, a_l)] = \max\left\{1 - \max\left[\frac{\Delta^{-1}(\overline{s}_l, \overline{a}_l) - \Delta^{-1}(\underline{s}_k, \underline{a}_k)}{D(s_k, a_k) + D(s_l, a_l)}, 0\right], 0\right\} \tag{7}$$

式中，$D(s_k, a_k) = \Delta^{-1}(\overline{s}_k, \overline{a}_k) - \Delta^{-1}(\underline{s}_k, \underline{a}_k)$；$D(s_l, a_l) = \Delta^{-1}(\overline{s}_l, \overline{a}_l) - \Delta^{-1}(\underline{s}_l, \underline{a}_l)$。

定义 7[2-3] 设 (s_k, a_k) 和 (s_l, a_l) 为任意 2 个区间二元语义信息，则它们之间的距离定义为

$$d((s_k, a_k), (s_l, a_l)) = \Delta\left[\sqrt[p]{\frac{|\Delta^{-1}(\underline{s}_k, \underline{a}_k) - \Delta^{-1}(\underline{s}_l, \underline{a}_l)|^p + |\Delta^{-1}(\overline{s}_k, \overline{a}_k) - \Delta^{-1}(\overline{s}_k, \overline{a}_k)|^p}{2}}\right] \tag{8}$$

当 $p=1$ 时，$d((s_k, a_k), (s_l, a_l))$ 为汉明距离；当 $p=2$ 时，$d((s_k, a_k), (s_l, a_l))$ 为欧氏距离。

定义 8[2-3] 设 $(s_j, a_j) = [(\underline{s}_j, \underline{a}_j), (\overline{s}_j, \overline{a}_j)] = \{(s_1, a_1), \cdots, (s_m, a_m)\}$ 是一组区间二元语义信息，$\boldsymbol{l} = (l_1, l_2, \cdots, l_m)$ 为相应的权重向量，且 $l_j \in [0, 1]$，$\sum_{j=1}^{m} l_j = 1$，则称式(9)为基于区间二元语义的加权算术平均算子。

$$(\tilde{s}, \tilde{a}) = \Delta\left\{\left[\sum_{j=1}^{m} l_j \Delta^{-1}(\underline{s}_j, \underline{a}_j)\right], \left[\sum_{j=1}^{m} l_j \Delta^{-1}(\overline{s}_j, \overline{a}_j)\right]\right\} \tag{9}$$

2 基于区间二元语义的动态灰色关联群决策方法

2.1 问题描述

设 $S=(E,U,A,V_{t_k})$ 为二元语义动态多指标问题。其中，$E=\{E_1,E_2,\cdots,E_s\}$ 为专家有限非空集；$U=\{U_1,U_2,\cdots,U_n\}$ 为方案有限非空集；$A=\{a_1,a_2,\cdots,a_m\}$ 为指标 a_j 的有限非空集；$V_{t_k}=\bigcup r_{ij}^{k,r}$ 为决策者 E_r 对于方案 U_i 在第 k 阶段指标 a_j 的值域，且 $r_{ij}^{k,r}\in S$，从而得到决策者 E_r 在 k 阶段的语言决策矩阵 $\boldsymbol{R}_r^k=(r_{ij}^{k,r})_{n\times m}$，利用式(2)将语言决策矩阵 $\boldsymbol{R}_r^k=(r_{ij}^{k,r})_{n\times m}$ 转化为二元语义决策矩阵 $\boldsymbol{R}_r^k=(v_{ij}^{k,r},0)_{n\times m}$；$\boldsymbol{\eta}=(\eta_1,\eta_2,\cdots,\eta_m)$ 为指标权重向量，且 $\eta_j\geqslant 0$，$\sum_{j=1}^{m}\eta_j=1$；$\boldsymbol{L}=(L_1,\cdots,L_t\cdots,L_s)$ 为专家权重向量，且 $\sum_{r=1}^{s}L_r^k=1$，$L_t^k\geqslant 0$；$\boldsymbol{w}=(w_1,\cdots,w_k,\cdots,w_p)$ 为时间权重向量，且 $w_k\geqslant 0\sum_{k=1}^{p}w_k=1$。

2.2 基于二元语义的综合评价信息集成

基于专家给出的语言评价信息，首先对其标准化处理，利用式(9)的加权平均算子集结，求得 k 时段群的矩阵 $\boldsymbol{R}^k=(r_{ij}^k,a_{ij}^k)_{n\times m}$。

$$(r_{ij}^k,a_{ij}^k)=\Delta\left[\sum_{r=1}^{s}w_r^k\Delta^{-1}(r_{ij}^k,a_{ij}^k)\right]=\Delta\left[\sum_{r=1}^{s}\frac{1}{s}\Delta^{-1}(r_{ij}^k,a_{ij}^k)\right] \tag{10}$$

式中，$r=1,2,\cdots,s$；$i=1,2,\cdots,n$；$j=1,2,\cdots,m$；$k=1,2,\cdots,p$；(r_{ij}^k,a_{ij}^k) 表示方案在专家 E_r 在 k 时段的关于指标 a_j 的群的综合评价值。

2.3 动态灰色关联群决策方法

灰色关联分析是基于灰关联空间而建立的一种分析方法，其基本思想是根据曲线间变化大小的接近性和相似程度来判断因素间的关联程度[38]。

定义 9 设

$$(r_j^{+k},a_j^{+k})=\max_i(r_{ij}^k,a_{ij}^k)=\max\{(r_{ij}^k,a_{ij}^k)\mid 1\leqslant i\leqslant n\} \tag{11}$$

$$(r_j^{-k},a_j^{-k})=\min_i(r_{ij}^k,a_{ij}^k)\min\{(r_{ij}^k,a_{ij}^k)\mid 1\leqslant i\leqslant n\} \tag{12}$$

称方案

$$\boldsymbol{U}_{t_k}^{+}=(r_j^{+k},a_j^{+k})=\{(r_1^{+k},a_1^{+k}),\cdots,(r_m^{+k},a_m^{+k})\} \tag{13}$$

和

$$\boldsymbol{U}_{t_k}^{-}=(r_j^{-k},a_j^{-k})=\{(r_1^{-k},a_1^{-k}),\cdots,(r_m^{-k},a_m^{-k})\} \tag{14}$$

分别为 k 阶段的正和负理想方案。

根据灰色关联方法，可计算在 k 阶段方案的正、负理想关联系数。利用式(7)可求得 k 时段各方案的正、负理想关联系数 $(\varepsilon_{ij}^{+k},\delta_{ij}^{+k})$ 和 $(\varepsilon_{ij}^{-k},\delta_{ij}^{-k})$，表示为

$$(\varepsilon_{ij}^{+k},\delta_{ij}^{+k})=\frac{\min_i\min_j D_{ij}^{+k}+\rho\max_i\max_j D_{ij}^{+k}}{D_{ij}^{+k}+\rho\max_i\max_j D_{ij}^{+k}} \tag{15}$$

$$(\varepsilon_{ij}^{-k},\delta_{ij}^{-k})=\frac{\min_i\min_j D_{ij}^{-k}+\rho\max_i\max_j D_{ij}^{-k}}{D_{ij}^{-k}+\rho\max_i\max_j D_{ij}^{-k}} \tag{16}$$

式中，$\rho\in[0,1]$ 为分辨系数，取 $\rho=0.5$。

$$D_{ij}^{+k}=|\Delta^{-1}(r_{ij}^k,a_{ij}^k)-\Delta^{-1}(r_j^{+k},a_j^{+k})|=\Delta\left[\sqrt{\frac{|\Delta^{-1}(\underline{r_{ij}^k},\underline{a_{ij}^k})-\Delta^{-1}(\underline{r_{ij}^{+k}},\underline{a_{ij}^{+k}})|^2+|\Delta^{-1}(\overline{r_{ij}^k},\overline{a_{ij}^k})-\Delta^{-1}(\overline{r_{ij}^{+k}},\overline{a_{ij}^{+k}})|^2}{2}}\right] \tag{17}$$

$$D_{ij}^{-k}=|\Delta^{-1}(r_{ij}^k,a_{ij}^k)-\Delta^{-1}(r_j^{-k},a_j^{-k})|=\Delta\left[\sqrt{\frac{|\Delta^{-1}(\underline{r_{ij}^k},\underline{a_{ij}^k})-\Delta^{-1}(\underline{r_{ij}^{-k}},\underline{a_{ij}^{-k}})|^2+|\Delta^{-1}(\overline{r_{ij}^k},\overline{a_{ij}^k})-\Delta^{-1}(\overline{r_{ij}^{-k}},\overline{a_{ij}^{-k}})|^2}{2}}\right] \tag{18}$$

根据 k 阶段正理想最优方案的区间二元语义关联系数矩阵 $(\varepsilon_{ij}^{+k},\delta_{ij}^{+k})$，正理想方案与其自身的灰色关联系数向量为 $(1,1,\cdots,1)$，可得 U_i 与正理想的综合关联度偏差之和的表达式为

$$pd_i(\boldsymbol{\eta})=\sum_{k=1}^{p}\sum_{j=1}^{m}[(1-(\varepsilon_{ij}^{+k},\delta_{ij}^{+k}))\eta_j]^2 \tag{19}$$

相应地，可建立优化模型为

$$\min\ pd_i(\eta)=\sum_{k=1}^{p}\sum_{j=1}^{m}[(1-(\varepsilon_{ij}^{+k},\delta_{ij}^{+k}))\eta_j]^2 \tag{20}$$

鉴于方案间都处于同一竞争水平，可将式(20)转化为

$$\min\ pd(\eta)=\sum_{i=1}^{n}pd_i(\eta)=\sum_{i=1}^{n}\sum_{k=1}^{p}\sum_{j=1}^{m}[(1-(\varepsilon_{ij}^{+k},\delta_{ij}^{+k}))\eta_j]^2$$
$$\text{s.t.}\quad\sum_{j=1}^{m}\eta_j=1 \tag{21}$$

对式(21)建立拉格朗日函数为

$$L(\eta_j,\lambda)=\sum_{i=1}^{n}\sum_{k=1}^{p}\sum_{j=1}^{m}[(1-(\varepsilon_{ij}^{+k},\delta_{ij}^{+k}))\eta_j]^2+2\lambda(\sum_{j=1}^{m}\eta_j-1) \tag{22}$$

由极值存在条件，可得

$$\begin{cases}\dfrac{\partial L}{\partial\eta_j}=2\sum_{i=1}^{n}\sum_{k=1}^{p}[(1-(\varepsilon_{ij}^{+k},\delta_{ij}^{+k}))^2\eta_j+(1-(\varepsilon_{ij}^{+k},\delta_{ij}^{+k}))]+2\lambda=0\\ \dfrac{\partial L}{\partial\lambda}=\sum_{j=1}^{m}\eta_j-1=0\end{cases} \tag{23}$$

解得

$$\begin{cases}\lambda=-\left[\sum_{j=1}^{m}\left(\sum_{k=1}^{p}\sum_{i=1}^{n}\left(1-\left(\varepsilon_{ij}^{+k},\delta_{ij}^{+k}\right)\right)^{2}\right)^{-1}\right]^{-1}\\ \eta_{j}=\left[\sum_{j=1}^{m}\left(\sum_{k=1}^{p}\sum_{i=1}^{n}\left(1-\left(\varepsilon_{ij}^{+k},\delta_{ij}^{+k}\right)\right)^{2}\right)^{-1}\right]^{-1}\left[\sum_{k=1}^{p}\sum_{i=1}^{n}\left(1-\left(\varepsilon_{ij}^{+k},\delta_{ij}^{+k}\right)\right)^{2}\right]^{-1}\end{cases}\tag{24}$$

由指标权重 $\boldsymbol{\eta}=(\eta_1,\eta_2,\cdots,\eta_m)$，利用式(9)计算 k 阶段各方案的正、负理想关联度 γ_i^{+k} 和 γ_i^{-k}，其计算公式为

$$\gamma_i^{+k}=(\varepsilon_i^{+k},\delta_i^{+k})=\Delta\left[\sum_{j=1}^{m}\eta_j\times\Delta^{-1}(\varepsilon_{ij}^{+k},\delta_{ij}^{+k})\right]\tag{25}$$

$$\gamma_i^{-k}=(\varepsilon_i^{-k},\delta_i^{-k})=\Delta\left[\sum_{j=1}^{m}\eta_j\times\Delta^{-1}(\varepsilon_{ij}^{-k},\delta_{ij}^{-k})\right]\tag{26}$$

当 γ_i^{+k} 的取值越大，方案 U_i 与正理想方案 $U_{t_k}^{+}$ 的接近水平越高，方案 U_i 则越好；γ_i^{-k} 越小，表明方案 U_i 越优。所以，被评价方案 U_i 应该与正理想方案的距离越近越好，与负理想方案的距离越大越好。设优属度 d_i 为方案 U_i 隶属于正理想方案 $U_{t_k}^{+}$ 的程度，则 $1-d_i$ 为方案 U_i 隶属于负理想方案的程度，相应地，我们可以构建求解优属度 d_i 的规划模型为

$$\min H(d)=\min\sum_{k=1}^{p}\{[(1-d_i)w_k\gamma_i^{+k}]^2+[d_iw_k\gamma_i^{-k}]^2\}\tag{27}$$

式中，$L=\{L_1,\cdots,L_t,\cdots,L_s\}$为专家权重集，$L_t>0$，$\sum_{t=1}^{s}L_t=1$；$\boldsymbol{d}=(d_1,d_2,\cdots,d_n)$为最优解的优属度向量；$\boldsymbol{w}=\{w_1,\cdots,w_t,\cdots,w_p\}$为时间权重向量，$w_k>0$。

对式(27)求导可得

$$\frac{\partial H}{\partial d_i}=2\sum_{k=1}^{p}(1-d_i)(w_k\gamma_i^{+k})(-w_k\gamma_i^{+k})+2\sum_{k=1}^{p}d_i(w_k\gamma_i^{-k})(-w_k\gamma_i^{-k})=0$$

则有

$$d_i=\frac{\sum_{k=1}^{p}(w_k\gamma_i^{+k})^2}{\sum_{k=1}^{p}(w_k\gamma_i^{+k})^2+\sum_{k=1}^{p}(w_k\gamma_i^{-k})^2}=\frac{\sum_{k=1}^{p}(w_k\Delta[\sum_{j=1}^{m}\eta_j\times\Delta^{-1}(\varepsilon_{ij}^{+k},\delta_{ij}^{+k})])^2}{\sum_{k=1}^{p}(w_k\Delta[\sum_{j=1}^{m}\eta_j\times\Delta^{-1}(\varepsilon_{ij}^{+k},\delta_{ij}^{+k})])^2+\sum_{k=1}^{p}(w_k\Delta[\sum_{j=1}^{m}\eta_j\times\Delta^{-1}(\varepsilon_{ij}^{-k},\delta_{ij}^{-k})])^2}\tag{28}$$

根据优属度 d_i 表示方案 U_i 隶属于正理想方案 $U_{t_k}^{+}$ 的程度可知，d_i 的值越大，U_i 越好，d_i 的值越小，U_i 越劣。

2.4 群集结和方案优选的步骤

综上所述，群集结和方案优选的步骤如下：

步骤 1 利用式(2)将语言决策矩阵转化为二元语义决策矩阵，并基于式(8)将二元语义形式的评价信息集结为群的综合评价信息；

步骤 2 利用式(7)确定 k 阶段的正、负理想方案，利用式(14)和式(15)，求解 k 时段方案的正、负理想关联系数，并利用式(24)确定指标权重；

步骤 3 并利用式(25)和式(26)，计算 k 时段方案的正、负理想关联度；

步骤 4 利用式(28)求解各方案的优势度值。

3 算例计算

设有一动态多属性群决策问题，其语言评价集设为 $S=\{s_0,s_1,s_2,s_3,s_4,s_5,s_6\}$，其中，$s_0=FC$(非常差)、$s_1=HC$(很差)、$s_2=C$(差)、$s_3=YB$(一般)、$s_4=Z$(重要)、$s_5=HZ$(很重要)、$s_6=FZ$(非常重要)；专家权重为 $\boldsymbol{L}=(0.25,0.25,0.20,0.30)$，各时间段的权重分别为 $\boldsymbol{w}=(0.200\ 0,0.350\ 0,0.450\ 00)$，而指标权重未知。为有效地解决该问题，设有 4 名专家从 4 个方面，对 5 个被评价对象近 3 个时期的运行状况展开评估。

步骤 1 根据收集评估数据，利用式(2)可将语言决策矩阵转化为二元语义决策矩阵 $\boldsymbol{R}_r^k=(v_{ij}^{k,r},0)_{n\times m}$。

$$\boldsymbol{R}_1^1=\begin{bmatrix}[(s_4,0),(s_5,0)] & [(s_5,0),(s_6,0)] & [(s_2,0),(s_3,0)] & [(s_2,0),(s_3,0)]\\ [(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_1,0),(s_2,0)] & [(s_2,0),(s_3,0)]\\ [(s_4,0),(s_5,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_3,0)] & [(s_3,0),(s_4,0)]\\ [(s_3,0),(s_4,0)] & [(s_5,0),(s_5,0)] & [(s_1,0),(s_2,0)] & [(s_2,0),(s_3,0)]\\ [(s_4,0),(s_5,0)] & [(s_4,0),(s_5,0)] & [(s_3,0),(s_4,0)] & [(s_3,0),(s_4,0)]\end{bmatrix}$$

$$\boldsymbol{R}_2^1=\begin{bmatrix}[(s_4,0),(s_5,0)] & [(s_3,0),(s_4,0)] & [(s_1,0),(s_2,0)] & [(s_4,0),(s_5,0)]\\ [(s_4,0),(s_5,0)] & [(s_5,0),(s_6,0)] & [(s_0,0),(s_1,0)] & [(s_3,0),(s_3,0)]\\ [(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_1.0),(s_2.0)] & [(s_1,0),(s_2,0)]\\ [(s_4,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_1,0),(s_2,0)] & [(s_2,0),(s_3,0)]\\ [(s_5,0),(s_5,0)] & [(s_5,0),(s_6,0)] & [(s_3,0),(s_4,0)] & [(s_4,0),(s_4,0)]\end{bmatrix}$$

$$\boldsymbol{R}_3^1=\begin{bmatrix}[(s_4,0),(s_4,0)] & [(s_5,0),(s_5,0)] & [(s_2,0),(s_2,0)] & [(s_4,0),(s_4,0)]\\ [(s_3,0),(s_4,0)] & [(s_5,0),(s_6,0)] & [(s_2,0),(s_3,0)] & [(s_4,0),(s_5,0)]\\ [(s_4,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_1,0),(s_2,0)] & [(s_2,0),(s_3,0)]\\ [(s_5,0),(s_6,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_2,0)] & [(s_3,0),(s_4,0)]\\ [(s_4,0),(s_5,0)] & [(s_5,0),(s_6,0)] & [(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)]\end{bmatrix}$$

$$\boldsymbol{R}_4^1=\begin{bmatrix}[(s_2,0),(s_3,0)] & [(s_4,0),(s_5,0)] & [(s_4,0),(s_4,0)] & [(s_3,0),(s_4,0)]\\ [(s_4,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_0,0),(s_1,0)] & [(s_3,0),(s_3,0)]\\ [(s_4,0),(s_5,0)] & [(s_5,0),(s_5,0)] & [(s_2,0),(s_2,0)] & [(s_4,0),(s_5,0)]\\ [(s_3,0),(s_4,0)] & [(s_5,0),(s_5,0)] & [(s_1,0),(s_2,0)] & [(s_3,0),(s_4,0)]\\ [(s_4,0),(s_4,0)] & [(s_5,0),(s_6,0)] & [(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)]\end{bmatrix}$$

$$\boldsymbol{R}_1^2=\begin{bmatrix}[(s_3,0),(s_3,0)] & [(s_4,0),(s_5,0)] & [(s_4,0),(s_4,0)] & [(s_3,0),(s_3,0)]\\ [(s_3,0),(s_4,0)] & [(s_5,0),(s_6,0)] & [(s_2,0),(s_2,0)] & [(s_3,0),(s_4,0)]\\ [(s_3,0),(s_3,0)] & [(s_4,0),(s_5,0)] & [(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)]\\ [(s_3,0),(s_4,0)] & [(s_5,0),(s_5,0)] & [(s_3,0),(s_4,0)] & [(s_1,0),(s_2,0)]\\ [(s_4,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_3,0)] & [(s_2,0),(s_3,0)]\end{bmatrix}$$

$$\boldsymbol{R}_2^2=\begin{bmatrix}[(s_4,0),(s_5,0)] & [(s_4,0),(s_5,0)] & [(s_0,0),(s_1,0)] & [(s_4,0),(s_5,0)]\\ [(s_4,0),(s_5,0)] & [(s_3,0),(s_4,0)] & [(s_2,0),(s_2,0)] & [(s_4,0),(s_4,0)]\\ [(s_3,0),(s_4,0)] & [(s_4,0),(s_4,0)] & [(s_2,0),(s_3,0)] & [(s_3,0),(s_4,0)]\\ [(s_4,0),(s_5,0)] & [(s_4,0),(s_5,0)] & [(s_0,0),(s_1,0)] & [(s_3,0),(s_3,0)]\\ [(s_4,0),(s_4,0)] & [(s_5,0),(s_6,0)] & [(s_3,0),(s_4,0)] & [(s_4,0),(s_4,0)]\end{bmatrix}$$

$$\boldsymbol{R}_3^2=\begin{bmatrix}[(s_4,0),(s_5,0)] & [(s_4,0),(s_5,0)] & [(s_3,0),(s_3,0)] & [(s_3,0),(s_3,0)]\\ [(s_4,0),(s_5,0)] & [(s_5,0),(s_5,0)] & [(s_1,0),(s_2,0)] & [(s_3,0),(s_4,0)]\\ [(s_4,0),(s_4,0)] & [(s_4,0),(s_4,0)] & [(s_3,0),(s_4,0)] & [(s_2,0),(s_3,0)]\\ [(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_3,0)] & [(s_3,0),(s_4,0)]\\ [(s_3,0),(s_4,0)] & [(s_4,0),(s_4,0)] & [(s_2,0),(s_3,0)] & [(s_2,0),(s_3,0)]\end{bmatrix}$$

$$\boldsymbol{R}_4^2=\begin{bmatrix}[(s_4,0),(s_5,0)] & [(s_5,0),(s_5,0)] & [(s_3,0),(s_3,0)] & [(s_3,0),(s_4,0)]\\ [(s_3,0),(s_4,0)] & [(s_5,0),(s_6,0)] & [(s_1,0),(s_2,0)] & [(s_4,0),(s_4,0)]\\ [(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_3,0)] & [(s_3,0),(s_4,0)]\\ [(s_3,0),(s_4,0)] & [(s_4,0),(s_4,0)] & [(s_2,0),(s_3,0)] & [(s_4,0),(s_5,0)]\\ [(s_5,0),(s_5,0)] & [(s_5,0),(s_6,0)] & [(s_2,0),(s_3,0)] & [(s_2,0),(s_3,0)]\end{bmatrix}$$

$$\boldsymbol{R}_1^3=\begin{bmatrix}[(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_2,0)] & [(s_3,0),(s_4,0)]\\ [(s_4,0),(s_4,0)] & [(s_3,0),(s_4,0)] & [(s_2,0),(s_3,0)] & [(s_2,0),(s_3,0)]\\ [(s_2,0),(s_3,0)] & [(s_4,0),(s_5,0)] & [(s_1,0),(s_2,0)] & [(s_2,0),(s_3,0)]\\ [(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_3,0)] & [(s_2,0),(s_3,0)]\\ [(s_5,0),(s_6,0)] & [(s_5,0),(s_5,0)] & [(s_3,0),(s_4,0)] & [(s_2,0),(s_3,0)]\end{bmatrix}$$

$$\boldsymbol{R}_2^3=\begin{bmatrix}[(s_3,0),(s_3,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_3,0)] & [(s_4,0),(s_4,0)]\\ [(s_4,0),(s_5,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_3,0)] & [(s_3,0),(s_4,0)]\\ [(s_2,0),(s_3,0)] & [(s_4,0),(s_4,0)] & [(s_1,0),(s_2,0)] & [(s_3,0),(s_4,0)]\\ [(s_3,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_1,0),(s_2,0)] & [(s_4,0),(s_5,0)]\\ [(s_4,0),(s_5,0)] & [(s_4,0),(s_5,0)] & [(s_3,0),(s_4,0)] & [(s_3,0),(s_4,0)]\end{bmatrix}$$

$$\boldsymbol{R}_3^3=\begin{bmatrix}[(s_4,0),(s_5,0)] & [(s_4,0),(s_5,0)] & [(s_3,0),(s_3,0)] & [(s_3,0),(s_3,0)]\\ [(s_3,0),(s_4,0)] & [(s_3,0),(s_4,0)] & [(s_1,0),(s_2,0)] & [(s_2,0),(s_3,0)]\\ [(s_4,0),(s_4,0)] & [(s_4,0),(s_5,0)] & [(s_2,0),(s_3,0)] & [(s_3,0),(s_4,0)]\\ [(s_3,0),(s_4,0)] & [(s_5,0),(s_5,0)] & [(s_2,0),(s_3,0)] & [(s_3,0),(s_4,0)]\\ [(s_4,0),(s_4,0)] & [(s_5,0),(s_6,0)] & [(s_3,0),(s_4,0)] & [(s_2,0),(s_3,0)]\end{bmatrix}$$

$$R_4^3=\begin{bmatrix}[(s_4,0),(s_5,0)] & [(s_4,0),(s_5,0)] & [(s_3,0),(s_4,0)] & [(s_2,0),(s_3,0)]\\ [(s_3,0),(s_4,0)] & [(s_5,0),(s_6,0)] & [(s_2,0),(s_2,0)] & [(s_3,0),(s_4,0)]\\ [(s_4,0),(s_5,0)] & [(s_4,0),(s_4,0)] & [(s_1,0),(s_2,0)] & [(s_3,0),(s_3,0)]\\ [(s_3,0),(s_4,0)] & [(s_3,0),(s_4,0)] & [(s_2,0),(s_3,0)] & [(s_2,0),(s_3,0)]\\ [(s_4,0),(s_5,0)] & [(s_5,0),(s_5,0)] & [(s_3,0),(s_4,0)] & [(s_1,0),(s_2,0)]\end{bmatrix}$$

步骤 2 由专家权重 L=(0.25,0.25,0.20,0.30),利用式(8)可得各阶段群综合评价信息为

$$D_1=\begin{bmatrix}[(s_3,0.30),(s_4,0.15)] & [(s_4,0.15),(s_5,0.00)] & [(s_2,0.45),(s_3,-0.05)] & [(s_3,-0.35),(s_4,-0.00)]\\ [(s_4,-0.40),(s_4,0.20)] & [(s_4,0.40),(s_5,0.40)] & [(s_1,-0.45),(s_2,-0.45)] & [(s_3,-0.35),(s_3,0.30)]\\ [(s_4,-0.25),(s_5,-0.40)] & [(s_4,0.35),(s_5,0.00)] & [(s_2,-0.40),(s_2,0.25)] & [(s_3,0.20),(s_4,0.20)]\\ [(s_4,-0.45),(s_4,0.30)] & [(s_4,0.15),(s_5,0.00)] & [(s_1,0.15),(s_2,0.00)] & [(s_2,0.50),(s_3,0.50)]\\ [(s_4,0.25),(s_5,-0.15)] & [(s_5,-0.25),(s_6,-0.25)] & [(s_3,0.00),(s_4,0.00)] & [(s_3,0.50),(s_4,0.50)]\end{bmatrix}$$

$$D_2=\begin{bmatrix}[(s_4,-0.4),(s_4,0.35)] & [(s_5,-0.25),(s_6,-0.15)] & [(s_2,-0.15),(s_2,0.5)] & [(s_4,-0.1),(s_5,-0.45)]\\ [(s_3,0.40),(s_4,0.40)] & [(s_4,0.50),(s_5,0.35)] & [(s_1,0.50),(s_2,0.00)] & [(s_4,-0.40),(s_4,0.00)]\\ [(s_3,0.45),(s_4,-0.15)] & [(s_4,0.00),(s_5,-0.40)] & [(s_2,0.40),(s_3,0.40)] & [(s_3,0.10),(s_4,0.10)]\\ [(s_3,0.25),(s_4,0.25)] & [(s_4,0.25),(s_5,-0.35)] & [(s_2,-0.25),(s_3,-0.25)] & [(s_3,-0.15),(s_4,-0.4)]\\ [(s_4,0.35),(s_5,-0.15)] & [(s_4,0.35),(s_5,0.00)] & [(s_3,-0.10),(s_3,0.30)] & [(s_3,0.00),(s_4,-0.15)]\end{bmatrix}$$

$$D_3=\begin{bmatrix}[(s_3,0.50),(s_4,0.10)] & [(s_4,0.15),(s_5,-0.15)] & [(s_2,0.50),(s_3,0.10)] & [(s_3,-0.40),(s_4,-0.40)]\\ [(s_4,-0.35),(s_4,0.25)] & [(s_4,0.25),(s_5,-0.20)] & [(s_1,0.15),(s_2,0.15)] & [(s_3,-0.25),(s_3,0.40)]\\ [(s_3,0.00),(s_4,-0.25)] & [(s_4,0.00),(s_4,0.25)] & [(s_2,0.40),(s_3,0.40)] & [(s_3,0.10),(s_4,0.10)]\\ [(s_3,0.1),(s_4,-0.15)] & [(s_4,-0.35),(s_5,-0.35)] & [(s_2,-0.25),(s_3,-0.25)] & [(s_3,-0.35),(s_4,-0.35)]\\ [(s_4,0.25),(s_5,0.10)] & [(s_5,-0.15),(s_5,0.25)] & [(s_3,0.00),(s_4,0.00)] & [(s_3,0.15),(s_4,-0.25)]\end{bmatrix}$$

由群综合评价信息可得各阶段的正、负理想方案为

$$U_{t_1}^{+}=([(s_4,0.25),(s_5,-0.15)],[(s_5,-0.25),(s_6,-0.25)],[(s_3,0.00),(s_4,0.00)],[(s_3,0.50),(s_4,0.50)])$$

$$U_{t_1}^{-}=([(s_3,0.30),(s_4,0.15)],[(s_4,0.15),(s_5,0.00)],[(s_1,-0.45),(s_2,-0.45)],[(s_2,0.50),(s_3,0.50)])$$

$$U_{t_2}^{+}=([(s_4,0.35),(s_5,-0.15)],[(s_5,-0.25),(s_6,-0.15)],[(s_2,0.40),(s_3,0.40)],[(s_4,-0.1),(s_5,-0.45)])$$

$$U_{t_2}^{-}=([(s_3,0.45),(s_4,-0.15)],[(s_4,0.00),(s_5,-0.40)],[(s_2,-0.25),(s_3,-0.25)],[(s_3,0.00),(s_4,-0.15)])$$

$$U_{t_3}^{+}=([(s_4,0.25),(s_5,0.10)],[(s_5,-0.15),(s_5,0.25)],[(s_3,0.00),(s_4,0.00)],[(s_3,0.10),(s_4,0.10)])$$

$$U_{t_3}^{-}=([(s_3,0.00),(s_4,-0.25)],[(s_4,0.00),(s_4,0.25)],[(s_1,0.15),(s_2,0.15)],[(s_3,-0.40),(s_4,-0.40)])$$

利用式(15)与式(16),可得 3 个时间段各方案的正、负理想关联系数,并基于与各阶段的正理想方案偏差最小优化模型,将各数据代入式(24)确定指标权重,其为 η=(0.257 1,0.262 5,0.254 2,0.226 3)。

步骤 3 由指标权重 η=(0.257 1,0.262 5,0.254 2,0.226 3),利用式(25)和式(26)可得各方案在各时段的正、负理想方案关联度为

$$\{\gamma_1^{+k}\}=\{(\varepsilon_1^{+1},\delta_1^{+1}),(\varepsilon_1^{+2},\delta_1^{+2}),(\varepsilon_1^{+3},\delta_1^{+3})\}=\{0.623\ 1,0.766\ 7,0.665\ 3\}$$

$$\{\gamma_1^{-k}\}=\{(\varepsilon_1^{-1},\delta_1^{-1}),(\varepsilon_1^{-2},\delta_1^{-2}),(\varepsilon_1^{-3},\delta_1^{-3})\}=\{0.793\ 3,0.458\ 4,0.609\ 9\}$$

$$\{\gamma_2^{+k}\}=\{(\varepsilon_2^{+1},\delta_2^{+1}),(\varepsilon_2^{+2},\delta_2^{+2}),(\varepsilon_2^{+3},\delta_2^{+3})\}=\{0.589\ 5,0.528\ 3,0.574\ 5\}$$

$$\{\gamma_2^{-k}\}=\{(\varepsilon_2^{-1},\delta_2^{-1}),(\varepsilon_2^{-2},\delta_2^{-2}),(\varepsilon_2^{-3},\delta_2^{-3})\}=\{0.876\ 4,0.677\ 6,0.673\ 8\}$$

$$\{\gamma_3^{+k}\}=\{(\varepsilon_3^{+1},\delta_3^{+1}),(\varepsilon_3^{+2},\delta_3^{+2}),(\varepsilon_3^{+3},\delta_3^{+3})\}=\{0.666\ 5,0.477\ 6,0.485\ 4\}$$

$$\{\gamma_3^{-k}\}=\{(\varepsilon_3^{-1},\delta_3^{-1}),(\varepsilon_3^{-2},\delta_3^{-2}),(\varepsilon_3^{-3},\delta_3^{-3})\}=\{0.714\ 6,0.763\ 8,0.866\ 8\}$$

$$\{\gamma_4^{+k}\}=\{(\varepsilon_4^{+1},\delta_4^{+1}),(\varepsilon_4^{+2},\delta_4^{+2}),(\varepsilon_4^{+3},\delta_4^{+3})\}=\{0.583\ 5,0.431\ 6,0.510\ 8\}$$

$$\{\gamma_4^{-k}\}=\{(\varepsilon_4^{-1},\delta_4^{-1}),(\varepsilon_4^{-2},\delta_4^{-2}),(\varepsilon_4^{-3},\delta_4^{-3})\}=\{0.832\ 4,0.751\ 9,0.843\ 4\}$$

$$\{\gamma_5^{+k}\}=\{(\varepsilon_5^{+1},\delta_5^{+1}),(\varepsilon_5^{+2},\delta_5^{+2}),(\varepsilon_5^{+3},\delta_5^{+3})\}=\{1.000\ 0,0.935\ 7,0.909\ 8\}$$

$$\{\gamma_5^{-k}\}=\{(\varepsilon_5^{-1},\delta_5^{-1}),(\varepsilon_5^{-2},\delta_5^{-2}),(\varepsilon_5^{-3},\delta_5^{-3})\}=\{0.528\ 9,0.525\ 9,0.548\ 7\}$$

步骤 4 各时间段的权重为 w=(0.200 0,0.350 0,0.450 00),根据所求的各阶段的灰色关联度,利用式(19)可得到各被评价对象的优势度分别为 $d_1=0.563\ 7$,$d_2=0.413\ 2$,$d_3=0.282\ 3$,$d_4=0.272\ 6$,$d_5=0.784\ 3$。根据优势度值,对被评价对象进行排序可得 $d_5>d_1>d_2>d_3>d_4$。可知,在所有被评价对象中,对象 x_5 是最优的,而对象 x_4 是最差的。

基于计算分析,与其他多属性动态评价方法相比可知,该方法可以很好地避免多阶段、多专家信息的信息集结带来的语言评价信息丢失,能够很好地实现对动态评价问题的有效处理,同时该方法计算简单,便于计算机操作。

3 结 论

为了避免区间语言评价信息集结过程中产生的信息损失和扭曲，构建了动态灰色关联群决策方法。针对决策属性值为区间语言评价信息、时间权重和专家权重已知而属性未知情形，本文利用灰色关联分析方法，建立了与正理想灰色关联度偏差最小的优化模型，从而给出一种确定指标权重的客观赋权方法。通过计算各时间段各方案对正、负理想方案的语义灰色关联度，利用构建的方案优属度优化模型，确定多阶段群决策方案的优劣，以实现对语言评价信息的处理。结果表明，该方法简洁直观、可操作性强，能够用于解决现实的群决策问题，具有广泛的应用价值。

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