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一类具有非局部扩散的时滞 Lotka-Volterra 竞争模型的行波解 *

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摘 要 本文研究一类具有非局部扩散的时滞 Lotka-Volterra 竞争模型

$$\begin{cases} \dfrac{\partial}{\partial t}u_1(x,t) = d_1[(J_1 * u_1)(x,t) - u_1(x,t)] \\ \qquad\qquad +r_1u_1(x,t)[1 - a_1u_1(x,t) - b_1u_1(x,t-\tau_1) - c_1u_2(x,t-\tau_2)], \\ \dfrac{\partial}{\partial t}u_2(x,t) = d_2[(J_2 * u_2)(x,t) - u_2(x,t)] \\ \qquad\qquad +r_2u_2(x,t)[1 - a_2u_2(x,t) - b_2u_2(x,t-\tau_3) - c_2u_1(x,t-\tau_4)] \end{cases}$$

行波解的存在性问题．通过利用交叉迭代技巧，我们可以把行波解的存在性转化为寻找一对适当的上下解，这篇文章中的结果推广了已有的一些结果.

关键词 Lotka-Volterra 竞争模型；行波解；上下解；非局部扩散；时滞

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1　引言

众所周知， Lotka-Volterra 竞争系统是数学生物学研究领域中最为典型和重要的系统之一．特别地，当研究对象为具有空间扩散能力的两种群时，该模型形如：

$$\begin{cases} \dfrac{\partial}{\partial t}u_1(x,t) = d_1\dfrac{\partial^2}{\partial x^2}u_1(x,t) + r_1u_1(x,t)[1 - a_1u_1(x,t) - b_1u_2(x,t)], \\ \dfrac{\partial}{\partial t}u_2(x,t) = d_2\dfrac{\partial^2}{\partial x^2}u_2(x,t) + r_2u_2(x,t)[1 - b_2u_1(x,t) - a_2u_2(x,t)], \end{cases} \tag{1}$$

其中 $u_1(x,t)$, $u_2(x,t)$ 分别表示两个竞争者在时刻 t 位置 x 处的种群密度； d_1, d_2 表示两个竞争者的扩散系数， d_i, r_i, a_i, b_i $(i=1,2)$ 都是正常数．在模型 (1) 的各种动力学行为的研究中，行波解由于其在数学理论及实际应用的重要作用，引起了越来越多学者的关注 [1−4].

在现实的种群中，由于妊娠、孵化和成熟等方面的影响，时间滞后往往是不可避免的．近年来，人们开始专注于时滞反应扩散方程行波解的研究． Schaaf[5] 是先驱研究工作之一，他应用相平面技术、抛物型泛函微分方程的最大值原理和泛函微分方程的一般理论研究了具有 Huxley 型和 Fisher 型的拟单调的反应扩散方程行波解的存在性和稳定性． [6–10] 等借助上下解技术，单调迭代和不动点定理等研究了系统的非线性项满足拟单调条件 (QM) 和指数拟单调条件 (EQM) 时行波解的存在性．由于 Lotka-Volterra 竞争系统一般不满足 (QM) 和 (EQM) 条件， Li 等 [11] 引进交互迭代格式，利用 Schauder 不动点定理建立了非线性项满足弱拟单调条件 (WQM) 和弱指数拟单调条件 (EWQM) 时行波解的存在性，并应用到下面竞争扩散系统

$$\begin{cases} \dfrac{\partial}{\partial t}u_1(x,t) = d_1\dfrac{\partial^2}{\partial x^2}u_1(x,t) + r_1u_1(x,t)[1 - a_1u_1(x,t-\tau_1) - b_1u_2(x,t-\tau_2)], \\ \dfrac{\partial}{\partial t}u_2(x,t) = d_2\dfrac{\partial^2}{\partial x^2}u_2(x,t) + r_2u_2(x,t)[1 - b_2u_1(x,t-\tau_3) - a_2u_2(x,t-\tau_4)]. \end{cases} \tag{2}$$

注意到系统 (1) 和 (2) 均用 Laplace 算子来描述种群在空间的扩散，然而 Laplace 算子是个局部算子，表示物种仅在局部范围内扩散．在生态学和流行病学等领域，例如胚胎发育过程， Laplace 反应扩散方程并不能准确地描述研究对象的时空行为 [12]．为此研究者引入卷积算子来描述空间扩散过程，模型形如

$$\frac{\partial u_i(x,t)}{\partial t} = d_i[(J_i * u_i)(x,t) - u_i(x,t)] + f_i(u_t(x)), \qquad x \in \mathbb{R}, \tag{3}$$

其中 $i \in I = \{1, 2, \cdots, n\}$， $d_i > 0$， $u_{i,\,t}(x)(\theta) = u_i(x,\,t+\theta)$， $-\tau \le \theta \le 0$， τ 表示系统出现的最大时滞． $(J_i * u_i)(x,t) = \int_{\mathbb{R}} J_i(x-y)u_i(y,t)\,\mathrm{d}y$， $J_i(x-y)$ 表示个体从空间 y 处到空间 x 处的概率分布函数，这类卷积的详细导出及合理性可参阅 [12–15]. 对于模型 (3) 行波解的研究，很多学者有着浓厚的兴趣． Pan 等 [13,16] 在 (QM) 和 (EQM) 条件下证明了模型 (3) 行波解的存在性．对于二维情况，最近， Yu, Yuan[14] 和 Zhang 等 [15] 在部分拟单调条件给出了行波解的存在性． Yu, Yuan[14] 在弱拟单调条件 (WQM) 和弱指数

拟单调条件 (EWQM) 下讨论了 (3) 的行波解，并且 [14] 将结果应用到如下非局部扩散的 Lotka-Volterra 竞争系统

$$\begin{cases} \dfrac{\partial}{\partial t}u_1(x,t) = d_1[(J_1 * u_1)(x,t) - u_1(x,t)] \\ \qquad\qquad +r_1u_1(x,t)[1 - a_1u_1(x,t-\tau_1) - b_1u_2(x,t-\tau_2)], \\ \dfrac{\partial}{\partial t}u_2(x,t) = d_2[(J_2 * u_2)(x,t) - u_2(x,t)] \\ \qquad\qquad +r_2u_2(x,t)[1 - b_2u_1(x,t-\tau_3) - a_2u_2(x,t-\tau_4)]. \end{cases} \tag{4}$$

本文将利用 [14] 建立的理论，基于 Hutchinson 方程更为现实和一般的模型 $x'(t) = rx(t)[1 - a_1x(t) - a_2x(t-\tau)]$, 考虑一个更为一般的非局部扩散的 Lotka-Volterra 竞争模型

$$\begin{cases} \dfrac{\partial}{\partial t}u_1(x,t) = d_1[(J_1 * u_1)(x,t) - u_1(x,t)] \\ \qquad\qquad +r_1u_1(x,t)[1 - a_1u_1(x,t) - b_1u_1(x,t-\tau_1) - c_1u_2(x,t-\tau_2)], \\ \dfrac{\partial}{\partial t}u_2(x,t) = d_2[(J_2 * u_2)(x,t) - u_2(x,t)] \\ \qquad\qquad +r_2u_2(x,t)[1 - a_2u_2(x,t) - b_2u_2(x,t-\tau_3) - c_2u_1(x,t-\tau_4)], \end{cases} \tag{5}$$

其中所有参数为非负数．显然，当 $a_1 = a_2 = 0$ 时，系统 (5) 变为系统 (4), 即 (4) 是我们所考虑的系统 (5) 的一种特殊情况．

2 预备知识

在本文中，对于向量 $u = (u_1, u_2)$, $v = (v_1, v_2)$. 若 $u_1 \le v_1$, $u_2 \le v_2$, 则称 $u \le v$. 进一步，若 $u \le v$, 但 $u \ne v$, 则称 $u < v$. 特别地，如果 $u \le v$, 但 $u_1 \ne v_1$, $u_2 \ne v_2$, 则称 $u \ll v$. 记 $(u,v] = \{w \in \mathbb{R}^2,\ u < w \le v\}$, $[u,v) = \{w \in \mathbb{R}^2, u \le w < v\}$,, $[u,\, v] = \{w \in \mathbb{R}^2,\ u \le w \le v\}$. $|\cdot|$ 表示 $\mathbb{R}^2$ 的欧氏范数， $\|\cdot\|$ 表示 $C([-\tau,\, 0], \mathbb{R}^2)$ 中的上确界范数.

具有时滞的非局部扩散系统的一般形式为 [14]

$$\begin{cases} \dfrac{\partial}{\partial t}u_1(x,t) = d_1[(J_1 * u_1)(x,t) - u_1(x,t)] + f_1(u_{1,\,t}(x), u_{2,\,t}(x)), \\ \dfrac{\partial}{\partial t}u_2(x,t) = d_2[(J_2 * u_2)(x,t) - u_2(x,t)] + f_2(u_{1,\,t}(x), u_{2,\,t}(x)), \end{cases} \tag{6}$$

式中， $t \ge 0$, $x \in \mathbb{R}$, $d_i > 0$, $u_{i,t}(x)(\theta) = u_i(x, t+\theta)$, $-\tau \le \theta \le 0$, 这里 τ 表示系统中出现的最大时滞． $f_i : \mathbb{R}^2 \to \mathbb{R}$ 是连续函数， $J_i : \mathbb{R} \to \mathbb{R}$ 是非负偶函数, 满足 $\int_{\mathbb{R}} J_i(y)\,\mathrm{d}y = 1$, $(J_i * u_i)(x,t) = \int_{\mathbb{R}} J_i(x-y)u_i(y,t)\,\mathrm{d}y$, $i = 1, 2$.

式 (6) 的行波解是形如 $u_1(x,t) = \phi(x+ct)$, $u_2(x,t) = \psi(x+ct)$ 的一对特解，其中 c 表示波速．将 ϕ, ψ 代入 (6) 中，并用 t 代替 $x+ct$, 则 ϕ, ψ 满足下面的泛函微分方程

$$\begin{cases} c\phi'(t) = d_1(J_1 * \phi)(t) - d_1\phi(t) + f_1^c(\phi_t,\, \psi_t), \\ c\psi'(t) = d_2(J_2 * \psi)(t) - d_2\psi(t) + f_2^c(\phi_t,\, \psi_t) \end{cases} \tag{7}$$

及渐近边界条件

$$\lim_{t\to-\infty}(\phi(t),\psi(t))=(0,0),\qquad \lim_{t\to\infty}(\phi(t),\psi(t))=(k_1,k_2),\tag{8}$$

其中 $(0,0)$ 和 (k_1,k_2) 是 (6) 的两个平衡点，且 $k_1>0$, $k_2>0$. 式 (7) 中， $f_i^c(\phi_t,\psi_t)$ 定义为 $f_i(\phi(t+cs),\psi(t+cs))$, $s\in[-\tau,0]$, $i=1,2$. $(J_1*\phi)(t)=\int_{\mathbb{R}}J_1(t-y)\phi(y)\,\mathrm{d}y$, $(J_2*\psi)(t)=\int_{\mathbb{R}}J_2(t-y)\psi(y)\,\mathrm{d}y$.

对于式 (6) 行波解的存在问题， [14] 中已建立了一个抽象结果，其中要求非线性项满足弱指数拟单调条件 (EWQM)

(**EWQM**) 存在两个常数 $\beta_1>0$, $\beta_2>0$, 使得

$$\begin{aligned}&f_1(\phi_1(s),\,\psi_1(s))-f_1(\phi_2(s),\,\psi_1(s))+(\beta_1-d_1)(\phi_1(0)-\phi_2(0))\ge0,\\&f_1(\phi_1(s),\,\psi_1(s))-f_1(\phi_1(s),\,\psi_2(s))\le0,\\&f_2(\phi_1(s),\,\psi_1(s))-f_2(\phi_1(s),\,\psi_2(s))+(\beta_2-d_2)(\psi_1(0)-\psi_2(0))\ge0,\\&f_2(\phi_1(s),\,\psi_1(s))-f_2(\phi_2(s),\,\psi_1(s))\le0,\end{aligned}$$

其中 $\phi_1(s),\,\phi_2(s),\,\psi_1(s),\,\psi_2(s)\in C([-\tau,0],\mathbb{R})$; 并且满足

(i) $0\le\phi_2(s)\le\phi_1(s)\le M_1$, $0\le\psi_2(s)\le\psi_1(s)\le M_2$, $s\in[-\tau,0]$, 其中 $M_i>k_i$ $(i=1,2)$;

(ii) $e^{\beta_1 s}[\phi_1(s)-\phi_2(s)]$ 与 $e^{\beta_2 s}[\psi_1(s)-\psi_2(s)]$ 关于 $s\in[-\tau,0]$ 都是不减的.

下面给出一些必要的假设：

(**A1**) $f_i(0,0)=f_i(k_1,\,k_2)=0$, $i=1,\,2$;

(**A2**) 存在两个常数 $L_i>0$ $(i=1,2)$, 使得

$$|f_i(\phi_1,\,\psi_1)-f_i(\phi_2,\,\psi_2)|\le L_i\|\Phi-\Psi\|,\qquad i=1,2.$$

式中： $\Phi=(\phi_1,\,\psi_1)$, $\Psi=(\phi_2,\,\psi_2)\in C([-\tau,0],\,\mathbb{R}^2)$ 满足 $0\le\phi_i(s)\le M_1$, $0\le\psi_i(s)\le M_2$, $s\in[-\tau,0]$, 其中 $M_i>k_i$ $(i=1,2)$.

(**A3**) 如果 $u,\,v\in C(\mathbb{R},\mathbb{R})$ 且 $u\ge v$, 则 $(J_i*u)(t)\ge(J_i*v)(t)$, $i=1,\,2$.

(**A4**) 对任意 $\lambda\in\mathbb{R}$, $\int_{\mathbb{R}}J_i(y)e^{\lambda y}\,\mathrm{d}y<\infty$, $i=1,\,2$.

定义 1[14]　如果存在常数 T_i, $i=1,\cdots,m$, 使得 $\overline{\Phi}=(\overline{\phi},\,\overline{\psi})$ 和 $\underline{\Phi}=(\underline{\phi},\,\underline{\psi})$ 在 $\mathbb{R}\backslash\{T_i:i=1,\cdots,m\}$ 上二阶连续可微且满足

$$\begin{cases}c\overline{\phi}'(t)\ge d_1(J_1*\overline{\phi})(t)-d_1\overline{\phi}(t)+f_1^c(\overline{\phi}_t,\,\underline{\psi}_t), & t\in\mathbb{R}\backslash\{T_i:i=1,\cdots,m\},\\ c\overline{\psi}'(t)\ge d_2(J_2*\overline{\psi})(t)-d_2\overline{\psi}(t)+f_2^c(\underline{\phi}_t,\,\overline{\psi}_t), & t\in\mathbb{R}\backslash\{T_i:i=1,\cdots,m\}\end{cases}$$

和

$$\begin{cases}c\underline{\phi}'(t)\le d_1(J_1*\underline{\phi})(t)-d_1\underline{\phi}(t)+f_1^c(\underline{\phi}_t,\,\overline{\psi}_t), & t\in\mathbb{R}\backslash\{T_i:i=1,\cdots,m\},\\ c\underline{\psi}'(t)\le d_2(J_2*\underline{\psi})(t)-d_2\underline{\psi}(t)+f_2^c(\overline{\phi}_t,\,\underline{\psi}_t), & t\in\mathbb{R}\backslash\{T_i:i=1,\cdots,m\},\end{cases}$$

则称 $\overline{\Phi}=(\overline{\phi},\,\overline{\psi})$ 和 $\underline{\Phi}=(\underline{\phi},\,\underline{\psi})\in C(\mathbb{R},\,\mathbb{R}^2)$ 为 (7) 的一对弱上、下解.

定理 2[14] 假设 (**A1**)–(**A4**) 和 (**EWQM**) 成立. 如果 (7) 存在一对弱上下解 $(\overline{\phi}(t), \overline{\psi}(t))$, $(\underline{\phi}(t), \underline{\psi}(t)) \in C_{[\mathbf{0},\mathbf{M}]}(\mathbb{R}, \mathbb{R}^2)$ 满足

(**P1**) $(0, 0) \le (\underline{\phi}(t), \underline{\psi}(t)) \le (\overline{\phi}(t), \overline{\psi}(t)) \le (M_1, M_2)$;

(**P2**) $\lim\limits_{t\to-\infty}(\overline{\phi}(t), \overline{\psi}(t)) = (0, 0)$, $\lim\limits_{t\to+\infty}(\underline{\phi}(t), \underline{\psi}(t)) = \lim\limits_{t\to+\infty}(\overline{\phi}(t), \overline{\psi}(t)) = (k_1, k_2)$;

(**P3**) $e^{\beta_1 t}[\overline{\phi}(t) - \underline{\phi}(t)]$ 和 $e^{\beta_2 t}[\overline{\psi}(t) - \underline{\psi}(t)]$ 关于 $t \in \mathbb{R}$ 是不减的,

则对于 $c \ge 1$, (7) 存在行波解满足渐近边界条件式 (8).

附注 受 [17] 中弱指数拟单调条件定义的启发，我们的 (**EWQM**) 也简化为:

存在两个常数 $\beta_1 > 0$, $\beta_2 > 0$ 使得

$$f_1(\phi_1(s), \psi_2(s)) - f_1(\phi_2(s), \psi_1(s)) + (\beta_1 - d_1)(\phi_1(0) - \phi_2(0)) \ge 0,$$
$$f_2(\phi_2(s), \psi_1(s)) - f_2(\phi_1(s), \psi_2(s)) + (\beta_2 - d_2)(\psi_1(0) - \psi_2(0)) \ge 0,$$

其中 $\phi_1(s), \phi_2(s), \psi_1(s), \psi_2(s) \in C([-\tau, 0], \mathbb{R})$; 并且满足

(i) $0 \le \phi_2(s) \le \phi_1(s) \le M_1$, $0 \le \psi_2(s) \le \psi_1(s) \le M_2$, $s \in [-\tau, 0]$, 其中 $M_i > k_i$ $(i = 1, 2)$;

(ii) $e^{\beta_1 s}[\phi_1(s) - \phi_2(s)]$ 与 $e^{\beta_2 s}[\psi_1(s) - \psi_2(s)]$ 关于 $s \in [-\tau, 0]$ 都是不减的.

3 行波解的存在性

本节我们假设 $a_1 + b_1 > c_2$, $a_2 + b_2 > c_1$, 则系统 (5) 有唯一的正平衡点 $E(k_1, k_2)$, 其中

$$k_1 = \frac{a_2 + b_2 - c_1}{(a_1 + b_1)(a_2 + b_2) - c_1 c_2}, \qquad k_2 = \frac{a_1 + b_1 - c_2}{(a_1 + b_1)(a_2 + b_2) - c_1 c_2}.$$

我们将建立连接平衡态 $\mathbf{O} = (\mathbf{0}, \mathbf{0})$ 和 $\mathbf{K} = (\mathbf{k_1}, \mathbf{k_2})$ 的行波解. 将 $u_1(x, t) = \phi(x + ct)$, $u_2(x, t) = \psi(x + ct)$, $s = x + ct$ 代入 (5) 中，并仍记 s 为 t, 则模型 (5) 变为

$$\begin{cases} c\phi'(t) = d_1(J_1 * \phi)(t) - d_1\phi(t) + r_1\phi(t)(1 - a_1\phi(t) - b_1\phi(t - c\tau_1) - c_1\psi(t - c\tau_2)), \\ c\psi'(t) = d_2(J_2 * \psi)(t) - d_2\psi(t) + r_2\psi(t)(1 - a_2\psi(t) - b_2\psi(t - c\tau_3) - c_2\phi(t - c\tau_4)) \end{cases} \tag{9}$$

满足渐近边界条件

$$\lim_{t\to-\infty}(\phi(t), \psi(t)) = (0, 0), \qquad \lim_{t\to\infty}(\phi(t), \psi(t)) = (k_1, k_2). \tag{10}$$

对于 $\phi, \psi \in C([-\tau, 0], \mathbb{R})$, 其中 $\tau = \max\{\tau_1, \tau_2, \tau_3, \tau_4\}$, 定义

$$f_1(\phi, \psi) = r_1\phi(0)(1 - a_1\phi(0) - b_1\phi(-\tau_1) - c_1\psi(-\tau_2)),$$
$$f_2(\phi, \psi) = r_2\psi(0)(1 - a_2\psi(0) - b_2\psi(-\tau_3) - c_2\phi(-\tau_4)).$$

显然, (f_1, f_2) 满足条件 (**A1**) 和 (**A2**).

引理 1 如果 τ_1 和 τ_3 充分小，则函数 (f_1, f_2) 满足 (**EWQM**).

证　令 $(\phi_1(s),\,\psi_1(s)),\,(\phi_2(s),\,\psi_2(s))\in C([-\tau,\,0],\mathbb{R}^2)$ 满足 (i) $0\le\phi_2(s)\le\phi_1(s)\le M_1,\ 0\le\psi_2(s)\le\psi_1(s)\le M_2,\ s\in[-\tau,\,0]$; (ii) $e^{\beta_1 s}[\phi_1(s)-\phi_2(s)]$ 及 $e^{\beta_2 s}[\psi_1(s)-\psi_2(s)]$ 关于 $s\in[-\tau,\,0]$ 都是不减的.

如果 τ_1 和 τ_3 充分小，则我们可选择 $\beta_1>0$ 和 $\beta_2>0$ 使得

$$r_1(1-2a_1M_1-b_1M_1-c_1M_2-b_1M_1e^{\beta_1\tau_1})-d_1\ge-\beta_1,$$
$$r_2(1-2a_2M_2-b_2M_2-c_2M_1-b_2M_2e^{\beta_2\tau_3})-d_2\ge-\beta_2.$$

于是，我们有

$$\begin{aligned}&f_1(\phi_1,\,\psi_2)-f_1(\phi_2,\,\psi_1)\\=&r_1\phi_1(0)\big[1-a_1\phi_1(0)-b_1\phi_1(-\tau_1)-c_1\,\psi_2(-\tau_2)\big]\\&-r_1\phi_2(0)\big[1-a_1\phi_2(0)-b_1\phi_2(-\tau_1)-c_1\,\psi_1(-\tau_2)\big]\\\ge&r_1(1-2a_1M_1-b_1M_1-c_1M_2-b_1M_1e^{\beta_1\tau_1})[\phi_1(0)-\phi_2(0)].\\\ge&(d_1-\beta_1)[\phi_1(0)-\phi_2(0)].\end{aligned}$$

所以,

$$f_1(\phi_1,\,\psi_2)-f_1(\phi_2,\,\psi_1)+(\beta_1-d_1)[\phi_1(0)-\phi_2(0)]\ge0.$$

类似地，我们可以证明

$$f_2(\phi_2(s),\,\psi_1(s))-f_2(\phi_1(s),\,\psi_2(s))+(\beta_2-d_2)(\psi_1(0)-\psi_2(0))\ge0.$$

引理证毕.

为应用上节定理 2, 只需构造一对弱上下解满足条件 (**P1**)–(**P3**). 为此定义

$$\Delta_1(\lambda,\,c)=d_1\int_{-\infty}^{+\infty}J_1(y)e^{-\lambda y}\,\mathrm{d}y-d_1-c\lambda+r_1,\tag{11}$$
$$\Delta_2(\lambda,\,c)=d_2\int_{-\infty}^{+\infty}J_2(y)e^{-\lambda y}\,\mathrm{d}y-d_2-c\lambda+r_2.\tag{12}$$

由函数的凸性易知下面的结论成立.

引理 2　存在 $c^*>0$, 使得对任意的 $c>c^*$, 式 (11) 和 (12) 分别有两个不同的正根 $\lambda_1,\,\lambda_2$ 和 $\lambda_3,\,\lambda_4$, 并且

$$\Delta_1(\lambda,c)=\begin{cases}>0, & \text{当 } \lambda>\lambda_2 \text{ 时};\\<0, & \text{当 } \lambda\in(\lambda_1,\lambda_2) \text{ 时};\\>0, & \text{当 } \lambda<\lambda_1 \text{ 时},\end{cases}\qquad\Delta_2(\lambda,c)=\begin{cases}>0, & \text{当 } \lambda>\lambda_4 \text{ 时};\\<0, & \text{当 } \lambda\in(\lambda_3,\lambda_4) \text{ 时};\\>0, & \text{当 } \lambda<\lambda_3 \text{ 时}.\end{cases}$$

取定常数 η 满足：

$$\eta\in\left(1,\,\min\left\{2,\,\frac{\lambda_2}{\lambda_1},\,\frac{\lambda_4}{\lambda_3},\,\frac{\lambda_1+\lambda_3}{\lambda_1},\,\frac{\lambda_2+\lambda_4}{\lambda_2}\right\}\right).$$

对于充分大的正常数 q, 考虑函数

$$l_1(t)=e^{\lambda_1 t}-qe^{\eta\lambda_1 t},\qquad l_2(t)=e^{\lambda_3 t}-qe^{\eta\lambda_3 t}.$$

易知存在负常数 t_1 及 t_3, 使得 $l_1(t)$ 和 $l_2(t)$ 分别在 t_1 和 t_3 取到全局极大值 $m_1>0$ 以及 $m_2>0$, 其中 $t_i=\frac{1}{\eta\lambda_i-\lambda_i}\ln\frac{1}{q\eta}$ $(i=1,3)$. 再取充分小的正常数 λ, 则存在 $\varepsilon_2\in(0,k_1)$, $\varepsilon_4\in(0,k_2)$, 使得 $k_1-\varepsilon_2e^{-\lambda t_1}=l_1(t_1)=m_1$, $k_2-\varepsilon_4e^{-\lambda t_3}=l_1(t_3)=m_2$. 由于 $a_1+b_1>c_2$, $a_2+b_2>c_1$, 则存在 ε_0, ε_1 和 ε_3 使得

$$\begin{cases}(a_1+b_1)\varepsilon_1-c_1\varepsilon_4>\varepsilon_0, & (a_2+b_2)\varepsilon_3-c_2\varepsilon_2>\varepsilon_0,\\ (a_1+b_1)\varepsilon_2-c_1\varepsilon_3>\varepsilon_0, & (a_2+b_2)\varepsilon_4-c_2\varepsilon_1>\varepsilon_0.\end{cases}\tag{13}$$

对于上面的参数，定义连续函数

$$\overline{\phi}(t)=\begin{cases}e^{\lambda_1 t}, & t\le t_2,\\ k_1+\varepsilon_1e^{-\lambda t}, & t\ge t_2,\end{cases}\qquad \overline{\psi}(t)=\begin{cases}e^{\lambda_3 t}, & t\le t_4,\\ k_2+\varepsilon_3e^{-\lambda t}, & t\ge t_4,\end{cases}$$
$$\underline{\phi}(t)=\begin{cases}e^{\lambda_1 t}-qe^{\eta\lambda_1 t}, & t\le t_1,\\ k_1-\varepsilon_2e^{-\lambda t}, & t\ge t_1,\end{cases},\qquad \underline{\psi}(t)=\begin{cases}e^{\lambda_3 t}-qe^{\eta\lambda_1 t}, & t\le t_3,\\ k_2-\varepsilon_4e^{-\lambda t}, & t\ge t_3,\end{cases}$$

其中 q 充分大， λ 充分小. 容易看出当 q 充分大时，$\min\{t_2,t_4\}-c\max\{\tau_1,\tau_2,\tau_3,\tau_4\}\ge\max\{t_1,t_3\}$. 而且，易知 $\overline{\phi}(t)$, $\overline{\psi}(t)$, $\underline{\phi}(t)$, $\underline{\psi}(t)$ 满足 (**P1**)–(**P3**). 事实上，对于充分小的 λ, $\beta>\lambda$ 时， (**P3**) 成立.

下面只需验证 $(\overline{\phi}(t),\overline{\psi}(t))$, $(\underline{\phi}(t),\underline{\psi}(t))$ 满足弱上下解的定义.

引理 3 假设 $a_1+b_1>c_2$, $a_2+b_2>c_1$, 且 (**A3**) 和 (**A4**) 成立. 如果 τ_1,τ_3 充分小，则 $(\overline{\phi}(t),\overline{\psi}(t))$ 是式 (9) 的弱上解， $(\underline{\phi}(t),\underline{\psi}(t))$ 是式 (9) 的弱下解.

证 对于 $(\overline{\phi}(t),\overline{\psi}(t))\in C(\mathbb{R},\mathbb{R}^2)$, (i) 如果 $t\le t_2$, 则 $\overline{\phi}(t)=e^{\lambda_1 t}$, $\overline{\phi}(t-c\tau_1)=e^{\lambda_1(t-c\tau_1)}$, $\underline{\psi}(t-c\tau_2)\ge 0$. 另外一方面， $t\in\mathbb{R}$, $\overline{\phi}(t)\le e^{\lambda_1 t}$. 因此,

$$\begin{aligned}&d_1(J_1*\overline{\phi})(t)-d_1\overline{\phi}(t)-c\bar{\phi}'(t)+r_1\overline{\phi}(t)[1-a_1\overline{\phi}(t)-b_1\overline{\phi}(t-c\tau_1)-c_1\underline{\psi}(t-c\tau_2)]\\ \le& d_1\int_{-\infty}^{+\infty}J_1(t-s)e^{\lambda_1 s}ds-d_1e^{\lambda_1 t}-c\lambda_1e^{\lambda_1 t}\\ &+r_1e^{\lambda_1 t}[1-a_1e^{\lambda_1 t}-b_1e^{\lambda_1(t-c\tau_1)}-c_1\underline{\psi}(t-c\tau_2)]\\ \le& d_1\int_{-\infty}^{+\infty}J_1(t-s)e^{\lambda_1 s}ds-d_1e^{\lambda_1 t}-c\lambda_1e^{\lambda_1 t}+r_1e^{\lambda_1 t}\\ =&e^{\lambda_1 t}\Delta_1(\lambda_1,c)=0.\end{aligned}$$

(ii) 如果 $t_2<t\le t_2+c\tau_1$, 则 $\overline{\phi}(t)=k_1+\varepsilon_1e^{-\lambda t}$, $\overline{\phi}(t-c\tau_1)=e^{\lambda_1(t-c\tau_1)}$, $\underline{\psi}(t-c\tau_2)=k_2-\varepsilon_4e^{-\lambda(t-c\tau_2)}$. 因为对 $t\in\mathbb{R}$, $0<\overline{\phi}(t)\le k_1+\varepsilon_1e^{-\lambda t_2}$, $e^{\lambda_1 t_2}=k_1+\varepsilon_1e^{-\lambda t_2}$, 所以

$$\begin{aligned}&d_1(J_1*\overline{\phi})(t)-d_1\overline{\phi}(t)-c\bar{\phi}'(t)+r_1\overline{\phi}(t)[1-a_1\overline{\phi}(t)-b_1\overline{\phi}(t-c\tau_1)-c_1\underline{\psi}(t-c\tau_2)]\\ \le& d_1(J_1*(k_1+\varepsilon_1e^{-\lambda t_2}))-d_1(k_1+\varepsilon_1e^{-\lambda t})+c\varepsilon_1\lambda e^{-\lambda t}\\ &+r_1(k_1+\varepsilon_1e^{-\lambda t})\{1-a_1(k_1+\varepsilon_1e^{-\lambda t})-b_1e^{\lambda_1(t_2-c\tau_1)}-c_1[k_2-\varepsilon_4e^{-\lambda(t-c\tau_2)}]\}\end{aligned}$$

$$=d_1(k_1+\varepsilon_1 e^{-\lambda t_2})-d_1(k_1+\varepsilon_1 e^{-\lambda t})+c\varepsilon_1\lambda e^{-\lambda t}$$
$$+r_1(k_1+\varepsilon_1 e^{-\lambda t})\{1-a_1(k_1+\varepsilon_1 e^{-\lambda t})-b_1 e^{-\lambda_1 c\tau_1}(k_1+\varepsilon_1 e^{-\lambda t_2})-c_1[k_2-\varepsilon_4 e^{-\lambda(t-c\tau_1)}]\}$$
$$=:I_1(\lambda),$$

易知 $I_1(\lambda)$ 关于 $\lambda\in[0,1]$ 是一致连续的，并且

$$\begin{aligned}I_1(0)=&r_1(k_1+\varepsilon_1)(1-a_1k_1-a_1\varepsilon_1-b_1k_1e^{-\lambda_1 c\tau_1}-b_1\varepsilon_1 e^{-\lambda_1 c\tau_1}-c_1k_2+c_1\varepsilon_4)\\=&r_1(k_1+\varepsilon_1)[b_1k_1-b_1(k_1+\varepsilon_1)e^{-\lambda_1 c\tau_1}-a_1\varepsilon_1+c_1\varepsilon_4].\end{aligned}$$

因为 τ_1 充分小，我们可选取 ε^* $(0<\varepsilon^*<\frac{\varepsilon_0}{b_1(k_1+\varepsilon_1)})$, 使得 $e^{-\lambda_1 c\tau_1}>1-\varepsilon^*$. 于是,

$$\begin{aligned}I_1(0)\le&r_1(k_1+\varepsilon_1)[b_1k_1-b_1(k_1+\varepsilon_1)(1-\varepsilon^*)-a_1\varepsilon_1+c_1\varepsilon_4]\\=&r_1(k_1+\varepsilon_1)[c_1\varepsilon_4-(a_1+b_1)\varepsilon_1+b_1(k_1+\varepsilon_1)\varepsilon^*]\\<&r_1(k_1+\varepsilon_1)[-\varepsilon_0+b_1(k_1+\varepsilon_1)\varepsilon^*]<0.\end{aligned}$$

上式是根据 (13) 得到的. 根据 $I_1(\lambda)$ 的连续性, 存在 $\lambda_1^*>0$, 使得对任意的 $\lambda\in(0,\lambda_1^*)$, 都有 $I_1(\lambda)<0$.

(iii) 如果 $t\ge t_2+c\tau_1$, $\overline{\phi}(t)=k_1+\varepsilon_1 e^{-\lambda t}$, $\overline{\phi}(t-c\tau_1)=k_1+\varepsilon_1 e^{-\lambda(t-c\tau_1)}$, $\underline{\psi}(t-c\tau_2)=k_2-\varepsilon_4 e^{-\lambda(t-c\tau_2)}$, 则

$$\begin{aligned}&d_1(J_1*\overline{\phi})(t)-d_1\overline{\phi}(t)-c\bar{\phi}'(t)+r_1\overline{\phi}(t)[1-a_1\overline{\phi}(t)-b_1\overline{\phi}(t-c\tau_1)-c_1\underline{\psi}(t-c\tau_2)]\\\le&d_1(J_1*(k_1+\varepsilon_1 e^{-\lambda t_2}))-d_1(k_1+\varepsilon_1 e^{-\lambda t})+c\varepsilon_1\lambda e^{-\lambda t}\\&+r_1(k_1+\varepsilon_1 e^{-\lambda t})\{1-a_1(k_1+\varepsilon_1 e^{-\lambda t})-b_1[k_1+\varepsilon_1 e^{-\lambda(t-c\tau_1)}]-c_1[k_2-\varepsilon_4 e^{-\lambda(t-c\tau_2)}]\}\\=&d_1(k_1+\varepsilon_1 e^{-\lambda t_2})-d_1(k_1+\varepsilon_1 e^{-\lambda t})+c\varepsilon_1\lambda e^{-\lambda t}\\&+r_1(k_1+\varepsilon_1 e^{-\lambda t})[-a_1\varepsilon_1 e^{-\lambda t}-b_1\varepsilon_1 e^{-\lambda(t-c\tau_1)}+c_1\varepsilon_4 e^{-\lambda(t-c\tau_2)}]\\=:&I_2(\lambda).\end{aligned}$$

由于 (13) 式 $(a_1+b_1)\varepsilon_1-c_1\varepsilon_4>\varepsilon_0$, 故 $I_2(0)=r_1(k_1+\varepsilon_1)(c_1\varepsilon_4-a_1\varepsilon_1-b_1\varepsilon_1)<-r_1(k_1+\varepsilon_1)\varepsilon_0<0$. 所以，存在 $\lambda_2^*>0$, 使得对任意的 $\lambda\in(0,\lambda_2^*)$, 都有 $I_2(\lambda)<0$. 取 $\lambda^*=\min\{\lambda_1^*,\lambda_2^*\}$, 综合 (i)–(iii), 有

$$d_1(J_1*\overline{\phi})(t)-d_1\overline{\phi}(t)-c\bar{\phi}'(t)+r_1\overline{\phi}(t)[1-a_1\overline{\phi}(t)-b_1\overline{\phi}(t-c\tau_1)-c_1\underline{\psi}(t-c\tau_2)]\le 0.$$

类似地，我们可证明

$$d_2(J_2*\overline{\psi})(t)-d_2\overline{\psi}(t)-c\bar{\psi}'(t)+r_2\overline{\psi}(t)[1-a_2\overline{\psi}(t)-b_2\overline{\psi}(t-c\tau_3)-c_2\underline{\phi}(t-c\tau_4)]\le 0.$$

对于 $(\underline{\phi}(t),\underline{\psi}(t))\in C(\mathbb{R},\mathbb{R}^2)$, (iv) 如果 $t\le t_1$, 则 $\underline{\phi}(t)=e^{\lambda_1 t}-qe^{\eta\lambda_1 t}$, $\underline{\phi}(t-c\tau_1)=$

$e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)}$, $\overline{\psi}(t-c\tau_2)=e^{\lambda_3(t-c\tau_2)}$,

$$\begin{aligned}
&d_1(J_1*\underline{\phi})(t)-d_1\underline{\phi}(t)-c\underline{\phi}'(t)+r_1\underline{\phi}(t)[1-a_1\underline{\phi}(t)-b_1\underline{\phi}(t-c\tau_1)-c_1\overline{\psi}(t-c\tau_2)]\\
=&d_1\Big(\int_{t_1}^{+\infty}J_1(t-s)\underline{\phi}(s)\,\mathrm{d}s+\int_{-\infty}^{t_1}J_1(t-s)\underline{\phi}(s)\,\mathrm{d}s\Big)-d_1\underline{\phi}(t)-c\underline{\phi}'(t)\\
&+r_1\underline{\phi}(t)\big[1-a_1\underline{\phi}(t)-b_1\underline{\phi}(t-c\tau_1)-c_1\overline{\psi}(t-c\tau_2)\big]\\
&=d_1\int_{-\infty}^{+\infty}J_1(t-s)(e^{\lambda_1 s}-qe^{\eta\lambda_1 s})\,\mathrm{d}s+\int_{t_1}^{\infty}J_1(t-s)\big[k_1-\varepsilon_2e^{-\lambda s}-(e^{\lambda_1 s}-qe^{\eta\lambda_1 s})\big]\,\mathrm{d}s\\
&-d_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})-c(\lambda_1e^{\lambda_1 t}-q\eta\lambda_1e^{\eta\lambda_1 t})\\
&+r_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})\big\{1-a_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})-b_1\big[e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)}\big]-c_1e^{\lambda_3(t-c\tau_2)}\big\}\\
\geq&d_1\int_{-\infty}^{+\infty}J_1(t-s)e^{\lambda_1 s}\,\mathrm{d}s-d_1e^{\lambda_1 t}-c\lambda_1e^{\lambda_1 t}+r_1e^{\lambda_1 t}\\
&-d_1\int_{-\infty}^{+\infty}J_1(t-s)qe^{\eta\lambda_1 s}\,\mathrm{d}s+d_1qe^{\eta\lambda_1 t}+cq\eta\lambda_1e^{\eta\lambda_1 t}-r_1qe^{\eta\lambda_1 t}\\
&+\int_{t_1}^{\infty}J_1(t-s)\big[k_1-\varepsilon_2e^{-\lambda t_1}-(e^{\lambda_1 t_1}-qe^{\eta\lambda_1 t_1})\big]\,\mathrm{d}s\\
&-r_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})\big\{a_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})+b_1\big[e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)}\big]+c_1e^{\lambda_3(t-c\tau_2)}\big\}\\
=&\Delta_1(\lambda_1,c)e^{\lambda_1 t}-\Delta_1(\eta\lambda_1,c)qe^{\eta\lambda_1 t}\\
&-r_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})\big\{a_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})+b_1[e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)}]+c_1e^{\lambda_3(t-c\tau_2)}\big\}\\
\geq&-\Delta_1(\eta\lambda_1,c)qe^{\eta\lambda_1 t}-r_1e^{\lambda_1 t}\big[(a_1+b_1)e^{\lambda_1 t}+c_1e^{\lambda_3 t}\big]\\
=&-qe^{\eta\lambda_1 t}\Big\{\Delta_1(\eta\lambda_1,c)+\frac{r_1}{q}\big[(a_1+b_1)e^{(2\lambda_1-\eta\lambda_1)t}+c_1e^{(\lambda_3+\lambda_1-\eta\lambda_1)t}\big]\Big\}\\
\geq&-qe^{\eta\lambda_1 t}\Big\{\Delta_1(\eta\lambda_1,c)+\frac{r_1}{q}\big[(a_1+b_1)e^{(2\lambda_1-\eta\lambda_1)t_1}+c_1e^{(\lambda_3+\lambda_1-\eta\lambda_1)t_1}\big]\Big\}\\
=&-qe^{\eta\lambda_1 t}\Big\{\Delta_1(\eta\lambda_1,c)+\frac{r_1}{q}\Big[(a_1+b_1)e^{(2\lambda_1-\eta\lambda_1)\frac{\ln\frac{1}{\eta q}}{\lambda_1(\eta-1)}}+c_1e^{(\lambda_3+\lambda_1-\eta\lambda_1)\frac{\ln\frac{1}{\eta q}}{\lambda_1(\eta-1)}}\Big]\Big\}\\
\geq&0.
\end{aligned}$$

上式是根据 q 和 η 的选取得到的.

(v) 如果 $t_1<t<t_1+c\tau_1$, 则 $\underline{\phi}(t)=k_1-\varepsilon_2e^{-\lambda t}$, $\underline{\phi}(t-c\tau_1)=e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)}$, $\overline{\psi}(t-c\tau_2)=e^{\lambda_3(t-c\tau_2)}$. 注意 $e^{\lambda_1 t_1}-qe^{\eta\lambda_1 t_1}=k_1-\varepsilon_2e^{-\lambda t_1}$, $e^{\lambda_3 t_4}=k_2+\varepsilon_3e^{-\lambda t_4}$.

$$\begin{aligned}
&d_1(J_1*\underline{\phi})(t)-d_1\underline{\phi}(t)-c\underline{\phi}'(t)+r_1\underline{\phi}(t)\big[1-a_1\underline{\phi}(t)-b_1\underline{\phi}(t-c\tau_1)-c_1\overline{\psi}(t-c\tau_2)\big]\\
\geq&d_1\int_{t_1}^{+\infty}J_1(t-s)(k_1-\varepsilon_2e^{-\lambda s})\,\mathrm{d}s-d_1(k_1-\varepsilon_2e^{-\lambda t})-c\lambda\varepsilon_2e^{-\lambda t}\\
&+r_1(k_1-\varepsilon_2e^{-\lambda t})\big[1-a_1(k_1-\varepsilon_2e^{-\lambda t})-b_1(e^{\lambda_1 t_1}-qe^{\eta\lambda_1 t_1})-c_1e^{\lambda_3 t_4}\big]\\
=&d_1\int_{t_1}^{+\infty}J_1(t-s)(k_1-\varepsilon_2e^{-\lambda s})\,\mathrm{d}s-d_1(k_1-\varepsilon_2e^{-\lambda t})-c\lambda\varepsilon_2e^{-\lambda t}\\
&+r_1(k_1-\varepsilon_2e^{-\lambda t})\big[a_1\varepsilon_2e^{-\lambda t}+b_1\varepsilon_2e^{-\lambda t_1}-c_1\varepsilon_3e^{-\lambda t_4}\big]\\
=:&I_3(\lambda).
\end{aligned}$$

如果 $q>1$ 充分大, 则 $-t_1$ 也充分大, 则对于 ε' $(0<\varepsilon'<\frac{r_1\varepsilon_0}{d_1})$, 存在充分大的 $T(q)>0$ 使得当 $t_1<-T(q)$ 时,

$$\int_{t_1}^{\infty} J_1(s)\,\mathrm{d}s > \int_{-\infty}^{\infty} J_1(s)\,\mathrm{d}s - \varepsilon' = 1-\varepsilon'.$$

因此,

$$\begin{aligned} I_3(0) =& d_1\int_{t_1}^{+\infty} J_1(t-s)(k_1-\varepsilon_2)\,\mathrm{d}s - d_1(k_1-\varepsilon_2) + r_1(k_1-\varepsilon_2)\big[(a_1+b_1)\varepsilon_2 - c_1\varepsilon_3\big] \\ \geq& d_1(k_1-\varepsilon_2)(1-\varepsilon') - d_1(k_1-\varepsilon_2) + r_1(k_1-\varepsilon_2)\varepsilon_0 \\ =&(k_1-\varepsilon_2)(r_1\varepsilon_0-\varepsilon' d_1)>0. \end{aligned}$$

这样，存在 $\lambda_3^*>0$ 使得对于 $\lambda\in(0,\lambda_3^*)$, $I_3(\lambda)>0$.

(vi) 如果 $t>t_1+c\tau_1$, 则 $\underline{\phi}(t)=k_1-\varepsilon_2 e^{-\lambda t}$, $\underline{\phi}(t-c\tau_1)=k_1-\varepsilon_2 e^{-\lambda(t-c\tau_1)}$, $\overline{\psi}(t-c\tau_2)\leq k_2+\varepsilon_3 e^{-\lambda t_4}$.

$$\begin{aligned} & d_1(J_1*\underline{\phi})(t) - d_1\underline{\phi}(t) - c\underline{\phi}'(t) + r_1\underline{\phi}(t)\big[1-a_1\underline{\phi}(t)-b_1\underline{\phi}(t-c\tau_1)-c_1\overline{\psi}(t-c\tau_2)\big] \\ \geq& d_1\int_{-\infty}^{t_1} J_1(t-s)(e^{\lambda_1 s}-qe^{\eta\lambda_1 s})ds + d_1\int_{t_1}^{+\infty} J_1(t-s)(k_1-\varepsilon_2 e^{-\lambda s})\,\mathrm{d}s \\ & - d_1(k_1-\varepsilon_2 e^{-\lambda t}) - c\varepsilon_2\lambda e^{-\lambda t} \\ & + r_1(k_1-\varepsilon_2 e^{-\lambda t})\big[1-a_1(k_1-\varepsilon_2 e^{-\lambda t}) - b_1(k_1-\varepsilon_2 e^{-\lambda(t-c\tau_1)}) - c_1(k_2+\varepsilon_3 e^{-\lambda t_4})\big] \\ \geq& d_1\int_{t_1}^{\infty} J_1(t-s)(k_1-\varepsilon_2 e^{-\lambda s})\,\mathrm{d}s - d_1(k_1-\varepsilon_2 e^{-\lambda t}) - c\varepsilon_2\lambda e^{-\lambda t} \\ & + r_1(k_1-\varepsilon_2 e^{-\lambda t})\big[a_1\varepsilon_2 e^{-\lambda t} + b_1\varepsilon_2 e^{-\lambda(t-c\tau_1)} - c_1\varepsilon_3 e^{-\lambda t_4}\big]. \\ =&: I_4(\lambda). \end{aligned}$$

于是,

$$\begin{aligned} I_4(0) =& d_1\int_{t_1}^{+\infty} J_1(t-s)(k_1-\varepsilon_2)\,\mathrm{d}s - d_1(k_1-\varepsilon_2) + r_1(k_1-\varepsilon_2)\big[(a_1+b_1)\varepsilon_2 - c_1\varepsilon_3\big] \\ \geq& d_1(k_1-\varepsilon_2)(1-\varepsilon') - d_1(k_1-\varepsilon_2) + r_1(k_1-\varepsilon_2)\varepsilon_0 \\ =&(k_1-\varepsilon_2)(r_1\varepsilon_0-\varepsilon' d_1)>0. \end{aligned}$$

于是，存在 $\lambda_4^*>0$ 使得对于 $\lambda\in(0,\lambda_4^*)$, $I_4(\lambda)>0$. 综合步骤 (iv)–(vi), 并取 $\lambda'=\min\{\lambda_3^*,\lambda_4^*\}$, 我们有

$$d_1(J_1*\underline{\phi})(t) - d_1\underline{\phi}(t) - c\underline{\phi}'(t) + r_1\underline{\phi}(t)[1-a_1\underline{\phi}(t)-b_1\underline{\phi}(t-c\tau_1)-c_1\overline{\psi}(t-c\tau_2)]\geq 0.$$

类似地，我们可以证明

$$d_2(J_2*\underline{\psi})(t) - d_2\underline{\psi}(t) - c\underline{\psi}'(t) + r_2\underline{\psi}(t)(1-a_2\underline{\psi}(t)-b_2\underline{\psi}(t-c\tau_3)-c_2\overline{\phi}(t-c\tau_4))\geq 0.$$

引理证毕.

通过上节定理 2 和本节引理 1 和 3, 我们得到本文的主要结果:

定理 2 假设 $a_1+b_1>c_2$, $a_2+b_2>c_1$ 和 (**A3**), (**A4**) 成立. 当 τ_1, τ_3 充分小时, 则对于引理 2 中的 c^*, 对任意的 $c>\max\{c^*, 1\}$, (9) 存在连接 $(0,0)$ 和 (k_1,k_2) 的行波解 $(\phi(x+ct), \psi(x+ct))$. 而且, $\lim\limits_{\xi\to-\infty}\phi(\xi)e^{-\lambda_1\xi}=\lim\limits_{\xi\to-\infty}\psi(\xi)e^{-\lambda_3\xi}=1$.

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Traveling Waves of a Competitive Lotka-Volterra Model with Nonlocal Diffusion and Time Delays

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Abstract In this paper, we consider the existence of traveling waves for a competitive Lotka-Volterra model with nonlocal diffusion and time delays

$$\begin{cases} \dfrac{\partial}{\partial t}u_1(x,t) = d_1[(J_1 * u_1)(x,t) - u_1(x,t)] \\ \qquad\qquad +r_1u_1(x,t)[1 - a_1u_1(x,t) - b_1u_1(x,t-\tau_1) - c_1u_2(x,t-\tau_2)], \\ \dfrac{\partial}{\partial t}u_2(x,t) = d_2[(J_2 * u_2)(x,t) - u_2(x,t)] \\ \qquad\qquad +r_2u_2(x,t)[1 - a_2u_2(x,t) - b_2u_2(x,t-\tau_3) - c_2u_1(x,t-\tau_4)]. \end{cases}$$

By a crossing interation technique, we reduce the existence of traveling waves to looking for a suitable upper-lower solutions. The result in the present paper extends some known results.

Key words competitive Lotka-Volterra model; traveling wave; upper-lower solution; nonlocal diffusion; time delay

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