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马尔科夫切换型中立型随机泛函微分方程 *

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摘 要 尽管具有马尔科夫切换型随机微分方程的稳定性受到了人们的关注，但是关于具有马尔科夫切换型中立型随机泛函微分方程的稳定性的研究则很少. 本文的主要目的是试图研究这一问题，我们证明了解的存在唯一性，并得到了 p- 阶指数稳定性和几乎处处指数稳定性的判据.

关键词 马尔科夫链；布朗运动；中立型随机泛函微分方程；指数稳定性

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1 引言

随机模型在科学和工程的许多领域中有着重要的应用，研究随机系统的稳定性始终是人们所关心的关键问题之一. 具有马尔科夫切换型随机微分方程的稳定性已经受到了人们的极大关注. 如 Mao[1] 研究了如下方程的稳定性：

$$\mathrm{d}x(t) = f(x(t), t, r(t))\mathrm{d}t + g(x(t), t, r(t))\,\mathrm{d}w(t), \qquad t \geq 0,$$

其中 $x(0) = x_0 \in R^n$ 为初值， $r(t)$ 是取值于集合 $S = \{1, 2, \cdots, N\}$ 上的马尔科夫链，$f : R^n \times R_+ \times S \to R^n$, $g : R^n \times R_+ \times S \to R^{n\times m}$ 为漂移系数和扩散系数. Mao[2−4] 考虑了如下马尔科夫切换型随机延迟微分方程

$$\mathrm{d}x(t) = f(x(t), x(t - \tau_1(t)), t, r(t))\mathrm{d}t + g(x(t), x(t - \tau_2(t)), t, r(t))\,\mathrm{d}w(t),$$

研究了其存在唯一性和稳定性. Kolmanovskii V[5] 研究了中立型随机延迟微分方程

$$\mathrm{d}[x(t) - u(x(t - \tau), r(t))] = f(x(t), x(t - \tau), t, r(t))\mathrm{d}t + g(x(t), x(t - \tau), t, r(t))\,\mathrm{d}w(t)$$

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的存在唯一性和指数稳定性．然而关于如下中立型随机泛函微分方程

$$\mathrm{d}[x(t)-u(x_t,r(t))]=f(x_t,t,r(t))\mathrm{d}t+g(x_t,t,r(t))\,\mathrm{d}w(t)$$

的工作则很少见．本文将主要是研究这一方程解的存在唯一性和指数稳定性.

2　存在唯一性

假定 $\{\mathcal{F}_t\}_{t\geq 0}$ 为满足通常条件的流，$(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},P)$ 是完备的概率空间．$w(t)=(w_1(t),\cdots,w_m(t))^T$ 是定义于概率空间上的 m 维的布朗运动，$\tau>0$, 记 $C([-\tau,0];R^n)$ 为 $\varphi:[-\tau,0]\to R^n$ 连续函数类, 其范数定义为 $\|\varphi\|=\sup\limits_{-\tau\leq\theta\leq 0}|\varphi(\theta)|$, 其中 $|\cdot|$ 是 R^n 上的欧几里得范数. 若 A 为向量，A^T 为其转置. 若 A 矩阵, 其迹范数为 $|A|=\sqrt{\mathrm{trace}\,(A^TA)}$, 算子范数为 $\|A\|=\sup\{|Ax|:|x|=1\}$. 记 $C^b_{\mathcal{F}_0}([-\tau,0];R^n)$ 是有界的 $\mathcal{F}_0$- 可测的 $C([-\tau,0];R^n)$-值随机变量．令 $p>0$, $t\geq 0$, 记 $L^p_{\mathcal{F}_t}([-\tau,0];R^n)$ 为 $\mathcal{F}_t$- 可测的，$C([-\tau,0];R^n)$- 值且使得 $\sup\limits_{-\tau\leq\theta\leq 0}E|\phi(\theta)|^p<\infty$ 的随机变量 $\phi=\big\{\phi(\theta):-\tau\leq\theta\leq 0\big\}$ 的全体.

令 $r(t)$ $(t\geq 0)$ 是右连续的马尔科夫链，$\Gamma=(\gamma_{ij})_{N\times N}$ 满足

$$P\{r(t+\Delta)=j|r(t)=i\}=\begin{cases}\gamma_{ij}\Delta+0(\Delta), & \text{if}\ \ i\neq j,\\ 1+\gamma_{ii}\Delta+0(\Delta), & \text{if}\ \ i=j,\end{cases}$$

其中 $\Delta>0$, $\gamma_{ij}\geq 0$ 是从 i 到 j $(i\neq j)$ 的转移概率，且 $\gamma_{ii}=-\sum\limits_{j\neq i}\gamma_{ij}$. 假定 $r(\cdot)$ 独立于 $w(\cdot)$, 而且 $r(t)$ 除了在有限个跳跃点外是几乎处处为常数，所有的样本路径是右连续的.

考虑如下马尔科夫型中立性型随机泛函微分方程

$$\mathrm{d}(x(t)-u(x_t,r(t)))=f(x_t,t,r(t))\mathrm{d}t+g(x_t,t,r(t))\mathrm{d}w(t),\qquad t\geq 0, \tag{1}$$

其中 $x_0=\xi$, $\xi\in C^b_{\mathcal{F}_0}([-\tau,0];R^n)$, $x_t=\{x(t+\theta):-\tau\leq\theta\leq 0\}$ 是 $C([-\tau,0];R^n)$- 值随机过程，并且

$$\begin{aligned}&f:C([-\tau,0];R^n)\times R_+\times S\longrightarrow R^n,\\&g:C([-\tau,0];R^n)\times R_+\times S\longrightarrow R^{n\times m},\\&u:C([-\tau,0];R^n)\times S\longrightarrow R^n.\end{aligned}$$

由伊藤引理可得

$$x(t)-u(x_t,r(t))=\xi(0)-u(x_0,r(0))+\int_0^t f(x_s,s,r(s))\mathrm{d}s+\int_0^t g(x_s,s,r(s))\mathrm{d}w(s). \tag{2}$$

下面局部李普希茨条件和线性增长条件将发挥着重要作用.

(H) 假定 f 和 g 满足局部李普希茨条件和线性增长条件，即对于 $t\geq 0$, 任何 $i=1,2,\cdots,S$ 和任何 $\varphi,\psi\in C([-\tau,0];R^n)$, $\|\varphi\|\vee\|\psi\|\leq k$, 存在 $L_R>0$ 使得

$$\big|f(\varphi,t,i)-f(\psi,t,i)\big|^2\vee\big|g(\varphi,t,i)-g(\psi,t,i)\big|^2\leq L_R\|\varphi-\psi\|^2$$

及某个常数 $K>0$, $\varphi\in C\big([-\tau,0];R^n\big)$, $t\geq 0$, $i\in S$ 使得

$$|f(\varphi,t,i)|^2+|g(\varphi,t,i)|^2\leq K(1+\|\varphi\|^2).$$

定理 1 假定条件 (H) 成立，若对于 $0<\kappa<1/5$, 有

$$|u(\varphi,i)|^2\leq\kappa(1+\|\varphi\|^2),\qquad 1\leq i\leq N,\tag{3}$$

则方程 (1) 存在唯一的解 $x(t)$, $t\geq-\tau$, 且满足

$$E\Big[\sup_{-\tau\leq s\leq\tau_{k+1}\wedge T}|x(s)|^2\Big]<\frac{c^{k+1}-c}{c-1}+c^{k+1}E\|\xi\|^2,\qquad t\geq 0.$$

证 设 $T>0$, 仅仅需要证明方程 (1) 在 $[-\tau,T]$ 上有唯一解. 由于 $r(t)$ 的每一个样本路径是 R_+ 上的任何有限子区间上是具有有限个简单跳的右连续的步函数 [5], 则存在一序列的停时 $0=\tau_0<\tau_1<\cdots<\tau_k\to\infty$ 使得几乎处处有

$$r(t)=\sum_{k=0}^{\infty}r(\tau_k)I_{[\tau_k.\tau_{k+1})}(t),$$

其中 I_A 为 A 的示性函数，即 $r(t)$ 在每个区间 $[\tau_k,\tau_{k+1})$, $k\geq 0$ 上是常数，

$$r(t)=r(\tau_k),\quad\text{on}\quad\tau_k\leq t<\tau_{k+1}.$$

首先考虑方程 (1) 在 $t\in[0,\tau_1\wedge T]$ 上存在初值为 $x_0=\xi$ 的唯一解 $x(t)$. 事实上，当 $t\in[0,\tau_1\wedge T]$ 时，方程 (1) 可以写成

$$\mathrm{d}(x(t)-u(x_t,r(0)))=f(x_t,t,r(0))\mathrm{d}t+g(x_t,t,r(0))\mathrm{d}w(t),\tag{4}$$

即是

$$x(t)-u(x_t,r(0))=\xi(0)-u(x_0,r(0))+\int_0^t f(x_s,s,r(0))\mathrm{d}s+\int_0^t g(x_s,s,r(0))\mathrm{d}w(s).\tag{5}$$

这是泛函微分方程，根据 Mao[6] 的存在唯一性定理，方程 (4) 在 $[-\tau,\tau_1\wedge T]$ 上有唯一的解 $x(t)$. 因此对于 $t\in[-\tau,\tau_1\wedge T]$, 由方程 (5), 得到

$$\begin{aligned}&E\Big[\sup_{-\tau\leq s\leq\tau_1\wedge T}|x(s)|^2\Big]\leq E\|\xi\|^2+E\Big[\sup_{0\leq s\leq\tau_1\wedge T}|x(s)|^2\Big]\\ \leq&E\|\xi\|^2+E\Big[\sup_{0\leq s\leq\tau_1\wedge T}\Big|u(x_s,r(0))+\xi(0)-u(x_0,r(0))\end{aligned}$$

$$
\begin{aligned}
&+\int_0^s f(x_s,s,r(0))\mathrm{d}s+\int_0^s g(x_s,s,r(0))\mathrm{d}w(s)\Big|^2\Big]\\
\leq& E\|\xi\|^2+5E\Big[\sup_{0\leq s\leq\tau_1\wedge T}|u(x_s,r(0))|^2\Big]+5E\|\xi\|^2+5E\big[|u(x_0,r(0))|^2\big]\\
&+5E\Big[\sup_{0\leq s\leq\tau_1\wedge T}\Big|\int_0^s f(x_s,s,r(0))\mathrm{d}s\Big|^2\Big]\\
&+5E\Big[\sup_{0\leq s\leq\tau_1\wedge T}\Big|\int_0^s g(x_s,s,r(0))\mathrm{d}w(s)\Big|^2\Big]. \qquad (6)
\end{aligned}
$$

根据条件 (3), 得到

$$
\begin{aligned}
&E\Big[\sup_{0\leq s\leq\tau_1\wedge T}|u(x_s,r(0))|^2\Big]\leq\kappa E\Big[\sup_{0\leq s\leq\tau_1\wedge T}(1+\|x_s\|^2)\Big]\\
\leq&\kappa E\Big[\sup_{-\tau\leq s\leq\tau_1\wedge T}(1+|x(s)|^2)\Big], \qquad (7)\\
&E\big[|u(x_0,r(0))|^2\big]\leq\kappa(1+E\|\xi\|^2). \qquad (8)
\end{aligned}
$$

利用 Hölder 不等式及线性增长条件，有

$$
\begin{aligned}
&E\Big[\sup_{0\leq s\leq\tau_1\wedge T}\Big|\int_0^s f(x_s,s,r(0))\,\mathrm{d}s\Big|^2\Big]\\
\leq& T\int_0^t E|f(x_s,s,r(0))|^2\,\mathrm{d}s\leq TK\int_0^t(1+E\|x_s\|^2)\,\mathrm{d}s. \qquad (9)
\end{aligned}
$$

由 Burkholder-Davis-Gundy 不等式及线性增长条件，可得

$$
\begin{aligned}
&E\Big[\sup_{0\leq s\leq\tau_1\wedge T}\Big|\int_0^s g(x_s,s,r(0))\,\mathrm{d}w(s)\Big|^2\Big]\\
\leq&4E\Big[\int_0^{\tau_1\wedge T}|g(x_s,s,r(0))|^2\,\mathrm{d}s\Big]\leq 4K\int_0^t(1+E\|x_s\|^2)\,\mathrm{d}s. \qquad (10)
\end{aligned}
$$

将 (7)–(10) 代入 (6), 可得

$$
\begin{aligned}
&E\Big[\sup_{-\tau\leq s\leq\tau_1\wedge T}|x(t)|^2\Big]\\
\leq&\frac{1}{1-5\kappa}\Big[10\kappa+(6+5\kappa)E\|\xi\|^2+5K(T+4)\int_0^t(1+E\|x_s\|^2)\,\mathrm{d}s\Big],
\end{aligned}
$$

根据 Gronwall 不等式可知，对于 $t\in[-\tau,\tau_1\wedge T]$,

$$
E\Big[\sup_{-\tau\leq s\leq\tau_1\wedge T}|x(s)|^2\Big]\leq\frac{6+5\kappa}{1-5\kappa}(1+E\|\xi\|^2)e^{\frac{5K(T+4)T}{1-5\kappa}}.
$$

若 $t\in[\tau_1\wedge T,\tau_2\wedge T]$, 则方程 (1) 变为

$$
\mathrm{d}(x(t)-u(x_t,r(\tau_1\wedge T)))=f(x_t,t,r(\tau_1\wedge T))\mathrm{d}t+g(x_t,t,r(\tau_1\wedge T))\mathrm{d}w(t), \qquad (11)
$$

其中 $x_{\tau_1\wedge T}$ 为方程 (4) 的解. 注意到 $x_{\tau_1\wedge T}\in L^2_{\mathcal{F}_{\tau_1\wedge T}}([-\tau,0];R^n)$, 因而在 $[\tau_1\wedge T-\tau,\tau_2\wedge T]$ 上方程 (11) 有唯一解. 重复这一过程，则可得方程 (1) 在 $[-\tau,T]$ 上有唯一解. 于 $t\in$

$[\tau_k\wedge T-\tau,\tau_{k+1}\wedge T]$, 由方程 (11) 可得

$$\begin{aligned}h_{k+1}(t)\equiv&E\Big[\sup_{-\tau\le s\le\tau_{k+1}\wedge T}|x(s)|^2\Big]\\\le&E\Big[\sup_{-\tau\le s\le\tau_k\wedge T}|x(s)|^2\Big]+E\Big[\sup_{\tau_k\wedge T\le s\le\tau_{k+1}\wedge T}|x(s)|^2\Big]\\\le&E\Big[\sup_{-\tau\le s\le\tau_k\wedge T}|x(s)|^2\Big]+E\Big[\sup_{\tau_k\wedge T\le s\le\tau_{k+1}\wedge T}\Big|u(x_s,r(s))+x(\tau_k\wedge T)\\&-u(x_{\tau_k\wedge T},r(\tau_k\wedge T))+\int_{\tau_k\wedge T}^s f(x_s,s,r(s))\mathrm{d}s\\&+\int_{\tau_k\wedge T}^s g(x_s,s,r(s))\mathrm{d}w(s)\Big|^2\Big].\end{aligned}\tag{12}$$

依据条件 (3) 和线性增长条件，则有

$$\begin{aligned}&E\Big[\sup_{\tau_k\wedge T\le s\le\tau_{k+1}\wedge T}|u(x_s,r(s))|^2\Big]\\\le&\kappa E\Big[\sup_{\tau_k\wedge T\le s\le\tau_{k+1}\wedge T}(1+\|x_s\|^2)\Big]\le\kappa E\Big[\sup_{-\tau\le s\le\tau_{k+1}\wedge T}(1+|x(s)|^2)\Big].\end{aligned}\tag{13}$$

类似地，可得到

$$E|u(x_{\tau_k\wedge T},r(\tau_k\wedge T))|^2\le\kappa(1+E\|x_{\tau_k\wedge T}\|^2),\tag{14}$$

$$\begin{aligned}&E\Big[\sup_{\tau_k\wedge T\le s\le\tau_{k+1}\wedge T}\Big|\int_{\tau_k\wedge T}^s f(x_s,s,r(s))\mathrm{d}s+g(x_s,s,r(s))\mathrm{d}w(s)\Big|^2\Big]\\\le&(T+4)K\int_0^t(1+E\|x_s\|^2)\,\mathrm{d}s.\end{aligned}\tag{15}$$

将 (13) 和 (15) 代入 (12), 得到

$$\begin{aligned}h_{k+1}(t)\le&E\Big[\sup_{-\tau\le s\le\tau_k\wedge T}|x(s)|^2\Big]+5\kappa E\Big[\sup_{-\tau\le s\le\tau_{k+1}\wedge T}(1+|x(s)|^2)\Big]+5E|x(\tau_k\wedge T)|^2\\&+5\kappa\big(1+E\|x_{\tau_k\wedge T}\|^2\big)+5K(T+4)\int_{\tau_k\wedge T}^{\tau_{k+1}\wedge T}(1+E\|x_s\|^2)\,\mathrm{d}s.\end{aligned}$$

因而

$$\begin{aligned}h_{k+1}\le&h_k+5\kappa+5\kappa h_{k+1}+5h_k+5\kappa(1+h_k)\\&+5K(T+4)\int_{\tau_k\wedge T}^{\tau_{k+1}\wedge T}(1+E\|x_s\|^2)\,\mathrm{d}s,\end{aligned}$$

即

$$\begin{aligned}h_{k+1}\le&\frac{6h_k+5\kappa+5\kappa(1+h_k)}{1-5\kappa}+\frac{5K(T+4)}{1-5\kappa}\int_0^t(1+E\|x_s\|^2)\,\mathrm{d}s\\\le&\frac{(6+5\kappa)(1+h_k)}{1-5\kappa}+\frac{5K(T+4)}{1-5\kappa}\int_0^t(1+h_{k+1})\,\mathrm{d}s.\end{aligned}$$

由 Gronwall 不等式，则

$$h_{k+1} \le \frac{(6+5\kappa)(1+h_k)}{1-5\kappa} e^{\frac{5K(T+4)}{1-5\kappa}T} \equiv c(1+h_k),$$

其中 $c = \frac{6+5\kappa}{1-5\kappa} e^{\frac{5K(T+4)T}{1-5\kappa}}$. 因而 $h_k \le c(1+h_{k-1}) \le \cdots \le c + c^2 + c^3 + \cdots + c^k + c^k E\|\xi\|^2$.

定理 2 假定 (H) 成立且存在常数 $\kappa_2(p\ln\kappa_2 + (p-1)\ln(\kappa_2+1) < (1-p)\ln 3)$ 使得对于初值 $\xi \in L^p_{\mathcal{F}_0}([-\tau,0];R^n)$, $p \ge 2$, 有 $|u(\varphi,i)| \le \kappa_2\|\varphi\|$, 则方程 (1) 的解是 p- 阶矩有界的，即 $E|x(t,\xi)|^p \le Ce^{\beta t}$, 其中

$$C = \frac{[1+3^{p-1}(1+\kappa_2^p)(1+\kappa_2)^{p-1}]E\|\xi\|^p + 2^{p-1}(\frac{\kappa_2}{1+\kappa_2})^{1-p}K^{\frac{p}{2}}(1+\kappa_2)^{\frac{p}{2}-1}[(\frac{p(p-1)}{2})^{\frac{p}{2}}T^{\frac{p}{2}} + T^p]}{1-3^{p-1}\kappa_2^p(1+\kappa_2)^{p-1}},$$

$$\beta = \frac{2^{p-1}(\frac{\kappa_2}{1+\kappa_2})^{1-p}K^{\frac{p}{2}}(1+\kappa_2)^{\frac{p}{2}-1}[(\frac{p(p-1)}{2})^{\frac{p}{2}-1}T^{\frac{p}{2}-1} + T^{p-1}]\kappa_2^{1-\frac{p}{2}}}{1-3^{p-1}\kappa_2^p(1+\kappa_2)^{p-1}}.$$

证　令 $H(t) \equiv \Big[\sup\limits_{-\tau\le s\le t}|x(s)|^p\Big]$, 则

$$\begin{aligned} H(t) \le& E\|\xi\|^p + E\Big(\sup_{0\le s\le t}|x(s)|^p\Big) \\ =& E\|\xi\|^p + E\Big(\sup_{0\le s\le t}\Big|u(x_s,r(s)) + \xi(0) - u(x_0,r(0)) \\ &+ \int_0^s f(x_s,s,r(s))\,\mathrm{d}s + \int_0^s g(x_s,s,r(s))\,\mathrm{d}w(s)\Big|^p\Big) \\ \le& E\|\xi\|^p + (1+\kappa_2)^{p-1}E\Big[\sup_{0\le s\le t}|u(x_s,r(s)) + \xi(0) - u(x_0,r(0))|^p\Big] \\ &+ \Big(\frac{\kappa_2}{1+\kappa_2}\Big)^{1-p}E\Big|\int_0^t f(x_s,s,r(s))\,\mathrm{d}s + \int_0^t g(x_s,s,r(s))\,\mathrm{d}w(s)\Big|^p. \end{aligned} \tag{16}$$

因此

$$\begin{aligned} &E\Big[\sup_{0\le s\le t}|u(x_s,r(s)) + \xi(0) - u(x_0,r(0))|^p\Big] \\ \le& 3^{p-1}E\Big[\sup_{0\le s\le t}|u(x_s,r(s))|^p\Big] + 3^{p-1}E\|\xi\|^p + 3^{p-1}E|u(x_0,r(0))|^p \\ \le& 3^{p-1}\kappa_2^pE\Big[\sup_{0\le s\le t}\|x_s\|^p\Big] + 3^{p-1}E\|\xi\|^p + 3^{p-1}\kappa_2^pE\|\xi\|^p \\ \le& 3^{p-1}(1+\kappa_2^p)E\|\xi\|^p + 3^{p-1}\kappa_2^pE\Big[\sup_{0\le s\le t}\|x_s\|^p\Big] \\ \le& 3^{p-1}(1+\kappa_2^p)E\|\xi\|^p + 3^{p-1}\kappa_2^pE\Big[\sup_{-\tau\le s\le t}|x(s)|^p\Big]. \end{aligned} \tag{17}$$

根据 Hölder 不等式，有

$$\begin{aligned} \int_0^t E(1+\|x_s\|^2)^{\frac{p}{2}}\mathrm{d}s \le& \int_0^t (1+\kappa_2)^{\frac{p}{2}-1}\big(1+\kappa_2^{1-\frac{p}{2}}E\|x_s\|^p\big)\mathrm{d}s \\ =& (1+\kappa_2)^{\frac{p}{2}-1}\int_0^t \big(1+\kappa_2^{1-\frac{p}{2}}E\|x_s\|^p\big)\mathrm{d}s \end{aligned}$$

$$\leq (1+\kappa_2)^{\frac{p}{2}-1}T+\Big(\frac{1+\kappa_2}{\kappa_2}\Big)^{\frac{p}{2}-1}\int_0^t E\|x_s\|^p\mathrm{d}s. \tag{18}$$

由 Hölder 不等式和 (18), 我们得到

$$\begin{aligned}
&E\Big|\int_0^t f(x_s,s,r(s))\,\mathrm{d}s+\int_0^t g(x_s,s,r(s))\,\mathrm{d}w(s)\Big|^p\\
\leq & 2^{p-1}\Big(E\Big|\int_0^t f(x_s,s,r(s))\,ds\Big|^p+E\Big|\int_0^t g(x_s,s,r(s))\,\mathrm{d}w(s)\Big|^p\Big)\\
\leq & 2^{p-1}\Big(T^{p-1}\int_0^t E|f(x_s,s,r(s))|^p\,\mathrm{d}s+\Big(\frac{p(p-1)}{2}\Big)^{\frac{p}{2}}T^{\frac{p}{2}-1}\int_0^t E|g(x_s,s,r(s))|^p\,\mathrm{d}s\Big)\\
\leq & 2^{p-1}K^{\frac{p}{2}}\Big(T^{p-1}+\Big(\frac{p(p-1)}{2}\Big)^{\frac{p}{2}}T^{\frac{p}{2}-1}\Big)\int_0^t \big(1+E\|x_s\|^2\big)^{\frac{p}{2}}\,\mathrm{d}s\\
\leq & 2^{p-1}K^{\frac{p}{2}}\Big(T^{p-1}+\Big(\frac{p(p-1)}{2}\Big)^{\frac{p}{2}}T^{\frac{p}{2}-1}\Big)\\
&\cdot\Big[(1+\kappa_2)^{\frac{p}{2}-1}T+\Big(\frac{1+\kappa_2}{\kappa_2}\Big)^{\frac{p}{2}-1}\int_0^t E\|x_s\|^p\,\mathrm{d}s\Big].
\end{aligned}\tag{19}$$

将 (17) 和 (19) 代入 (16), 可得

$$\begin{aligned}
E\Big[\sup_{-\tau\leq s\leq t}|x(s)|^p\Big]\leq&\big[1+3^{p-1}(1+\kappa_2^p)(1+\kappa_2)^{p-1}\big]E\|\xi\|^p+3^{p-1}\kappa_2^p(1+\kappa_2)^{p-1}\\
&\cdot E\Big[\sup_{-\tau\leq s\leq t}|x(s)|^p\Big]+2^{p-1}\Big(\frac{\kappa_2}{1+\kappa_2}\Big)^{1-p}K^{\frac{p}{2}}(1+\kappa_2)^{\frac{p}{2}-1}\\
&\cdot\Big[\Big(\frac{p(p-1)}{2}\Big)^{\frac{p}{2}}T^{\frac{p}{2}-1}+T^{p-1}\Big]\Big(T+\kappa_2^{1-\frac{p}{2}}\int_0^t E\|x_s\|^p\,\mathrm{d}s\Big),
\end{aligned}$$

因而

$$E\Big[\sup_{-\tau\leq s\leq t}|x(s)|^p\Big]\leq C+\beta\int_0^t E\|x_s\|^p\,\mathrm{d}s,$$

利用 Gronwall 不等式，则 $E\big[\sup\limits_{-\tau\leq s\leq t}|x(s)|^p\big]\leq Ce^{\beta t}$.

3 指数稳定性

定义 1 方程 (1) 被称为是 p- 阶矩指数稳定的，如果对于 $\xi\in C^b_{\mathcal{F}_0}([-\tau,0];R^n)$, 有

$$\limsup_{t\to\infty}\frac{1}{t}\log(E|x(t;\xi)|^p)<0.$$

方程 (1) 被称为是几乎处处 p- 阶指数稳定的，如果对于 $\xi\in C^b_{\mathcal{F}_0}([-\tau,0];R^n)$,

$$\limsup_{t\to\infty}\frac{1}{t}\log(|x(t;\xi)|)<0\qquad\text{a.s.}$$

令 $C^{2,1}(R^n\times[-\tau,\infty]\times S;R_+)$ 为 $R^n\times[-\tau,\infty]\times S$ 上关于 x 二次可微，关于 t 一次可微的所有非负函数 $V(x,t,i)$ 全体，若 $V(x,t,i)\in C^{2,1}(R^n\times[-\tau,\infty]\times S;R_+)$, 定义算

子 $\mathcal{L}V: C([-\tau,0];R^n)\times R^n\times R_+\times S\to R$ 如下：

$$\begin{aligned}LV(\tilde{x},\varphi,t,i) =& V_t(\tilde{x},t,i)+V_x(\tilde{x},t,i)f(\varphi,t,i)\\ &+\frac{1}{2}\mathrm{trace}\,[g^T(\varphi,t,i)V_{xx}(\tilde{x},t,i)g(\varphi,t,i)]+\sum_{j=1}^{N}\gamma_{ij}V(\tilde{x},t,i),\end{aligned}$$

其中 $\tilde{x}=x-u(x_t,r(t))$, $V_t=\frac{\partial V(x,t,i)}{\partial t}$, $V_x=\big(\frac{\partial V(x,t,i)}{\partial x_1},\ \frac{\partial V(x,t,i)}{\partial x_2},\cdots,\frac{\partial V(x,t,i)}{\partial x_n}\big)$, $V_{xx}=\big(\frac{\partial^2 V(x,t,i)}{\partial x_i\partial x_j}\big)_{n\times n}$.

定理 3 设 (H) 成立，设 $\lambda,\alpha_1,\alpha_2$ 是正数，$p\ge 2$，假设存在函数 $V(x,t,i)\in C^{2,1}(R^n\times[-\tau,\infty]\times S;R_+)$ 使得

$$\begin{aligned}&\alpha_1|x|^p\le V(x,t,i)\le\alpha_2|x|^p,\qquad 1\le i\le N,\\ &|u(\varphi,i)|\le\kappa_2\|\varphi\|,\qquad 1\le i\le N,\quad 0<\kappa_2<1,\\ &\mathrm{LV}\,(\tilde{x},\varphi,s,i)\le-\lambda_1|\tilde{x}|^p+\lambda_2\|\varphi\|^p+\beta,\end{aligned}\tag{20}$$

其中 $\lambda_1\ge\frac{\lambda_2}{\kappa_2(1+\kappa_2)^{p-1}}$, 则对于所有的 $\xi\in C^b_{\mathcal{F}_0}([-\tau,0];R^n)$, 有

$$E|x(t;\xi)|^p\le\frac{\beta}{\alpha_1\gamma(1-\kappa_2)^p}+\Big(\frac{\kappa_2}{1-\kappa_2}+\frac{C_0}{(1-\kappa_2)^p}\Big)e^{-\gamma t}E\|\xi\|^p,$$

其中 $\gamma\in(0,\gamma_0),\gamma_0=\frac{\lambda_1}{\alpha_2}-\frac{\lambda_2}{\alpha_2\kappa_2(1+\kappa_2)^{p-1}}$, 即方程 (1) 的平凡解是 p- 阶矩指数稳定的.

证　对于给定的 $\xi\in C^b_{\mathcal{F}_0}([-\tau,0];R^n)$, 记 $x(t;\xi)=x(t)$, 则

$$\begin{aligned}e^{\gamma t}E|x(t)|^p =& e^{\gamma t}E|x(t)-u(x_t,r(t))+u(x_t,r(t))|^p\\ \le& e^{\gamma t}E\Big|1\times u(x_t,r(t))+\varepsilon^{\frac{p-1}{p}}\frac{x(t)-u(x_t,r(t))}{\varepsilon^{\frac{p-1}{p}}}\Big|^p\\ \le& e^{\gamma t}(1+\varepsilon)^{p-1}E|u(x_t,r(t))|^p+e^{\gamma t}\Big(\frac{1+\varepsilon}{\varepsilon}\Big)^{p-1}E|x(t)-u(x_t,r(t))|^p\\ \le& e^{\gamma t}\kappa_2^{1-p}E|u(x_t,r(t))|^p+e^{\gamma t}(1-\kappa_2)^{1-p}E|x(t)-u(x_t,r(t))|^p\\ \le& e^{\gamma t}\kappa_2E\|x_t|^p+e^{\gamma t}(1-\kappa_2)^{1-p}E|x(t)-u(x_t,r(t))|^p,\end{aligned}$$

取 $\varepsilon=\frac{1-\kappa_2}{\kappa_2}$. 由 (20), 则得

$$\begin{aligned}&\alpha_1e^{\gamma t}E|x(t)-u(x_t,r(t))|^p\\ \le& e^{\gamma t}EV(x(t)-u(x_t,r(t)),t,r(t))\\ \le& EV(\xi(0)-u(x_0,r(0)),0,r(0))\\ &+\int_0^t e^{\gamma s}E\big[\gamma V(x(s)-u(x_s,r(s)),s,r(s))+LV(x(s)-u(x_s,r(s)),x_s,s,r(s))\big]\,\mathrm{d}s\\ \le&\alpha_2E|\xi(0)-u(x_0,r(0))|^p\\ &+\int_0^t e^{\gamma s}\big[\alpha_2\gamma E|x(s)-u(x_s,r(s))|^p-\lambda_1E|x(s)-u(x_s,r(s))|^p+\lambda_2E\|x_s\|^p+\beta\big]\,\mathrm{d}s\\ \le&\alpha_2(1+\kappa_2)^pE\|\xi\|^p+\int_0^t e^{\gamma s}\big[(\alpha_2\gamma-\lambda_1)E|x(s)-u(x_s,r(s))|^p+\lambda_2E\|x_s\|^p+\beta\big]\,\mathrm{d}s\end{aligned}$$

$$
\begin{aligned}
&\le \alpha_2(1+\kappa_2)^p E\|\xi\|^p + \frac{\beta e^{\gamma t}}{\gamma} + (\alpha_2\gamma - \lambda_1)\int_0^t e^{\gamma s} E|x(s) - u(x_s, r(s))|^p \, \mathrm{d}s \\
&\quad + \lambda_2 \int_0^t e^{\gamma s} E\|x_s\|^p \, \mathrm{d}s. \qquad (21)
\end{aligned}
$$

因而

$$
\begin{aligned}
E|x(s) - u(\varphi(s), r(s))|^p &\le (1+\kappa_2)^{p-1}(E|x|^p + \kappa_2^{1-p} E|u(x_t, r(t))|^p) \\
&\le (1+\kappa_2)^{p-1}(E|x|^p + \kappa_2^{1-p}\kappa_2^p E\|x_t\|^p) \\
&\le (1+\kappa_2)^{p-1}(E|x|^p + \kappa_2 E\|x_t\|^p). \qquad (22)
\end{aligned}
$$

将 (22) 代入 (21), 结果是

$$
\begin{aligned}
&\alpha_1 e^{\gamma t} E|x(t) - u(x_t, r(t))|^p \\
\le& \alpha_2(1+\kappa_2)^p E\|\xi\|^p + \frac{\beta e^{\gamma t}}{\gamma} + \lambda_2 \int_0^t e^{\gamma s} E\|x_s\|^p \mathrm{d}s \\
&+ (\alpha_2\gamma - \lambda_1)\int_0^t e^{\gamma s}\big[(1+\kappa_2)^{p-1} E|x(s)|^p + (1+\kappa_2)^{p-1}\kappa_2 E\|x_s\|^p\big] \, \mathrm{d}s \\
\le& \alpha_2(1+\kappa_2)^p E\|\xi\|^p + \frac{\beta e^{\gamma t}}{\gamma} + (\alpha_2\gamma - \lambda_1)(1+\kappa_2)^{p-1} \int_0^t e^{\gamma s} E|x(s)|^p \, \mathrm{d}s \\
&+ (\lambda_2 - (\lambda_1 - \alpha_2\gamma)(1+\kappa_2)^{p-1}\kappa_2)\int_0^t e^{\gamma s} E\|x_s\|^p \mathrm{d}s. \qquad (23)
\end{aligned}
$$

由于

$$
\begin{aligned}
\int_0^t e^{\gamma s} E\|x_s\|^p \, \mathrm{d}s &\le \sup_{-\tau\le\theta\le 0} \int_0^t e^{\gamma s} E|x(s+\theta)|^p \mathrm{d}s \\
&\le \sup_{-\tau\le\theta\le 0} \int_0^t e^{\gamma\tau} E\big[e^{\gamma(s+\theta)}|x(s+\theta)|^p\big] \mathrm{d}s \\
&\le e^{\gamma\tau}\int_{-\tau}^t e^{\gamma s} E|x(s)|^p \, \mathrm{d}s \\
&\le e^{\gamma\tau}\Big(\tau E\|\xi\|^p + \int_0^t e^{\gamma s} E|x(s)|^p \mathrm{d}s\Big).
\end{aligned}
$$

将此代入 (23), 得到

$$
\begin{aligned}
&e^{\gamma t} E|x(t) - u(x_t, r(t))|^p \\
\le& \frac{1}{\alpha_1}\big[(1+\kappa_2)^p\alpha_2 + \tau e^{\gamma\tau}(\lambda_2 - (\lambda_1 - \alpha_2\gamma)\kappa_2(1+\kappa_2)^{p-1})\big] E\|\xi\|^p + \frac{\beta e^{\gamma t}}{\alpha_1\gamma} \\
&+ \frac{1}{\alpha_1}\big[\lambda_2\tau e^{\gamma\tau} + \alpha_2\gamma(1+\kappa_2)^{p-1}(1+\kappa_2\tau e^{\gamma\tau}) \\
&- \lambda_1(1+\kappa_2)^{p-1}(1+\kappa_2\tau e^{\gamma\tau})\big]\int_0^t e^{\gamma s}|x(s)|^p \, \mathrm{d}s.
\end{aligned}
$$

因此对于 $\gamma \in (0, \gamma_0)$, 我们有

$$\begin{aligned}&\lambda_2 e^{\gamma\tau}+\alpha_2\gamma(1+\kappa_2)^{p-1}(1+\kappa_2 e^{\gamma\tau})-\lambda_1(1+\kappa_2)^{p-1}(1+\kappa_2 e^{\gamma\tau})\\=&[\lambda_2+(\alpha_2\gamma-\lambda_1)(1+\kappa_2)^{p-1}\kappa_2]e^{\gamma\tau}+(\alpha_2\gamma-\lambda_1)(1+\kappa_2)^{p-1}\le 0,\\&e^{\gamma t}E|x(t)-u(x_t,r(t))|^p\le C_0E\|\xi\|^p+\frac{\beta e^{\gamma t}}{\alpha_1\gamma},\end{aligned}$$

其中

$$C_0=\frac{1}{\alpha_1}\big[(1+\kappa_2)^p\alpha_2+\tau e^{\gamma\tau}(\lambda_2-(\lambda_1-\alpha_2\gamma)\kappa_2(1+\kappa_2)^{p-1})\big].$$

由 (21), 则

$$\begin{aligned}E\Big[\sup_{0\le s\le t}e^{\gamma s}|x|^p\Big]\le&\kappa_2E\Big[\sup_{0\le s\le t}e^{\gamma s}\|x_s\|^p\Big]+(1-\kappa_2)^{1-p}E\Big[\sup_{0\le s\le t}e^{\gamma s}|x(s)-u(x_s,r(s))|^p\Big]\\\le&\kappa_2\Big(E\|\xi\|^p+E\Big[\sup_{0\le s\le t}e^{\gamma s}|x(s)|^p\Big]\Big)+(1-\kappa_2)^{1-p}\Big(C_0E\|\xi\|^p+\frac{\beta e^{\gamma t}}{\alpha_1\gamma}\Big),\end{aligned}$$

因此对于 $t\ge 0,\ \gamma=\min\{\gamma_0,\frac{1}{2\tau}\log\frac{1}{\kappa_2}\}$,

$$E|x(t)|^p\le\Big(\frac{\kappa_2}{1-\kappa_2}+\frac{C}{(1-\kappa_2)^p}\Big)e^{-\gamma t}E\|\xi\|^p+\frac{\beta}{\alpha_1\gamma(1-\kappa_2)^p}.$$

定理 4 假定定理 3 的条件成立，令 $\beta=0$, 则对于初值 $\xi\in C^b_{\mathcal{F}_0}([-\tau,0];R^n)$, 方程 (1) 的解是 p- 阶矩指数稳定的，即

$$\limsup_{t\to\infty}\frac{1}{t}\log\left(E|x(t;\xi)|^p\right)<-\gamma.$$

定理 5 假定定理 3 的条件成立，且 $\beta=0$, 则存在正数 K_p,κ_2,c_2 使得对于 $p\ge 2$ 有

$$|f(\varphi,t.i)|^p\vee|g(\varphi,t.i)|^p\le K_p\|\varphi\|^p,\tag{24}$$

$$E|x(t)|^p\le c_2e^{-\gamma t},\qquad |u(\varphi,t)|\le\kappa_2\|\varphi\|,\tag{25}$$

其中 $\gamma>0$ 如定理 3, 则对于 $p\ge 2$ 和任意初值 $\xi\in C^b_{\mathcal{F}_0}([-\tau,0];R^n)$, 有

$$\limsup_{t\to\infty}\frac{1}{t}\log\big(|x(t;\xi)|\big)<-\frac{\gamma-\epsilon}{p},\qquad \text{a.s.},$$

即 p 阶矩指数稳定蕴含了几乎处处指数稳定.

证 对于每一个 $k\ge 2$, 我们有

$$\begin{aligned}&E\|x_{k\tau}\|^p=E\Big(\sup_{-\tau\le\theta\le 0}|x(k\tau+\theta)|^p\Big)\\=&E\Big(\sup_{0\le h\le\tau}|x((k-1)\tau+h)|^p\Big)\\=&E\Big(\sup_{0\le h\le\tau}\Big|x((k-1)\tau)-u(x_{(k-1)\tau},r((k-1)\tau))+u(x_{(k-1)\tau+h},r((k-1)\tau+h))\end{aligned}$$

$$
\begin{aligned}
&+\int_{(k-1)\tau}^{(k-1)\tau+h}(f(x_s,s,r(s))\,\mathrm{d}s+g(x_s,s,r(s))\mathrm{d}w(s))\Big|^p\Big)\\
\le&5^{p-1}\Big(E\Big[\sup_{0\le h\le\tau}|u(x_{(k-1)\tau+h},r((k-1)\tau+h))|^p\Big]+E|x((k-1)\tau)|^p\\
&+E\big[|u(x_{(k-1)\tau},r((k-1)\tau))|^p\big]+\Big|\int_{(k-1)\tau}^{k\tau}f(x_s,s,r(s))\,\mathrm{d}s\Big|^p\\
&+E\Big[\Big|\int_{(k-1)\tau}^{k\tau}g(x_s,s,r(s))\,\mathrm{d}w(s)\Big|^p\Big]\Big)\\
\le&5^{p-1}\Big(E\Big[\sup_{0\le h\le\tau}\kappa_2^p|x_{(k-1)\tau+h}|^p\Big]+E|x((k-1)\tau)|^p+E[\kappa_2|x_{(k-1)\tau}|^p]\\
&+\Big|\int_{(k-1)\tau}^{k\tau}f(x_s,s,r(s))\,\mathrm{d}s\Big|^p+E\Big[\Big|\int_{(k-1)\tau}^{k\tau}g(x_s,s,r(s))\,\mathrm{d}w(s)\Big|^p\Big]\Big).
\end{aligned}
$$

由 (25), 可得

$$
\begin{aligned}
&E\Big(\sup_{0\le h\le\tau}|x_{(k-1)\tau+h}|^p\Big)\le E\Big(\sup_{(k-1)\tau\le s\le k\tau}|x_s|^p\Big)\\
\le&E\Big(\sup_{(k-1)\tau\le s\le k\tau,-\tau\le\theta\le0}|x(s+\theta)|^p\Big)\\
\le&E\Big(\sup_{(k-2)\tau\le s+\theta\le k\tau}ce^{-\gamma(s+\theta)}\Big)\le c_2e^{-\gamma(k-2)\tau}.
\end{aligned}\tag{26}
$$

易得

$$
E\|x_s\|^p\le E\Big[\sup_{-\tau\le\theta\le0}|x(s+\theta)|^p\Big]\le c_2e^{-\gamma(s-\tau)}.\tag{27}
$$

根据 Hölder 不等式，有

$$
\begin{aligned}
&\Big|\int_{(k-1)\tau}^{k\tau}(f(x_s,s,r(s))\,\mathrm{d}s\Big|^p\le\tau^{p-1}\int_{(k-1)\tau}^{k\tau}E|f(x_s,s,r(s)|^p\,\mathrm{d}s\\
\le&\tau^{p-1}\int_{(k-1)\tau}^{k\tau}\sum_{1\le i\le N}E|f(x_s,s,i)|^p\,\mathrm{d}s\le\tau^{p-1}NK_p\int_{(k-1)\tau}^{k\tau}\|x_s\|^p\,\mathrm{d}s\\
\le&\tau^{p-1}NK_P\int_{(k-1)\tau}^{k\tau}ce^{-\gamma(s-\tau)}\,\mathrm{d}s\le\tau^{p-1}c_2\gamma^{-1}NK_pe^{-\gamma(k-2)\tau};
\end{aligned}
$$

同理

$$
E\Big|\int_{(k-1)\tau}^{k\tau}(g(x_s,s,r(s))\,\mathrm{d}w(s)\Big|^p\le\tau^{p/2-1}\Big(\frac{p(p-1)}{2}\Big)^{p/2}c_2\gamma^{-1}NK_pe^{-\gamma(k-2)\tau}.
$$

因而

$$
\begin{aligned}
E|x_{k\tau}|^p\le&5^{p-1}\Big[c\kappa_2e^{-\gamma(k-2)\tau}+c_2e^{-\gamma(k-1)\tau}+c_2\kappa_2e^{-\gamma(k-1)\tau}\\
&+\Big(\tau^{p-1}+\Big(\frac{p(p-1)}{2}\Big)^{\frac p2}\tau^{\frac p2-1}\Big)\frac{c_2NK_p}{\gamma}e^{-\gamma(k-2)\tau}\Big]
\end{aligned}
$$

$$\begin{aligned}&\le 5^{p-1}\Big[c_2\kappa_2 e^{2\gamma\tau}+c(1+\kappa_2)e^{\gamma\tau}\\&\quad+\Big(\tau^{p-1}+\Big(\frac{p(p-1)}{2}\Big)^{\frac{p}{2}}\tau^{p/2-1}\Big)\frac{c_2NK_P}{\gamma}e^{-2\gamma\tau}\Big]e^{-\gamma k\tau}\\&\equiv c_2e^{-\gamma k\tau},\end{aligned}$$

其中 $c_2=5^{p-1}[c_2\kappa_2e^{2\gamma\tau}+c_2(1+\kappa_2)e^{\gamma\tau}+(\tau^{p-1}+(\frac{p(p-1)}{2})^{p/2}\tau^{p/2-1})\frac{c_2NK_p}{\gamma}e^{-2\gamma\tau}]$. 根据定理 3 的证明过程可得

$$E|x(k\tau)-u(x_{k\tau},r(k\tau))|^p\le C_0e^{-\gamma k\tau}E\|\xi\|^p,$$

其中 C_0 是独立于 k 的正常数. 因而可得

$$\begin{aligned}E|x(k\tau)|^p&=E\big|x(k\tau)-u(x_{k\tau},r(k\tau))+u(x_{k\tau},r(k\tau))\big|^p\\&\le 2^{p-1}E\big|x(k\tau)-u(x_{k\tau},r(k\tau))\big|^p+2^{p-1}E\big|u(x_{k\tau},r(k\tau))\big|^p\\&\le 2^{p-1}C_0e^{-\gamma k\tau}E\|\xi\|^p+2^{p-1}c_1\kappa_2^pe^{-\gamma k\tau}\\&=\big(2^{p-1}C_0E\|\xi\|^p+2^{p-1}c_1\kappa_2^p\big)e^{-\gamma k\tau}.\end{aligned}$$

因而对于任意 $\varepsilon\in(0,\gamma)$,

$$P\big\{|x(k\tau)|^p>e^{-(\gamma-\epsilon)k\tau}\big\}\le e^{(\gamma-\epsilon)k\tau}E|x(k\tau)|^p\le 2^{p-1}(C_0E\|\xi\|^p+\kappa_2^pc_1)e^{-\epsilon k\tau}.$$

根据 Borel Cantelli[7] 引理，则对于几乎所有的 $w\in\Omega$ 和除有限多个 k, 有 $|x(k\tau)|^p\le e^{-(\gamma-\epsilon)k\tau}$. 因此对于除零测集的所有的 $w\in\Omega$, 存在整数 $k_0=k_0(w)$ 使得当 $k\ge k_0$, 有

$$|x(k\tau)|^p\le e^{-(\gamma-\epsilon)k\tau},$$

即对于所有的 $w\in\Omega$, 若 $(k-1)\tau\le t\le k\tau,\ k\ge k_0$, 则

$$\frac{1}{t}\log|x(t)|\le-\frac{k\tau(\gamma-\epsilon)}{p(k-1)\tau},\qquad t\ge k_0\tau,$$

因此

$$\limsup_{t\to\infty}\frac{1}{t}\log|x(t)|\le-\frac{\gamma-\epsilon}{p}\qquad \text{a.s.}$$

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Neutral Stochastic Functional Differential Equations with Markovian Switchings

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Abstract The stability of stochastic functional differential equation with Markovian switching have an increasing attention, but there is almost no work on the stability of the neutral stochastic functional differential equations with Markovian switching. The main aim of this paper is to close this gap. We establishes the existence and uniqueness of the neutral stochastic functional differential equations with Markovian switching, and obtain criteria for p-th moment exponential stability and almost surely exponential stability for the solutions.

Key words Markovian chain; Brownian motion;
neutral stochastic functional differential equation; exponential stability

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