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Implementing W state of remote atoms trapped in separated cavities

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Abstract: A scheme was proposed for implementing the entangled W state of three atoms trapped in distant cavities connected by single-mode fibers. The scheme is robust to atomic spontaneous decay, cavity decay and photon leaking out of the fiber due to that the atomic system, all the modes of cavity fields and fibers are only virtually excited. Compared to the previous schemes, the significant advantage is that the adiabatic passage is applied in the scheme. It does not need precise control of the Rabi frequency, pulse duration and is insensitive to moderate fluctuations of experimental parameters. In principle, the n-atom W state can be prepared by using such a method.

Key words: quantum optics; W state; remote atoms; separated cavities

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基于分离腔系统远程制备 W 态方案

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摘 要： 提出了一个由两光纤连接的三个分离腔中远程制备三原子 W 态的方案。制备过程中，由于原子系统、腔模和光纤模均处于非激发态，该方案能有效地抑制原子的自发辐射、腔衰减以及光纤泄露。相比于其他方案，此方案的优点是所用的绝热演化方法对实验参数的变化不敏感。另外，该方案可以简单推广到制备 n 个原子的 W 态。

关键词： W 态；远程原子；分离腔

1 Introduction

Quantum entanglement, as one of the remarkable features of quantum mechanics, not only provides a basic ingredient for testing quantum nonlocality against local hidden theory[1~3], but also plays a crucial part in many quantum information processing task, such as quantum teleportation[4], quantum secret sharing[5], and distributed quantum computation[6] etc. Therefore, the entangled states have drawn more and more people's attention both in theory and experiment. It goes without saying that the generation of the entangled states becomes increasingly hard with the number of relevant particles increase. In terms of three-particle entangled

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states, there are two typical forms: the GHZ state and the W state[7]. Compared with GHZ state, the distinct characteristic of W state is that when one of the three particles is traced out the rest of two particles remain a relatively high entanglement degree[8]. In other words, the W state is more superior to GHZ state in resist particle losing[9].

It is no doubt that the cavity quantum electrodynamics (QED), one of the possible candidates for implementing quantum information process, always attract much attention. It is due to the case that the atomic systems are well suit for storing the quantum information over a long time scale and the photons are suitable to communicate over long distance for distributing quantum information[10]. On the other hand, entanglement between separate subsystems play an important role in many quantum information processes[11] and distributed quantum computation[12]. Therefore, recently, based on cavity QED system, various schemes have been proposed to prepare entanglement state of distant atoms trapped in distant optical cavities connected by fibers[13~19]. Ye *et al.*[15] put forward a scheme to generate a three-dimension entangled state of two atoms separately trapped in two cavities which is connected by an optical fiber. Zheng[16] presented a scheme for generating entangled GHZ states of multiple atoms trapped in distant cavities connected by fibers. Very recently, Lü *et al.*[17] proposed a scheme for achieving multipartite entanglement of distant atoms through selective photon emission and absorption process and used the method of numerical simulate. However, the deterministically generated is relying on the appropriate time. Inspired by the above-mentioned proposals, in this paper, we present a scheme for generating the entangled W state for spatially separated atoms in separated cavities through adiabatic passage along dark state. Compared with the paper[18], our scheme take advantage of adiabatic passage for generating entangled state and it does not need precise control of the Rabi frequency and pulse duration. What is more, neither the atomic system nor the modes of cavities and fibers are excited during the whole interaction process so that the scheme is robust to atomic spontaneous emission, cavity decay and fiber-losing.

2 Generation of W state for atoms trapped in separated cavities

We consider that three distant cavities (set C_1, C_2 and C_3) connected by two optical fibers A and B, as depicted in Fig.1. Three atoms 1, 2 and 3 are individually trapped in cavities C_1, C_2 and C_3, which was respectively driven by three classical fields with Rabi frequencies Ω_1, Ω_2 and Ω_3. In addition, their corresponding atom-field coupling constants are denoted by g_1, g_2 and g_3. Δ_1, Δ_2 and Δ_3 are the frequency detuning and satisfy the atom-photon resonance conditions, which can be found in Fig.2 clearly.

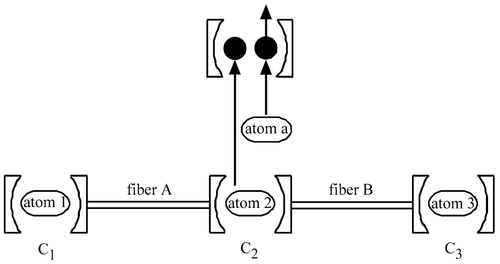

Fig.1 Experimental setup of preparing entangled W state. Atom 1, 2 and 3 are trapped in three spatially separated cavities C_1, C_2 and C_3, respectively. The cavities are linked by optical fibers A and B

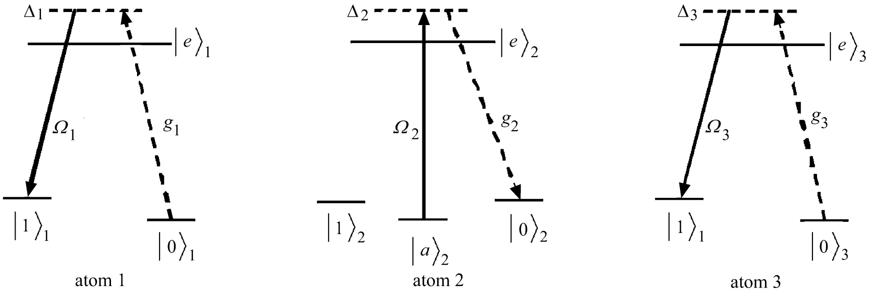

Fig.2 The level configuration of each atom in the scheme

In the interaction picture, the total Hamiltonian of the cavity-fiber-cavity system under the dipole and rotating-wave approximation can be written by (setting $\hbar = 1$)[15,19,20]

$$\begin{aligned} H= & \sum_{n=1,2,3}(-\Delta_n|e\rangle_n\langle e|) + (\Omega_1|e\rangle_1\langle 1| + g_1a_1|e\rangle_1\langle 0| + \\ & \Omega_2|e\rangle_2\langle a| + g_2a_2|e\rangle_2\langle 0| + \Omega_3|e\rangle_3\langle 1| + g_3a_3|e\rangle_3\langle 0| + h.c.) + \\ & [\eta_\mathrm{A}b_\mathrm{A}(a_1^\dagger + a_2^\dagger) + \eta_\mathrm{B}b_\mathrm{B}(a_2^\dagger + a_3^\dagger) + h.c.], \end{aligned} \tag{1}$$

where η_A and η_B are the corresponding cavity-fiber coupling constants. b_A and b_B are the annihilation operator for the fibers A and B. $a_1^\dagger, a_2^\dagger$ and $a_3^\dagger$ are the creation operator for the cavity (C_1, C_2 and C_3) mode, respectively. In the above calculation, we assume the short fiber limit, that is, $\frac{2L\bar{\nu}}{2\pi c} \ll 1$. Under the condition of large-detuning, $|\Delta_n| \gg |\Omega_n|, |g_n| (n = 1, 2, 3)$, the excited states of atoms can be adiabatically eliminated. Therefore, the effective Hamiltonian of the total system is given as

$$\begin{aligned} H_\mathrm{eff}= & [\Omega_{e1}a_1|1\rangle_1\langle 0| + \Omega_{e2}a_2|a\rangle_2\langle 0| + \Omega_{e3}a_3|1\rangle_3\langle 0| + h.c.] + \\ & \eta_\mathrm{A}b_\mathrm{A}(a_1^\dagger + a_2^\dagger) + \eta_\mathrm{B}b_\mathrm{B}(a_2^\dagger + a_3^\dagger) + h.c.], \end{aligned} \tag{2}$$

where $\Omega_{en} = g_n\Omega_n/\Delta_n (n = 1, 2, 3)$ are the effective Rabi frequencies relevant to the Raman transitions of $|0\rangle_1 \to |e\rangle_1 \to |1\rangle_1, |a\rangle_2 \to |e\rangle_2 \to |0\rangle_2$ and $|0\rangle_3 \to |e\rangle_3 \to |1\rangle_3$. Now, we will elaborate the generation of W states of atoms based on the above model. Suppose that atom 1, 2, 3 are initially in the state $|0a0\rangle_{123}$ and all the modes of cavities and fibers are in the vacuum state $|000\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}}$. Then, the cavity-fiber-cavity system remains in the subspace spanned by $\{|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle, |\phi_4\rangle, |\phi_5\rangle, |\phi_6\rangle, |\phi_7\rangle, |\phi_8\rangle\}$, where

$$\begin{aligned} & |\phi_1\rangle = |0a0\rangle_{123}|000\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}}, \quad |\phi_2\rangle = |000\rangle_{123}|010\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}}, \\ & |\phi_3\rangle = |000\rangle_{123}|000\rangle_{\mathrm{c_1c_2c_3}}|10\rangle_{f_\mathrm{A}f_\mathrm{B}}, \quad |\phi_4\rangle = |000\rangle_{123}|000\rangle_{\mathrm{c_1c_2c_3}}|01\rangle_{f_\mathrm{A}f_\mathrm{B}}, \\ & |\phi_5\rangle = |000\rangle_{123}|100\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}}, \quad |\phi_6\rangle = |000\rangle_{123}|001\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}}, \\ & |\phi_7\rangle = |100\rangle_{123}|000\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}}, \quad |\phi_8\rangle = |001\rangle_{123}|000\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}}. \end{aligned} \tag{3}$$

The dark state with null eigenvalue is

$$\begin{aligned} |\Psi\rangle = & N[-\Omega_{e1}\Omega_{e3}(\eta_\mathrm{A} + \eta_\mathrm{B})|0a0\rangle_{123}|000\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}} + \Omega_{e1}\Omega_{e2}\Omega_{e3}|000\rangle_{123}\times \\ & |000\rangle_{\mathrm{c_1c_2c_3}}|10\rangle_{f_\mathrm{A}f_\mathrm{B}} + \Omega_{e1}\Omega_{e2}\Omega_{e3}|000\rangle_{123}|000\rangle_{\mathrm{c_1c_2c_3}}|01\rangle_{f_\mathrm{A}f_\mathrm{B}} - \Omega_{e2}\Omega_{e3}\eta_\mathrm{A}\times \\ & |100\rangle_{123}|000\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}} - \Omega_{e1}\Omega_{e2}\eta_\mathrm{B}|001\rangle_{123}|000\rangle_{\mathrm{c_1c_2c_3}}|00\rangle_{f_\mathrm{A}f_\mathrm{B}}], \end{aligned} \tag{4}$$

here

$$N = \sqrt{\frac{1}{\Omega_{e1}^2\Omega_{e3}^2(\eta_\mathrm{A} + \eta_\mathrm{B})^2 + 2\Omega_{e1}^2\Omega_{e2}^2\Omega_{e3}^2 + \Omega_{e2}^2\Omega_{e3}^2\eta_\mathrm{A}^2 + \Omega_{e1}^2\Omega_{e2}^2\eta_\mathrm{B}^2}}. \tag{5}$$

At the beginning of the stage we set $\eta_{\rm A} = \eta_{\rm B} = \eta$ and $\Omega_{e1} = \Omega_{e2} = \Omega_{e3} = \Omega_e$. Slowly decreasing Ω_e while increasing η until it satisfy the condition of $\eta \gg \Omega_e$. Then the dark state will given by

$$|\varPsi\rangle = \frac{1}{\sqrt{6}}(2|0a0\rangle_{123} + |100\rangle_{123} + |001\rangle_{123})|000\rangle_{{\rm c}_1{\rm c}_2{\rm c}_3}|00\rangle_{f_{\rm A}f_{\rm B}}. \tag{6}$$

Perform a single-qubit rotation $|a\rangle_2 \to |1\rangle_2$ on atom 2, the state of system becomes the following form

$$|\varPsi\rangle = \frac{1}{\sqrt{6}}(2|010\rangle_{123} + |100\rangle_{123} + |001\rangle_{123})|000\rangle_{{\rm c}_1{\rm c}_2{\rm c}_3}|00\rangle_{f_{\rm A}f_{\rm B}}. \tag{7}$$

In this case, we obtain the entangled W-like state of three separated atoms and meanwhile all the modes of cavity fields and fibers are both in the vacuum state. Then, we design the course that change the W-like state into the standard form of W state as

$$\frac{1}{\sqrt{3}}(|010\rangle_{123} + |100\rangle_{123} + |001\rangle_{123}). \tag{8}$$

Consider two identical two-level atoms simultaneously interacting with a single mode cavity field. Under the large-detuning condition, the effective Hamiltonian in interaction picture is[21]

$$H = \lambda \sum_{m(m=p,q)} (|1_m\rangle\langle 1_m|aa^\dagger - |0_m\rangle\langle 0_m|a^\dagger a) + (|1_p\rangle\langle 0_p||0_q\rangle\langle 1_q| + |0_p\rangle\langle 1_p||1_q\rangle\langle 0_q|), \tag{9}$$

where $\lambda = g^2/\delta$. If cavity field is initially in the vacuum state and the atoms are initially in the state $|10\rangle_{pq}$. Then, the state evolution of the system is

$$|10\rangle_{pq} \to {\rm e}^{-{\rm i}\lambda t}[\cos(\lambda t)|10\rangle_{pq} - {\rm i}\sin(\lambda t)|01\rangle_{pq}], \tag{10}$$

while the interaction Hamiltonian takes no effect on state $|00\rangle_{pq}$.

After atom 2 be released from cavity C$_2$, and atom a are simultaneously sent to another cavity. In the condition of large-detuning and choosing the appropriate interaction time satisfy $\lambda t = \pi/3$, the state of system is

$$\begin{aligned}|\varPsi\rangle_{12{\rm a}3} = \sqrt{\frac{8}{5}} \times \frac{1}{\sqrt{6}}\Big[({\rm e}^{-{\rm i}\frac{\pi}{3}}|010\rangle_{123} + |100\rangle_{123} + |001\rangle_{123})|0\rangle_{\rm a} - {\rm i}\frac{\sqrt{3}}{2}|000\rangle_{123}|1\rangle_{\rm a}\Big] = \\ \sqrt{\frac{4}{5}} \times \frac{1}{\sqrt{3}}\Big[({\rm e}^{-{\rm i}\frac{\pi}{3}}|010\rangle_{123} + |100\rangle_{123} + |001\rangle_{123})|0\rangle_{\rm a} - {\rm i}\frac{\sqrt{3}}{2}|000\rangle_{123}|1\rangle_{\rm a}\Big].\end{aligned} \tag{11}$$

Then, perform a rotation on atom 2 with $|1\rangle_2 \to {\rm e}^{{\rm i}\frac{\pi}{3}}|1\rangle_2$.

We perform a measurement on atom a. If the result of the state is $|0\rangle_{\rm a}$, we obtain the maximum entangled W state $\frac{1}{\sqrt{3}}(|010\rangle_{123} + |100\rangle_{123} + |001\rangle_{123})$ of atoms with the probability of 4/5.

3 Discussion and conclusion

Now we briefly discuss the experimental feasibility of this scheme under the practical situations. Firstly, the hyperfine-split levels for the D lines of cold alkali-metal atoms can be chosen in our scheme on the bases of Ref. [14]. In order to experimentally realize, first we set $\eta_{\rm A} = \eta_{\rm B} = \eta$ and $\Omega_{e1} = \Omega_{e2} = \Omega_{e3} = \Omega_e$ and then slowly decreasing Ω_e while increasing η until it satisfy the condition of adiabatically evolutes, $\eta \gg \Omega_e$. In addition, under the large-detuning condition of $|\Delta_n| \gg |\Omega_n|, |g_n| (n = 1, 2, 3)$ and $\eta \gg \Omega_e$, the atomic excited states and fiber modes are only virtually excited, so the influence of atomic decay rate and fiber decay rate can be well suppressed during the dark state evolution[22]. In addition, the cavity-fiber coupling strength

$\eta=\sqrt{4\pi\bar{\nu}c/L}$ relys on L and $\bar{\nu}$. Here L is the length of the optical fiber and $\bar{\nu}$ is the decay rate of cavity field into a continuum of fiber mode. In the short fiber limit $\frac{2L\bar{\nu}}{2\pi c}\ll 1, L\leq 1$ m, $\eta\approx 25\ \Omega_e$ and $\bar{\nu}\approx 1$ GHz[13,17] are the most realistic experimental conditions. Through decreasing the reflectivity R of the cavity mirror connected to the fiber, we can increase the coupling strength η. That is to say, the coupling strength η can be controlled by regulating fiber length L and the reflectivity R of the cavity mirror expediently. The effect of photon leaking out of the cavities and fibers have been discussed in Refs [17,23]. The decay rates of cavity and fiber, according to the recent experimental parameters[24,25], might satisfy the demand of our system to implement W state with high fidelity. Lastly, the required time in the process of W-like state into normal style W state is about 2×10^{-4}s with the coupling constant $g=2\pi\times 24$ kHz and δ=10 g[21]. The decay time of photo is about $T_c\approx 10^{-3}$ s[26], which is much longer than the require time. Therefore, on the basis of the above analysis, our scheme might be realized in current experiment setup of cavity QED technique.

We note that the proposal can be generalized to generate W state for n atoms trapped in n separated cavities connected by $n-1$ fibers. Assume that the atoms are initially in the state $|0\rangle_1|a\rangle_2|0\rangle_3\cdots|0\rangle_n$ and meanwhile the modes of cavities and fibers are initially in the vacuum states. Under the condition of $\eta\gg\Omega_e$ ($\eta_A=\eta_B=\cdots=\eta_{n-1}=\eta$ and $\Omega_{e1}=\Omega_{e2}=\Omega_{e3}=\cdots=\Omega_{en}=\Omega_e$), we get the n-atoms entanglement W-like state during the adiabatic evolution passage, then sent atom 2 and another atom a into another cavity experience the large-detuning interaction, thus we obtain the normal form of n-atom W state as $|\Psi\rangle_n=\frac{1}{\sqrt{n}}(|100\cdots0\rangle+|001\cdots0\rangle+\cdots+|000\cdots1\rangle)$.

In conclusion, we propose a scheme for preparing a three-particle entangled state of spatially separated atoms. During the operation, the atomic system, the modes of cavity fields and fibers are only virtually excited. Thus our scheme is robust to atomic spontaneous decay, cavity decay, and photon leaking out of the fiber and insensitive to moderate fluctuations of experimental parameters. In addition, taking advantage of adiabatic passage, it does not need precise control of the Rabi frequency and pulse duration. Also we can extend the scheme to prepare the n-particle entanglement state of distant atoms. The techniques of trapping atom, releasing atom, manipulating and transforming quantum states in the system have been improved according to the recent proposed schemes. Therefore, our scheme would be realized with current techniques and potentially be used for the distribute quantum computation.

Reference:

[1] Einstein A, *et al.* Can quantum-mechanical description of physical reality be considered complete? [J]. *Phys. Rev.,* 1935, 47: 777-780.

[2] Greenberger D M, Horne M A, Shimony A,*et al.* Bell's theorem without inequalities [J]. *Am. J. Phys.,* 1990, 58: 1131-1143.

[3] Bell J S. On the Eienstein-Podolsky-Rosen Paradox [J]. *Physics,* 1964, 1: 195.

[4] Bennett C H, Brassard G, Crepeau C, *et al.* Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels [J]. *Phys. Rev. Lett.,* 1993, 70: 1895.

[5] Cleve R, Gottesman D, Lo H-K. How to share a quantum secret [J]. *Phys. Rev. Lett.,* 1999, 83: 648.

[6] Cirac J I, Ekert A K, Huelga S F, *et al.* Distributed quantum computation over noisy channels [J]. *Phys. Rev. A,* 1999, 59: 4249.

[7] Dür W, Vidal G, Cirac J I. Three qubits can be entangled in two inequivalent ways [J]. *Phys. Rev. A,* 2000, 62: 062314.

[8] Roos C F, Riebe M, Haffner H, *et al.* Control and measurement of three-qubit entangled states [J]. *Science,* 2004, 304: 1478-1480.

[9] Wang X Q, Lu H X, Zhao J Q. Entanglement and non-locality of three-qubit W states [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2012, 29(5): 542-546 (in Chinese).

[10] Zhang D Y, Wang X W, Tang S Q, *et al.* Generation of $2n$-mode photon GHZ states based on cavity quantum electrodynamics [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2012, 29(5): 591-596 (in Chinese).

[11] Chen L B, Ye M Y, Lin G W, *et al.* Generation of entanglement via adiabatic passage [J]. *Phys. Rev. A,* 2007, 76: 062304.

[12] Mancini S, Bose S. Engineering an interaction and entanglement between distant atoms [J]. *Phys. Rev. A,* 2004, 70: 022307.

[13] Serafini A, Mancini S, Bose S. Distributed quantum computation via optical fibers [J]. *Phys. Rev. Lett.,* 2006, 96: 010503.

[14] Peng P, Li F L. Entangling two atoms in spatially separated cavities through both photon emission and absorption processes [J]. *Phys. Rev. A,* 2007, 75: 062320.

[15] Ye S Y, Zhong Z R, Zheng S B. Deterministic generation of three-dimensional entanglement for two atoms separately trapped in two optical cavities [J]. *Phys. Rev. A,* 2008, 77: 014303.

[16] Zheng S B. Generation of Greenberger-Horne-Zeilinger states for multiple atoms trapped in separated cavities [J]. *Eur. Phys. J. D,* 2009, 54: 719-722.

[17] Lü X Y, Si L G, Hao X Y. Achieving multipartite entanglement of distant atoms through selective photon emission and absorption processes [J]. *Phys. Rev. A,* 2009, 79: 052330.

[18] Ye L, Xiong W, Li A X, *et al.* Implementing ancilla-free phase covariant quantum cloning with atoms trapped in cavities [J]. *Sci. China Phys. Mech. Astron.,* 2011, 54: 262-267.

[19] Lü X Y, Liu J B, Ding C L, *et al.* Dispersive atom-field interaction scheme for three-dimensional entanglement between two spatially separated atoms [J]. *Phys. Rev. A,* 2008, 78: 032305.

[20] Wu Y, Saldana J, Zhu Y. Large enhancement of four-wave mixing by suppression of photon absorption from electromagnetically induced transparency [J]. *Phys. Rev. A,* 2003, 67: 013811.

[21] Zheng S B, Guo G C. Efficient scheme for two-atom entanglement and quantum information processing in cavity QED [J]. *Phys. Rev. Lett.,* 2000, 85: 2392-2395.

[22] Duan L M, Kuzmich A, Kimble H J. Cavity QED and quantum-information processing with “ hot ” trapped atoms [J]. *Phys. Rev. A,* 2003, 67: 032305.

[23] Savukov I M, Berry H G. Laser gas-discharge absorption measurements of the ratio of two transition rates in neutral argon [J]. *Phys. Rev. A,* 2003, 67: 032505.

[24] Trupke M, Goldwin J, Darquié B, *et al.* Atom detection and photon production in a scalable, open, optical microcavity [J]. *Phys. Rev. Lett.,* 2007, 99: 063601.

[25] Blatt R, Wineland D. Entangled states of trapped atomic ions [J]. *Nature,* 2008, 453: 1008-1015.

[26] Osnaghi S, Bertet P, Auffeves A, *et al.* Coherent control of an atomic collision in a cavity [J]. *Phys. Rev. Lett.,* 2001, 87: 037902.