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Creation of cluster coherent states via cavity QED

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Abstract: A scheme is proposed for the creation of cluster-type entangled coherent states via cavity QED. Based on the large detuning condition interaction between a three-level Λ type atom driven by two classical fields and two bimodal cavities, the spontaneous emission of the atom can be ignored. Moreover, the initial states of the two cavity fields are all prepared in vacuum. In this scheme, it is shown that the cluster-type entangled coherent states can be generated after the atomic measurement, and the experimental feasibility is also discussed.

Key words: quantum optics; cluster state; entangled coherent state; bimodal cavity; cavity QED

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腔 QED 中 cluster 相干态的产生

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摘 要： 提出一个在腔 QED 中产生 cluster- 型纠缠相干态的方案。基于三能级Λ型原子和双模腔场之间的大失谐相互作用，原子的自发辐射可以忽略。此外，腔场的初态是真空态。在这个方案中，对原子进行测量后，能够产生腔场 cluster- 型纠缠相干态，并讨论了实验的可行性。

关键词： 量子光学； cluster 态；纠缠相干态；双模腔；腔 QED

1 Introduction

Quantum entanglement is the foundation of quantum mechanics, and is regarded as the key resource of quantum systems. As a valuable resource, entanglement has been applied in quantum information processing such as quantum cryptography[1] and quantum teleportation[2]. Due to its potential of quantum states to exhibit relationships that cannot be accounted for classically, much theoretical and experimental effort have been focused on the creation of entangled states. In the past few years, the proposal of quantum entanglement with many qubits have attracted much attention. For example, the schemes of Greenbery-Horne-Zeilinger

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(GHZ) state[3], W state[4] and cluster state[5] have been presented. Recently, another class of entangled states, i.e., the cluster states with a large persistently of entanglement, have been proposed by Briegel and Raussendorf. Cluster states are important resources for quantum error correction[6] and multi-particle entanglement[7], so there are great interest in the creation of cluster states.

A number of theoretical schemes for generating cluster states have been presented in all kinds of physical models, such as cavity QED[8~11], superconductor charge qubits[12] and quantum dot system[13]. For instance, Zou *et al.*[8] present two experimental schemes to generate the cluster states in microwave cavity QED. In addition, Tanamoto *et al.*[12] propose a proposal for producing entangled cluster states by a one-touch entanglement in charge qubits. Guo *et al.*[13] present an efficient one-step generation scheme for cluster states in quantum dot molecules. With the development of technique, cluster states have been realized in experiments[14,15]. In Ref.[14], Kiesel *et al.* experimentally analyse a four-qubit photon cluster state. Recently, there are great interest in the generation of cluster-type entangled coherent states[9,16] where resonant interaction between cavity fields and atom is occured.

In this paper, we propose a scheme to generate cluster-type entangled coherent states. Our scheme is performed under the large detuning condition, thus the spontaneous emission of atom can be ignored. In our scheme, we do not need prepare the initial coherent states due to initial vacuum states of cavities. The paper is organized as follow. In sect. 2, we introduce the three-level Λ type atom model, and calculate the evolution of the state. In sect. 3, we describe the creation of the cluster-type coherent states of two bimodal cavities. In sect. 4, we show the discussion and summary.

2 Theoretical model

We consider a three-level atom in Λ configuration crossing a two-mode cavity fields. The atomic level configuration of the model is described in Fig.1(b), where the atomic levels $|r\rangle \leftrightarrow |g\rangle$ and $|r\rangle \leftrightarrow |s\rangle$ are coupled to the jth cavity mode ($j = 1, \cdots, 4$) with coupling strengthes g_1 and g_2 and detunings δ_1 and δ_2. At the same time, two classical fields with Rabi frequencies Ω_2 and Ω_1 drive the same atomic levels. The coupling constant $g_1(g_2)$ and the Rabi frequency $\Omega_1(\Omega_2)$ are assumed to be the same for each cavity. In the interaction picture, the Hamiltonian can be given by ($\hbar = 1$)

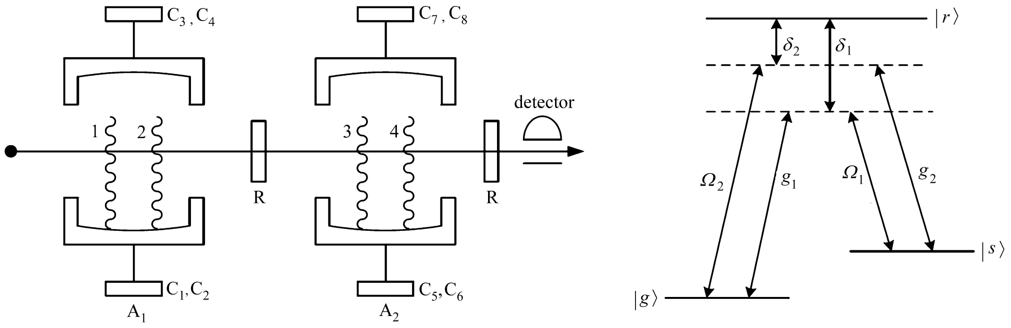
$$H = g_1 a_j^+ \mathrm{e}^{-\mathrm{i}\delta_1 t}|g\rangle\langle r| + g_2 a_j^+ \mathrm{e}^{-\mathrm{i}\delta_2 t}|s\rangle\langle r| + \Omega_1 \mathrm{e}^{-\mathrm{i}\delta_1 t}|s\rangle\langle r| + \Omega_2 \mathrm{e}^{-\mathrm{i}\delta_2 t}|g\rangle\langle r| + h.c.\ , \tag{1}$$

where a_j^+ and a are the creation and annihilation operators for the cavity field, respectively. Under the large detuning condition when $\left\{\frac{g_1}{\delta_1}, \frac{\Omega_1}{\delta_1}, \frac{g_2}{\delta_2}, \frac{\Omega_2}{\delta_2}\right\} \ll 1$, we can eliminate adiabatically the excited level $|r\rangle$. Thus the effective Hamiltonian can be obtained as

$$H_{\text{eff}} = -(\nu a_j^+ + \nu^* a_j)(\sigma^+ + \sigma), \tag{2}$$

where $\nu = \frac{g_1 \Omega_1^*}{\delta_1} = \frac{g_2 \Omega_2^*}{\delta_2}$, $\sigma^+ = |s\rangle\langle g|$ and $\sigma = |g\rangle\langle s|$ are the rasing and lowering atomic operators, respectively. We change the atomic bare-state basis into dressed-state basis, i.e., $|\pm\rangle = \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle)$. So the effective Hamiltonian is rewritten as

$$H_{\text{eff}} = -(\nu a_j^+ + \nu^* a_j)(|+\rangle\langle +| - |-\rangle\langle -|)\ . \tag{3}$$



(a) (b)

Fig.1 (a) The setup for the creation of cluster-type entangled coherent state. Here an atom is sent through two bimodal cavities A$_1$ and A$_2$, and is finally detected by a field ionization detector D. (b) The sketch of a three-level atom in Λ configuration. The atomic levels $|r\rangle \leftrightarrow |g\rangle$ and $|r\rangle \leftrightarrow |s\rangle$ with detuning δ_1 and δ_2

3 Creation of cluster-type entangled coherent states

In this section, we propose a scheme to create cluster states via cavity QED. Now let us describe our scheme. Fig.1(a) displays the setup for the creation of cluster-type entangled coherent state. A_1 and A_2 denote two identical bimodal cavities and $C_i(i=1,2,\cdots,8)$ are eight classical fields. The two bimodal cavities consist of two modes, respectively, modes 1 and 2 in A_1, 3 and 4 in A_2, as shown in Fig.1(a). We assume that the initial states of the two bimodal cavities are in the vacuum states, and the atom is initially in the ground state $|g\rangle$. Hence the initial state of the system is

$$|\varphi\rangle_0 = |g\rangle|0\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4 = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)|0\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4, \tag{4}$$

and the atom is driven by two external classical fields C_1 and C_2 in cavity A_1 for an interaction time t_1. The other mode 2 of cavity A_1 remains unaffected. The evolution of the system under the Hamiltonian can be drived as

$$|\varphi(t_1)\rangle_1 = U|\varphi(0)\rangle = \frac{1}{\sqrt{2}}(|+\rangle|\alpha\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4 + |-\rangle|-\alpha\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4), \tag{5}$$

with $\alpha = \mathrm{i}\nu t_1$. We then turn off the classical fields C_1, C_2 and turn on the another classical fields C_3, C_4. The atom interacts with the mode 2. After an interaction time t_2 (assume $t_2 = t_1 = t$), the state of the system is

$$|\varphi(t_2)\rangle_2 = U|\varphi(t_1)\rangle = \frac{1}{\sqrt{2}}(|+\rangle|\alpha\rangle_1|\alpha\rangle_2|0\rangle_3|0\rangle_4 + |-\rangle|-\alpha\rangle_1|-\alpha\rangle_2|0\rangle_3|0\rangle_4). \tag{6}$$

After leaving cavity A_1, we let the atom go through the first Ramsey zone R. The interaction will transform at $|+\rangle \rightarrow \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ and $|-\rangle \rightarrow \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$. Thus Eq.(6) becomes

$$\begin{aligned} |\varphi(t_2)\rangle_2 = & \frac{1}{2}[|+\rangle(|\alpha\rangle_1|\alpha\rangle_2|0\rangle_3|0\rangle_4 + |-\alpha\rangle_1|-\alpha\rangle_2|0\rangle_3|0\rangle_4) + \\ & |-\rangle(|\alpha\rangle_1|\alpha\rangle_2|0\rangle_3|0\rangle_4 + |-\alpha\rangle_1|-\alpha\rangle_2|0\rangle_3|0\rangle_4)]. \end{aligned} \tag{7}$$

Next the atom crosses cavity A_2 and interacts with the modes 3 and 4, and the classical fields C_5, C_6 and C_7, C_8, just like the cavity A_1, then the whole state of the system is

$$\begin{aligned}|\varphi(t)\rangle_{\rm tot}=&\frac{1}{2}[(|+\rangle(|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4+|-\alpha\rangle_1|-\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4)+\\&|-\rangle(|\alpha\rangle_1|\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4-|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4)]=\\&\frac{1}{2\sqrt{2}}[|g\rangle(|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4+|-\alpha\rangle_1|-\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4+\\&|\alpha\rangle_1|\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4-|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4)+\\&|e\rangle(|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4+|-\alpha\rangle_1|-\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4-\\&|\alpha\rangle_1|\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4+|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4)].\end{aligned}\tag{8}$$

We now measure the state of the atom, if the measurement result is $|g\rangle$, then the field collapses into the state

$$\begin{aligned}|\varphi_f\rangle_1=&\frac{1}{2}(|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4+|-\alpha\rangle_1|-\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4+\\&|\alpha\rangle_1|\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4-|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4),\end{aligned}\tag{9}$$

which is a cluster-type entangled coherent state, however, if the measurement outcome is $|e\rangle$, the system state becomes

$$\begin{aligned}|\varphi_f\rangle_2=&\frac{1}{2}(|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4+|-\alpha\rangle_1|-\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4-\\&|\alpha\rangle_1|\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4+|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4).\end{aligned}\tag{10}$$

4 Discussion and conclusion

We give a brief discussion on the experimental feasibility of our program. To create the cluster state, the enough long lifetimes of the atom and cavities should be needed. In our scheme, due to the large detuning condition, we can ignore the atomic spontaneous emission and only consider the lifetime of the cavities. In a recent experiment[17], cavity decay time $T_{\rm d}$ is about 10^{-1} s. The decoherence time $T_{\rm d}$ of cluster state is about 10^{-2} s[9]. According to the experimental parameters[18], we choose $\alpha=2, g=2.2\pi\times10^8, \mathit{\Omega}=10^7$ Hz, and can obtain the interaction time between atom and one cavity mode is $t_i\sim10^{-7}$ s. While the atom cross one Ramsey zone is $T_{\rm R}\sim10^{-8}$ s. To our scheme, the required time T of preparing a cluster state is about $4\times(4\times10^{-7}+4\times10^{-8})\sim10^{-6}$ s. It can be seen that T is much smaller than the decoherence time of the cluster states. Our scheme might be feasible in experiments along with the process of the technology.

In conclusion, we present a scheme to create cluster-type entangled coherent states of cavities. In our scheme, the high-level of the atom is eliminated adiabatically under the large detuning condition. Moreover, the cavity fields are initially in vacuum states, we do not need to prepare the initial coherent states.

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