

Some Examples to Show That Objects Be Presented by Mathematical Equations

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ABSTRACT

We have often heard remarks such as “We can plot graphs from the mathematical equations”, including equations of lines, equations of curves, and equations of invisible and visible objects. Actually, we can present each object by mathematical equation and we can plot graphs from equations. Equations not only show visible objects but also can show invisible objects such as wave equations in differential equations. In fact, the change of equations is also to conduce the change of objects and phenomena. This paper presents mathematical equations, methods to plot the graphs in 2D and 3D space. The paper is also a small proof of this conclusion have been provided and addressing visualization problem for any object. The novelty of this paper presents some special equations of objects and shows the ideas to build objects from equations.

Keywords: Graph, Equation; Visible and Invisible Object

1. Introduction

In this section, we will briefly review some basic and special equations in 2D and 3D space. In 2D space, we will plot graphs of simple linear equations, equation of circles, and some special equations as equation of heart in Cartesian coordinates and Polar coordinate to show the attitude of objects. In 3D space, we have a change from equations in two variables in 2D space to equations in three variables in 3D space. We use some special functions in Mathematica to plot graph of objects, including sphere, hearts, apple, and wave equations. In addition, we can plot an object in two different coordinate systems to present mathematical methods and functions in Mathematica.

1.1. In 2D Space

1.1.1. In Cartesian Coordinates

We can plot linear equation as follows:

$$y = ax + b$$

The graph of this equation is a line.

We also plot the graph of higher order equations such as Quadratic equation:

$$y = ax^2 + bx + c \quad (a \neq 0)$$

Cubic equation:

$$y = ax^3 + bx^2 + cx + d \quad (a \neq 0)$$

And higher order equations. Graph of these equations are

curves.

1.1.2. In Polar Coordinate

We can plot some graphs in this coordinate. We will use two variables in the equation.

Generally, we will introduce the equation of circles with the form:

$$(x-a)^2 + (y-b)^2 = r^2$$

We will vary from Cartesian coordinate to Polar coordinate by using form:

$$\begin{cases} x = r \cos(\theta) \\ y = r \sin(\theta) \end{cases}$$

1.2. In 3D Space

In this section, we will plot equations in two coordinate systems and will show equation of sphere in the two mathematical methods

1.2.1. In Cartesian Coordinates

We will plot some objects in 3D space to see a relationship between equations and graphs. We will use equation in three variables.

Generally, we rewrite equation of sphere with the following form:

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2,$$

where $I(-a, -b, -c)$ is center of the sphere.

1.2.2. In Polar Coordinate

We can vary equation of sphere to polar coordinates as follows :

$$\begin{cases} x = \sin(u) \cos(v) \\ y = \sin(u) \sin(v) \\ z = \cos(u) \end{cases}$$

where $u, v \in [0, 2\pi]$:

2. Some Equations in Cartesian Coordinates

2.1. Linear Equation

We see that linear equation with the following form:

$$y = ax + b.$$

With $a = 0$, we have the equation $y = b$, with $b = 2$ or $b = -2$.

Using Mathematica software to plot these graphs
In Mathematica

Input: Plot $[\{2, -2\}, \{x, -10, 10\}]$

Press “**Shift-Enter**” at the end of the command line.

See **Figure 1**.

We have the graphs of the equations:

Where $a > 0$, we will plot the equation $y = 2x$.

Input: Plot $[2x, \{x, -10, 10\}]$

Press “**Shift-Enter**” at the end of the command line.

See **Figure 2**.

Where $a < 0$, we will plot the equation $y = -2x$.

Input: Plot $[-2x, \{x, -10, 10\}]$

Press “**Shift-Enter**” at the end of the command line.

See **Figure 3**.

2.2. Quadratic Equation

We know that the graph of the quadratic equation has the form:

$$y = ax^2 + bx + c \quad (a \neq 0)$$

In Mathematica

Where $a > 0$, we can plot the equation

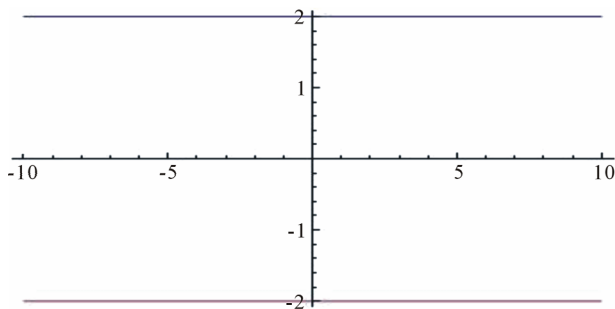


Figure 1. The graphs of the equations $y = 2$ and $y = -2$.

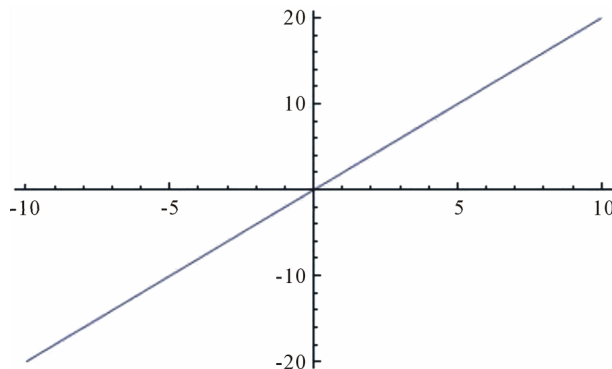


Figure 2. The graph of the equation $y = 2x$.

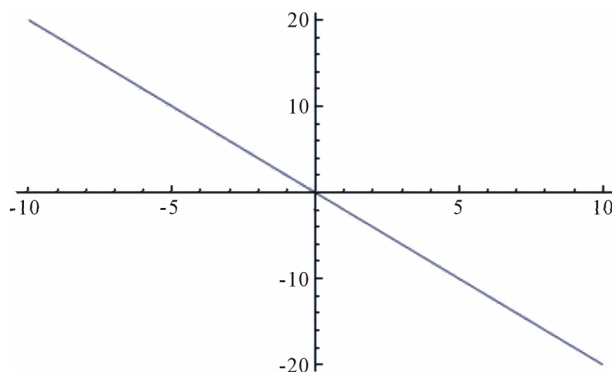


Figure 3. The graph of the equation $y = -2x$.

$y = 2x^2 + 3x + 5$.

Input: Plot $[2x^2 + 3x + 5, \{x, -11, 10\}]$

Press “**Shift-Enter**” at the end of the command line.

See **Figure 4**.

Where $a < 0$, we can plot the equation

$y = -2x^2 + 3x + 5$.

Input: Plot $[-2x^2 + 3x + 5, \{x, -10, 12\}]$

Press “**Shift-Enter**” at the end of the command line.

See **Figure 5**.

2.3. Cubic Equation

We know that the graph of the cubic equation:

$$y = ax^3 + bx^2 + cx + d \quad (a \neq 0)$$

is a curve.

In Mathematica

Where $a > 0$, we can plot the equation

$y = 2x^3 + 3x^2 + 4x + 5$.

Input: Plot $[2x^3 + 3x^2 + 4x + 5, \{x, -10, 10\}]$

Press “**Shift-Enter**” at the end of the command line.

See **Figure 6**.

Where $a < 0$, we have the graph of

$y = -2x^3 + 3x^2 + 4x + 5$.

Input: Plot $[-2x^3 + 3x^2 + 4x + 5, \{x, -10, 10\}]$

Press “**Shift-Enter**” at the end of the command line.

See **Figure 7**.

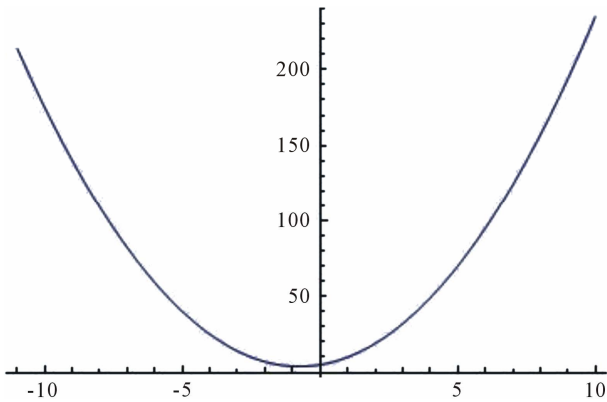


Figure 4. The graph of the equation $y = 2x^2 + 3x + 5$.

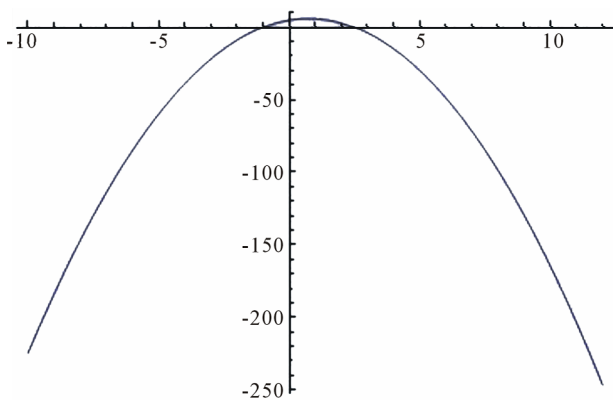


Figure 5. The graph of the equation $y = -2x^2 + 3x + 5$.

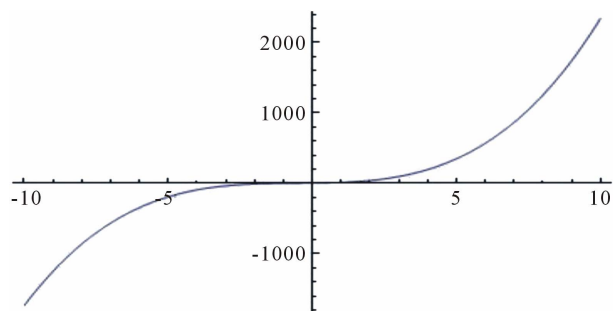


Figure 6. The graph of the equation $y = 2x^3 + 3x^2 + 4x + 5$.

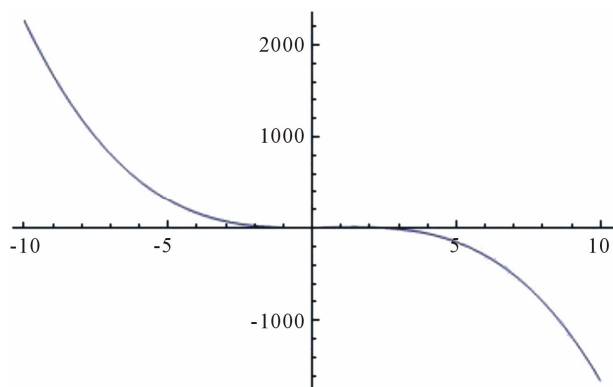


Figure 7. The graph of the equation $y = -2x^3 + 3x^2 + 4x + 5$.

2.4. Some Higher Order Equations

Quartic equation [1]

$$y = ax^4 + bx^3 + cx^2 + dx + e \quad (a \neq 0)$$

Plot graph of the equation:

$$y = -6x^4 + 12x^3 + cx^2 - 3x + 13$$

Input: Plot $[-6x^4 + 12x^3 + cx^2 - 3x + 13, \{x, -2, 2\}]$

Press “**Shift-Enter**”} at the end of the command line.

See **Figure 8**.

Quintic equation

$$y = ax^5 + bx^4 + cx^3 + dx^2 + ex + f \quad (a \neq 0)$$

To plot graph of the equation:

$$y = 2x^5 + 6x^4 - 8x^2 + x - 3$$

Input: Plot $[2x^5 + 6x^4 - 8x^2 + x - 3, \{x, -1.8, 1.8\}]$

Press “**Shift-Enter**”} at the end of the command line.

See **Figure 9**.

Plot graph of the equation:

$$y = x^{100} + 6x^{50} - 8x^{10} + x - 3$$

Input: Plot $[x^{100} + 6x^{50} - 8x^{10} + x - 3, \{x, -1, 1\}]$

Press “**Shift-Enter**”} at the end of the command line.

See **Figure 10**.

3. Some Equation in Polar Coordinates

3.1. Circle Equations

Generally, we will introduce the equation of circles

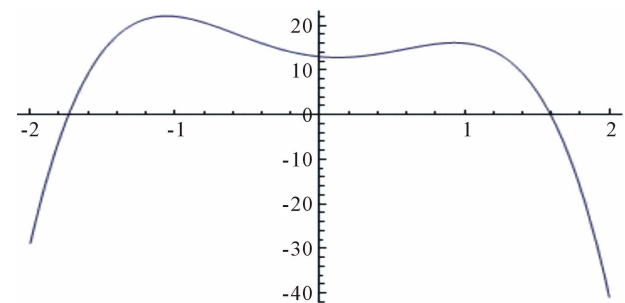


Figure 8. The graph of $y = -6x^4 + 12x^3 + cx^2 - 3x + 13$ [1].

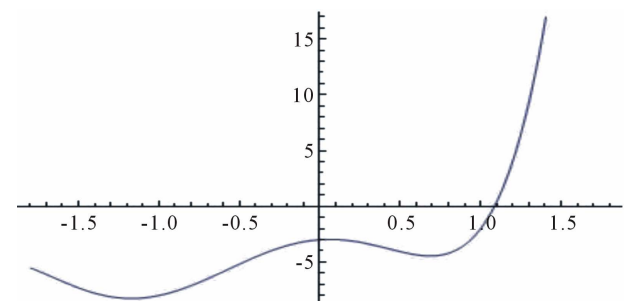


Figure 9. The graph of $y = 2x^5 + 6x^4 - 8x^2 + x - 3$ 100th order equation [2].

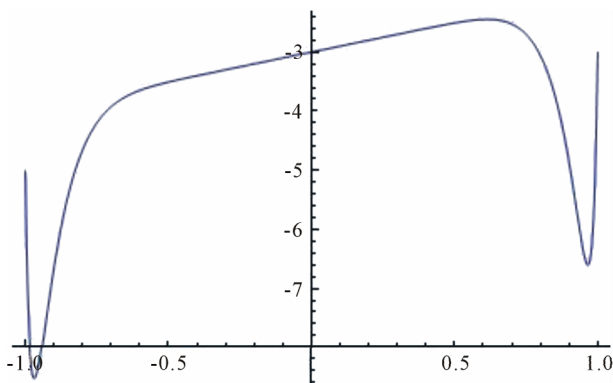


Figure 10. The graph of $y = x^{100} + 6x^{50} - 8x^{10} + x - 3$.

as follows:

$$(x-a)^2 + (y-b)^2 = r^2$$

Choose $r = 2, a = b = 0$. We have the equation $x^2 + y^2 = 4$. We can directly plot with the command.

Input: ContourPlot

```
[x^2 + y^2 = 4, {x, -2, 2}, {y, -2, 2}, Plot Label
→ "THE CIRCLE x^2 + y^2 = 4"]
```

Press “**Shift-Enter**” at the end of the command line. See **Figure 11**.

We can vary to polar coordinates, where $x \in [0, 2\pi]$:
Set:

$$\begin{cases} x = 2 \cos(x) \\ y = 2 \sin(x) \end{cases}$$

Plot graph of the equation in Mathematica.

Input: ParametricPlot

```
[{2 cos[x], 2 sin[x]}, {x, 0, 2π}]
```

Press “**Shift-Enter**” at the end of the command line. See **Figure 12**.

3.2. Elliptic Equations

In this section, we will plot the equations of three circles $x = 3 \cos t, y = 3 \sin t; x = 2 \cos t, y = 2 \sin t; x = \cos t, y = \sin t$ and the equation of two ellipses:

$$x = 4 \cos t, y = \sin t; x = \cos t, y = 4 \sin t$$

Input: ParametricPlot

```
[{{3 cos[t], 3 sin[t]}, {2 cos[t], 2 sin[t]},
{4 cos[t], sin[t]}, {cos[t], 4 sin[t]},
{cos[t], sin[t]}}, {t, 0, 2π}]
```

Press “**Shift-Enter**” at the end of the command line. See **Figure 13**.

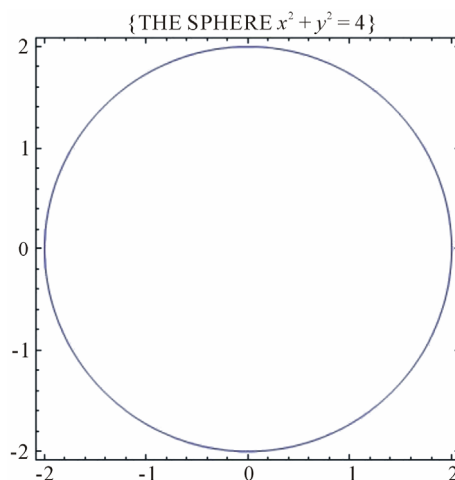


Figure 11. The graph of the circle in Cartesian coordinates.

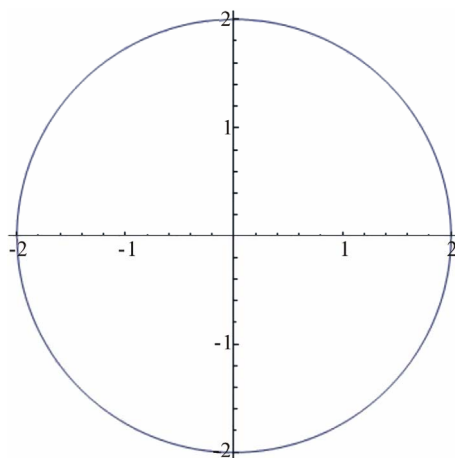


Figure 12. The graph of the circle in polar coordinates.

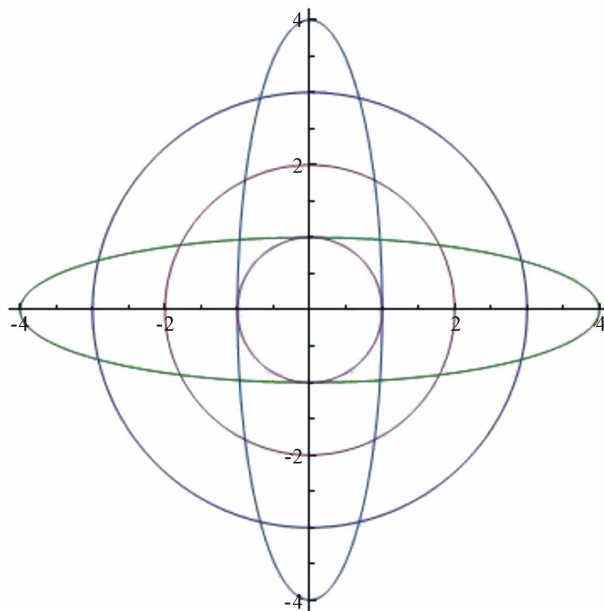


Figure 13. The graph of the elliptic equations.

3.3. The Equations of Heart [3]

From the general form, where $x \in [0, 2\pi]$:

$$\begin{cases} x = r \cos(x) \\ y = r \sin(x) \end{cases}$$

We can revise the equation system to plot graph of heart

$$\begin{cases} x = (2 - 2 \sin u) \cos u \\ y = (2 - 2 \sin u) \sin u \end{cases}$$

where $x \in [0, 2\pi]$.

Input: ParametricPlot

$$\left[\left\{ (2 - 2 \sin[u]) \cos[u], (2 - 2 \sin[u]) \sin[u] \right\}, \{u, 0, 2\pi\} \right],$$

PlotStyle \rightarrow Thick, Color Function \rightarrow Function

$$\left[\{x, y, u\}, \text{Hue}[u/(2\pi)] \right],$$

Color Function Scaling \rightarrow False]

Press “**Shift-Enter**” at the end of the command line. See **Figure 14**.

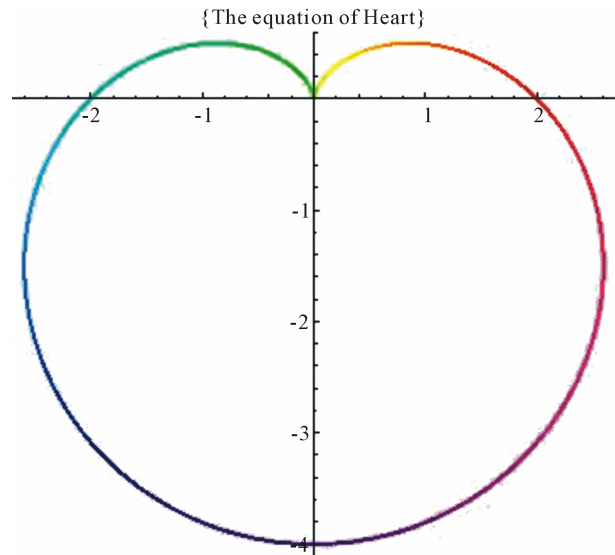


Figure 14. The graph of the heart.

3.4. The Equation of Wedding [4]

From Eugen Beutel equation :

$$(x^2 + y^2 - 1)^3 - x^2 y^3 = 0$$

We will create a new equation in a command line to build two intersectional hearts in the graph.

We can set a name of the equation with title: “Song Hy Equation”.

Input: ContourPlot

$$\left[\left\{ (x^2 + y^2 - 1)^3 - x^2 y^3, ((x - 1.5)^2 + y^2 - 1)^3 \right. \right.$$

$$\left. - (x - 1.5)^2 y^3 \{x, -2, 3\}, \{y, -2, 3\} \right\},$$

PlotLabel \rightarrow "SONG HY-喜喜"]

ContourStyle \rightarrow Red]

Press “**Shift-Enter**” at the end of the command line. See **Figure 15**.

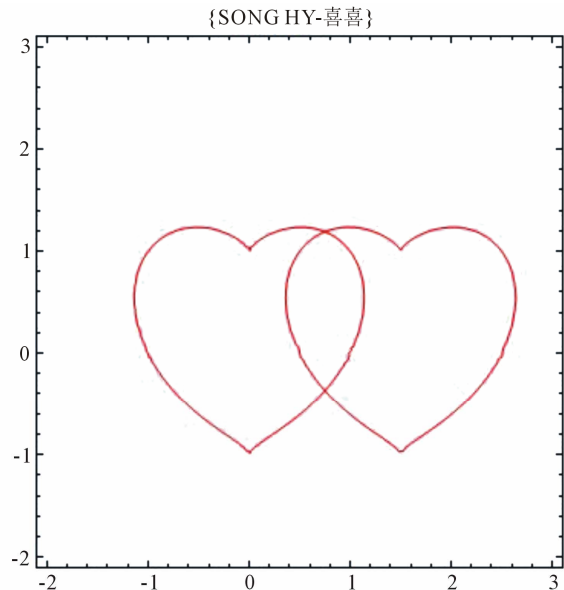


Figure 15. The graph of wedding (SONG HY-喜喜).

4. Some Equations in 3D Space

4.1. The Equation of Sphere

Generally, we will write the equation of sphere with form $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$, where $I(-a, -b, -c)$ is centre of the sphere.

Choose $r = 2, a = b = c = 0$. We have the equation: $x^2 + y^2 + z^2 = 4$

We can directly plot with the command.

Input: ContourPlot3D

$$\left[x^2 + y^2 + z^2 = 4, \{x, -2, 2\}, \{y, -2, 2\}, \{z, -2, 2\} \right],$$

PlotLabel \rightarrow {"THE SPHERE $x^2 + y^2 + z^2 = 4$ "}

Press “**Shift-Enter**” at the end of the command line. See **Figure 16**.

We can vary to polar coordinates, where $u, v \in [0, 2\pi]$:

$$\begin{cases} x = \sin(u) \cos(v) \\ y = \sin(u) \sin(v) \\ z = \cos(u) \end{cases}$$

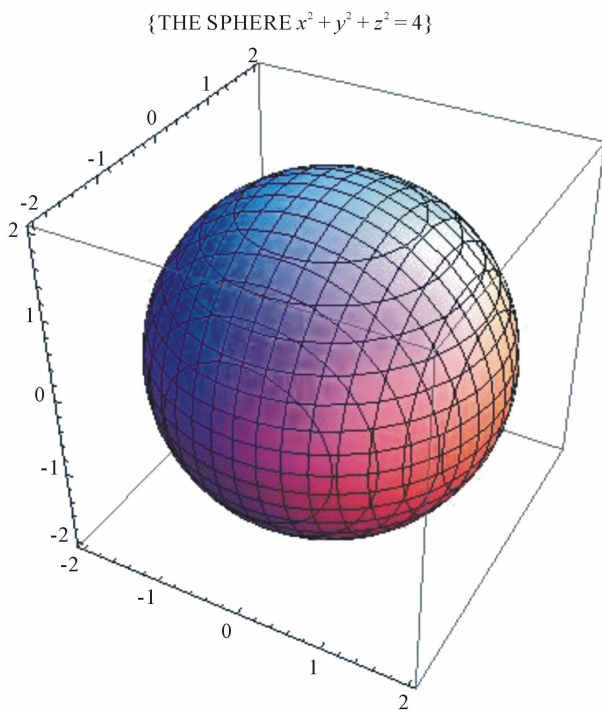


Figure 16. The graph of sphere in Cartesian coordinates.

Plot graph of the equation in Mathematica.

Input: ParametricPlot3D

$$\left[\left\{ \sin[u] \sin[v], \sin[u] \cos[v], \cos[u] \right\}, \left\{ u, 0, 2\pi \right\}, \left\{ v, 0, 2\pi \right\} \right]$$

Press “**Shift-Enter**” at the end of the command line. See **Figure 17**.

4.2. The Equation of Apple

From the equation of sphere in polar coordinates as follows:

$$\begin{cases} x = \sin(u) \cos(v) \\ y = \sin(u) \sin(v) \\ z = \cos(u) \end{cases}$$

where $u, v \in [0, 2\pi]$

We will vary the coordinates of x and y to create new coordinates system in 3D space.

$$\begin{cases} x = (2 - 2 \sin(v)) \cos(u) \\ y = (2 - 2 \sin(v)) \sin(u) \\ z = 2 - 3.5 \cos(v) \end{cases}$$

where $u, v \in [0, 2\pi]$.

The change of the equation shows that a new object has been created. The graph of new equation can be observed in the figure below. It is the same an apple and is called

with title: “The equation of Apple”.

Input: ParametricPlot3D

$$\left[\left((2 - 2 \sin[v]) \cos[u], (2 - 2 \sin[v]) \sin[u], 2 - 3.5 \cos[v] \right), \left\{ u, 0, 2\pi \right\}, \left\{ v, 0, 2\pi \right\}, \text{PlotLabel} \rightarrow \left\{ \text{"The equation of Apple"} \right\} \right]$$

Press “**Shift-Enter**” at the end of the command line. See **Figure 18**.

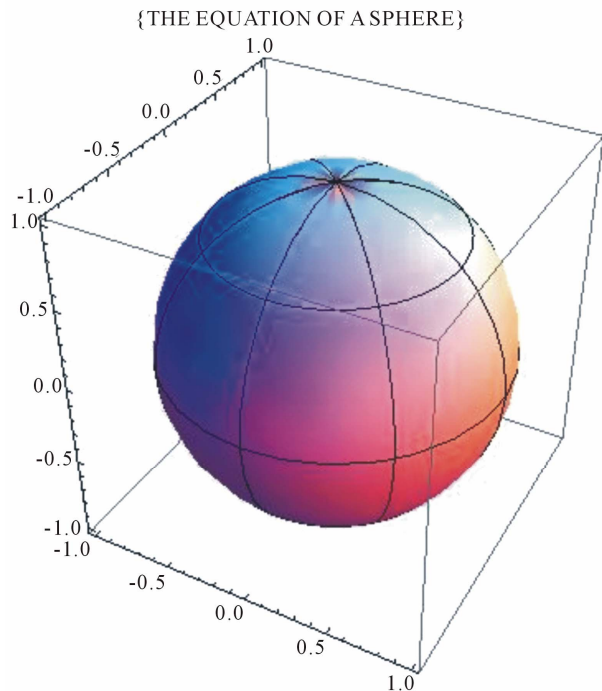


Figure 17. The graph of sphere in polar coordinates.

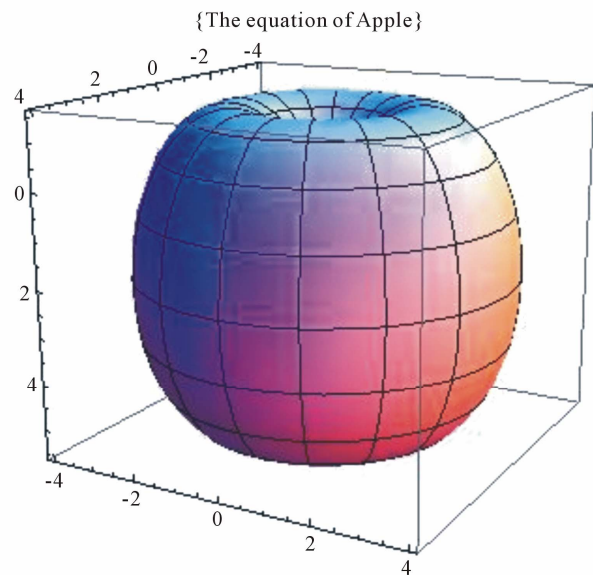


Figure 18. The graph of apple.

4.3. The Equation of Donuts Cakes [3]

According to Polar coordinates in 3D space, we can be plotted an object. By the change of the coordinates of equations, we can built the different attitude of objects. From the combine of equations, we have plotted the rings in different colors in 3D space. These figures have called with title: "Donuts cakes".

Input: ParametricPlot3D

```
[{ {4 + (3 + Cos[v]) Sin[u], 4 + (3 + Cos[v]) Cos[u],
4 + Sin[v]}, {8 + (3 + Cos[v]) Cos[u], 3 + Sin[v],
4 + (3 + Cos[v]) Sin[u]}, {12 + (3 + Cos[v]) Sin[u],
4 + (3 + Cos[v]) Cos[u], 4 + Sin[v]}
{16 + (3 + Cos[v]) Cos[u], 3 + Sin[v],
4 + (3 + Cos[v]) Sin[u]}}, {u, 0, 2π}, {v, 0, 2π},
PlotStyle -> {Red, Green, Blue, Yellow} ]
```

Press "Shift-Enter" at the end of the command line. See **Figure 19**.

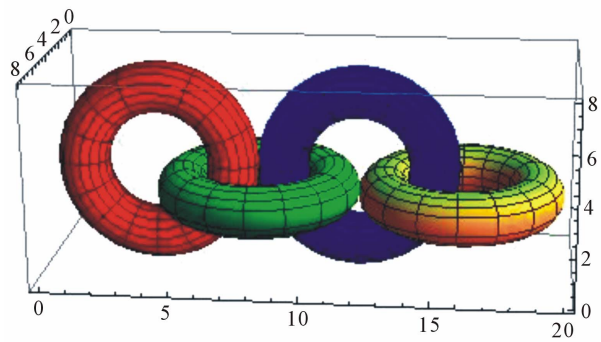


Figure 19. The graph of donuts cakes.

4.4. The Wedding Equation [4]

From Gabriel Taubin equation:

$$(2x^2 + y^2 + z^2 - 1)^3 - \frac{1}{10}x^2z^3 - y^2z^3 = 0$$

We can vary and combine two equations in a command line to build a new object with the figure of two intersectional hearts.

We can set a name of this equation with title: "Wedding Equation".

Input: ContourPlot3D

```
[{ { (2x^2 + y^2 + z^2 - 1)^3 - 1/10 x^2 z^3 - y^2 z^3 },
{ (2x^2 + (y-1)^2 + z^2 - 1)^3 - 1/10 x^2 z^3 - (y-1)^2 z^3 }
{x, -0.9, 2.5}, {y, -1.2, 2.5}, {z, -1.2, 2.5},
Contours -> {0}, ContourStyle -> Pink,
Axes -> False, ViewPoint -> {2, 1, 5},
Mesh -> None, BoxRatios -> {.6, .6, .8},
PlotLabel -> {"MY WEDDING"} ]
```

Press "Shift-Enter" at the end of the command line. See **Figure 20**.

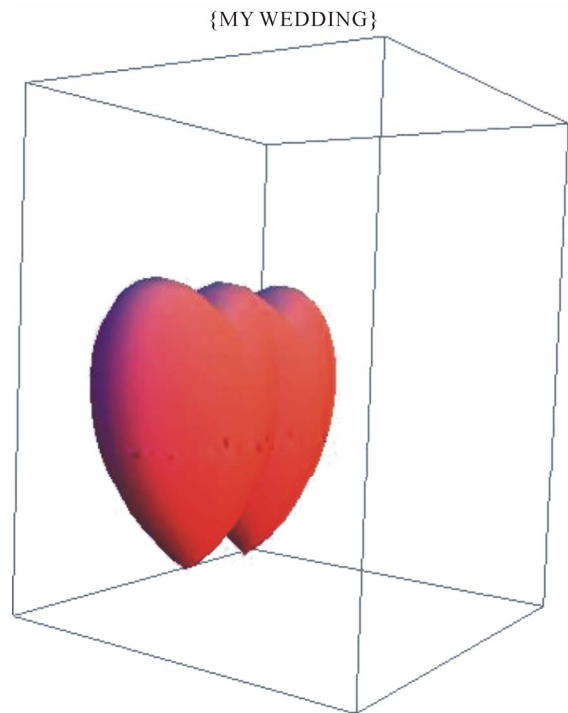


Figure 20. The graph of the wedding in space.

4.5. The Wave Equation in Plane [3]

We have the following differential equation:

$$y''(x) + \tan(y(x)) = 0, \text{ with initial condition } y(0) = 1, y'(0) = 0.$$

Using Mathematica software to plot the wave form of the equation as follows:

Input:

```
wave1 = {y''[x] + Tan[y[x]] = 0, y[0] = 1, y'[0] = 0};
sol = NDSolve[wave1, y, {x, -10, 10}]
```

Input: Plot [y[x]/. sol, {x, -10, 10}]

Press "Shift-Enter" at the end of the command line. See **Figure 21**.

4.6. The Wave Equation in Space [3]

We have the differential equation [5]

$$u_{tt}(t, x) = u_{xx}(t, x) \text{ with initial boundary condition}$$

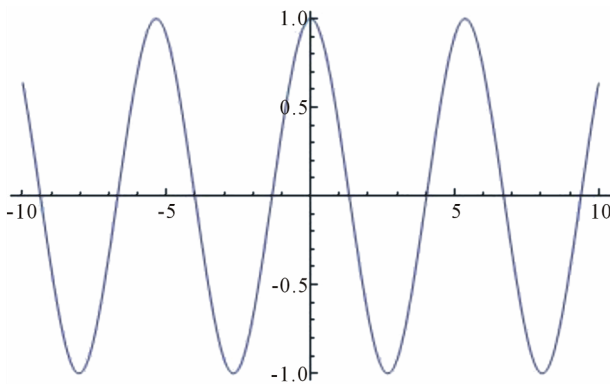


Figure 21. The graph of the wave equation in plane.

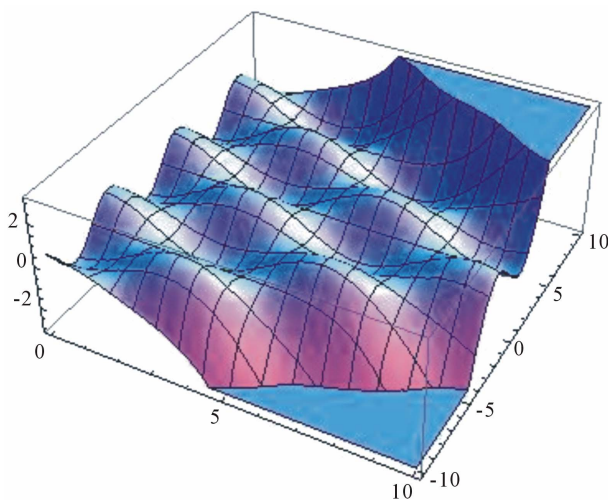


Figure 22. The graph of the wave equation in space.

$$u(0, x) = \sin(x), \quad u'_x(0, x) = 0.$$

Using Mathematica to plot the wave form of this equation.

Input:

```
wave2 = NDSolve[ { D[u[t, x], t, t] = D[u[t, x], x, x],
u[0, x] = sin[x], Derivative[1, 0][u][0, x] = 0 },
u, {t, 0, 10}, {x, -10, 10} ]
```

```
Plot3D[Evaluate[u[t, x]/.wave2],
{t, 0, 10}, {x, -10, 10}]
```

Press “**Shift-Enter**” at the end of the command line. See Figure 22.

5. Conclusions

This paper has shown some visible and invisible figures of the equations. In the paper, we have also discussed the equations and have presented the graphs of mathematical equations in 2D and 3D space. Each change of an equation is shown the change of object, so we will have a stronger understanding in relationship between equations and objects.

Indeed, we can plot any graphs from equations; contradictorily, from the objects are known, we can also find equations of these objects and how we can find these equations. I think, that is the future of work, which we can do to determine equations by softwares, mathematical modeling.

This paper also kindles new ideas about the change of equation. The use of Mathematica in this paper illustrates the important role of technology in research in mathematical equations. It not only help in providing a computing platform but also serves as a useful tool for plotting visual images resulting from the equation.

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