

# Rejoinder

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## 1. INTRODUCTION

We are grateful to the discussants for their positive and interesting comments. In an area moving so rapidly it is to be expected that our review overlooks some work, and all the contributions helpfully supplement our paper. Our remarks focus on points of possible disagreement or where expansion seems useful.

## 2. COOLEY AND SAIN

Cooley and Sain bring to the discussion wide experience of statistical applications in atmospheric science, in addition to innovative methodological work. We entirely agree with them that the analysis of annual or seasonal maxima is often unsatisfactory from the statistical point of view: it fails to make full use of the available data, which typically comprise numerous simultaneous time series, and by reducing daily or even hourly data to annual maxima does not allow detailed modeling of the underlying process. In some cases it is useful to follow Stephenson and Tawn (2005) and to incorporate information on the occurrence times of annual maxima; Davison and Gholamrezaee (2012) show that this is quite feasible in the present context, and find some improvement in precision of estimation from doing so. There is a close relationship between models for annual maxima, as considered in our paper, and those for peaks

over thresholds (Smith (1989); Davison and Smith (1990)), and max-stable models of both types share the deficiencies mentioned at the end of Section 8 of our paper. Huser and Davison (2012) extend the ideas used for annual maxima in the present paper to a space-time treatment of extreme hourly rainfall data using the threshold approach. The use of pairwise likelihood poses some tricky issues in that context, however, because of the multiplicity of pairs, which can correspond to simultaneous events in different time series, events at different times in a single series, or at different times in different series. The application considered by Huser and Davison (2012) involves 10 hourly rainfall time series for 27 summers, around 580,000 observations giving 7 billion possible pairs, of which a subset of only around 30 million were used! Although heavy computational burdens arise also in other spatial modeling contexts, better approaches are clearly needed to deal with larger settings for spatial extremes, as Cooley and Sain remark. As an aside, the choice of subsets of observations that contribute to the composite likelihood can be more subtle than at first appears: Huser and Davison (2012) find that although one might think it best to include only strongly-dependent pairs, it can be preferable to include some for which observations are independent or nearly so, in order to get reasonable estimates of the ranges of extremal phenomena.

We entirely agree that the goals of analysis may differ, and that it may not be worthwhile to fit a spatial (or space-time) extremal model when a map of quantiles is the intended output. However, naive use of a latent variable model that ignores the correlations between the events may provide uncertainty measures that are overly precise, as pointed out in the discussion contribution by Gabda et al. Thus, building some form of spatial dependence between events, and not merely between model parameters, seems wise. A pragmatic way to do this may be the use of a Gaussian copula, as in Sang and Gelfand (2010).

Cooley and Sain's final comment concerns a crucial part of extremal modeling, namely, the incorporation of subject-matter knowledge. While the generalized extreme-value distribution, max-stable process and the like rest on elegant and mathemati-

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cally compelling theory, the real world is a messy place to which the relevance of that theory may be unclear. Very often extremal data show much greater variation than a simplistic view of the theory might suggest, perhaps due to unsuspected dependencies or to underlying heterogeneous phenomena. Garavaglia et al. (2010) and Süveges and Davison (2012) suggest two approaches to modeling in such cases, the first based on a decomposition into weather types, with a different extremal model fitted for each, and the second using a more conventional statistical approach based on a mixture model. If sufficiently full data are available, the first approach incorporates substantive information more fully and therefore seems preferable, but the second may be a useful backstop.

### 3. GABDA, TOWE, WADSWORTH AND TAWN

Tawn and his co-workers have made many innovative contributions to statistics of extremes, and their discussion contribution does not disappoint. They are quite correct to say that despite its mathematical attraction, the max-stability of classical multivariate extreme-value distributions is often inappropriate for data, and for that reason we welcome their suggested diagnostic, which is useful beyond the spatial setting. It is related to plots such as Figure 5 of the paper, which compares observed maxima for selected groups of stations with corresponding maxima for simulations from a fitted max-stable model. The main difference is that the proposed diagnostic compares maxima for data from many more groups of stations, all of the same sizes, directly with a fitted Gumbel distribution. One might therefore expect it to have greater power, and to get a feeling for this we applied it to our data. These have both fewer replicates and fewer stations than used in the simulations of Gadba et al., who gen-

erate 1000 replications at 100 stations on a grid, whereas we have 47 replications at 36 irregularly-spaced stations in the data we use for fitting. The resulting diagnostic plots, with  $|D| = 2, 3, 4, 36$ , are shown in Figure 1. We use all 630 pairs of sites for  $|D| = 2$ , 620 randomly-chosen triplets and quadruplets for  $|D| = 3$  and 4, and, of course, just one set when  $|D| = 36$ . Although the power of the diagnostic will be much lower than in the simulations of Gadba et al., the figure does not suggest that the max-stability assumption is unreasonable for our data—indeed, it seems to give a surprisingly good fit. In other settings we have mixed experience with rainfall data: in a very detailed but short-term data set from a high Alpine watershed, max-stability seems appropriate for stations just a few hundred meters apart, while in a longer-term data set from South Africa, near-independence seems to apply, though at longer distances.

There is clearly scope for further investigation here, and for the construction of different diagnostics, for example, developing ideas of Naveau et al. (2009) beyond max-stability.

### 4. SEGERS

Segers has made some important theoretical contributions, and his discussion nicely supplements our rather incomplete treatment of the copula approach to modeling extremes. While we agree that the non-parametric methods he describes are valuable for exploratory analysis and for assessing the quality of fit of parametric models, we feel that further development is likely to be needed before they can be routinely used in settings like that discussed in our paper. One reason for this is the apparent restriction to max-stable models. Although such models seem to fit our data, they may be unrealistic in other settings, and spatial models for near-independence (Wadsworth and Tawn (2012)) are clearly an im-

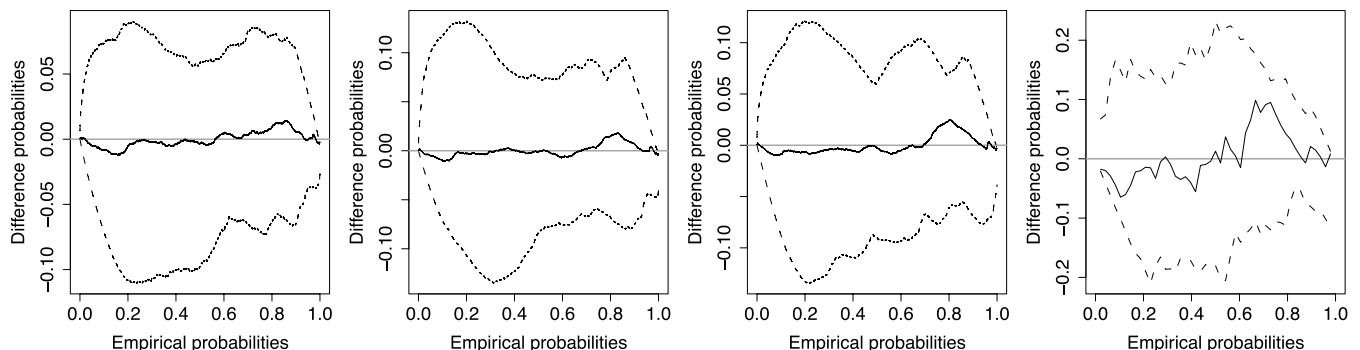


FIG. 1. Rescaled  $P$ - $P$  plot for  $Z_D$  derived from the annual maximum rainfall data, with bootstrap pointwise 95% confidence sets. From left to right:  $k = 2, 3, 4, 36$ .

portant development that greatly enlarges practical modeling possibilities. More fundamentally, we can't see how the nice theory for estimating the Pickands dependence function that Segers describes deals with the spatial element: how does one make predictions at ungauged stations, or simulate realizations for entire regions, based on nonparametric fits for the data at gauged stations? Moreover, and since the available time series in environmental applications are often much shorter than the periods for which extrapolation is required, why would it be useful to avoid modeling the marginal behavior, since in that case extrapolation beyond the data would not be possible? Finally, we suspect that a nonparametric estimate of a 35-dimensional extremal dependence function based on 47 independent observations, as would be necessary in our application, is unlikely to be useful—it seems clear that some sort of strong structural constraints would have to be applied, perhaps using ideas of empirical likelihood (Einmahl and Segers (2009)).

## 5. SHABY AND REICH

These authors' Bayesian approach to fitting the Smith model is a nice contribution, and its appearance in print will certainly stimulate work on fitting more general models—as mentioned above, fitting methods that can deal with the complexities and size of climatic and atmospheric data sets are badly needed for applications. We entirely agree that viewing extreme-value problems, where moments are of doubtful utility, through Gaussian spectacles, is not typically helpful, and that scale-invariant quantities such as the Brier score are more valuable. Our feelings about quantile regression are mixed: Northrop and Jonathan (2011) suggest using this approach to specify a covariate-dependent threshold, but as mentioned in its discussion (Chavez-Demoulin, Davison and Frossard (2011)), it seems to us that in many cases it will be preferable to base inference on the largest few order statistics at each site; this implicitly specifies a threshold but without having to fit a quantile regression model separate from a peaks over threshold model, and then having to pull together uncertainties from these two estimators. Using a Bayesian approach for individual series seems reasonable, though expanding a model using any available substantive knowledge seems preferable to naive use of a nonparametric Bayes approach, but the difficulty in specifying asymptotically-justified joint densities for extremes at different sites again raises its ugly head, if a model consistent with known theory for extremes is required.

## 6. CONCLUSION

Given the degree of current research activity in the area, it seems reasonable to hope that some of the problems raised above will be solved fairly soon. This is devoutly to be wished, since flexible but mathematically justified inferences for spatial and spatio-temporal extremes are urgently needed in applications.

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