Statistical Science
2012, Vol. 27, No. 2, 197–198
DOI: 10.1214/12-STS376D
Main article DOI: 10.1214/11-STS376

Discussion of "Statistical Modeling of Spatial Extremes" by A. C. Davison, S. A. Padoan and M. Ribatet

Benjamin Shaby and Brian J. Reich

The review paper on spatial extremes by Davison, Padoan and Ribatet is a most welcome contribution. The authors cover quite a lot of ground, making connections between different approaches while highlighting important differences. In particular, we applaud their careful attention to model checking, which can be difficult in general but particularly so for spatial extreme value models.

1. PREDICTION AND MODEL VALIDATION

With its extensive set of diagnostics for evaluating model fit, this paper provides a nice template for practitioners to follow. However, perhaps the most important feature of spatial models is their ability to predict at unobserved sites. The account presented here does not address the prediction problem, which is both a critical task in its own right and a tool for comparing models. More traditional spatial analyses typically include various performance metrics to evaluate prediction at a withheld test set of observation locations. While spatial prediction is difficult for the max-stable process models described in the paper, computational tools to accomplish this task do exist (Wang and Stoev (2011)). Spatial prediction for copula models is considerably more straightforward.

However, we note that even with predictions at hold-out locations in hand, evaluating model skill at reproducing extremal quantities requires some care. Clearly, the metrics used in traditional geostatistical analysis such as mean squared prediction error are unsatisfying for block-maximum data. Rather, we recommend the quantile score and the Brier score for threshold exceedences, as discussed and justified by Gneiting and Raftery (2007). These metrics are specifically tailored to evaluate the tail of the predictive distribution and therefore seem more appropriate in this context.

2. TOWARD HIERARCHICAL BAYESIAN MAX-STABLE MODELS

In their discussions of the relative merits of various approaches, the authors highlight the ability of hierarchical Bayesian models to represent richly flexible structures for underlying marginal parameters. As they point out, however, the conditional independence assumption made in the Bayesian analyses they discuss hamstrings the model's ability to produce spatial association in process realizations. Indeed, others have also shown that failing to properly account for spatial dependence can lead to dramatic underestimation of uncertainty, and thus undercoverage of posterior intervals, for the GEV parameters and return levels (Fuentes, Henry and Reich (2011)).

The authors correspondingly laud the ability of max-stable processes to capture joint behavior across spatial locations, but lament the restriction to relatively simple underlying structures that pairwise likelihood fitting of max-stable process models imposes. The trade-off between flexible marginal modeling and realistic spatial dependence modeling is almost treated as an inherent conundrum, almost analogous to a Heisenberg's uncertainty principle for spatial extremes. But we want to have it both ways!

The authors rightly note that the unavailability of joint likelihoods for max-stable process models appears to render their inclusion in hierarchical Bayesian models problematic. We view surmounting this

Benjamin A. Shaby is Postdoctoral Researcher, Department of Statistics, University of California, Berkeley, USA e-mail: bshaby@stat.berkeley.edu. Brian J. Reich is Assistant Professor, Department of Statistics, North Carolina State University e-mail: brian_reich@ncsu.edu.

This is an electronic reprint of the original article published by the Institute of Mathematical Statistics in *Statistical Science*, 2012, Vol. 27, No. 2, 197–198. This reprint differs from the original in pagination and typographic detail.

obstacle as a welcome challenge! As they note, progress has already been made. For example, Ribatet, Cooley and Davison (2012) specifies such a hierarchical model, but replaces the joint likelihood with a pairwise likelihood and modifies the resultant MCMC sampler using an asymptotic argument. The resultant sample from the "posterior" distribution appears to have desirable frequentist properties. While this approach may not be completely satisfying in that it is computationally intensive and it does not pass the Bayesian purity test, it represents nice out-of-the-box thinking and is a clear step in the right direction.

The authors also mention our recent manuscript, which describes an approach to hierarchical max-stable process modeling that we really like because it is fully Bayesian, straightforwardly produces predictions at unobserved locations, and can be fit to large data sets. Our approach suffers a bit because it does not readily generalize to most of the max-stable process models mentioned here by the authors. While possibilities certainly exist to expand on our approach, and efforts are underway to do just that, we expect that completely novel angles and insights will be brought to bear on the rapidly-evolving field of Bayesian analysis for spatial extremes.

3. SPATIAL MODELING OF HIGH QUANTILES

Finally, we note that while the authors focus exclusively on the asymptotic extreme value thoery, other approaches do exist. Asymptotic arguments and parametric assumptions are clearly needed to estimate very extreme quantities, such as the 10,000 year return level required by the Dutch Delta Commission. However, there are many important applications where the focus is less extreme. For example, one may be interested in determining the effect of climate change on the 10-year return level of daily maximum temperature. Given the vast amount of meteorological data collected in the past century, there may be sufficient data to justify a less restrictive and more robust approach such as quantile regression. Classical quantile regression (Koenker (2005)) gives a nonparametric estimate of covariate effects on a quantile (equivalent to a return level) of interest. Recently, semiparametric Bayesian quantile regression models have been proposed, including methods for spatial data (Lum (2010); Reich, Fuentes and Dunson (2011); Reich (2012); Tokdar and Kadane (2012)). An advantage of the Bayesian approach to quantile regression is the possibility of centering the prior of a flexible quantile model on a parametric extreme value distribution, and thus hopefully exploiting asymptotic arguments as the data deem appropriate. Comparing, and ideally merging, quantile regression with extreme value analysis may be a promising line of future research.

REFERENCES

Fuentes, M., Henry, J. and Reich, B. (2011). Nonparametric spatial models for extremes: Applications to exterme temperature data. *Extremes*. To appear.

Gneiting, T. and Raftery, A. E. (2007). Strictly proper scoring rules, prediction, and estimation. *J. Amer. Statist. Assoc.* **102** 359–378. MR2345548

Koenker, R. (2005). *Quantile Regression. Econometric Society Monographs* **38**. Cambridge Univ. Press, Cambridge. MR2268657

Lum, K. (2010). Bayesian spatial quantile regression. Ph.D. thesis, Dept. Statistical Science, Duke Univ.

Reich, B. (2012). Spatiotemporal quantile regression for detecting distributional changes in environmental processes. *J. Roy. Statist. Soc. Ser. C*. To appear.

Reich, B. J., Fuentes, M. and Dunson, D. B. (2011). Bayesian spatial quantile regression. *J. Amer. Statist. Assoc.* **106** 6–20. MR2816698

Ribatet, M., Cooley, D. and Davison, A. C. (2012). Bayesian inference from composite likelihoods, with an application to spatial extremes. *Statist. Sinica* **22** 813–845.

Tokdar, S. and Kadane, J. (2012). Simultaneous linear quantile regression: A semiparametric Bayesian approach. *Bayesian Anal.* **7** 51–70.

Wang, Y. and Stoev, S. A. (2011). Conditional sampling for spectrally discrete max-stable random fields. *Adv. in Appl. Probab.* **43** 461–483. MR2848386